Fundamentals Physics

Tenth Edition

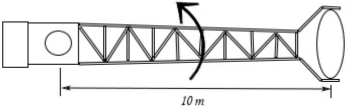
Halliday

Chapter 10_2

Rotation

10-3 Relating the Linear and Angular Variables, Example on Module 10 - 3

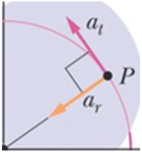
An astronaut is tested in a centrifuge with radius 10 m and rotating according to $\vartheta = 0.30t^2$. At t = 5.0 s, what are the magnitudes of the:
(a) angular velocity, (b) linear velocity, (c) tangential acceleration, and (d) radial acceleration?



(a) Using Eq. 10-6, the angular velocity at t = 5.0s is $\omega = \left.\frac{d\theta}{dt}\right|_{t=5.0} = \frac{d}{dt}\left(0.30t^2\right)\Big|_{t=5.0} = 2(0.30)(5.0) = 3.0 \text{ rad/s}.$

(b) Equation 10-18 gives the linear speed at t = 5.0s: $v = \omega r = (3.0 \text{ rad/s})(10 \text{ m}) = 30 \text{ m/s}.$

(c) We find first the angular acceleration (α, because $a_t = r\alpha$) from Eq. 10-8,



$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}(0.60t) = 0.60 \text{ rad / s}^2.$$

Then, the tangential acceleration at t = 5.0s is, using Eq. 10-22,

$$a_t = r\alpha = (10 \text{ m})\left(0.60 \text{ rad/s}^2\right) = 6.0 \text{ m/s}^2.$$

(d) The radial (centripetal) acceleration is given by Eq. 10-23: $a_r = \omega^2 r = (3.0 \text{ rad/s})^2 (10 \text{ m}) = 90 \text{ m/s}^2.$

10-4 Kinetic Energy of Rotation (2 of 6)

- Apply the kinetic energy formula for a point particle and sum over all the particles

$$K = \sum \tfrac{1}{2} m_i v_i^2$$

- different linear velocities (same angular velocity for all particles but possibly different radii)
- Then write velocity in terms of angular velocity:

$$K = \sum \tfrac{1}{2} m_i \left(\omega r_i\right)^2 = \tfrac{1}{2}\left(\sum m_i r_i^2\right)\omega^2, \quad \textbf{Equation (10-32)}$$

We call the quantity in parentheses on the right side the **rotational inertia**, or **moment of inertia**, $I = \sum m_i r_i^2 \quad (\text{rotational inertia})$

10-4 Kinetic Energy of Rotation (3 of 6)

- It is a constant for a rigid object and given rotational axis
- Caution: the axis for *I* must always be specified:

$$I = \sum m_i r_i^2 \quad (\text{rotational inertia})$$ **Equation (10-33)**

10-4 Kinetic Energy of Rotation (4 of 6)

- And rewrite the kinetic energy as:

$$K = \frac{1}{2} I \omega^2 \quad \text{(radian measure)}$$ **Equation (10-34)**

- Use these equations for a finite set of rotating particles
- Rotational inertia corresponds to how difficult it is to change the state of rotation (speed up, slow down or change the axis of rotation)

10-4 Kinetic Energy of Rotation (5 of 6)

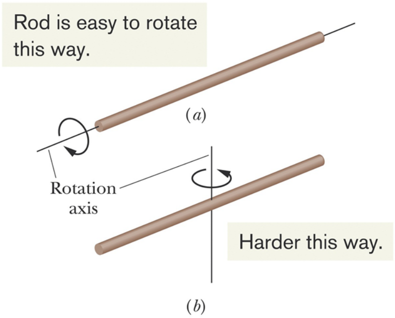
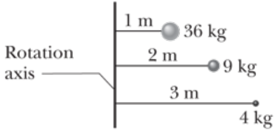


Figure 10-11

10-4 Kinetic Energy of Rotation (6 of 6)

Checkpoint 4

The figure shows three small spheres that rotate about a vertical axis. The perpendicular distance between the axis and the center of each sphere is given. Rank the three spheres according to their rotational inertia about that axis, greatest first.



Answer:

They are all equal!

