

4.1

$$\begin{aligned}
 17.87 \text{ m} &\rightarrow 4 \text{ s.f.} \\
 0.4730 \text{ km} &\rightarrow 4 \text{ s.f.} \\
 17.9 \text{ sec} &\rightarrow 3 \text{ s.f.} \\
 0.473 &\rightarrow 3 \text{ s.f.} \\
 18 \text{ ns} &\rightarrow 2 \text{ s.f.} \\
 0.472 &\rightarrow 2 \text{ s.f.} \\
 1.34 \times 10^2 \frac{\text{m}}{\text{s}} &\rightarrow 3 \text{ s.f.} \\
 2.567 \times 10^5 \text{ cm} &\rightarrow 4 \text{ s.f.}
 \end{aligned}$$

$$\begin{aligned}
 2.0 \times 10^{10} \text{ J} &\rightarrow 2 \text{ s.f.} \\
 1.001 \text{ sec} &\rightarrow 4 \text{ s.f.} \\
 1.000 \text{ sec} &\rightarrow 4 \text{ s.f.} \\
 107 &\rightarrow 1 \text{ s.f.} \\
 1000 \text{ liters} &\rightarrow 1 \text{ s.f.} \\
 1001 \text{ cm}^2 &\rightarrow 4 \text{ s.f.}
 \end{aligned}$$

4.2 a) $231.3 + 5.8 \times 10^2 \Rightarrow$ (You must write the both in scientific notation)

$$\begin{aligned}
 (2.313 + 5.8) \times 10^2 &= 8.113 \times 10^2 = \boxed{8.1 \times 10^2} \\
 &\quad \begin{array}{l} 3 \text{ decimal pt} \quad 1 \text{ decimal point} \end{array}
 \end{aligned}$$

b) $\frac{24.7}{3 \text{ s.f.}} \times \frac{2.7 \times 10^4}{2 \text{ s.f.}} = 66.69 \times 10^4 = \boxed{67 \times 10^4}$

c) $\frac{217.0}{1 \text{ decimal point}} - \frac{141.65}{2 \text{ decimal point}} = 75.35 = \boxed{75.4}$

4.3 $3.13 / 3.22 / 3.32 / 3.21 / 3.17$ { By using calculator? }
 $\bar{h} \pm \sigma_m = (3.21 \pm 0.03) \text{ m}$
 SD

4.4 a) $\bar{L} = 87.414 \text{ cm}$

b) Sample standard deviation

$$\sigma_s = 0.24103 \sim \boxed{0.2 \text{ cm}}$$

c) $\sigma_m = \text{uncertainty in the length} = \frac{\sigma_s}{\sqrt{7}} = \frac{0.24103}{\sqrt{7}}$

$$\sigma_m = 0.0911 \sim \boxed{0.09 \text{ cm}}$$

$$\bar{L} \pm \sigma_m = (87.41 \pm 0.09) \text{ cm}$$

4.5 9.87, 9.75, 9.63, 9.80, 9.72, 9.73, 9.78, 9.67

a) $\sigma_s = 0.07936$

b) $\bar{g} \pm \sigma_m = (9.74 \pm 0.03) \frac{m}{s^2}$

c) $g_{true} = 9.81$

$D = |g_{exp} - g_{true}| = |9.74 - 9.81| = 0.07 > 0.06$

$2\sigma_g = 2 \times 0.03 = 0.06$ not accepted

✱

4.6 a) $3.565 \pm 0.035 \rightarrow 3.56 \pm 0.04$

b) $4.02 \pm 0.02315 \rightarrow 4.02 \pm 0.02$

c) $1.75143 \pm 1 \rightarrow 2 \pm 1$

d)

e) $0.000,000,432 \pm 0.000,000,07 \rightarrow 0.000000432 \pm 0.00000007$

f) $2.362 \times 10^3 \pm 21 \rightarrow 2.362 \times 10^3 \pm 0.02 \times 10^3$
 $(2.36 \pm 0.02) \times 10^3$

✱

4.7 a) $g = 10.4 \pm 1.1 \frac{m}{s^2}$, $g = 9.8 \frac{m}{s^2}$ $[2 \times 1.1 = 2.2]$

$D = |10.4 - 9.8| = 0.6 < 2.2$ accepted

b) $\tau = 2.5 \pm 0.2 \text{ sec}$, $\tau = 3.1 \text{ sec}$ $[2 \times 0.2 = 0.4]$

