Birzeit University Mechanical & Mechatronics Engineering Department Heat Transfer ENME 431 Final exam formula sheet

Instructor: Dr. Afif Akel Hasan 1st. semester 2018/2019

Conduction

$$q_{x}'' = -k\frac{dT}{dx} \qquad q_{x}'' = k\frac{T_{1} - T_{2}}{L} = k\frac{\Delta T}{L}$$

$$R_{t,\mathrm{cond}} \equiv \frac{T_{s,1} - T_{s,2}}{q_x} = \frac{L}{kA}$$

$$q'' = h(T_s - T_\infty)$$

$$R_{t,\text{conv}} \equiv \frac{T_{z} - T_{\infty}}{q} = \frac{1}{hA}$$

$$q_{\rm rad}'' = \frac{q}{A} = \varepsilon E_b(T_s) - \alpha G = \varepsilon \sigma (T_s^4 - T_{\rm sur}^4)$$

$$R_{t,\mathrm{rad}} = \frac{T_s - T_{\mathrm{sur}}}{q_{\mathrm{rad}}} = \frac{1}{h_r A}$$

$$\dot{E}_{\rm st} = \frac{dE_{\rm st}}{dt} = \dot{E}_{\rm in} - \dot{E}_{\rm out} + \dot{E}_{\rm g}$$

$$\alpha = \frac{k}{\rho c_p}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

Steady state conduction

$$q = rac{\Delta T_{overall}}{R_{th}}$$
 $q_{x} = rac{T_{\infty,1} - T_{\infty,4}}{\Sigma R_{t}}$

$$R_{\text{tot}} = \sum R_t = \frac{\Delta T}{q} = \frac{1}{UA}$$

Table 3.3 One-dimensional, steady-state solutions to the heat equation with no generation

	Plane Wall	Cylindrical Wall ^a	Spherical Wall ^a
Heat equation	$\frac{d^2T}{dx^2} = 0$	$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$	$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$
Temperature distribution	$T_{s,1} - \Delta T \frac{x}{L}$	$T_{s,2} + \Delta T \frac{\ln{(r/r_2)}}{\ln{(r_1/r_2)}}$	$T_{s,1} - \Delta T \left[\frac{1 - (r_1/r)}{1 - (r_1/r_2)} \right]$
Heat flux $(q^{''})$	$k rac{\Delta T}{L}$	$\frac{k\Delta T}{r\ln\left(r_2/r_1\right)}$	$\frac{k\Delta T}{r^2[(1/r_1)-(1/r_2)]}$
Heat rate (q)	$kA\frac{\Delta T}{L}$	$\frac{2\pi Lk\Delta T}{\ln\left(r_2/r_1\right)}$	$\frac{4\pi k \Delta T}{(1/r_1) - (1/r_2)}$
Thermal resistance $(R_{t,cond})$	$rac{L}{kA}$	$\frac{\ln\left(r_2/r_1\right)}{2\pi Lk}$	$\frac{(1/r_1) - (1/r_2)}{4\pi k}$
$T(x) = \frac{\dot{q}L^2}{1 - \frac{x^2}{1 - \frac{x^2}}{1 - \frac{x^2}{1 - \frac{x^2}{1$	$+\frac{T_{s,2}-T_{s,1}}{2}\frac{x}{t}+\frac{T_{s,2}-T_{s,1}}{2}$	$T(x) = \frac{\dot{q}L}{1}$	$\frac{x^2}{1}\left(1-\frac{x^2}{r^2}\right)+T_s$

$$T(x) = \frac{\dot{q}L^2}{2k} \left(1 - \frac{x^2}{L^2} \right) + \frac{T_{s,2} - T_{s,1}}{2} \frac{x}{L} + \frac{T_{s,1} + T_{s,2}}{2} \qquad T(x) = \frac{\dot{q}L^2}{2k} \left(1 - \frac{x^2}{L^2} \right) + T_s$$

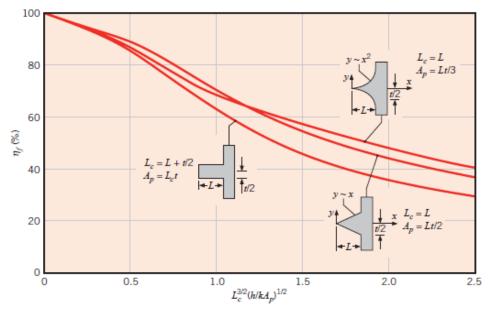
$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{q}}{k} = 0$$