Example on module 10-4:

What is I of a wheel with K of 24400 J, rotating at 602 rev/min?

$$\omega = \frac{(602\ \text{rev/min})(2\pi\,\text{rad/rev})}{60\ \text{s/min}} = 63.0\ \text{rad/s},$$

$$K = \tfrac{1}{2} I\omega^2, \quad \Rightarrow \quad I = \frac{2K}{\omega^2} = \frac{2(24400\ \text{J})}{(63.0\ \text{rad}/\text{s})^2} = 12.3\ \text{kg}\cdot\text{m}^2.$$

10-5 Calculating the Rotational Inertia (2 of 8)

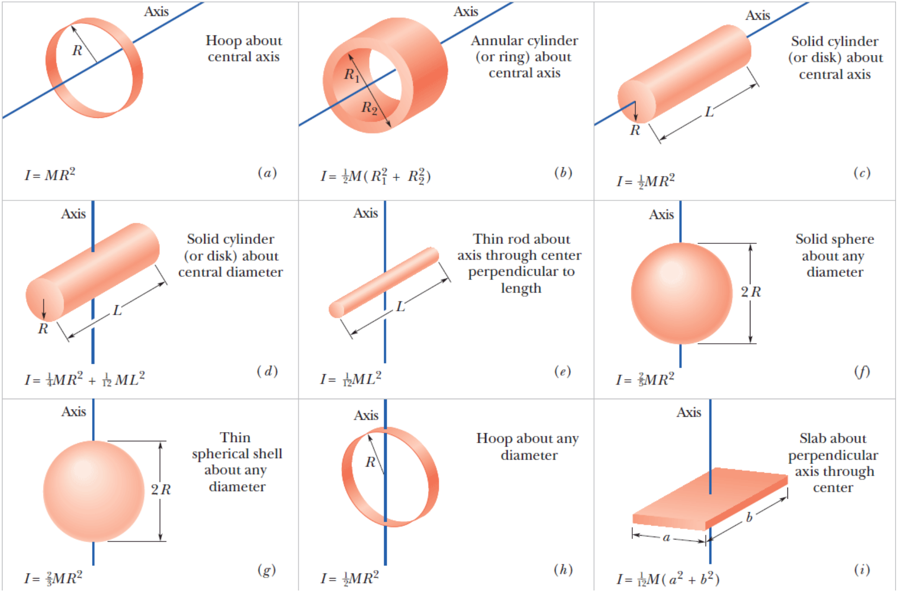
- Integrating Equation.10-33 **over a continuous body:**

$$I = \int r^2 \, dm \quad \text{(rotational inertia, continuous body).}$$ **Equation (10-35)**

- In principle we can always use this equation
- But there is a set of common shapes for which values have already been calculated (Table 10-2) for common axes

10-5 Calculating the Rotational Inertia (3 of 8)

Table 10-2 Some Rotational Inertias



Hoop about central axis — $I = MR^2$ (a)	Annular cylinder (or ring) about central axis — $I = \frac{1}{2}M(R_1^2 + R_2^2)$ (b)	Solid cylinder (or disk) about central axis — $I = \frac{1}{2}MR^2$ (c)
Solid cylinder (or disk) about central diameter — $I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$ (d)	Thin rod about axis through center perpendicular to length — $I = \frac{1}{12}ML^2$ (e)	Solid sphere about any diameter — $I = \frac{2}{5}MR^2$ (f)
Thin spherical shell about any diameter — $I = \frac{2}{3}MR^2$ (g)	Hoop about any diameter — $I = \frac{1}{2}MR^2$ (h)	Slab about perpendicular axis through center — $I = \frac{1}{12}M(a^2 + b^2)$ (i)

- If we know the moment of inertia for the **center of mass** axis, we can find the moment of inertia for a parallel axis with the **parallel-axis theorem:**

$$I = I_{com} + Mh^2$$ **Equation (10-36)**

- Note the axes must be parallel, and the first must go through the center of mass
- This does not relate the moment of inertia for two arbitrary axes

We need to relate the rotational inertia around the axis at *P* to that around the axis at the com.

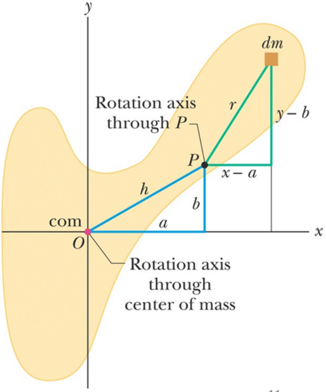
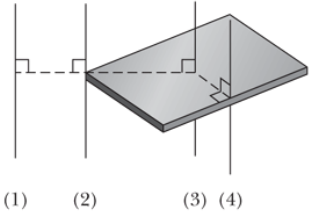


Figure 10-12

10-5 Calculating the Rotational Inertia (6 of 8)

Checkpoint 5

The figure shows a book-like object (one side is longer than the other) and four choices of rotation axes, all perpendicular to the face of the object. **Rank** the choices according to the **rotational inertia** of the object about the **axis**, **greatest** first.

