تلخيص مادة الاقتصاد المالي
ECON435
اعداد موقع BZU-HUB

ملاحظة: تم اعداد التلخيص في السنة الدراسية ٢٠٢٣/٢٠٢٤ الفصل الاول، كان المساق يُدرّس من قِبَل د.بيان عرقاوي، يعني جميع الملاحظات من شرح الدكتورة

Chapter 1: The Investment Environment

* Corporate Finance Decisions
  - Investment Decision ⇒ real assets , القرار الاستثماري
  - Financing Decision ⇒ القرار التمويلي
  - Dividend policy ⇒ سياسة توزيع الأرباح

• Investment in Financial assets

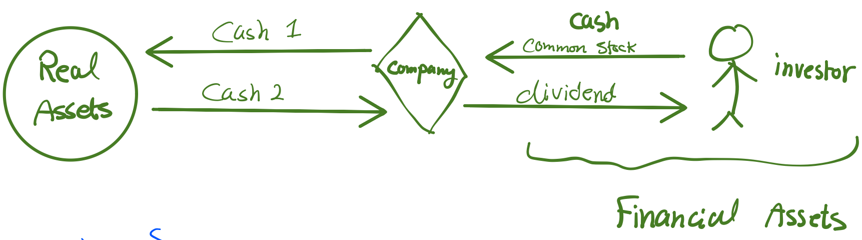
• Real Assets VS Financial Assets

* Real Assets :
  Determine the productive capacity and net income of the economy
  → Examples: Land, buildings, machines, Knowledge used to produce goods and services

* Financial Assets
  Claims on real assets
  (حصة ملكية)

* العلاقة بين ال real assets وال Financial assets *

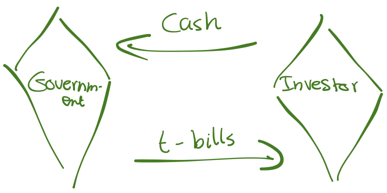
مثلاً: إذا ملكت 1٪ من الشركة (عن طريق الأسهم) هاد كأنه ملكت 1٪ من ال Real Assets



• Financial Assets ٣ أنواع :-

[1] Fixed Income or debt : Payments fixed or determined by a formula

* Money market debt : Short-term, highly marketable usually low credit risk (Treasury bills, C.D)
  أذونات الخزينة   شهادات الإيداع



* Capital market debt : long-term bonds, Can be safe or risky (Bonds → t-bonds من الحكومة
  → Corporate bonds من الشركات)

[2] Common Stock and Equity : is ownership in a Corporation
  - Payments to Stockholders are not fixed, but depend on the success of the firm (risky than bonds)

3 Derivative Securities : Value derives from prices of other Securities, such as Stocks and bonds.

المشتقات المالية : أصول قيمتها مشتقة من أصول أخرى

- Used to transfer risk (risk management) ⇒ Speculation في ناس بعتمدوا عالتخمين

- Options
  - calls → إذا كان سعر السهم أعلى أو أقل من السعر المعروض → يعطي الحق في الشراء
  - puts → يعطي الحق في البيع
- Futures / forward العقود المستقبلية / الآجلة
- SWAPS عقود التبادل

• Financial Markets and the Economy

* Allocation of Risk : Investors can select securities consistent with their tastes for risk

* Seperation of ownership and Management : with Stability comes agency Problems

• The Investment Process

* Asset allocation
  • Choice among broad asset classes
    * مثال : 30% equity , 20% Debt, 50% real estate

* Security Selection
  • Choice of which securities to hold <u>within</u> asset Class
    * مثال : Equity → google, facebook, paltel   Debt → t-bonds, Ford, Cacacolla
  • Security analysis to value securities and determine investment attractiveness

* Approaches :

1 Top - Down approach : 1) Asset allocation
2) Security Selection

2 Bottom - up approach : 1) Security Selection
2) Asset allocation

لحدهون بس مطلوب من الشابتر الأول

Chapter 2: Asset classes and Financial Instruments

- The money market
  - ↳ large denomination: قيمتها عالية

1 T-bill:

- Issuer: Government (Fedral Government)
- Denomination: min \$100, \$1,000 commonly \$10,000
- maturity: 4, 13, 26, 52 weeks (أقل من سنة)
- liquidity: very high
  - ↳ easily converted into cash with low transaction cost and with no much price risk
- Default Risk: none, free-Risk
- Taxation: Fedral ✓ , local/State ✗
- Interest Type: Discounted Security

now — later: يعني مثلاً

9,820 — 10,000

- Listing Yields rather than price
  - ↳ The ask price: the price you would have to pay to buy a T-bill from a securities dealer.
  - ↳ The bid price: the Slightly lower price you would receive if you wanted to sell a bill to a dealer
  - ↳ The bid-ask spread: الفرق بين سعرهم، وهو ربح التاجر
  - ↳ bid yield > ask yield لأنه علاقة ال price وال yield عكسية
    - ask price > bid price

مثال: من موقع Wall Street عند قسم ال T-bills

from 5.Oct.2023 to 28.Dec.2023

1. Days to maturity = 84 days
2. Bank Discount Method:
   - Asked yield = 5.305% → from 5.Oct.2023

$$\therefore \text{Discount} = 5.305 \times \frac{84}{360} = 1.237\%$$

market convention

So the bill with \$10,000 face value will be traded at a discount price of: 10,000 (1 − 0.01237)

= \$ 9,876.3

You purchase at this price

Dealer sells at this price

now — 84 days

P_0 = \$9,876.3 — P = \$10,000

3 Rate of Return = $\frac{10,000 - 9,876.3}{9,876.3}$ = 1.25%

4 annual return = 1.25% $\times \frac{365}{84}$ → الأهمية

= 5.44% → asked Yield

2 Certificates of Deposit

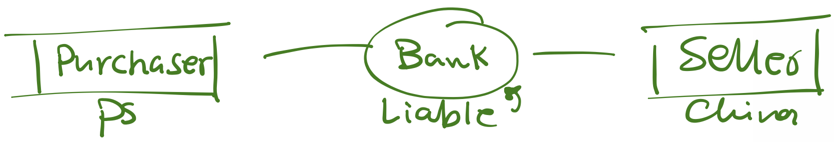
- Issuer: Depository Institution
- Denomination: $100,000 (most common), any > $100,000
- maturity: Vary, min of 14 Days
- liquidity: High for CDs < 3 months
- Default Risk: The first $250,000 Insurance from FDIC
  Fedral Deposit Insurance Corporation
- Taxation: owed (لازم يندفع ضريبة)
- Interes type: Add on (للجدين), Cannot be withdrawn on demand (يعني ما بنقدر نسحبهم وينتاما بدنا)

3 Commercial Paper

- Issuer: Large, Credit worthy Corporation / Financial Institute
- Denomination: min $100,000
- maturity: max 270 days, 1-2 months (usually)
- liquidity: High if maturity < 3 months
- Default Risk: unsecured, rated, mostly high quality
  ما في asset مرتبط بهاي المديونية
- Taxation: Owed
- Interest Type: Discount