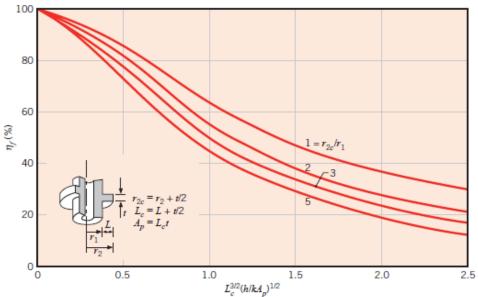
$$T(r) = -\frac{\dot{q}}{4k}r^2 + C_1 \ln r + C_2 \qquad T(r) = \frac{\dot{q}r_o^2}{4k} \left(1 - \frac{r^2}{r_o^2}\right) + T_s$$

$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b}$$
 $\eta_f \equiv \frac{q_f}{q_{\text{max}}} = \frac{q_f}{hA_f\theta_b}$
 $\eta_f = \frac{\tanh mL_c}{mL_c}$

TABLE 3.4 Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition $(x = L)$	Temperature Distribution θ/θ_b	i	Fin Heat Transfer Rate	q_f
A	Convection heat transfer: $h\theta(L) = -kd\theta/dx _{x=L}$	$\frac{\cosh m(L-x) + (h/mk)\sinh mL + (h/mk)\sinh mL}{\cosh mL + (h/mk)\sinh mL}$		$M\frac{\sinh mL + (h/mk)}{\cosh mL + (h/mk)}$	cosh <i>mL</i> sinh <i>mL</i> (3.77)
В	Adiabatic: $d\theta/dx _{x=L} = 0$	$\frac{\cosh m(L-x)}{\cosh mL}$	(3.80)	$M \tanh mL$, ,
С	Prescribed temperature: $\theta(L) = \theta_L$	$\frac{(\theta_L/\theta_b)\sinh mx + \sinh m}{\sinh mL}$		$M \frac{(\cosh mL - \theta)}{\sinh mL}$	
_			(3.82)		(3.83)
D	Infinite fin $(L \to \infty)$: $\theta(L) = 0$	$e^{-m\alpha}$	(3.84)	M	(3.85)
	T_{∞} $m^2 \equiv hP/kA_c$ = $T_b - T_{\infty}$ $M \equiv \sqrt{hPkA_c}\theta_b$				





$$q_t = N\eta_f h A_f \theta_b + h A_b \theta_b$$

$$\eta_o = 1 - \frac{NA_f}{A_t}(1 - \eta_f)$$

External flow

$$\overline{h}_m = \frac{1}{A_s} \int_{A_s} h_m dA_s$$

$$Re_{x,c} \equiv \frac{\rho u_{\infty} x_c}{\mu} = 5 \times 10^5$$

$$\overline{Nu}_L = C \operatorname{Re}_L^m \operatorname{Pr}^n$$

$$\overline{Nu}_{x} \equiv \frac{\overline{h}_{x}x}{k}$$

$$\textit{Re}_{\textit{D}} \equiv \frac{\rho \textit{VD}}{\mu} = \frac{\textit{VD}}{\nu}$$

