جامعة بيرزيت

BIRZEIT UNIVERSITY

ANSWER BOOKLET

Student: Digital		Number 3	
Course:	Department:	Number:	
	Division:	Instructor:	
Date:	Day	Month	Year

Dr. Abdellatif Abu Issa

For Instructor's Use

Question	Grade
1	
2	
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11	
12	
Total	

Examples of Algebraic Manipulation

1. $x(x'+y) = xx' + xy = 0 + xy = xy$
2. $x + x'y = (x+x')(x+y) = 1(x+y) = x+y$
3. $(x+y)(x+y')$
$= x + xy + xy' + yy' = x(1+y+y') = x$
4. $xy + x'z + yz = xy + x'z + yz(x+x')$
$= xy + x'z + xyz + x'yz$
$= xy(1+z) + x'z(1+y)$
$= xy + x'z$

5. $(x+y)(x'+z)(y+z) = (x+y)(x'+z)$
$= (xx' + xz + x'y + yz)(y+z)$
$= xyz + \cancel{xz} + x'y + x'yz + yz$
$= yz(x+x') + xz + x'y + yz$
$= yz + xz + x'y$

$\cancel{= yz + x'y}$
$\cancel{= (x+y)(}$
$\cancel{(x+y)(y+z)(x+z)(z+z) + x'y}$

$\cancel{x + x' +}$

$= yz(x+x') + xz + x'y =$
$xyz + x'yz + xz + x'y = xz(y+1) + x'y(z+1)$
$= xz + x'y$ $\cancel{= (x+x')(x+y)(x'+z)}$
$\cancel{(x+y)(x'+z)}$

Complement of a function

Demorgan's theorems :-

$(A+B+C+\cdots+Z)' = A'B'C'D'\cdots Z'$

$(ABCDE\cdots Z)' = A'+B'+C'+D'+E'\cdots+Z'$

Example

$F = x'yz' + x'y'z$

$\rightarrow F' = (x'yz' + x'y'z)'$

$= (x+y'+z)(x+y+z')$

Ex.

$F = x(x'z' + yz)$

$F' = (x(\cancel{x}y'z' + yz))'$

$= x' + (y'z' + yz)'$

$= x' + (y'z')' \cdot (yz)'$

$= x' + (y+z)(y'+z')$

2-5

Canonical and Standard Forms

- a binary variable (X) may appear in its normal form (X) or in its complement form (x').

- for two binary variable (x, y) combined with AND operation → there are 2^2 forms: $x'y'$, $x'y$, xy', xy.

- these terms are called minterms or a standard product

- for n variable → 2^n forms (minterms).

- for two binary variables (x, y), combined with OR operation → there are 2^n forms: $x+y$, $x+y'$, $x'+y$, $x'+y'$

- these terms are called maxterms or standard sums

		Minterm		Maxterm	
x	y	Term	Designation	Term	Designation
0	0	$x'y'$	m_0	$x+y$	M_0
0	1	$x'y$	m_1	$x+y'$	M_1
1	0	xy'	m_2	$x'+y$	M_2
1	1	xy	m_3	$x'+y'$	M_3

@ sum of all minterms (for 2 variables x, y)

$\Rightarrow x'y' + x'y + xy' + xy = 1$ (always)

@ product of all maxterms (for 2 variables x, y)

$\Rightarrow (x+y).(x+y').(x'+y).(x'+y') = 0$ (always)

Ex.

x	y	z	minterms	maxterms	f_1	f_2
0	0	0	$x'y'z'$ (m_0)	$x+y+z$	0	0
0	0	1	$x'y'z$ (m_1)	$x+y+z'$	1	0
0	1	0	$x'yz'$ (m_2)	$x+y'+z$	0	0
0	1	1	$x'yz$ (m_3)	$x+y'+z'$	0	1
1	0	0	$xy'z'$ (m_4)	$x'+y+z$	1	0
1	0	1	$xy'z$ (m_5)	$x'+y+z'$	0	1
1	1	0	xyz' (m_6)	$x'+y'+z$	0	1
1	1	1	xyz (m_7)	$x'+y'+z'$	1	1