4 Bankers' Acceptances

→ المشتري بيوكل البنك يدفع عنه لما بده يعطي أموال للبائع



- Can be traded
- Sell at a discount

5 Euro dollars

→ large Sums: خوانشأ كل

6 Repos and Reverses

- اتفاقيات إعادة الشراء
  dealer needs cash so he sells the government Security (for 1-day) → الهدف هو السيولة

• Reverse: dealer الشخص الي اشترى من ال العكس

7 Fed Funds

- rate ال

8 Brokers' call

- very risk margin
- بجيب من البنك
- ال rate من البنك أقل من الأفراد

9 The LIBOR Market

- London Interbank offered Rate
- الجزء من ال interest متغير

* Yields on money market Instruments
  - spread → CD rate − t-bill rate > 0
    لما يزيد الفرق بيفسر أزمات اقتصادية ، كل ما قل بيكون الاقتصاد أحسن

• Capital Market Investments (the Bond Market)

- longer term
- fixed income capital market

1 Treasury Notes and Bonds

- Issuer: U.S government
- maturity: T-notes: up to 10 years
  Bond: from 10 to 30 years
- denominations: $100, commonly $1,000
- Interest: semiannual interest → Coupon payments
- Yield to maturity: determining the semiannual yield and then doubling it, quoted on an annual percentage rate (APR) basis rather than as an effictive annual yield
  also called the bond equivalent yield

2 Inflation - Protected Treasury Bonds

- Issuer: Governments around the world
- linked to an Index of the cost of living
- Called TIPS
- The principle amount: adjusted in proportion to increases in the Consumer Price Index
- Yields interpreted as real or inflation-adjusted interest rates

3 Fedral Agency Debt

- Issuer: Some government agencies, to finance their activities
- For Example: Federal Home Loan Bank (FHLB), Fannie Mae, Ginnie Mae, Freddie Mac
- FHLB borrows money by issuing securities and lends this money to savings and loan institutions to be lent in turn to individuals borrowing for home mortgages
- Government agency issue debt and channel (حوّل) the money to a particular sector of the economy that government believes might not recieve adequate credit through normal private source

4 International Bonds

- For Example:
  1. Euro dollar Bonds
  2. Euro Yen Bonds
  3. Yankee Bond: Sold in U.S by a non-U.S issuer (dollar-denominated)
  4. Samurai Bond: Yen-denominated bonds sold in Japan by non-Japanese issuers

5 Municipal Bonds سندات بلدية

- Issuer: State and local governments
- Interest: Is exempt from Federal income tax and sometimes from State and local tax
- Types:
  1. General Obligation Bonds: Backed by taxing power of issuer
  2. Revenue Bonds: Backed by project's revenues or by the municipal agency operating the project

6 Corporate Bonds

- Issuer: Private Firms
- Interest: Semi-annual
- Default Risk: larger than government securities
- options in Corporate bonds:
  1. Callable: repurchase the bond at call price
  2. Convertible: convert bond to stocks

7 Mortgage-Backed Securities

- Proportional ownership of a mortgage pool or a specified obligation secured by a pool
- Produced by securitizing mortgages
- Mortgage-backed securities are called pass-throughs because the cash flows produced by homeowners paying off their mortgages are passed through to investors.
- Most mortgage-backed securities were issued by Fannie Mae and Freddie Mac.
- Traditionally, pass-throughs were comprised of conforming mortgages, which met standards of credit worthiness.
- Eventually, "Private-label" issuers securitized large amounts of subprime mortgages, made to financially weak borrowers.
- Finally, Fannie and Freddie were allowed and even encouraged to buy subprime mortgage pools.
- September, 2008: Fannie and Freddie got taken over by the federal government.

• Equity Securities

1 Common Stock

- Ownership
- Residual Claim
- Limited liability

2 Preferred Stock

- Perpetuity
- Fixed dividends
- Priority Over Common
- Tax Treatment

3 American Depositary Receipt (ADR)

- Certificate representing Shares of a foreign Security

• Stock Market Indexes

1 Dow Jones Industrial Average

- Includes 30 large blue-Chip Corporations
- Computed Since 1896
- Price-Weighted average

مثال :

Table 2.3

Data to construct stock price indexes

Stock	Initial Price	Final Price	Shares (million)	Initial Value of Outstanding Stock ($ million)	Final Value of Outstanding Stock ($ million)
ABC	$ 25	$30	20	$500	$600
XYZ	100	90	1	100	90
Total				$600	$690

* Portfolio :
- Initial Value: 25 + 100 = \$125
- Final Value: 30 + 90 = \$120
- Percentage Change in portfolio Value: $\frac{120 - 125}{125} = -4\%$

* Index :
- Initial index value : $(25 + 100) \div 2 = 62.5$
- Final index value : $(30 + 90) \div 2 = 60$
- Percentage change in index: $\frac{60 - 62.5}{62.5} = -4\%$

2 Standard & Poor's Indexes

- S&P 500
  - Broadly based index of 500 firms
  - Market-Value-weighted index

- Investors Can base their portfolios on an Index :
  - Buy an Index mutual fund
  - Buy exchange traded funds (ETFs)

* Other Indexes

U.S Indexes
- NYSE Composite
- NASDAQ Composite
- Wilshire 5000

Foreign Indexes
- Nikkei (Japan)
- FTSE (U.K)
- DAX (Germany)
- Hang Seng (Hong Kong)
- TSX (Canada)

• Derivatives Markets

- Options and futures provide payoffs that depend on the values of other assets such as commodity prices, bond and stock prices, or market index values.

- A derivative is a security that gets its value from the values of another asset.

1 Options عقود الخيارات

- Call: Right to buy underlying asset at the strike or exercise price.
- Value of calls decrease as strike price increases

- Put: Right to sell underlying asset at the strike or exercise price.
- Value of puts increase with strike price

- Value of both calls and puts increase with time until expiration.

2 Futures Contracts العقود الآجلة/المستقبلية

- A futures contract calls for delivery of an asset (or in some cases, its cash value) at a specified delivery or maturity date for an agreed-upon price, called the futures price, to be paid at contract maturity.

- Long position: Take delivery at maturity

- Short position: Make delivery at maturity

* Comparison

Option	Futures Contract
• Right, but not obligation, to buy or sell ; Option is exercised only when it is Profitable	• Obliged to make or take delivery. Long position <u>must</u> buy at the futures price, Short Position <u>must</u> sell at futures price
• Options must be Purchased	• Futures Contracts are entered into Without Cost
• The <u>Premium</u> is the Price of the option itself	

* Chapter 2 Questions *

①

11. Consider the three stocks in the following table. P_t represents price at time t, and Q_t represents shares outstanding at time t. Stock C splits two for one in the last period.

	P_0	Q_0	P_1	Q_1	P_2	Q_2
A	90	100	95	100	95	100
B	50	200	45	200	45	200
C	100	200	110	200	55	400

a. Calculate the rate of return on a price-weighted index of the three stocks for the first period ($t = 0$ to $t = 1$).

* Index Value of $t=0$: $(90+50+100) \div 3 = 80$

* Index Value of $t=1$: $(95+45+110) \div 3 = 83.3$

* rate of return : $\frac{83.3 - 80}{80} = 4.17\%$

②

12. Using the data in the previous problem, calculate the first-period rates of return on the following indexes of the three stocks:

a. A market-value-weighted index.
b. An equally weighted index.