Table 7.7 Summary of convection heat transfer correlations for external flow a,b

Correlation		Geometry	Conditions
$Nu_{x} = 0.332 Re_{x}^{1/2} Pr^{1/3}$	(7.23)	Flat plate	Laminar, local, T_f , $Pr \gtrsim 0.6$
$\overline{Nu_x} = 0.664 Re_x^{1/2} Pr^{1/3}$	(7.30)	Flat plate	Laminar, average, T_f , $Pr \gtrsim 0.6$
$Nu_{\rm x} = 0.564 Pe_{\rm x}^{1/2}$	(7.32)	Flat plate	Laminar, local, T_f , $Pr \lesssim 0.05$, $Pe_x \gtrsim 100$
$Nu_x = 0.0296 \ Re_x^{4/5} \ Pr^{1/3}$	(7.36)	Flat plate	Turbulent, local, T_f , $Re_x \lesssim 10^8$, $0.6 \lesssim Pr \lesssim 60$
$\overline{Nu}_{L} = (0.037 Re_{L}^{4/5} - 871) Pr^{1/3}$	(7.38)	Flat plate	Mixed, average, T_f , $Re_{x,c} = 5 \times 10^5$, $Re_L \lesssim 10^8$, $0.6 \lesssim Pr \lesssim 60$
$\overline{Nu}_D = C Re_D^m Pr^{1/3}$ (Table 7.2)	(7.52)	Cylinder	Average, T_f , $0.4 \le Re_D \le 4 \times 10^5$, $Pr \ge 0.7$
$\overline{Nu_D} = C Re_D^m Pr^n (Pr/Pr_s)^{1/4}$ (Table 7.4)	(7.53)	Cylinder	Average, T_{∞} , $1 \leq Re_D \leq 10^6$, $0.7 \leq Pr \leq 500$
$\overline{Nu_D} = 0.3 + [0.62 Re_D^{1/2} Pr^{1/3}]$ $\times [1 + (0.4/Pr)^{2/3}]^{-1/4}]$ $\times [1 + (Re_D/282,000)^{5/8}]^{4/5}$	(7.54)	Cylinder	Average, T_f , $Re_D Pr \approx 0.2$
$ \overline{Nu_D} = 2 + (0.4 Re_D^{1/2} + 0.06 Re_D^{2/3})Pr^{0.4} \times (\mu/\mu_s)^{1/4} $	(7.56)	Sphere	Average, T_{∞} , $3.5 \lesssim Re_D \lesssim 7.6 \times 10^4$, $0.71 \lesssim Pr \lesssim 380$, $1.0 \lesssim (\mu/\mu_S) \lesssim 3.2$
$\overline{Nu_D} = 2 + 0.6 \ Re_D^{1/2} \ Pr^{1/3}$	(7.57)	Falling drop	Average, T_{∞}
$\overline{Nu}_D = C_1 C_2 Re_{D,\text{max}}^m P r^{0.36} (P r / P r_s)^{1/2}$ (Tables 7.5, 7.6)	(7.58), (7.59)	Tube bank ^d	Average, \overline{T} , $10 \leq Re_D \leq 2 \times 10^6$, $0.7 \leq Pr \leq 500$
$V_{\max} = \frac{S_T}{S_T - D} V$ $V_{\max} = \frac{S_T}{S_T - D} V$	$= \frac{S_T}{2(S_D - D)} V$		
$S_D = \left[S_L^2 + \left(\frac{S_T}{2}\right)^2\right]^{1/2} < \frac{S_T}{2}$	$\frac{+D}{2}$		

$$S_D = \left[S_L^2 + \left(\frac{S_T}{2} \right)^2 \right]^{1/2} < \frac{S_T + D}{2}$$

Table 7.6 Correction factor C_2 of Equation 7.59 for $N_L < 20$ $(Re_{D,\max} \gtrsim 10^3)$ [16]

N_L	1	2	3	4	5	7	10	13	16
Aligned	0.70	0.80	0.86	0.90	0.92	0.95	0.97	0.98	0.99
Staggered	0.64	0.76	0.84	0.89	0.92	0.95	0.97	0.98	0.99

TABLE 7.2 Constants of Equation 7.52 for the circular cylinder in cross flow [11, 12]

Re_D	С	m
0.4-4	0.989	0.330
4-40	0.911	0.385
40-4000	0.683	0.466
4000-40,000	0.193	0.618
40,000-400,000	0.027	0.805

TABLE 7.4 Constants of Equation 7.53 for the circular cylinder in cross flow [17]

Re_{D}	С	m
1-40	0.75	0.4
40-1000	0.51	0.5
$10^3 - 2 \times 10^5$	0.26	0.6
$2 \times 10^5 - 10^6$	0.076	0.7

$$Nu_x = \frac{Nu_x|_{\xi=0}}{[1 - (\xi/x)^{3/4}]^{1/3}}$$

with unheated section laminar local

$$Nu_x = \frac{Nu_x|_{\xi=0}}{[1-(\xi/x)^{9/10}]^{1/9}}$$

with unheated section turbulent local

Internal flow

$$Re_D = \frac{4\dot{m}}{\pi D\mu}$$

$$\left(\frac{x_{\mathrm{fd},t}}{D}\right)_{\mathrm{lam}} \approx 0.05 \ Re_D \ Pr \qquad (x_{\mathrm{fd},t}/D) = 10.$$

