Chapter 7
Kinetic Energy and Work
+some selected problems

2021 11 09

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7-3 Work Done by the Gravitational Force (7 of 7) Sample Problem 7.05

Being lowered down in an elevator, m=500 kg, v_i = 4.0 m/s, supporting cable begins to slip, allowing it to fall with constant acceleration $\vec{a} = \vec{g}/5$
(Tension does negative work, gravity does positive work)

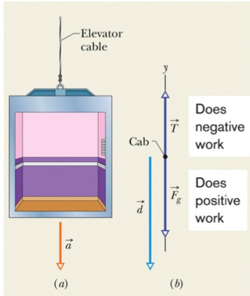


Figure 7-9

(a) During the fall through a distance d = 12 m, what is the work done on the cab by the gravitational force $\vec{F}_g$?

Angle between the directions of $\vec{F}_g$ and the cab's displacement $\vec{d}$ is 0˚

$$W_g = mgd \cos 0° = (500 \text{ kg})(9.8 \text{ m/s}^2)(12 \text{ m})(1)$$
$$= 5.88 \times 10^4 \text{ J} \approx 59 \text{ kJ}. \qquad \text{(Answer)}$$

(b) During the 12 m fall, what is the work W_T done on the cab by the upward pull $\vec{T}$ of the elevator cable?

Eq. 7-7 $W = Fd \cos \phi$ (180° angle between $\vec{d}$ and $\vec{F}$)

$T - F_g = ma$ ($a = -g/5$ downward)

$$W_T = Td \cos \phi = m(a + g)d \cos \phi.$$

$$W_T = m\left(-\frac{g}{5} + g\right)d \cos \phi = \frac{4}{5} mgd \cos \phi$$
$$= \frac{4}{5}(500 \text{ kg})(9.8 \text{ m/s}^2)(12 \text{ m}) \cos 180°$$
$$= -4.70 \times 10^4 \text{ J} \approx -47 \text{ kJ}. \quad \text{(Answer)}$$

(c) What is the net work W done on the cab during the fall?

Sum of the works done by the forces acting on the cab:

$$W = W_g + W_T = 5.88 \times 10^4 \text{ J} - 4.70 \times 10^4 \text{ J}$$
$$= 1.18 \times 10^4 \text{ J} \approx 12 \text{ kJ}. \quad \text{(Answer)}$$

(d) What is the cab's kinetic energy at the end of the 12 m fall?

Eq. 7-11 ($K_f = K_i + W$), and Eq. 7-1 $K_i = 1/2\, mv_i^2$.

$$K_f = K_i + W = \tfrac{1}{2}mv_i^2 + W$$
$$= \tfrac{1}{2}(500 \text{ kg})(4.0 \text{ m/s})^2 + 1.18 \times 10^4 \text{ J}$$
$$= 1.58 \times 10^4 \text{ J} \approx 16 \text{ kJ}.$$

7-4 Work Done by a Spring Force (3 of 8)

- A **spring force** is the variable force from a spring

- Figure (a) shows the spring in its **relaxed state**: since it is neither compressed nor extended, no force is applied
- If we stretch or extend the spring it resists, and exerts a restoring force that attempts to return the spring to its relaxed state

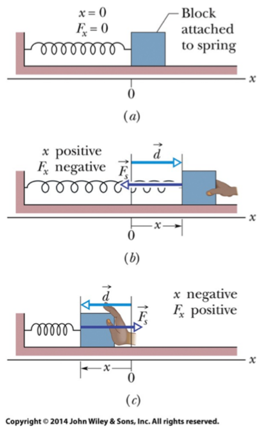


Figure 7-10

7-4 Work Done by a Spring Force (4 of 8)