ⓐ Market Capitilization — حسب ال market

	at $t=0$	at $t=1$
A :	$90 * 100 = 9{,}000$	$95 * 100 = 9{,}500$
B :	$50 * 200 = 10{,}000$	$45 * 200 = 9{,}000$
C :	$100 * 200 = 20{,}000$	$110 * 200 = 22{,}000$
	total = 39,000	total = 40,500

* rate of return $= \frac{40{,}500 - 39{,}000}{39{,}000} = 3.85\%$

بدون وزن بالوزن
Just return

ⓑ $r_a = (9{,}500 - 9{,}000) \div 9{,}000 = 5.56\%$
$r_b = (9{,}000 - 10{,}000) \div 10{,}000 = -10\%$
$r_c = (22{,}000 - 20{,}000) \div 20{,}000 = 10\%$

* Average of return $= (5.56 - 10 + 10) \div 3 = 1.85\%$

3

3. Which of the following *correctly* describes a repurchase agreement?

a. The sale of a security with a commitment to repurchase the same security at a specified future date and a designated price.

b. The sale of a security with a commitment to repurchase the same security at a future date left unspecified, at a designated price.

c. The purchase of a security with a commitment to purchase more of the same security at a specified future date.

The answer is : a

4

4. What would you expect to happen to the spread between yields on commercial paper and Treasury bills if the economy were to enter a steep recession?

The spread will Increase

Chapter (4): Compining Individual Securities Into Portfolios

- The Return is a \$ amount

- The Rate of Return is a %

- Assumptions: الفرضيات الي رح نتعامل على أساسها

  * على فرض أن الـ Portfolio مقتصر على:
  - Two Stocks, A and B
  - $E(r)$
  - W_A, W_B
  - σ_{AB} (S.D)
  - ρ_{AB} (Correlation)

- Portfolio Rate of Return = $\dfrac{\text{amount purchased (+) or Sold-Short (-)}}{\text{Total Investment in the Portfolio}}$

- **Purchased or bought Security:**
  - Held in the long position
  - owned by the Investor
  - appears on the balance Sheet as an asset
  - The weight held is positive
  - The Price is expected to increase

- **Short-Selling of Security:**
  - The investor does not own the asset
  - Borrowing the security
  - On Balance Sheet as a liability
  - The weight held is negative
  - The price is expected to decrease

- **Why Short-Sell?**

  Shortsellers will profit by selling today at a higher price and then buying the security at a lower future price.

- Expected Return of a Portfolio:

$$E(r_p) = w_A E(r_A) + w_B E(r_B)$$

* Example:

$\boxed{\text{Wealth} = \$1,000}$

	Stock A	Stock B	Wealth
Time = 0	\$ 400	\$600	\$ 1,000
Return on Time = 2 months	\$ 40	\$ 36	\$ 1,076

1. The amount of Return:

   $40 + 36 = \$76$

2. Rate of Return $= \dfrac{1,076 - 1,000}{1,000} = \boxed{7.6\ \%}$

3. Return of A:

   $r_A = \dfrac{40}{400} = 10\%$

4. Return of B:

   $r_B = \dfrac{36}{600} = 6\%$

5. Another Method (الطريقة المعتمدة لنهاية الفصل)

   - Return of Portfolio% $= w_A r_A + w_B r_B$
   - $w_A = \dfrac{400}{1,000} = \%40$, $w_B = \%60$
   - $r_p = 0.4(10\%) + 0.6(6\%)$
     $= \boxed{7.6\%}$

• The Risk of the Portfolio (σ_P)

- $\sigma_P^2 = W_A^2 \sigma_A^2 + W_B^2 \sigma_B^2 + 2 W_A W_B \underbrace{Cov(r_A, r_B)}_{= \sigma_{r_A} \sigma_{r_B} \rho_{AB}}$ $\sigma \geq 0$

- $-1 \leq \rho \leq +1$

$-1 \rightarrow$ Perfect negative Correlation (علاقة عكسية)

$+1 \rightarrow$ Perfect Positive Correlation (علاقة طردية)

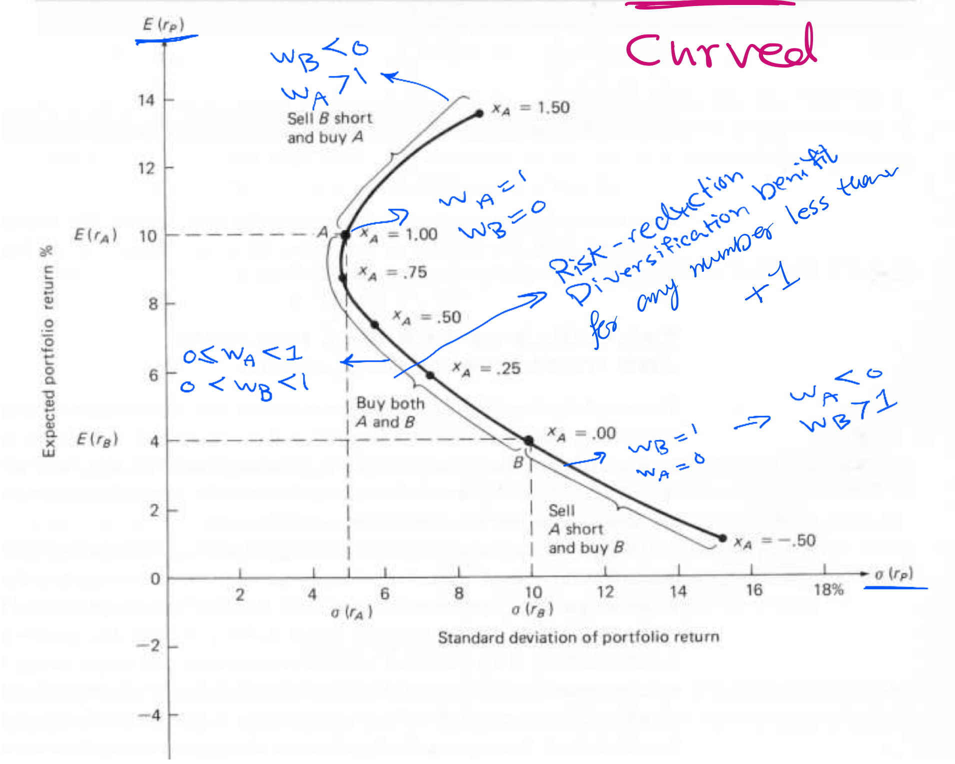
• Portfolio Standard Deviation (SD)

- $S.D = \sqrt{\sigma_P^2} = \sigma_P$
- Variance $= \sigma_P^2$

• Combination Lines

- S.D in (x-axis) $= \sigma_{r_P}$
- $E(r_P)$ in (Y-axis)
- possible weights of 2 Securities
- we can see how SD and E(r) vary

FIGURE 4.4 COMBINATION LINE BETWEEN SECURITIES A AND B FOR THE CASE OF ZERO CORRELATION.