$$D_{h} \equiv \frac{4A_{c}}{P}$$

Correlation		Conditions
$f = 64/Re_D$	(8.19)	Laminar, fully developed
$Nu_D = 4.36$	(8.53)	Laminar, fully developed, uniform $q_z^{\prime\prime}$
$Nu_D = 3.66$	(8.55)	Laminar, fully developed, uniform T_s
$\overline{Nu}_D = 3.66 + \frac{0.0668 Gz_D}{1 + 0.04 Gz_D^{2/3}}$		Laminar, thermal entry (or combined entry with $Pr \gtrsim 5$), uniform T_s , $Gz_D = (D/x) Re_D Pr$
$ \overline{Nu}_{D} = \frac{\frac{3.66}{\tanh[2.264 Gz_{D}^{-1/3} + 1.7 Gz_{D}^{-2/3}]} + 0.0499 Gz_{D} \tan(2.432 Pr^{1/6} Gz_{D}^{-1/6})}{\tanh(2.432 Pr^{1/6} Gz_{D}^{-1/6})} $	$ \frac{\sinh(Gz_D^{-1})}{2} $ (8.58)	Laminar, combined entry, $Pr \ge 0.1$, uniform T_z $Gz_D = (D/x) Re_D Pr$
$\frac{1}{\sqrt{f}} = -2.0 \log \left[\frac{e/D}{3.7} + \frac{2.51}{Re_D \sqrt{f}} \right]$	(8.20) ^c	Turbulent, fully developed
$f = (0.790 \ln Re_D - 1.64)^{-2}$	(8.21) ^c	Turbulent, fully developed, smooth walls, $3000 \lesssim Re_D \lesssim 5 \times 10^6$
$Nu_D = 0.023 Re_D^{4/5} Pr^n$		Turbulent, fully developed, $0.6 \le Pr \le 160$, $Re_D \ge 10,000$, $(L/D) \ge 10$, $n = 0.4$ for $T_s > T_m$ and $n = 0.3$ for $T_s < T_m$
$Nu_D = 0.027 Re_D^{4/5} Pr^{1/3} \left(\frac{\mu}{\mu_s}\right)^{0.14}$		Turbulent, fully developed, $0.7 \le Pr \le 16,700$, $Re_D \ge 10,000$, $L/D \ge 10$
$Nu_D = \frac{(f/8)(Re_D - 1000) Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}$		Turbulent, fully developed, $0.5 \le Pr \le 2000$, $3000 \le Re_D \le 5 \times 10^6$, $(L/D) \ge 10$
$Nu_D = 4.82 + 0.0185(Re_D Pr)^{0.827}$		Liquid metals, turbulent, fully developed, uniform q_2^r , $3.6 \times 10^3 \lesssim Re_D \lesssim 9.05 \times 10^5$, $3 \times 10^{-3} \lesssim Pr \lesssim 5 \times 10^{-2}$, $10^2 \lesssim Re_D Pr \lesssim 10^4$
$Nu_D = 5.0 + 0.025(Re_D Pr)^{0.8}$		Liquid metals, turbulent, fully developed, uniform T_s , $Re_D Pr \approx 100$
$q = \dot{m} c_p (T_{\text{out}} - T_{\text{in}})$		
$q_{ m conv} = \overline{h} A_s \Delta T_{ m lm}$ $\Delta T_{ m lm} \equiv rac{\Delta T_o}{\ln{(\Delta t)}}$	$\frac{-\Delta T_i}{T_o/\Delta T_i)}$	
$T_m(x) = T_{m,i} + \frac{q_s''P}{\dot{m}c_p}x \qquad \frac{T_s - T_m(x)}{T_s - T_{m,i}}$	$= \exp\left(-\frac{Px}{\dot{m}c_n}\overline{h}\right)$	$\frac{\Delta T_o}{\Delta T_i} = \frac{T_{\infty} - T_{m,o}}{T_{\infty} - T_{m,i}} = \exp\left(-\frac{\overline{U}A_s}{\dot{m}c_p}\right)$
$q=\overline{U}\!A_{s}\Delta T_{ m lm}$	P	

