Ch3

Saturday, March 20, 2021 9:21 AM

$x<0$ $x>0$ 0

(4) $f(x) = \begin{cases} \sin 2x & x \le 0 \\ a x & x > 0 \end{cases}$ Find a s.t

(i) f cont. at $x=0$

(ii) f diff at $x=0$

(i) $f(0) = \boxed{\lim_{x \to 0} f(x)}$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$

$a(0) = \sin 0$

$\boxed{0 = 0}$

a is any constant

$a \in \mathbb{R}$

$f' = \begin{cases} 2\cos 2x & x < 0 \\ a & x > 0 \end{cases}$

$\lim_{x \to 0^+} f'(x) = \lim_{x \to 0^-} f'(x)$

$f'_+(0) = f'_-(0)$

$a = 2\cos(0)$

$a = (2)(1)$

$a = 2$

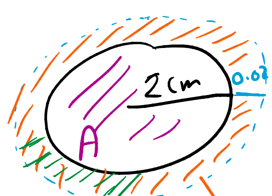
(7) $r_1 = 2$ cm $\Rightarrow$ $r_2 = 2.02$ cm

$\Delta r = r_2 - r_1$

$= 2.02 - 2$

$= 0.02$ cm

$= dr$



$A = r^2 \pi$

$dA = 2r\pi\, dr$

$= (2)(2)\pi\,(0.02)$

$= 0.08\pi$

(1) Estimate the resulting change in area $\Longrightarrow$

(1) Estimate the resulting change in area $\Rightarrow = 0.08\pi$

dA : Estimated

تقريبي DA : True Change $= A_2 - A_1$

$= A(r_2) - A(r_1)$

$= r_2^2\pi - r_1^2\pi$

$= (2.02)^2\pi - 2^2\pi$

$= 4.0804\pi - 4\pi$

$= \boxed{0.0804\pi}$

تقريبي Error $= |DA - dA|$

$= |0.0804\pi - 0.08\pi|$

$= 0.0004\pi$

(b) Express the estimate as % of the circle's original Area

$dA = 0.08\pi$

$A = r^2\pi$
$= 2^2\pi$
$= 4\pi$

$$\frac{dA}{A} \times 100\% = \frac{0.08\pi}{4\pi} \times 100\%$$

$\bar{A}$ 4 ''

$= 2\ \%$

⑥ ⓐ Find linearization for $f(x) = \tan x$ at $x_0 = \frac{\pi}{4}$

$L(x) = f(x_0) + m\,(x - x_0)$

$m = f'(x_0)$

$f'(x) = \sec^2 x$

$f'(\frac{\pi}{4}) = \sec^2 \frac{\pi}{4} = \left(\sec\frac{\pi}{4}\right)^2 = \left(\frac{1}{\cos\frac{\pi}{4}}\right)^2 = \left(\frac{1}{\frac{1}{\sqrt{2}}}\right)^2 = (\sqrt{2})^2 = 2$

$f(x_0) = \tan x_0$

$f(\frac{\pi}{4}) = \tan\frac{\pi}{4} = 1$

$= 1 + \boxed{2}\left(x - \frac{\pi}{4}\right)$

$= 1 + 2x - \frac{\pi}{2}$

$\frac{2}{2} = 1 - \frac{\pi}{2} + 2x$

$$L(x) = \frac{2-\pi}{2} + 2x$$

$\Rightarrow L\left(\frac{\pi}{4}\right) = 1$

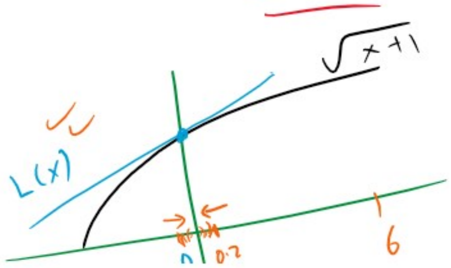
$\frac{2-\pi}{2} + 2\,\frac{\pi}{4} =$

$\frac{2-\pi}{2} + \frac{\pi}{2} = \frac{2 - \cancel{\pi} + \cancel{\pi}}{2} = \frac{2}{2} = 1$