الرسومات والشرح عليهم مهم جداً جداً واهم شي فهمهم

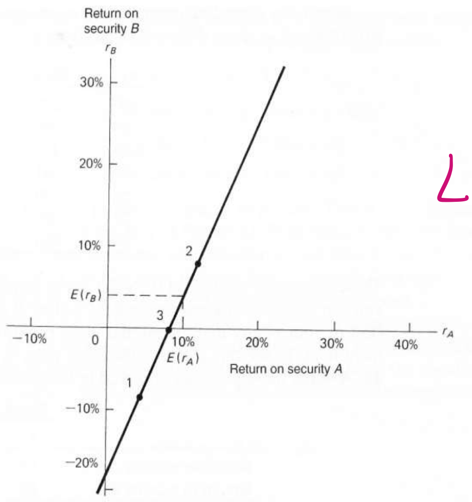
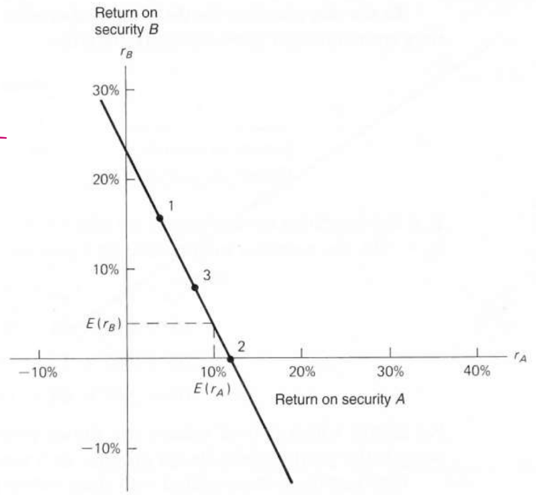


FIGURE 4.6 RELATIONSHIP BETWEEN SECURITIES A AND B WITH PERFECT POSITIVE CORRELATION.

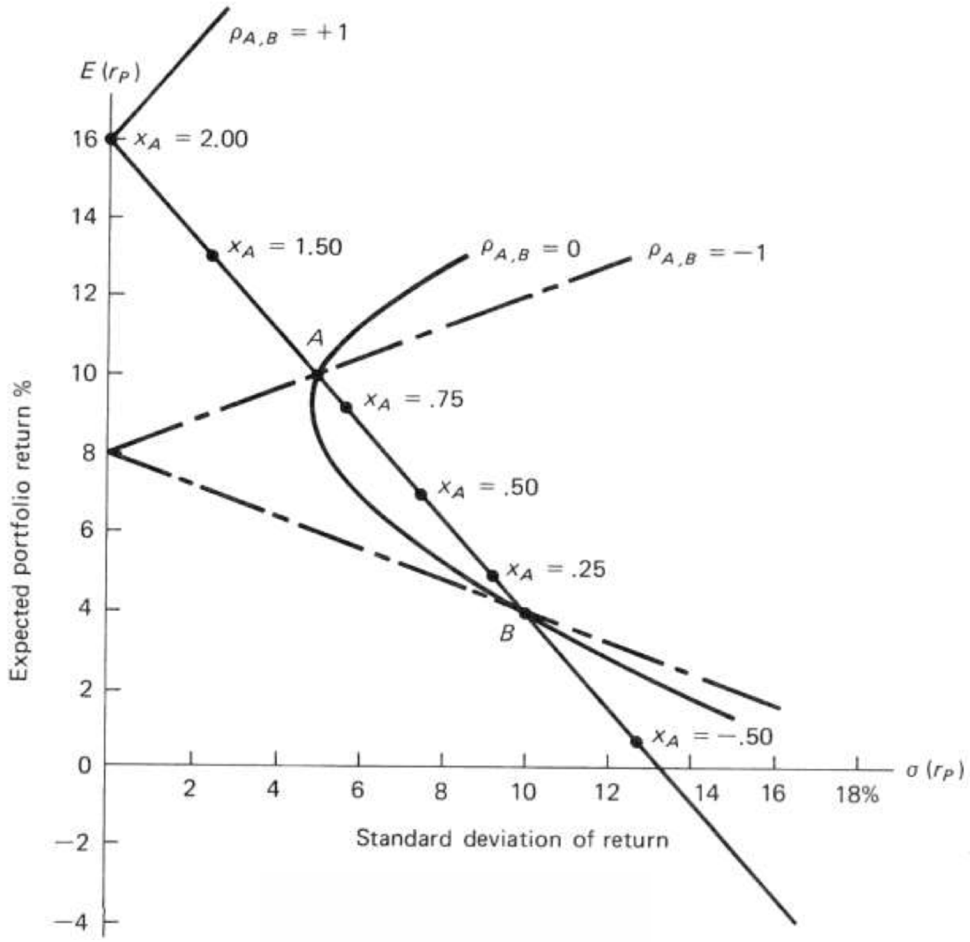
Linear

FIGURE 4.8 RELATIONSHIP BETWEEN SECURITIES A AND B WITH PERFECT NEGATIVE CORRELATION



أي Compinations بين ال $-1<\rho<1$ بيكون Curved

FIGURE 4.7 COMBINATIONS FOR THE CASES OF PERFECT POSITIVE, PERFECT NEGATIVE, AND ZERO CORRELATION.



- Introducing the Risk-Free Security
  - The risk-free Security results in a Linear efficient Set

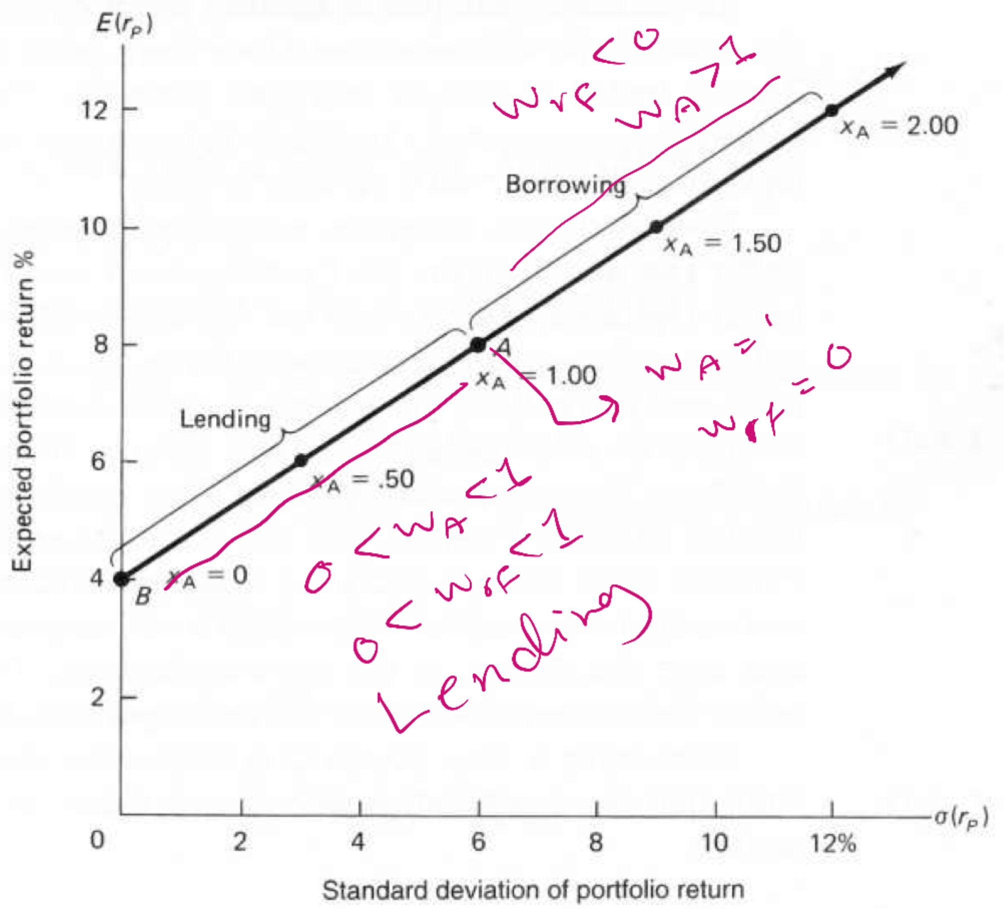
- Variance and S.D Portfolio with a risk-free Security