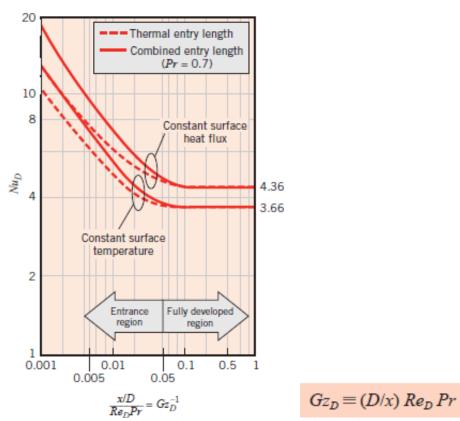


Table 8.1 Nusselt numbers and friction factors for fully developed laminar flow in tubes of differing cross section

		Nu_D		
Cross Section	$\frac{b}{a}$	(Uniform q'' _s)	(Uniform T _s)	Re_{D_h}
	_	4.36	3.66	64
a	1.0	3.61	2.98	57
a	1.43	3.73	3.08	59
a	2.0	4.12	3.39	62
a	3.0	4.79	3.96	69
a	4.0	5.33	4.44	73
a	8.0	6.49	5.60	82
	00	8.23	7.54	96
Heated /// // // // // // // Insulated	00	5.39	4.86	96
	_	3.11	2.49	53

Table 8.2 Nusselt number for fully developed laminar flow in a circular tube annulus with one surface insulated and the other at constant temperature

D_i/D_o	Nu_i	Nu_o	Comments		
0	_	3.66	See Equation 8.55		
0.05	17.46	4.06			
0.10	11.56	4.11			
0.25	7.37	4.23		, D	
0.50	5.74	4.43		$Nu_i \equiv \frac{h_i D_h}{L}$	$h_a D_h$
≈1.00	4.86	4.86	See Table 8.1, $b/a \rightarrow \infty$	k	$Nu_o \equiv \frac{h_o D_h}{k}$

Free convection

$$\beta = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_{p} = \frac{1}{\rho} \frac{p}{RT^{2}} = \frac{1}{T}$$

$$Gr_{L} \equiv \frac{g\beta(T_{s} - T_{\infty})L^{3}}{\nu^{2}}$$

$$Ra_{L} = Gr_{L} Pr = \frac{g\beta(T_{s} - T_{\infty})L^{3}}{\nu^{\alpha}}$$

Vertical wall

$$\overline{Nu}_{L} = \frac{\overline{h}L}{k} = \frac{4}{3} \left(\frac{Gr_{L}}{4} \right)^{1/4} g(Pr)$$

$$\overline{Nu}_{L} = \frac{\overline{h}L}{k} = CRa_{L}^{n}$$

Typically, n=1/4, and 1/3 for laminar and turbulent flows, respectively.

$$\overline{Nu}_{L} = \left\{ 0.825 + \frac{0.387 Ra_{L}^{1/6}}{\left[1 + (0.492/Pr)^{9/16}\right]^{8/27}} \right\}^{2}$$

Horizontal plate

$$L \equiv \frac{A_s}{P}$$

Upper Surface of Hot Plate or Lower Surface of Cold Plate [19]:

$$\overline{Nu}_L = 0.54 \, Ra_L^{1/4} \quad (10^4 \lesssim Ra_L \lesssim 10^7, Pr \gtrsim 0.7)$$

$$\overline{Nu}_L = 0.15 \, Ra_L^{1/3} \quad (10^7 \lesssim Ra_L \lesssim 10^{11}, \text{ all } Pr)$$

Lower Surface of Hot Plate or Upper Surface of Cold Plate [20]:

$$\overline{Nu}_L = 0.52 \, Ra_L^{1/5} \quad (10^4 \lesssim Ra_L \lesssim 10^9, Pr \gtrsim 0.7)$$

Horizontal Cylinder

$$\overline{Nu}_{D} = \frac{\overline{h}D}{k} = CRa_{D}^{n}$$

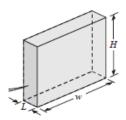
TABLE 9.1 Constants of Equation 9.33 for free convection on a horizontal circular cylinder [22]