- The spring force is given by **Hooke's law:**

$$\vec{F}_s = -k\vec{d}$$ **Equation (7-20)**

- The negative sign represents that the force always opposes the displacement
- The **spring constant** k is a measure of the stiffness of the spring
- This is a variable force (function of position) and it exhibits a linear relationship between F and d
- For a spring along the x-axis we can write:

$$F_x = -kx$$ **Equation (7-21)**

7-4 Work Done by a Spring Force (5 of 8)

- We can find the work by integrating:

$$W_s = \int_{x_i}^{x_f} -F_x dx.$$ **Equation (7-23)**

- Plug kx in for F_x:

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$ **Equation (7-25)**

- The work:
  - Can be positive or negative
  - Depends on the net energy transfer

7-4 Work Done by a Spring Force (6 of 8)

Work W_s is positive if the block ends up closer to the relaxed position ($x = 0$) than it was initially.

It is negative if the block ends up farther away from $x = 0$. It is zero if the block ends up at the same distance from $x = 0$.

7-4 Work Done by a Spring Force (7 of 8)

- For an initial position of $x = 0$:

$$W_s = -\frac{1}{2}kx^2$$ **Equation (7-26)**

- For an applied force where the initial and final kinetic energies are zero:

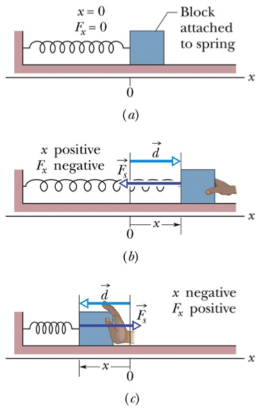
$$W_a = -W_s.$$ **Equation (7-28)**

If a block that is attached to a spring is stationary before and after a displacement, then the work done on it by the applied force displacing it is the negative of the work done on it by the spring force.

7-4 Work Done by a Spring Force (8 of 8)

Checkpoint 2

For three situations, the **initial and final positions**, respectively, along the *x* axis for the block in the Figure are (a) −3 cm, 2 cm; (b) 2 cm, 3 cm; and (c) −2 cm, 2 cm. In each situation, is the **work** done by the spring force on the block **positive, negative, or zero?**



Answer:

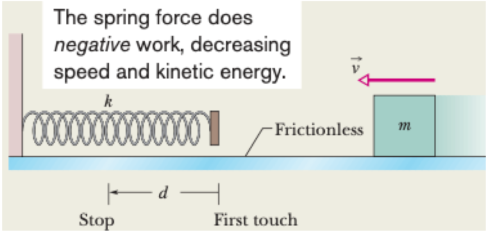
(a) positive (المكان النهائي أقرب)

(b) negative (المكان النهائي أبعد)

(c) Zero

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

Sample Problem 7.06



Canister of mass $m = 0.40$ kg, slides horizontally with speed $v = 0.50$ m/s.

When the canister is momentarily stopped by the spring, by what distance d is the spring compressed?

We use eq. 7-10: $K_f - K_i = W$

$$K_f - K_i = -\tfrac{1}{2}kd^2$$

Using the relation of KE and spring energy to find d

$$0 - \tfrac{1}{2}mv^2 = -\tfrac{1}{2}kd^2$$

Solving for d:

$$d = v\sqrt{\frac{m}{k}} = (0.50\ \text{m/s})\sqrt{\frac{0.40\ \text{kg}}{750\ \text{N/m}}}$$
$$= 1.2 \times 10^{-2}\ \text{m} = 1.2\ \text{cm}.$$

7-5 Work Done by a General Variable Force (3 of 4)

- We take a one-dimensional example
- We need to integrate the work equation (which normally applies only for a constant force) over the change in position
- Graph of F(x)
- We can show this process by an approximation with rectangles under the curve