$f(x) = \sqrt{x+1} \Rightarrow$ Find $L(x)$ at $x = 0$

$f(0) = \sqrt{0+1} = \sqrt{1} = 1$

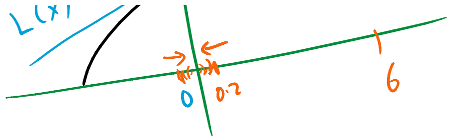
$L(x) = \frac{x}{2} + 1 \Rightarrow L(0) = \frac{0}{2} + 1 = 1$



Find $f(0.2) = \sqrt{0.2+1} = \sqrt{1.2} = 1.095$

$L(0.2) \approx f(0.2)$

$\frac{0.2}{2} + 1 = 0.1 + 1 = 1.1$



$\frac{0.2}{2} + 1 = 0.1 + 1 = \underline{1.1}$

ch 2 [1] h

$$\lim_{x\to\infty} \frac{\sqrt[3]{x} - \sqrt[5]{x}}{\sqrt[3]{x} + \sqrt[5]{x}}$$

$\infty - \infty$

$\frac{1}{3} > \frac{1}{5}$

$\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$

$\frac{1}{5} - \frac{1}{3}$

$$\lim_{x\to\infty} \frac{\frac{x^{1/3} - x^{1/5}}{x^{1/3}}}{\frac{x^{1/3} + x^{1/5}}{x^{1/3}}} = \lim_{x\to\infty} \frac{1 - x^{\frac{1}{5}-\frac{1}{3}}}{1 + x^{\frac{1}{5}-\frac{1}{3}}}$$

$$\lim_{x\to\infty} \frac{1 - x^{\frac{-2}{15}}}{1 + x^{\frac{-2}{15}}} = \frac{1 - \lim_{x\to\infty} \frac{1}{x^{\frac{2}{15}}}}{1 + \lim_{x\to\infty} \frac{1}{x^{\frac{2}{15}}}}$$

$x^{\frac{2}{15}} \to \infty$, $\frac{1}{x^{2/15}} \to 0$

$= \frac{1}{1} = 1$

(j) $$\lim_{x\to 3^+} \frac{\lfloor t \rfloor}{t} = \frac{\lfloor 3.0001 \rfloor}{3} = \frac{3}{3} = 1$$

$\lfloor 3 \rfloor = 3$

$\lfloor 4 \rfloor = 4$

$\lfloor -2 \rfloor = -2$

$\lfloor 3.5 \rfloor = 3$

$\lfloor 3.9 \rfloor = 3$

$\lfloor 3.0001 \rfloor = 3$

ch 3
Q4

$$f(x) = \begin{cases} \sin 2x & x \le 0 \\ ax & x > 0 \end{cases}$$

Find a s.t

... $x = 0$ ...

(i) f cont. at $x=0$

$$f(0) = \lim_{x\to 0} f(x) \qquad \lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$$

$$a(0) = \sin 2(0)$$

$$0 = 0$$

we can take any value for $a \in \mathbb{R}$ so that f cont at $x=0$

(ii) f diff at $x=0$

$$f'_+(0) = f'_-(0)$$

$$f'(x) = \begin{cases} 2\cos 2x, & x < 0 \\ \underline{\underline{a}}, & x > 0 \end{cases}$$

$$\lim_{x\to 0^+} f'(x) = \lim_{x\to 0^-} f'(x)$$

$$a = 2\cos 2(0)$$

$$a = 2\cos 0$$

$$a = 2(1)$$

$$\boxed{a = 2}$$

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ch3 Q7

$r_1 = 2$ cm

$r_2 = 2.02$ cm

$$\Delta r = r_2 - r_1 = 2.02 - 2$$
$$= 0.02 \text{ cm}$$
$$= \underline{\underline{dr}}$$

