1.1 Solutions of linear Equations and Inequalities in One Variable.

An equation is a statement that two algebraic expressio are equal

Example: ① $x+1=-6$

② $6(3-x) = 5 + \frac{x-1}{2}$

③ $x^2 + x = 2x + 6$

The values of x that makes the statements true are called solutions of the equation

Example:- $x=-7$ is a solution of the equation $x+1=-6$

✓ $-7+1=-6$

Solving the equation means Finding the solutions

Procedure:

1) If the equation contains fractions, multiply both sides by LCD

2) Remove the parentheses.

3) Perform any additions or subtractions in the equation

4) Divide ~~#~~ both sides of the equation by the coefficient of the variable.

6) check the solution

$$x \cancel{+1} = 6 \rightarrow x = 5$$
$$\quad \cancel{-1} \quad -1$$

Example : Solve $4x - 7 = 8x + 2$.

$$4\cancel{x} - 7 = 8x + 2 \rightarrow -7 = 4x \cancel{+2} \rightarrow$$
$$\cancel{-4x} \qquad -4x \qquad\qquad -2 \qquad \cancel{-2}$$

$$\rightarrow -9 = 4x \rightarrow x = \frac{-9}{4}$$

Example : $x + 8 = 8(x + 1)$.

$$x + 8 = 8(x + 1)$$
$$x + 8 = 8x + 8 \rightarrow 8 = 7x + 8 \rightarrow 0 = 7x$$
$$-x \qquad -x \qquad\qquad -8 \qquad -8$$

$$\rightarrow x = \frac{0}{7} = 0$$

Example : $\frac{2x+1}{x-3} = 4 + \frac{5}{x-3}$

LCD : $(x-3)$

$$(x-3) \cdot \frac{2x+1}{(x-3)} = 4 \cdot (x-3) + \frac{5}{(x-3)} \cdot (x-3)$$

$$2x + 1 = 4(x-3) + 5$$
$$2x + 1 = 4x - 12 + 5 \rightarrow 2x + 1 = 4x - 7$$
$$\qquad\qquad\qquad -2x \qquad -2x$$

$$1 = 2x - 7 \rightarrow 2x = 8 \rightarrow x = \frac{8}{2} = 4$$
$$+7 \qquad +7$$

Check:- $\frac{2(4)+1}{4-3} \stackrel{??}{=} 4+\frac{5}{4-3}$

$\frac{9}{1} \stackrel{??}{=} 4+\frac{5}{1}$

$9 = 9$ ✓ the solution $x=4$

Example: $\frac{2x}{3} - 1 = \frac{x-2}{2}$

LCD $= \cancel{6}(2)\cdot(3) = 6$

$\frac{2x}{3}\cdot 6 - 1\cdot 6 = \frac{(x-2)}{2}\cdot 6$

$2x\cdot 2 - 6 = (x-2)\cdot 3$

$4x - 6 = 3x - 6$

$-3x \qquad -3x$

$x - 6 = -6 \rightarrow x = 0$

$+6 \qquad +6$

Check $\frac{2(0)}{3} - 1 \stackrel{??}{=} \frac{0-2}{2}$

$-1 = -1$ ✓

Example: Solve $\frac{3x}{2x+10} = 1 + \frac{1}{x+5}$

$\frac{3}{2(x+5)} = 1 + \frac{1}{x+5}$

LCD $= 2(x+5)$

$$\frac{3x}{2(x+5)} \cdot 2(x+5) = 1 \cdot 2(x+5) + \frac{1}{(x+5)} \cdot 2(x+5)$$

$$3x = 2(x+5) + 2$$

$$3x = 2x + 10 + 2$$

$$\begin{aligned} 3x &= 2x + 12 \quad \rightarrow \quad x = 12 \\ -2x & \quad -2x \end{aligned}$$