express $f1$ in term of minterms

$\Rightarrow f1 = x'y'z + xy'z' + xyz$

$= m_1 + m_4 + m_7 = \Sigma(1,4,7)$

in the same way $f_2 = m_3 + m_5 + m_6 + m_7$

$= \Sigma(3,5,6,7)$

express $f1$ in term of maxterms

$f_1 = (x+y+z).(x+y'+z).(x'+y+z')(x'+y'+z)$

$= M_0 . M_2 . M_3 . M_5 . M_6 = \Pi(0,2,3,5,6)$

$f_2 = M_0 M_1 M_2 M_4$

(*) Any Boolean function can be expressed as a sum of ~~...~~ minterms or as a product of max terms.

Ex: Express $f = A + B'C$ ~~in~~ a sum of minterms

$\Rightarrow f = A(B+B') + B'C(A+A')$

$= AB + AB' + AB'C + A'B'C$

$= AB(C+C') + AB'(C+C') + AB'C + A'B'C$

$= ABC + ABC' + AB'C + AB'C' + \cancel{AB'C} + A'B'C$

$= m_7 + m_6 + m_5 + m_4 + m_1$

$\rightarrow F(A,B,C) = \sum(1,4,5,6,7)$

Ex. Express $f = xy + x'z$ in a product of maxterms

– change it into OR terms using distributive law

$\Rightarrow f = xy + x'z$

$= (xy + x')\cdot(xy + z)$

$= (x+x')\cdot(y+x')(x+z)(y+z)$

$= (x'+y)(x+z)(y+z)$

$= (x'+y+zz')(x+z+yy')(y+z+xx')$

$= (x'+y+z)(x'+y+z')(x+z+y)(x+z+y')\cdot$

$\cancel{(y+z+x)(y+z+x')}$

$= (x'+y+z)(x'+y+z')(x+y+z)(x+y'+z)$

$= M_4 \cdot M_5 \cdot M_0 \cdot M_2 = \prod(0,2,4,5)$

Conversion between Canonical forms

- complement of a function:

e.g. $F(A,B,C) = \Sigma(1,4,5,6,7)$

$\Rightarrow F'(A,B,C) = \Sigma(0,2,3) = m_0 + m_2 + m_3$

- The complement of a function expressed as the sum of minterms equals the sum of minterms missing from the original function.

* use Demorgan's theorem $\Rightarrow$

$F(A,B,C) = (m_0 + m_2 + m_3)'$

$= m_0' \cdot m_2' \cdot m_3' = M_0 \cdot M_2 \cdot M_3$

$= \Pi(0,2,3)$

- $m_j' = M_j$

- Using truth table instead of algebra theorems ($F = xy + x'z$)

x	y	z	x'	xy	x'z	xy+x'z
0	0	0	1	0	0	0
0	0	1	1	0	1	1
0	1	0	1	0	0	0
0	1	1	1	0	1	1
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	0	1	0	1
1	1	1	0	1	0	1

$\Rightarrow F = m_1 + m_3 + m_6 + m_7$

$= \Sigma(1,3,6,7)$

$\Rightarrow$

$F = \Pi(0,2,4,5)$

(same results as using algebra)

Standard forms

- two types of standard forms: sum of products and products of sums
- sum of products