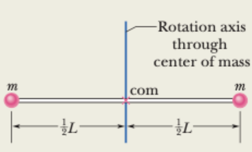


Answer:

(1), (2), (4), (3)

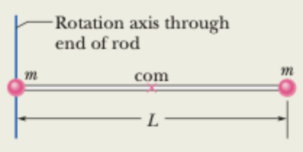
10-5 Calculating the Rotational Inertia (7 of 8)

Example (10.6) (a) What is the rotational inertia I_{com} about an axis through the center of mass, perpendicular to the rod as shown?



$$I = \sum m_i r_i^2 = (m)(\tfrac{1}{2}L)^2 + (m)(\tfrac{1}{2}L)^2 = \tfrac{1}{2}mL^2.$$

(b) Calculate the moment of inertia for 2nd Figure



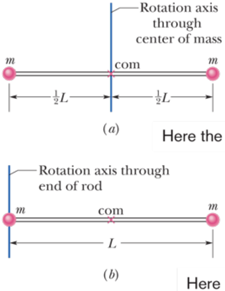
- Summing by particle:

$$I = m(0)^2 + mL^2 = mL^2.$$

- Use the parallel-axis theorem

$$\begin{aligned} I &= I_{com} + Mh^2 = \tfrac{1}{2}mL^2 + (2m)(\tfrac{1}{2}L)^2 \\ &= mL^2. \end{aligned}$$

10-5 Calculating the Rotational Inertia (8 of 8)

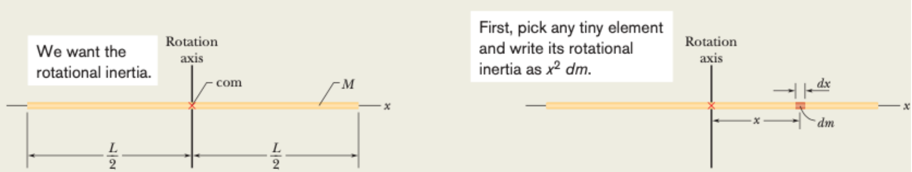


Here the rotation axis is through the com.

Here it has been shifted from the com without changing the orientation. We can use the parallel-axis theorem.

Figure 10-13

Sample Problem 10.07 Rotational inertia of a uniform rod, integration



$$I = \int_{x=-L/2}^{x=+L/2} x^2 \left(\frac{M}{L}\right) dx$$
$$= \frac{M}{3L}\left[x^3\right]_{-L/2}^{+L/2} = \frac{M}{3L}\left[\left(\frac{L}{2}\right)^3 - \left(-\frac{L}{2}\right)^3\right]$$
$$= \tfrac{1}{12} ML^2.$$

(b) What is the rod's rotational inertia I about a new rotation axis that is perpendicular to the rod and **through the left end?**

$$I = I_{com} + Mh^2 = \tfrac{1}{12} ML^2 + (M)(\tfrac{1}{2}L)^2$$
$$= \tfrac{1}{3} ML^2. \qquad \text{(Answer)}$$

10-6 Torque (3 of 6)

- The force necessary to rotate an object depends on the angle of the force and where it is applied
- We can resolve the force into components to see how it affects rotation

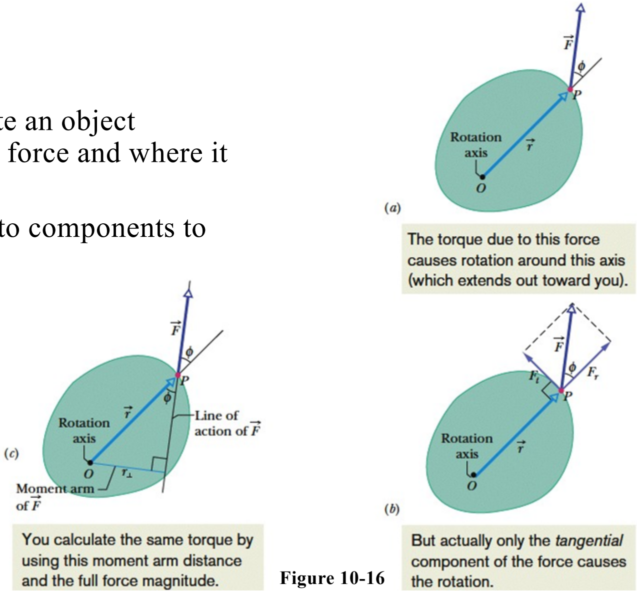


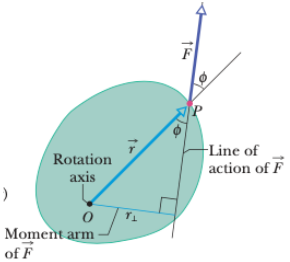
Figure 10-16

10-6 Torque (4 of 6)