$D = |2.5 - 3.1| = 0.6 < 0.4$ accepted

c) Ignore it

4.8 $L = 8.27 \pm 0.05 \text{ cm}$, $W = 5.12 \pm 0.02 \text{ cm}$

$$A = LW = 8.27 \times 5.12 = 42.3424 \text{ cm}^2$$

$$\frac{\Delta A}{A} = \frac{\Delta L}{L} + \frac{\Delta W}{W} \Rightarrow \Delta A = A \left[\frac{\Delta L}{L} + \frac{\Delta W}{W} \right]$$

$$\Delta A = 42.3424 \left[\frac{0.05}{8.27} + \frac{0.02}{5.12} \right]$$

$$\Delta A = 0.4214 = \underline{\underline{0.4 \text{ cm}^2}}$$

$$A = (42.3 \pm 0.4) \text{ cm}^2$$

4.9 a) $a + b - c = R$ $\begin{cases} R = 5, \Delta R = 4 \\ R = 5 \pm 4 \end{cases}$
 $\Delta R = \Delta a + \Delta b + \Delta c$

b) $R = ct^2$, $\frac{\Delta R}{R} = \frac{\Delta c}{c} + 2 \frac{\Delta t}{t}$

$$\Delta R = ct^2 \left[\frac{\Delta c}{c} + \frac{2\Delta t}{t} \right]$$

c) $R = \frac{mb}{t} \rightarrow \frac{\Delta R}{R} = \frac{\Delta m}{m} + \frac{\Delta b}{b} + \frac{\Delta t}{t}$

$$\Delta R = \frac{mb}{t} \left[\frac{\Delta m}{m} + \frac{\Delta b}{b} + \frac{\Delta t}{t} \right]$$

d) $\ln\left(\frac{c}{a}\right) = R$ let $\frac{c}{a} = x \Rightarrow R = \ln x$, $\Delta R = \frac{\Delta x}{x}$

$$x = \frac{c}{a} \Rightarrow \frac{\Delta x}{x} = \frac{\Delta c}{c} + \frac{\Delta a}{a}$$

$$\therefore \Delta R = \frac{\Delta c}{c} + \frac{\Delta a}{a}$$

e) $R = \exp\left(\frac{b}{ac}\right)$ "use general Rule" $\Rightarrow R = \exp[b a^{-1} c^{-1}]$

$$\Delta R = \left| \frac{\partial R}{\partial b} \right| \Delta b + \left| \frac{\partial R}{\partial a} \right| \Delta a + \left| \frac{\partial R}{\partial c} \right| \Delta c$$

$$\Delta R = \left(\frac{1}{ac} \Delta b + \frac{b}{a^2 c} \Delta a + \frac{b}{a c^2} \Delta c \right) \exp\left[\frac{b}{ac}\right]$$

4.10 $y = v_i t + \frac{1}{2} g t^2$
 $v_i = (2.4 \pm 0.2) \frac{m}{s}$
 $g = (9.80 \pm 0.02) \frac{m}{s^2}$
 $t = (3.45 \pm 0.05) s$

a) $y \pm \Delta y$, $y = 66.60225$

$y = (67 \pm 3) m$

Let $y = A + B$
 $\Delta y = \Delta A + \Delta B$
 $\Rightarrow A = v_i t$
 $\Delta A = v_i \Delta t + t \Delta v_i$
 $\Delta A = 0.81 m$

$\Rightarrow B = \frac{1}{2} g t^2 \Rightarrow B = 58.3223$

$\frac{\Delta B}{B} = \frac{\Delta g}{g} + 2 \frac{\Delta t}{t}$
 $\Delta B = 58.3223 \left[\frac{0.02}{9.8} + \frac{(0.05)2}{3.45} \right]$
 $\Delta B = 1.8095 m$

$\Delta y = \Delta A + \Delta B$
 $= 0.81 + 1.8095 = 2.6 m$

$\Delta y = 3 m$

b) most Important effect on y 's uncertainty?

$\Delta y = \Delta A + \Delta B = v_i \Delta t + t \Delta v_i + \frac{1}{2} t^2 \Delta g + g t \Delta t$

$\Delta y = t \Delta v_i + (v_i + g t) \Delta t + \frac{1}{2} t^2 \Delta g$
 $= 0.69 + 1.8105 + 0.11903 = 2.6 m$

(t) has the Important effect on y 's uncertainty since the terms that includes (Δt) is the largest term

• Least effect on y 's uncertainty?

g

the term that includes Δg is the smallest one.