$$\sigma_p^2 = W_A^2 \sigma_A^2 + \boxed{W_{rf}^2 \sigma_{rf}^2} + 2 W_A W_{rf} \boxed{Cov(r_A, r_{rf})}$$

The RF has Zero Risk and is uncorrelated with other asset returns

$$\therefore \sigma_p^2 = W_A^2 \sigma_A^2 \longrightarrow \sigma_p = W_A \sigma_A$$

FIGURE 4.9 EFFECT OF BORROWING AND LENDING ON RISK AND EXPECTED RETURN.



★ Chapter (11) Questions ★

8. Consider two securities, A and B, which have the following characteristics:

	A	B
$E(r)$	.12	.06
Std. dev.	.12	.06

Correlation coefficient of A with $B = -1.0$. Compute the expected returns and standard deviations of each of the following portfolios of A and B. Also plot securities and the portfolios of A and B on a graph with expected return and standard deviation on the axes.

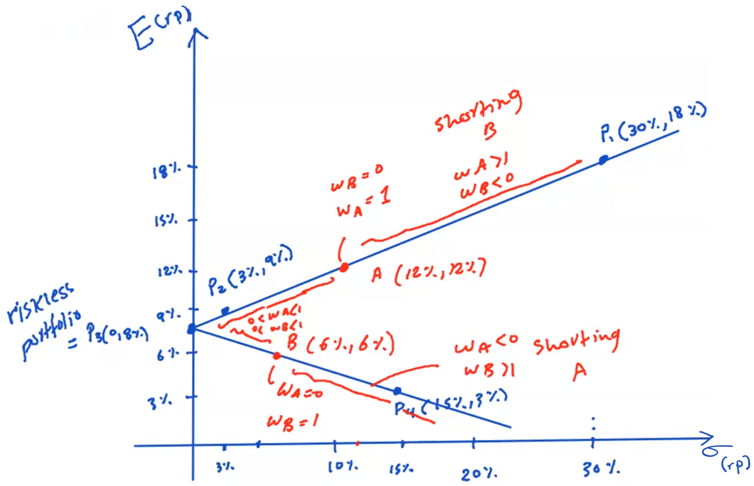
Portfolio 1: $x_A = 2, x_B = -1$
Portfolio 2: $x_A = .5, x_B = .5$
Portfolio 3: $x_A = \frac{1}{3}; x_B = \frac{2}{3}$
Portfolio 4: $x_A = -.5, x_B = 1.5$

	A	B
E(r)	12%	6%
σ	12%	6%

Assume the correlation between A and B is -1

W(A)	W(B)	E(rp)	σ(rp)
2	-1	18.00%	30.00%
0.5	0.5	9.00%	3.00%
0.30	0.70	8.00%	0 %
-0.5	1.50	3.00%	15.00%

max Benifit



9. Consider two securities with the following characteristics:

	Security X	Security Y
Expected return	.10	.14
Standard deviation	.25	.30

$w_A = w_B = 0.5$

Suppose you build a portfolio with equal dollar amounts in the two securities. Compute the expected return and variance of the portfolio under each of the following assumptions about the correlation between returns on X and Y:

Correlation = 1
Correlation = 0
Correlation = –1

★ Correlation = +1 $\quad \sigma_{rp} = w_x \sigma_{r_x} + w_y \sigma_{r_y}$

- $E(r_p) = 0.5 * 0.1 + 0.5 * 0.14 = 0.12$
- Variance $= \sigma_p^2 = (0.5)^2 (0.25)^2 + (0.5)^2 (0.3)^2 + 2(0.5)^2 (0.3)(0.25)(1)$
  $= 0.0756$
  $\sigma_p = 27.5\%$

* Correlation $= 0$ $\sigma_{rp} = \sqrt{w_x^2 \sigma_x^2 + w_y \sigma_y^2}$

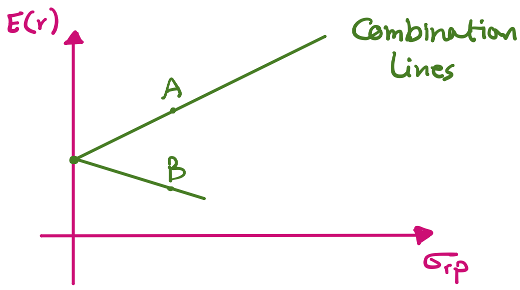
- $E(rp) = 0.5 * 0.1 + 0.5 * 0.14 = 0.12$
- Variance $= (0.5)^2 (0.25)^2 + (0.5)^2 (0.3)^2 + 0$
  $= 0.0381$
  $\sigma_{rp} = 19.52\%$

* Correlation $= -1$ $\sigma_{rp} = \sqrt{w_x \sigma_x - w_y \sigma_y}$

- $E(rp) = 0.12$
- Variance $= 0.0381 + 2(0.5)^2 (0.3)(0.25)(-1)$
  $= 0.0381 - 0.0375$
  $= 0.0006$
  $\sigma_{rp} = 2.5\%$

10. Assume that two securities have a correlation coefficient of –1.0.
    a. What would be the lowest possible standard deviation that could be achieved by constructing a portfolio of these two securities?
    b. Use your answer to part (a) and Equation (4.1) to derive an expression for the lowest standard deviation portfolio weights for the securities. (The weights for the securities will be a function of the standard deviations of the two securities.)