True 0.0804π

estimate 0.08π

$$A = r^2 \pi$$

$$dA = 2r\pi \, \underline{dr}$$
$$= (2)(2)\pi(0.02)$$
$$= \underline{0.08\pi}$$

$= \underline{dr}$

(i) Estimate the resulting change in area. $= 0.08\pi$

Estimated Change $dA = 0.08\pi$

True Change $= \Delta A = A_2 - A_1 = A(r_2) - A(r_1)$

$= A(2.02) - A(2)$

$= (2.02)^2\pi - (2)^2\pi$

$= 4.0804\pi - 4\pi$

$= 0.0804\pi$

True $\downarrow$ est. $\downarrow$

$\text{Error} = |\Delta A - dA| = |0.0804\pi - 0.08\pi| = 0.0004$

(b) Express the estimate as % of the original area

$A = 4\pi$

differential $\rightarrow$ $dA = 0.08\pi$

$$\frac{dA}{A} \times 100\% = \frac{\cancel{0.08}^{\,2}\cancel{\pi}}{\cancel{4}\cancel{\pi}} \times \cancel{100}\%$$

$= 2\%$

$dA \sim \Delta A$

Q6 (c) $h(x) = \dfrac{x^2}{x^2+1}$, $x = 0$ Find $L(x)$

$L(x) \sim h(x)$ near x_0 $\Downarrow$ 0

$L(x) = h(x_0) + h'(x_0)(x - x_0)$

$y = h(x_0) = h(0)$

$$L(x) = h(x_0) + h'(x_0)(x - x_0)$$

$$\underbrace{L(x)}_{y} = \underbrace{h(x_0)}_{y_0} + \underbrace{h'(x_0)}_{m}(x - x_0)$$