Ra_D	С	n
10 ⁻¹⁰ -10 ⁻²	0.675	0.058
10^{-2} – 10^{2}	1.02	0.148
10 ² -10 ⁴	0.850	0.188
10 ⁴ –10 ⁷	0.480	0.250
10 ⁷ –10 ¹²	0.125	0.333

$$\overline{Nu}_D = \left\{ 0.60 + \frac{0.387 \, Ra_D^{1/6}}{\left[1 + (0.559/Pr)^{9/16}\right]^{8/27}} \right\}^2 \qquad Ra_D \lesssim 10^{12}$$

Sphere

$$\overline{Nu}_D = 2 + \frac{0.589 \, Ra_D^{1/4}}{\left[1 + (0.469/Pr)^{9/16}\right]^{4/9}}$$



$$Ra_L = \frac{g\beta(T_1 - T_2)L^3}{\alpha\nu}$$
 $\overline{Nu}_L = 0.42 Ra_L^{1/4} Pr^{0.012} \left(\frac{H}{L}\right)^{-0.3}$

Heat exchanger

$$q = \dot{m}_h c_{p,h} \left(T_{h,i} - T_{h,o} \right)$$

$$q = \dot{m}_c c_{p,c} \left(T_{c,o} - T_{c,i} \right)$$

$$q = UA\Delta T_m$$

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln \Delta T_2 / \Delta T_1} = \frac{\Delta T_1 - \Delta T_2}{\ln \Delta T_1 / \Delta T_2}$$

$$\Delta T_{lm} = F\Delta T_{lm,CF}$$

$$\frac{1}{UA} = R_{\text{conv},h} + R_w + R_{\text{conv},c}$$

$$\frac{1}{UA} = \left(\frac{1}{hA}\right)_h + R_w + \left(\frac{1}{hA}\right)_c$$

$$\varepsilon \equiv \frac{q}{q_{\mathrm{max}}}$$

$$q_{\rm max} = C_{\rm min}(T_{h,i} - T_{c,i})$$

$$\varepsilon = \frac{C_h(T_{h,i} - T_{h,o})}{C_{\min}(T_{h,i} - T_{c,i})} \qquad \qquad \varepsilon = \frac{C_c(T_{c,o} - T_{c,i})}{C_{\min}(T_{h,i} - T_{c,i})}$$

$$q = \varepsilon C_{\min}(T_{h,i} - T_{c,i})$$

$$\mathrm{NTU} \equiv \frac{\mathit{UA}}{\mathit{C}_{\mathrm{min}}}$$

TABLE 11.3 Heat Exchanger Effectiveness Relations [5]

Flow Arrangement	Relation	
Parallel flo	$\varepsilon = \frac{1 - \exp\left[-\text{NTU}(1 + C_r)\right]}{1 + C_r}$	
Counterflo	$\varepsilon = \frac{1 - \exp\left[-\text{NTU}(1 - C_r)\right]}{1 - C_r \exp\left[-\text{NTU}(1 - C_r)\right]}$	$(C_r < 1)$
	$\varepsilon = \frac{\text{NTU}}{1 + \text{NTU}}$	$(C_{\tau}=1)$
Shell-and-tube		
One shell pass (2, 4, tube passes)	$\varepsilon_1 = 2 \left\{ 1 + C_r + (1 + C_r^2)^{1/2} \times \frac{1 + ex}{1 - ex} \right\}$	$\frac{\exp\left[-(\text{NTU})_{1}(1+C_{r}^{2})^{1/2}\right]}{\exp\left[-(\text{NTU})_{1}(1+C_{r}^{2})^{1/2}\right]}\right\}^{-1}$
n shell passes $(2n, 4n, \dots$ tube passes)	$\varepsilon = \left[\left(\frac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1} \right)^n - 1 \right] \left[\left(\frac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1} \right)^n \right]$	$\left[-C_r\right]^{-1}$
Cross-flow (single pass		
Both fluids unmixed	$\varepsilon = 1 - \exp\left[\left(\frac{1}{C_r}\right)(NTU)^{0.22} \left\{\exp\left[-\frac{1}{C_r}\right]\right]\right]$	$C_r(\text{NTU})^{0.78}] - 1\}$
C_{max} (mixed), C_{min} (unmixed)	$\varepsilon = \left(\frac{1}{C_r}\right)(1 - \exp\left\{-C_r[1 - \exp\left(-N\right)\right\}]$	TU)]})
C_{\min} (mixed), C_{\max} (unmixed)	$\varepsilon = 1 - \exp(-C_r^{-1}\{1 - \exp[-C_r(NT)]\})$	U)]})
All exchangers $(C_r = 0)$	$\varepsilon = 1 - \exp(-NTU)$	