ⓐ Lowest $\sigma = 0$



ⓑ if $\rho = -1$ for the

$$\sigma_p = w_x \sigma_{rx} - w_y \sigma_{ry} \quad --- (1)$$

$$w_x + w_y = 1 \Longrightarrow w_y = 1 - w_x \quad --- (2)$$

$$\sigma_p = w_x \sigma_{rx} - (1 - w_x) \sigma_{ry}$$

$$0 = w_x \sigma_{rx} - \sigma_{ry} + w_x \sigma_{ry}$$

$$\sigma_{ry} = w_x (\sigma_{rx} + \sigma_{ry})$$

$$\therefore w_x = \frac{\sigma_{ry}}{\sigma_{rx} + \sigma_{ry}}$$

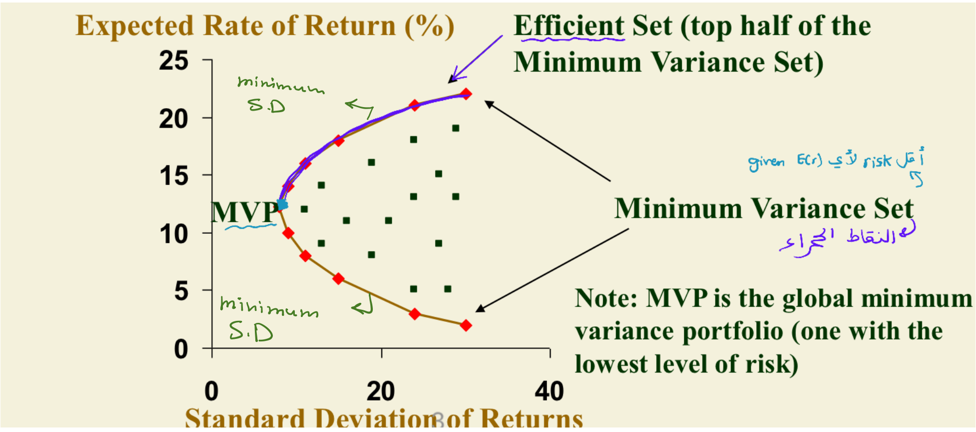
Chapter(5): Finding the efficient Set

* Feasible Portfolios : 3 or more securities

* Minimum Variance Set and the efficient Set :

↳ Portfolios that have the lowest level of risk for a given E(r)

Portfolios that have the highest E(r) for a given level of risk



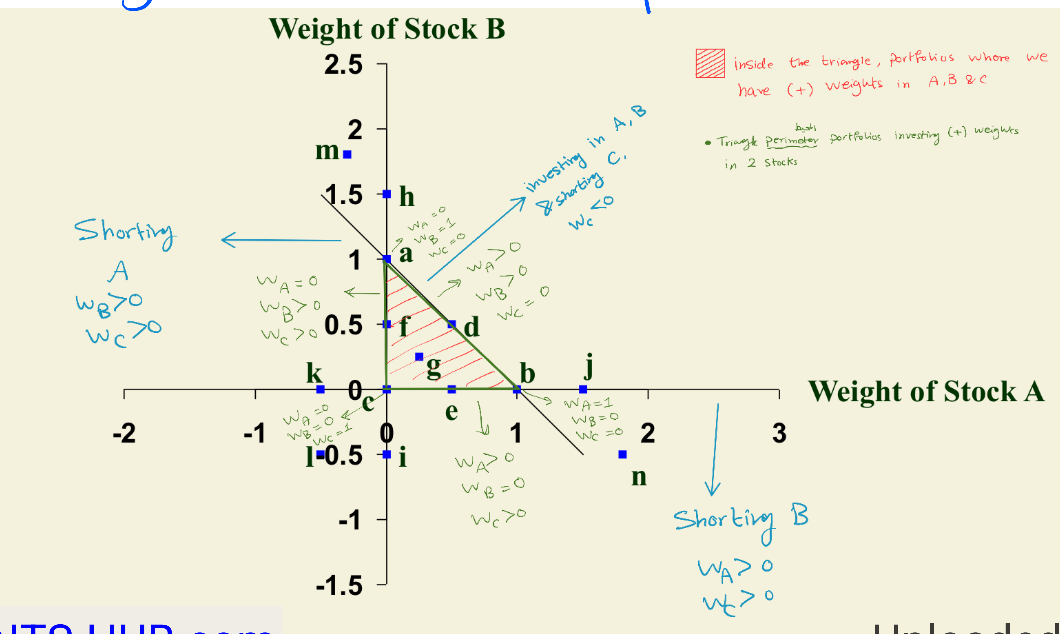
* Finding the Efficient Set → Risk, Return

1. Weights in three-Stock portfolio
2. Iso- Expected Return Lines
3. Iso- Variance Ellipses
4. The Critical Line

* held by wealth-maximising, risk averse investors

* "Supine" Segment of the MVS

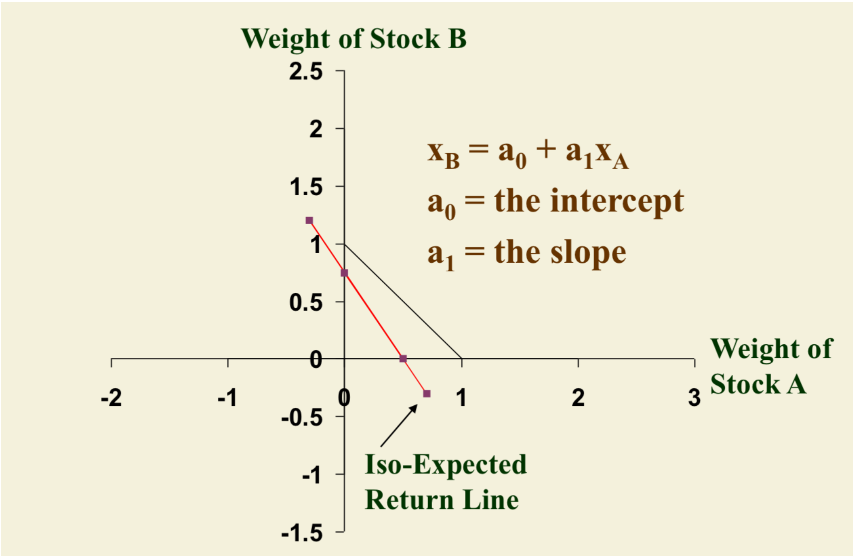
* Weights in 3-Stock Portfolio



* Iso-Expected Return Lines

- All portfolios have the same expected return
- $X_B = a_0 + a_1 X_A$

The Intercept (a_0) — The Slope (a_1)



* Computing the Intercept and Slope of the line:

$$E(r_p) = W_A E(r_A) + W_B E(r_B) + (1 - W_A - W_B) E(r_c)$$

- Rearranging: $W_B = \dfrac{E(r_p) - E(r_c)}{E(r_B) - E(r_c)} + \dfrac{E(r_c) - E(r_B)}{E(r_B) - E(r_c)} (W_A)$

* When $E(r_p)$ is changed, the intercept (a_0) changes but the slope (a_1) remains unchanged

* Iso-Variance Ellipse

- A set of portfolios with equal variances
- معادلة تربيعية : $a x_B^2 + b x_B + c = 0 = \sigma_p^2$

$a = \sigma_{r_B}^2 - \sigma_{r_c}^2 - 2\,Cov(r_B, r_c)$

$Cov(r_B, r_B)$

$= \sigma_B \sigma_B \rho_B = \sigma_B^2 (1) = \sigma_B^2$

$b = 2 x_a \left(Cov(r_A, r_B) + \sigma_{r_c}^2 - Cov(r_A, r_c) - Cov(r_B, r_c) \right) + 2 \left(Cov(r_B, r_c) - \sigma_{r_c}^2 \right)$

c: $x_a^2 (\sigma_a^2 + \sigma_c^2 - 2\,Cov(r_A, r_c)) + 2 x_a (Cov(r_A, r_c) - \sigma_c^2) + \sigma_c^2 - \sigma_p^2$

* Example:

$x_A = 0.5$, $\sigma_p^2 = 0.21$, $Cov(r_A, r_A) = 0.25$, $Cov(r_A, r_B) = 0.15$, $Cov(r_A, r_c) = 0.17$, $Cov(r_B, r_B) = 0.21$, $Cov(r_B, r_c) = 0.09$, $Cov(r_c, r_c) = 0.28$

$$X_B = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- $a = 0.21 - 0.28 - 2(0.09) = 0.31$
- $b = 2x_a\left(0.15 + 0.28 - 0.17 - 0.09\right) + 2\left(0.09 - 0.28\right)$

$$= 0.34\,x_a - 0.38 = 0.34(0.5) - 0.38) = -0.21$$

- $c = x_a^2\left(0.25 + 0.28 - 2(0.17)\right) + 2x_a\left(0.17 - 0.28\right) + 0.28 - 0.21$

$$= 0.19\,x_a^2 - 0.22x_a + 0.07 = 0.19(0.5)^2 - 22(0.5) + 0.07 = 0.0075$$

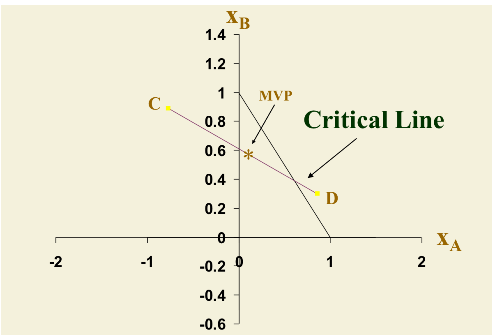
$$* \; X_B = \frac{-(-0.21) + \sqrt{(-0.21)^2 - 4(0.31)(0.0075)}}{2(0.31)} = 0.64$$

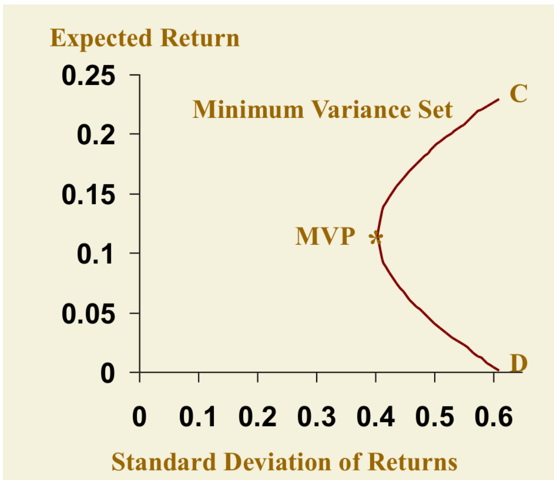
$$* \; X_B = \frac{-(-0.21) - \sqrt{(-0.21)^2 - 4(0.31)(0.0075)}}{2(0.31)} = 0.038$$

* The Critical Line :

↳ Shows the portfolio weights for the portfolios in the minimum Variance Set

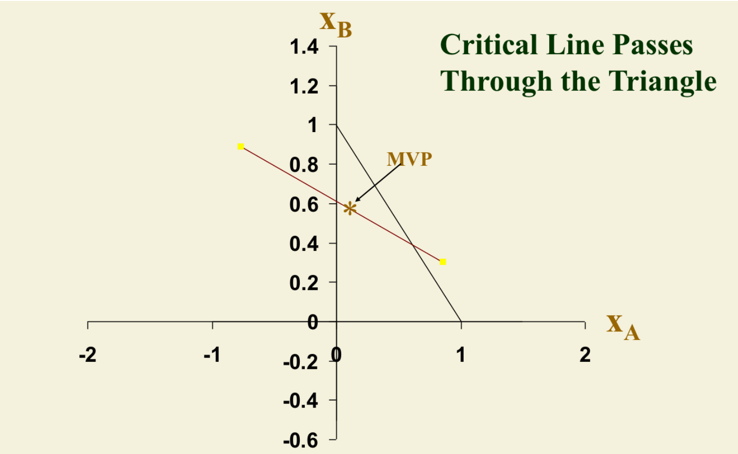
* Relationship between the critical line and MVS (set)

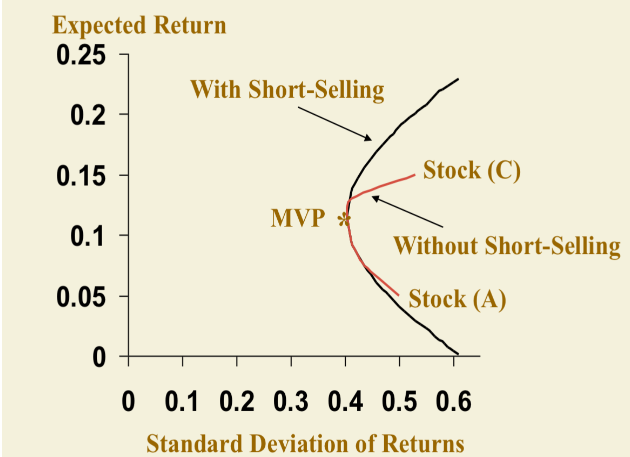




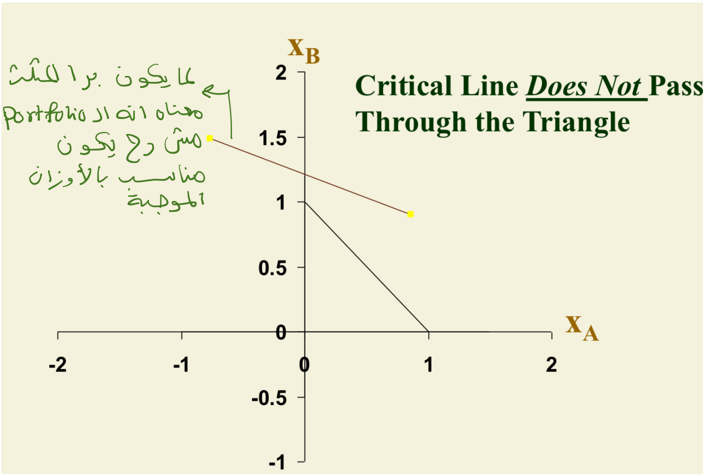
* Minimum Variance Set : lowest variance for a fixed E(r)
  - bullet shape

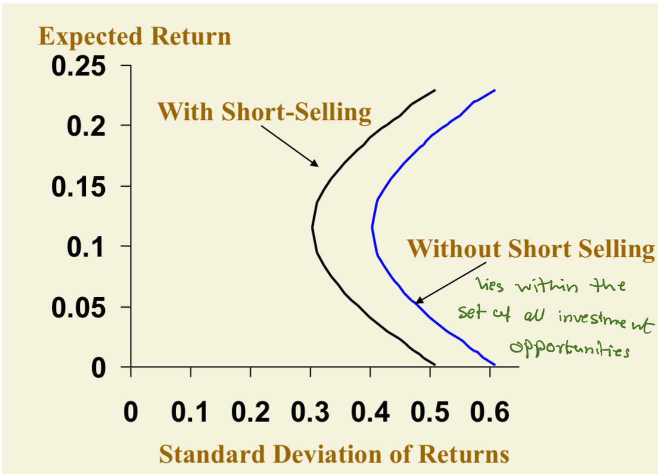
* MVS when Short-Selling is not allowed (يمر من المثلث)