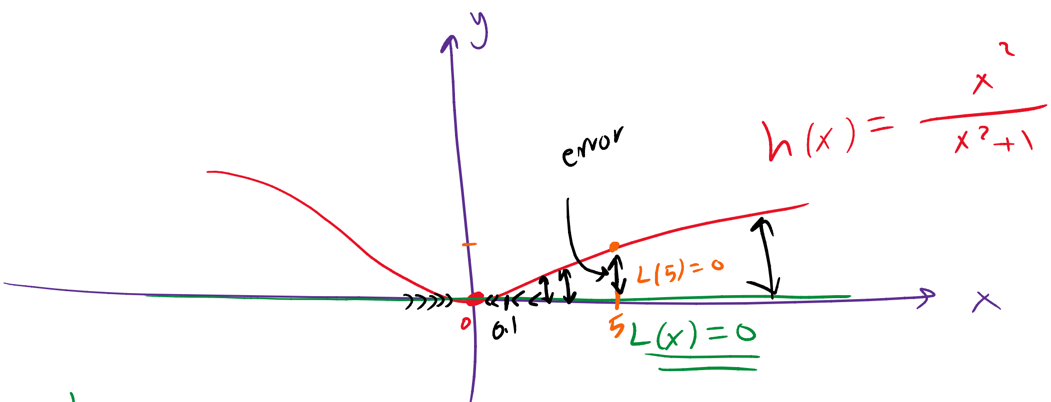
$$= 0 + \boxed{0}(x - 0)$$

$$\boxed{L(x) = 0} \rightarrow x\text{-axis}$$

$$y_0 = h(x_0) = h(0) = \frac{0^2}{0^2+1} = \frac{0}{1} = 0$$

$$h'(x) = \frac{(x^2+1)(2x) - x^2(2x)}{(x^2+1)^2}$$

$$h'(0) = \frac{0 - 0}{(0+1)^2} = \frac{0}{1} = 0$$



$$L(x) \sim h(x)$$

لما صحيح ودقيق
افترضنا ان
$X_0 = 0$

Estimat $h(0.1) \approx L(0.1) = 0$

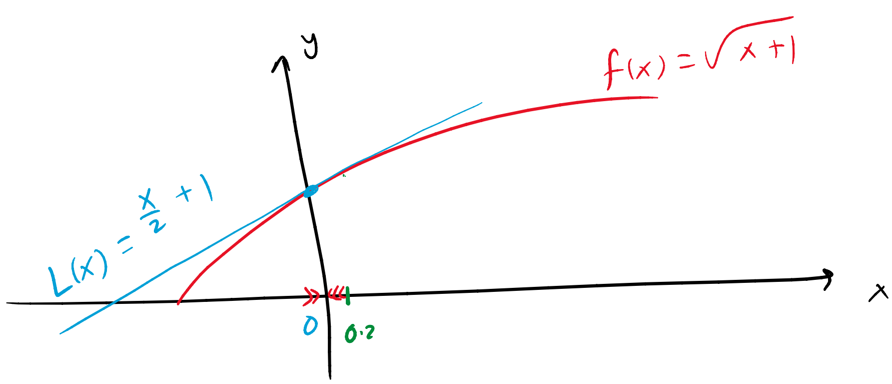
True value $h(0.1) = \frac{(0.1)^2}{(0.1)^2+1} = \frac{0.01}{0.01+1} = \frac{0.01}{1.01} \approx 0.0099$

Exp $f(x) = \sqrt{x+1}$

Find $L(x)$ at $x = 0$

$$L(x) = f(x_0) + f'(x_0)(x - x_0)$$

$$= \frac{x}{2} + 1$$



$$f(0.2) \sim L(0.2) = \frac{0.2}{2} + 1$$

$$L(x) \sim f(x)$$

Estimat $f(0.2) \sim L(0.2) = \frac{0.2}{2} + 1$

$= 0.1 + 1$

$= 1.1$

$L(x) \sim f(x)$

True Valu $f(0.2) = \sqrt{0.2 + 1} = \sqrt{1.2} \approx 1.095$

ch2 Q1 (g) $\lim_{x \to -\infty} \frac{\sqrt{x^2+1}}{x+1}$

$\frac{0}{0}$ $\frac{-1}{}$ $\frac{1}{1}$ $\frac{1}{-1}$

$x \to \infty \Rightarrow 1$

$\sqrt{x^2} = |x|$

$$\lim_{x \to -\infty} \frac{\sqrt{x^2+1}}{x+1} = \lim_{x \to -\infty} \frac{\frac{\sqrt{x^2+1}}{\sqrt{x^2}}}{\frac{x+1}{\sqrt{x^2}}}$$

$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ $\frac{a}{b} > 0$, $b \neq 0$

$$= \lim_{x \to -\infty} \frac{\sqrt{\frac{x^2+1}{x^2}}}{\frac{x+1}{|x|}} = \lim_{x \to -\infty} \frac{\sqrt{1 + \frac{1}{x^2}}}{\frac{x+1}{-x}}$$

$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

$\sqrt{\ }$ cont

$\lim \sqrt{\ } \Rightarrow \sqrt{\lim}$

$$\frac{\lim_{x \to -\infty} \sqrt{1 + \frac{1}{x^2}}}{\lim_{x \to -\infty} \left(-1 - \frac{1}{x}\right)} = \frac{1}{-1} = -1$$

$\frac{1}{x^2} \to 0$, $\frac{1}{x} \to 0$

$$\lim_{x \to \pm\infty} \frac{\sqrt{x^4+1}}{x^2+1} = 1$$

$\sqrt{x^4} = |x^2| = x^2$

$\frac{\sqrt{x^4+1}}{\sqrt{x^4}}$ $\lim \sqrt{\frac{x^4+1}{x^4}}$

$\sqrt{x^4+1}$

$x \to \pm\infty$ $x+1$

$$\lim_{x \to \pm\infty} \frac{\frac{\sqrt{x^4+1}}{\sqrt{x^4}}}{\frac{x^2+1}{\sqrt{x^4}}} = \lim \frac{\sqrt{\frac{x^4+1}{x^4}}}{\frac{x^2+1}{x^2}} = \lim \frac{\sqrt{\frac{x^4+1}{x^4}}}{1+\frac{1}{x^2}}$$

$$= \lim_{x \to \pm\infty} \frac{\sqrt{1+\frac{1}{x^4}}}{1+\frac{1}{x^2}} \qquad \frac{1}{x^4} \to 0 \qquad \frac{1}{x^2} \to 0 \qquad = \frac{1}{1}$$