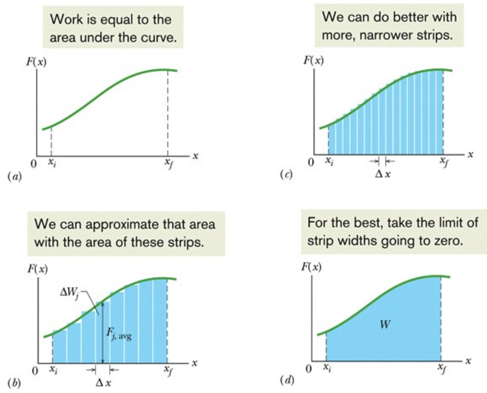


Figure 7-12

7-5 Work Done by a General Variable Force (4 of 4)

- Our sum of rectangles would be:

$$W = \lim_{\Delta x \to 0} \sum F_{j,\text{avg}} \Delta x.$$ **Equation (7-31)**

- As an integral this is:

$$W = \int_{x_i}^{x_f} F(x)\, dx$$ **Equation (7-32)**

- In three dimensions, we integrate each separately:

$$W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz.$$ **Equation (7-36)**

- The work-kinetic energy theorem still applies!

7-6 Power (2 of 5)

- **Power** is the time rate at which a force does work
- A force does W work in a time Δt; the **average power** due to the force is:

$$P_{\text{avg}} = \frac{W}{\Delta t}$$

Equation (7-42)

- The **instantaneous power** at a particular time is:

$$P = \frac{dW}{dt}$$

Equation (7-43)

7-6 Power (3 of 5)

- The SI unit for power is the watt (W): 1 W = 1 J/s
- Therefore work-energy can be written as (power) × (time) e.g. kWh, the kilowatt-hour

7-6 Power (4 of 5)

- Solve for the instantaneous power using the definition of work:

$$P = \frac{dW}{dt} = \frac{F \cos \phi \, dx}{dt} = F \cos \phi \left(\frac{dx}{dt} \right),$$

$$P = Fv \cos \phi. \qquad \textbf{Equation (7-47)}$$

- Or:

$$P = \vec{F} \cdot \vec{v} \qquad \textbf{Equation (7-48)}$$

7-6 Power (5 of 5)

Checkpoint 3

A block moves with uniform circular motion because a cord tied to the block is anchored at the center of a circle. Is the power due to the force on the block from the cord positive, negative, or zero?

Answer:

$$\text{Zero}\left(\text{consider } P = Fv\cos\phi, \text{ and note that } \phi = 90°\right)$$

F at an angle to direction of the travel

P.5 A **father racing his son has **half the kinetic energy of the son**, who has **half the mass of the father**. The **father speeds up by 1.0 m/s** and **then** has the **same kinetic energy** as the son.

What are the original speeds of (a) the father and (b) the son?

We denote the mass of the father as *m* and his initial speed v_i. The initial kinetic energy of the father is $K_i = ½ K_{son}$ and his final kinetic energy (when his speed is $v_f = v_i + 1.0$ m/s) is $K_f = K_{son}$.

(a) We see from the above that $K_i = ½ K_f$, which (with SI units understood) leads to $\frac{1}{2}mv_i^2 = \frac{1}{2}\left[\frac{1}{2}m\ (v_i + 1.0\text{ m/s})^2\right]$

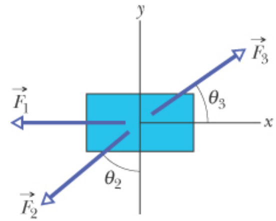
The mass cancels and we find a second-degree equation for v_i : $\frac{1}{2}v_i^2 - v_i - \frac{1}{2} = 0.$

The positive root (from the quadratic formula) yields $v_i = 2.4$ m/s.

(b) From the first relation above ($K_i = ½ K_{son}$), we have $\frac{1}{2}mv_i^2 = \frac{1}{2}\left(\frac{1}{2}\ (m/2)\ v_{son}^2\right)$

and (after canceling *m* and one factor of ½) are led to $v_{son} = 2v_i = 4.8\ \text{m/s}.$

••**P. 14** Three horizontal forces acting on a canister, initially stationary, but now moves across a frictionless floor. The force magnitudes are F_1 = 3.00 N, F_2 = 4.00 N, and F_3 = 10.0 N, and the indicated angles are θ_2 = 50.0° and θ_3 = 35.0°.