- **Torque** takes these factors into account:

$$\tau = (r)(F \sin \phi). \qquad \textbf{Equation (10-39)}$$

- A line extended through the applied force is called the **line of action** of the force
- The perpendicular distance from the line of action to the axis is called the **moment arm**
- The unit of torque is the newton-meter, N m
- Note that 1 J = 1 N m, but torques are never expressed in joules, torque is not energy



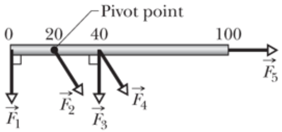
10-6 Torque (5 of 6)

- Again, torque is positive if it would cause a counterclockwise rotation, otherwise negative
- For several torques, the **net torque** or **resultant torque** is the sum of individual torques

10-6 Torque (6 of 6)

Checkpoint 6

The figure shows an overhead view of a meter stick that can pivot about the dot at the position marked 20 (for 20 cm). All five forces on the stick are horizontal and have the same magnitude. **Rank** the forces according to the **magnitude of the torque** they produce, **greatest** first.



Answer:

F_1 & F_3, F_4, F_2 & F_5

10-7 Newton's Second Law for Rotation (2 of 3)

- Rewrite $F = ma$ with rotational variables:

$$\tau_{\text{net}} = I\alpha \qquad \textbf{Equation (10-42)}$$

- It is torque that causes angular acceleration

The torque due to the tangential component of the force causes an angular acceleration around the rotation axis.

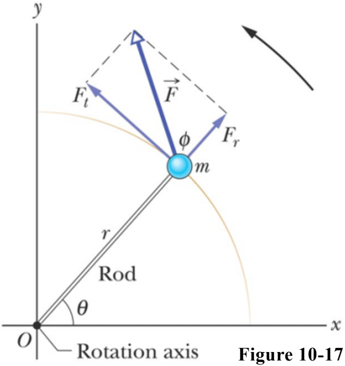
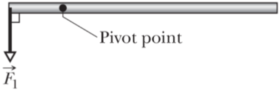


Figure 10-17

10-7 Newton's Second Law for Rotation (3 of 3)

Checkpoint 7

The figure shows an overhead view of a meter stick that can pivot about the point indicated, which is to the left of the stick's midpoint. Two horizontal forces, $\vec{F}_1$ and $\vec{F}_2$, are applied to the stick. Only $\vec{F}_1$ is shown. Force $\vec{F}_2$ is perpendicular to the stick and is applied at the right end. If the stick is not to turn, (a) what should be the direction of $\vec{F}_2$, and (b) should F_2 be greater than, less than, or equal to F_1?



Answer:

(a) F_2 should point downward, and

(b) should have a smaller magnitude than F_1

10-8 Work and Rotational Kinetic Energy (2 of 5)

- The rotational work-kinetic energy theorem states:

$$\Delta K = K_f - K_i = \tfrac{1}{2} I\omega_f^2 - \tfrac{1}{2} I\omega_i^2 = W$$ **Equation (10-52)**

- The work done in a rotation about a fixed axis can be calculated by:

$$W = \int_{\theta_i}^{\theta_f} \tau \, d\theta$$ **Equation (10-53)**

- Which, for a constant torque, reduces to:

$$W = \tau\left(\theta_f - \theta_i\right)$$ **Equation (10-54)**

10-8 Work and Rotational Kinetic Energy (3 of 5)

- We can relate work to power with the equation:

$$P = \frac{dW}{dt} = \tau\omega$$ **Equation (10-55)**

- Table 10-3 shows corresponding quantities for linear and rotational motion:

10-8 Work and Rotational Kinetic Energy (4 of 5)

Table 10-3 Some Corresponding Relations for Translational and Rotational Motion

Pure Translation (Fixed Direction)		Pure Rotation (Fixed Axis)	
Position	x	Angular position	θ
Velocity	$v = \frac{dx}{dt}$	Angular velocity	$\omega = \frac{d\theta}{dt}$
Acceleration	$a = \frac{dv}{dt}$	Angular acceleration	$\alpha = \frac{d\omega}{dt}$
Mass	m	Rotational inertia	I
Newton's second law	$F_{net} = ma$	Newton's second law	$\tau_{net} = I\alpha$
Work	$W = \int F\, dx$	Work	$W = \int \tau\, d\theta$

10-8 Work and Rotational Kinetic Energy (5 of 5)

Pure Translation (Fixed Direction)		Pure Rotation (Fixed Axis)	
Kinetic energy	$K = \frac{1}{2}mv^2$	Kinetic energy	$K = \frac{1}{2}I\omega^2$
Power (constant force)	$P = Fv$	Power (constant torque)	$P = \tau\omega$
Work–kinetic energy theorem	$W = \Delta K$	Work–kinetic energy theorem	$W = \Delta K$