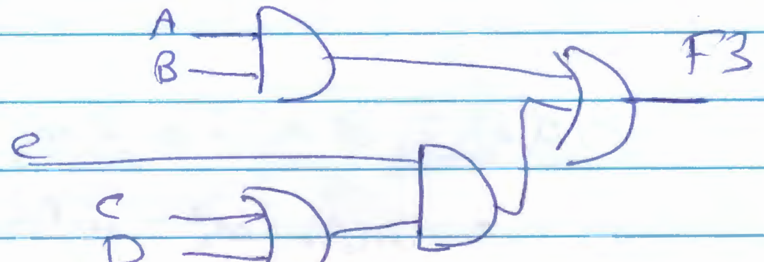
$$F1 = y' + xy + x'yz'$$

- products of sums

$$F2 = x(y' + z)(x' + y + z')$$

- nonstandard form

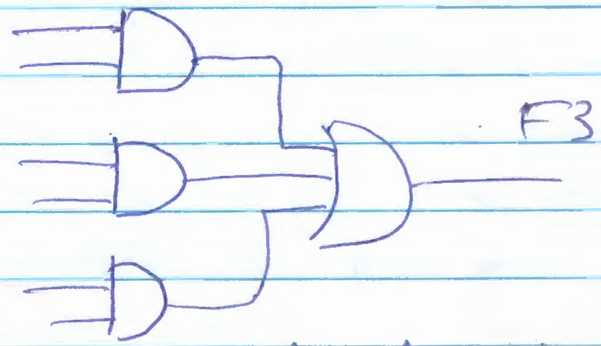
$$F3 = AB + C(D + E)$$



3-level implementation

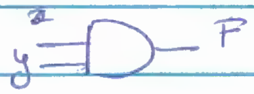
- change nonstandard form to a standard form

$$\Rightarrow F3 = AB + CD + CE$$

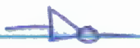


2-level implementation (better).

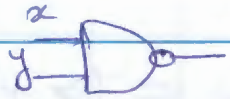
Digital Logic Gates

① AND 

② OR 

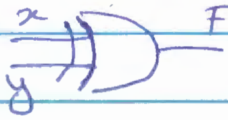
③ Inverter 

④ Buffer 

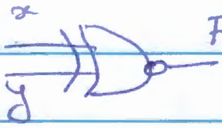
⑤ NAND 

$F = (xy)'$ $\uparrow$

⑥ NOR  $F = (x+y)'$ $\downarrow$

⑦ Exclusive-OR XOR 

$F = xy' + x'y$
$= x \oplus y$

⑧ Exclusive-Nor or equivalence 

$F = xy + x'y'$
$= (x \oplus y)'$

x	y	F
0	0	1
0	1	0
1	0	0
1	1	1

notes

$(x \downarrow y) \downarrow z \neq x \downarrow (y \downarrow z)$

$[(x+y)' + z]' \neq [x + (y+z)']'$

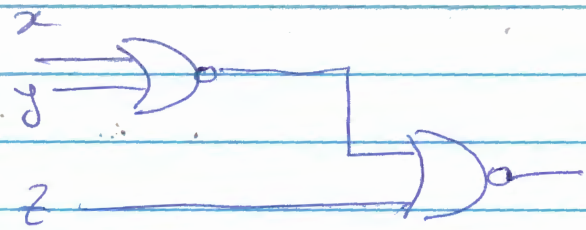
$xz' + yz' \neq x'y + x'z$

⇒ By definition

$x \downarrow y \downarrow z = (x+y+z)'$

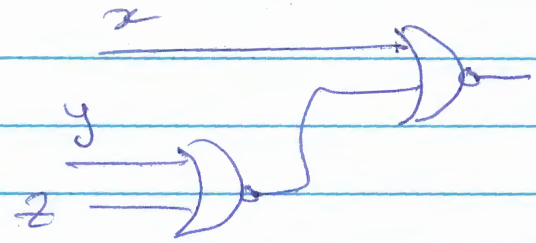
$x \uparrow y \uparrow z = (xyz)'$

1.



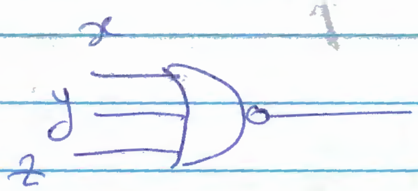
$(x \downarrow y) \downarrow z = (x+y) z'$

2.



$x \downarrow (y \downarrow z) = x'(y+z)$

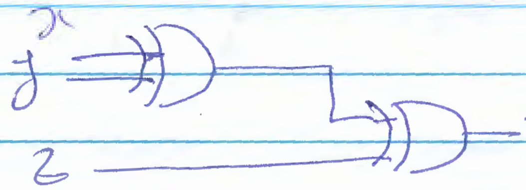
3.