What is the net work done on the canister by the three forces during the first 4.00 m of displacement?

Total work done by forces is given by $W=F_{net}\,\Delta x$, where F_{net} is the magnitude of the net force and Δx is the magnitude of the displacement. We add the three vectors, finding the *x* and *y* components of the net force:

$$F_{\text{net}\,x} = -F_1 - F_2 \sin 50.0° + F_3 \cos 35.0° = -3.00\,\text{N} - (4.00\,\text{N}) \sin 35.0° + (10.0\,\text{N}) \cos 35.0°$$
$$= 2.13\,\text{N}$$

$$F_{\text{net}\,y} = -F_2 \cos 50.0° + F_3 \sin 35.0° = -(4.00\,\text{N})\ \cos 50.0° + (10.0\,\text{N}) \sin 35.0°$$
$$= 3.17\,\text{N}.$$

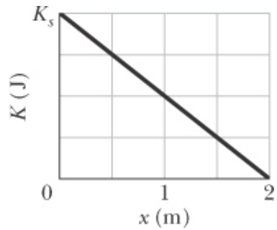
The magnitude of the net force is

$$F_{\text{net}} = \sqrt{F_{\text{net}\,x}^2 + F_{\text{net}\,y}^2} = \sqrt{(2.13\ \text{N})^2 + (3.17\ \text{N})^2} = 3.82\,\text{N}.$$

The work done by the net force is $W = F_{\text{net}} d = (3.82\,\text{N})(4.00\,\text{m}) = 15.3\ \text{J}$

where we have used the fact that $\vec{d}\,||\,\vec{F}_{net}$

••**20** A block is sent up a frictionless ramp along which an *x* axis extends upward. The figure gives the kinetic energy of the block as a function of position *x*; the scale of the figure's vertical axis is set by K_s = 40.0 J.



If the block's initial speed is 4.00 m/s, what is the normal force on the block?

From the figure, one may write the K.E.(x) as: $K = K_s - 20x = 40 - 20x$ (Line equation)

Since W = ΔK = $\vec{F}_x$. Δx, the component of the force along the force along +*x* is

$$F_x = dK/dx = -20 \text{ N}.$$

F_N on the block is F_N=F_y

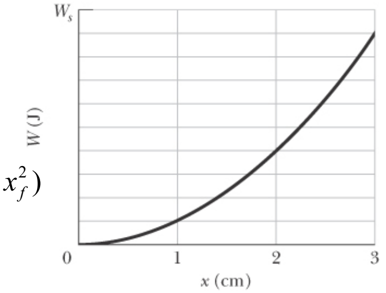
(Note that F_N points in the opposite direction of the component of the gravitational force.) With an initial kinetic energy K_s = 40.0 J and v_0 = 4.00 m/s, the mass of the block is

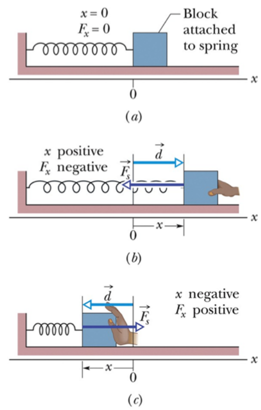
$$m = \frac{2K_s}{v_0^2} = \frac{2(40.0 \text{ J})}{(4.00 \text{ m/s})^2} = 5.00 \text{ kg}.$$

Thus, the normal force is

$$F_y = \sqrt{(mg)^2 - F_x^2} = \sqrt{(5.0 \text{ kg})^2 (9.8 \text{ m/s}^2)^2 - (20 \text{ N})^2} = 44.7 \text{ N} \approx 45 \text{ N}.$$

••29 In the arrangement of the Figure. 7-10, we gradually pull the block from *x* = 0 to *x* = +3.0 cm, where it is stationary. Figure 7-35 gives the work that our force does on the block. The scale of the figure's vertical axis is set by W_s = 1.0 J. We then pull the block out to *x* = +5.0 cm and release it from rest. How much work does the spring do on the block when the block moves from x_i = +5.0 cm to (a) *x* = +4.0 cm, (b) *x* = −2.0 cm, and (c) *x* = −5.0 cm?