Check :- $\frac{3(12)}{2(12)+10} \overset{??}{=} 1 + \frac{1}{12+5}$

$$\frac{36}{34} \overset{??}{=} 1 + \frac{1}{17}$$

$$\frac{18}{17} = \frac{18}{17} \checkmark$$

solution is $x = 12$

Example : Solve $\frac{2x}{x-3} = 4 + \frac{6}{x-3}$

LCD $= (x-3)$

$$\frac{2x}{x-3} \cdot (x-3) = 4 \cdot (x-3) + \frac{6}{(x-3)} \cdot (x-3)$$

$$2x = 4(x-3) + 6$$

$$2x = 4x - 12 + 6 \quad \rightarrow \quad \begin{aligned} 2x &= 4x - 6 \\ -4x & \quad -4x \end{aligned}$$

$$-2x = -6 \quad \rightarrow \quad x = \frac{-6}{-2} = 3$$

Check :- $\frac{2(3)}{3-3} \overset{??}{=} 4 + \frac{6}{3-3}$

$\frac{6}{0} \overset{??}{=} 4 + \frac{6}{0}$ undefined expressions

no solution

Example: Solve $2X + 5 = -3 + 2X$

$$2X + 5 = -3 + 2X$$
$$-2X \qquad\qquad -2X$$

$\rightarrow \quad 5 = -3$ no Solution

خطأ دائماً

Example:- $2X + 6 = 2(3 + X)$

$2X + 6 = 6 + 2X \rightarrow 6 = 6$ infinitely many solutions

$-2X \qquad\qquad -2X$

صحيح دائماً

Solve linear equation of two variables :

Example: Solve the following equation for y:-

$4X + 3y = 12 \rightarrow 3y = 12 - 4X$

$-4X \qquad -4X$

$y = \dfrac{12 - 4X}{3}$ infinite number of solutions.

2) Solve $9X + \frac{3}{2}y = 11$ for y

$\cancel{-9X} \qquad \cancel{-9X}$

LCD = 2

$\cancel{\tfrac{3}{2}y} = \cancel{11} - 9X \rightarrow$

$9X \cdot (2) + \frac{3}{2}y \cdot (2) = 11 \cdot (2)$

$18X + 3y = 22 \rightarrow 3y = 22 - 18X$

$-18X \qquad -18X$

$$y = \frac{22 - 18X}{3}$$

infinite number of solutions

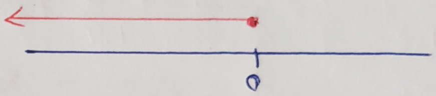
* Linear Inequalities. المتباينات

Inequality is a statement that one quantity is greater than or less than another quantity.

Example: Solve $3X + 2 \le 2$

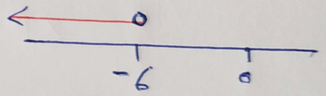
$$\underset{-2}{3X + 2} \le \underset{-2}{2} \rightarrow 3X \le 0 \rightarrow 3X \le \frac{0}{3}$$

$\rightarrow X \le 0$



Example: Solve $\underset{-2X}{2X - 1} > \underset{-2X}{3X + 5}$

$$\underset{-5}{-1} > \underset{-5}{X + 5} \longrightarrow -6 > X$$

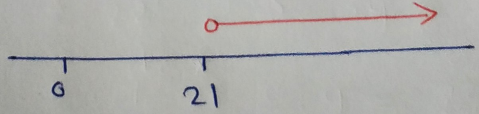


تذكر انه عند ضرب المتباينة بعدد سالب تقلب اشارة المتباينة

وعند قسمة المتباينة على عدد سالب تقلب اشارة المتباينة

Example: $\underset{-17}{17} \cancel{8} - X < \underset{-17}{-4} \rightarrow -X < -21$

$$X > \frac{-21}{-1} \rightarrow X > 21$$



Example: Solve $\frac{3x}{4} - \frac{1}{6} \geq x - \frac{2(x-1)}{3}$

Find LCD:-
$$\left.\begin{aligned} 4 &= 2(2) \\ 6 &= 2(3) \\ 3 &= 3(1) \end{aligned}\right\} \quad LCD = 2(2)(3) = 12$$

$$\frac{3x}{4}\cdot(12) - \frac{1}{6}\cdot(12) \geq x\cdot(12) - \frac{2(x-1)}{3}\cdot(12)$$

$$3x(3) - \cancel{\frac{1}{6}}\cdot(2) \geq 12x - 2(x-1)\cdot(4)$$

$$9x - 2 \geq 12x - 8(x-1)$$

$$9x - 2 \geq 12x - 8x + 8$$

$$9x - 2 \geq 4x + 8$$

$$-4x \qquad\qquad -4x$$

$$5x - 2 \geq 8 \rightarrow 5x \geq 10$$

$$+2 \qquad +2$$

$$x \geq \frac{10}{5} \rightarrow x \geq 2$$