TABLE 11.4 Heat Exchanger NTU Relations

Flow Arrangement	Relation		
Parallel flo	$NTU = -\frac{\ln\left[1 - \varepsilon(1 + C_r)\right]}{1 + C_r}$		
Counterflo	$NTU = \frac{1}{C_r - 1} \ln \left(\frac{\varepsilon - 1}{\varepsilon C_r - 1} \right)$	$(C_r < 1)$	
	$NTU = \frac{\varepsilon}{1 - \varepsilon}$	$(C_r=1)$	
Shell-and-tube			
One shell pass (2, 4, tube passes)	$(NTU)_1 = -(1 + C_r^2)^{-1/2} \ln \left(\frac{E}{E} \right)^{-1/2}$ $E = \frac{2/\varepsilon_1 - (1 + C_r)}{(1 + C_r^2)^{1/2}}$	$\frac{-1}{+1}$	
n shell passes $(2n, 4n, \ldots$ tube passes)	Use Equations 11.30b and 11.30c with $\varepsilon_1 = \frac{F - 1}{F - C_r} F = \left(\frac{\varepsilon C_r - 1}{\varepsilon - 1}\right)^{1/n} \text{NTU} = n(\text{NTU})$		
Cross-flow (single pass			
C_{\max} (mixed), C_{\min} (unmixed)	$NTU = -\ln\left[1 + \left(\frac{1}{C_r}\right)\ln(1 - \epsilon)\right]$	$\varepsilon C_{7})$	
C_{\min} (mixed), C_{\max} (unmixed)	$NTU = -\left(\frac{1}{C_r}\right) \ln[C_r \ln(1-\varepsilon)]$	+ 1]	
All exchangers $(C_t = 0)$	$NTU = -\ln(1-\varepsilon)$		

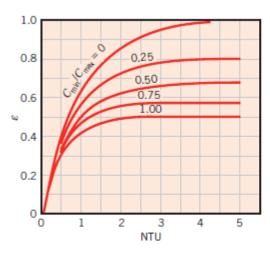


FIGURE 11.10 Effectiveness of a parallelflow heat exchanger (Equation 11.28).

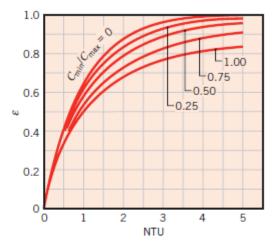


FIGURE 11.11 Effectiveness of a counterflow heat exchanger (Equation 11.29).

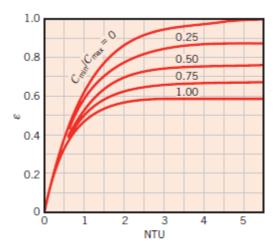


FIGURE 11.12 Effectiveness of a shell-andtube heat exchanger with one shell and any multiple of two tube passes (two, four, etc. tube passes) (Equation 11.30).

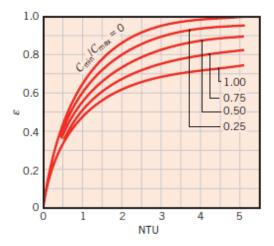


FIGURE 11.14 Effectiveness of a singlepass, cross-flow heat exchanger with both fluids unmixed (Equation 11.32).

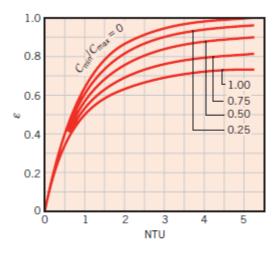


FIGURE 11.13 Effectiveness of a shell-andtube heat exchanger with two shell passes and any multiple of four tube passes (four, eight, etc. tube passes) (Equation 11.31 with n = 2).

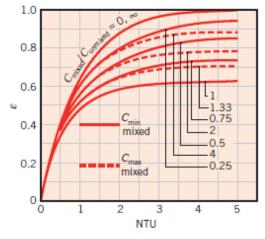


FIGURE 11.15 Effectiveness of a singlepass, cross-flow heat exchanger with one fluid mixed and the other unmixed (Equations 11.33, 11.34).