The work done by the spring force is given by Eq. 7-25: $W_s = \frac{1}{2}k(x_i^2 - x_f^2)$

The spring constant *k* can be deduced from the figure which shows the amount of work done to pull the block from 0 to *x* = 3.0 cm. The parabola W_a= $(kx^2)/2$ contains (0,0), (2.0 cm, 0.40 J) and (3.0 cm, 0.90 J). Thus, we may infer from the data that k = 2.0 x 10^3 N/m (direct substitution in equation)

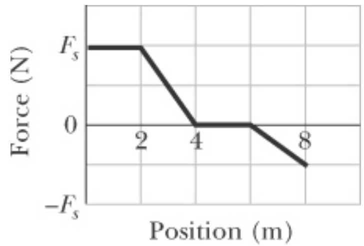
(a) When the block moves from x_i=+5.0 cm to x= + 4.0 cm $W_s = \frac{1}{2}(2.0\times10^3 \text{ N/m})[(0.050 \text{ m})^2 - (0.040 \text{ m})^2] = 0.90 \text{ J}.$

(b) Moving from x_i=+5.0 cm to x= – 2.0 cm $W_s = \frac{1}{2}(2.0\times10^3 \text{ N/m})[(0.050 \text{ m})^2 - (-0.020 \text{ m})^2] = 2.1 \text{ J}.$

(c) Moving from x_i=+5.0 cm to x= – 5.0 cm $W_s = \frac{1}{2}(2.0\times10^3 \text{ N/m})[(0.050 \text{ m})^2 - (-0.050 \text{ m})^2] = 0 \text{ J}.$

*36 A 5.0 kg block moves in a straight line on a horizontal frictionless surface under the influence of a **force that varies with position** as shown in the figure. The scale of the figure's vertical axis is set by F_s = 10.0 N.

How much work is done by the force as the block moves from the origin to x = 8.0 m?



From Eq. 7-32, we see that the "area" in the graph is equivalent to the work done. Finding that area (in terms of rectangular [length × width] and triangular [1/2 base x height] area s) we obtain

$$W = W_{0<x<2} + W_{2<x<4} + W_{4<x<6} + W_{6<x<8} = (20+10+0-5)\text{ J} = 25\text{ J}.$$

*45 A 100 kg block is pulled at a constant speed of 5.0 m/s across a horizontal floor by an applied force of 122 N directed 37° above the horizontal.

What is the **rate** at which the **force** does work on the block?

A block is pulled at a constant speed by a force directed at some angle with respect to the direction of motion. The quantity we're interested in is the **power**, or the time rate at which work is done by the applied force.

The power associated with force $\vec{F}$ is given by $P=\vec{F}.\vec{v}=Fv\cos\phi$ where $\vec{v}$ is the velocity of the object on which the force acts, and ϕ is the angle between $\vec{F}$ and $\vec{v}$

With F=122N, v= 5m/s and $\phi=37^\circ$, we find the power to be

$$P = Fv\cos\phi = (122\ \text{N})(5.0\ \text{m/s})\cos 37.0^\circ = 4.9\times 10^2\ \text{W}.$$

The power is at a maximum when $\vec{F}$ and $\vec{v}$ are in the same direction ($\phi=0$). We're told that the block moves at a constant speed, so $\Delta K=0$, and the net work done on it must also be zero by the work-kinetic energy theorem. Thus, the applied force here must be compensating another force (e.g., friction) for the net rate to be zero.