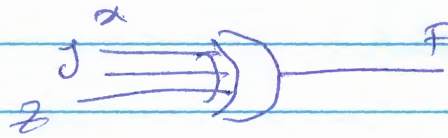
$x \downarrow y \downarrow z = (x+y+z)'$

XOR

$x \oplus (y \oplus z) = (x \oplus y) \oplus z = x \oplus y \oplus z$



$F = (x \oplus y) \oplus z$



$F = x \oplus y \oplus z$

CHAPTER 3

- Two variable map

- three variable map

- four variable map

$wx \backslash yz$	00	01	11	10
00				
01				
11				
10				

- If we have n variables
  - Two adjacent squares represent a term of all variables except 1
- 4 adjacent squares " " " "
  variables except 2 $\Rightarrow$ no. of literals $= n-2$

- 2^m adjacent squares " " " "
  variables except m
  $\Rightarrow$ no. of literals $= n-m$

Ex: $F(w,x,y,z) = \sum(0,1,2,4,5,6,8,9,12,13,14)$

wx \ yz	00	01	11	10
00	1	1		1
01	1	1		1
11	1	1		1
10	1	1		

w	x	y	z	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
⋮				

$\Rightarrow F = y' + w'z' + xz'$

~~Prime~~

yz \ wx	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11				
10	1	1	1	

$F = y' + w'z' + xz'$

Prime Implicants

- A prime implicant is a product term obtained by combining the maximum possible number of adjacent squares in the map.

- if a minterm in a square is covered by only one prime implicant, that prime implicant is said to be essential.

Ex.

$F(A,B,C,D) = \Sigma(0,2,3,5,7,8,9,10,11,13,15)$

AB \ CD	00	01	11	10
00	1		1	1
01		1	1	
11		1	1	
10	1	1	1	1

$F = BD + B'D' + CD + AD$

$= BD + B'D' + CD + AB'$

$= BD + B'D' + B'C + AD$

$= BD + B'D' + B'C + AB'$

Ex. $F(A,B,C) = \Sigma(0,1,2,5,7)$

A \ BC	00	01	11	10
0	1	1		1
1		1	1	

$F = A'C' + AC + A'B'$

$= A'C' + AC + B'C$

Five Variables Map

AB \ CDE	000	001	011	010	110	111	101	100
00								
01								
11								
10								

Ex.

$F(A,B,C,D,E) = \sum(0,2,4,6,9,13,21,23,25,29,31)$

AB \ CDE	000	001	011	010	110	111	101	100
00	1			1	1			1
01		1					1	
11		1				1	1	
10						1	1	

A = 0

BC \ DE	00	01	11	10
00	1			1
01	1			1
11		1		
10		1		

A = 1

BC \ DE	00	01	11	10
00				
01		1	1	
11		1	1	
10		1		

$F = ACE + A'B'E' + BD'E$

Product of Sums simplification

$F(A,B,C,D) = \sum(0,1,2,5,8,9,10)$

AB \ CD	00	01	11	10
00	1	1		1
01		1		
11				
10	1	1		1

$F = B'C' + B'D' + A'C'D$

$F' = AB + CD + BD'$

$\Rightarrow F = (A'+B')(C'+D')(B'+D)$

Don't Care Conditions

- Sometimes, for some combinations of the inputs, the output has not a specified value, this is called don't care condition and designated by X.

- Don't care conditions help to simplify the function.

Ex. $F(w,x,y,z) = \sum(1,3,7,11,15)$

$d(w,x,y,z) = \sum(0,2,5)$

wx \ yz	00	01	11	10
00	X	1	1	X
01		X	1	
11			1	
10			1	

$F = yz + w'x'$

$F = yz + w'z$

* note: they don't have an identical truth table.

Ex. $F(w,x,y,z) = \sum(1,3,7)$

$d(w,x,y,z) = \sum(0,5)$

wx \ yz	00	01	11	10
00	X	1	1	
01		X	1	
11				
10				

$F = w'z$