* MVS when Short-Selling is allowed (لا يمر من المثلث)





*The Minimum Variance Set (Property I)

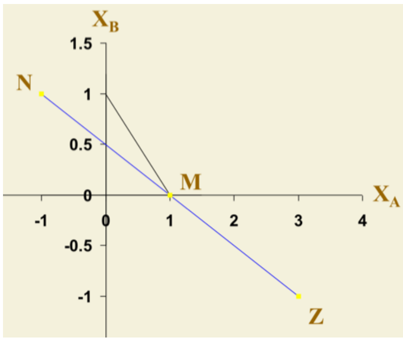
- If we combine two or more portfolios on the minimum variance set, we get another portfolio on the minimum variance set.
- *Example:* Suppose you have \$1,000 to invest. You sell portfolio (N) short \$1,000 and invest the total \$2,000 in portfolio (M). What are the security weights for your new portfolio (Z)?
- *Portfolio N:* $x_A = -1.0$, $x_B = 1.0$, $x_C = 1.0$

  Portfolio M: $x_A = 1.0$, $x_B = 0$, $x_C = 0$

  Portfolio Z: $x_A = -1(-1.0) + 2(1.0) = 3.0$

  $x_B = -1(1.0) + 2(0) = -1.0$

  $x_C = -1(1.0) + 2(0) = -1.0$



* Property II

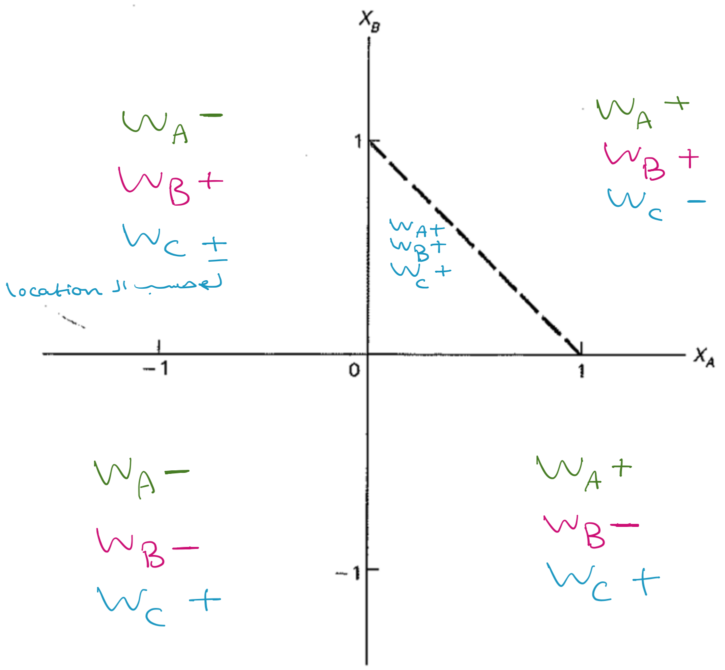
- **Given a population of securities, there will be a simple linear relationship between the beta factors of different securities and their expected (or average) returns if and only if the betas are computed using a minimum variance market index portfolio.**

* $\beta_{market} = 1$

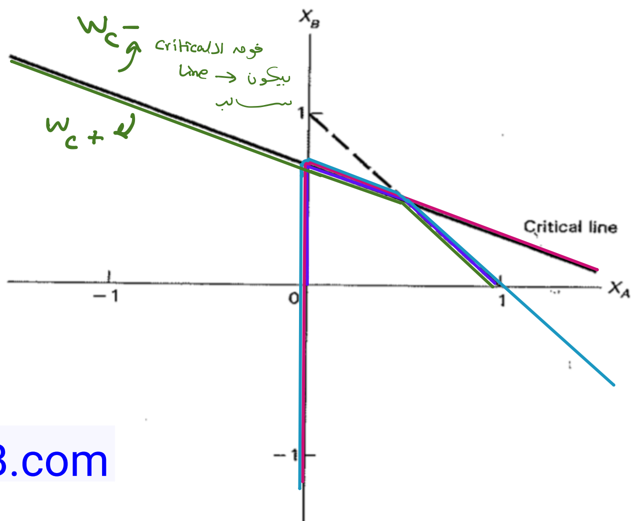
* $\beta_{rf} = 0$

* Chapter (5) questions *

1. The following figure depicts in X_A, X_B space the possible portfolio weights in a three-stock portfolio. Indicate the areas of positive, negative, and zero portfolio weights for each of the three securities.



2. The following figure shows the critical line for a portfolio containing stocks A, B and C when there are no restrictions on short selling. What would the critical lii look like in each of the following cases?
   a. No short selling allowed. $W_A, W_B, W_C \rightarrow +$
   b. Short selling not allowed in stock A. $W_A +, W_B \pm, W_C \pm$
   c. Short selling not allowed in stocks A and C. $W_A +, W_B \pm, W_C +$
   d. Short selling not allowed in stocks B and C. $W_A \pm, W_B +, W_C +$



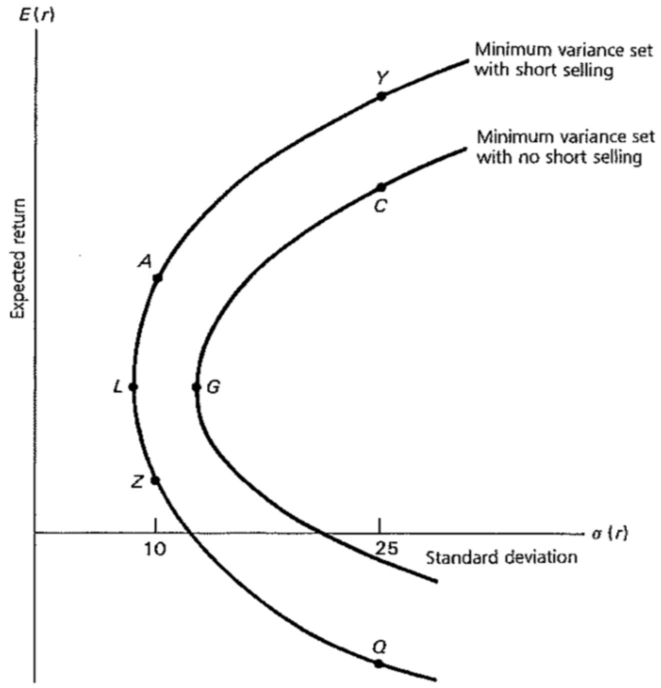
4. Suppose that we have two portfolios known to be on the minimum variance set for a population of three stocks, A, B, and C. There are no restrictions on short sales. The weights for each of the two portfolios are as follows:

	X_A	X_B	X_C
Portfolio 1	.24	.52	.24
Portfolio 2	−.36	.72	.64

on efficient Set

a. What would the stock weights be for a portfolio constructed by investing \$2,000 in portfolio 1 and \$1,000 in portfolio 2?

P3 = \$3,000



b. Plot portfolios 1 and 2 and the combined portfolio in X_A, X_B space. Is the combined portfolio on the critical line?

c. Suppose you invest \$1,500 of the \$3,000 in stock A. How will you allocate the remaining \$1,500 between stocks A and B to ensure that your portfolio is on the minimum variance set?

$\boxed{a}$

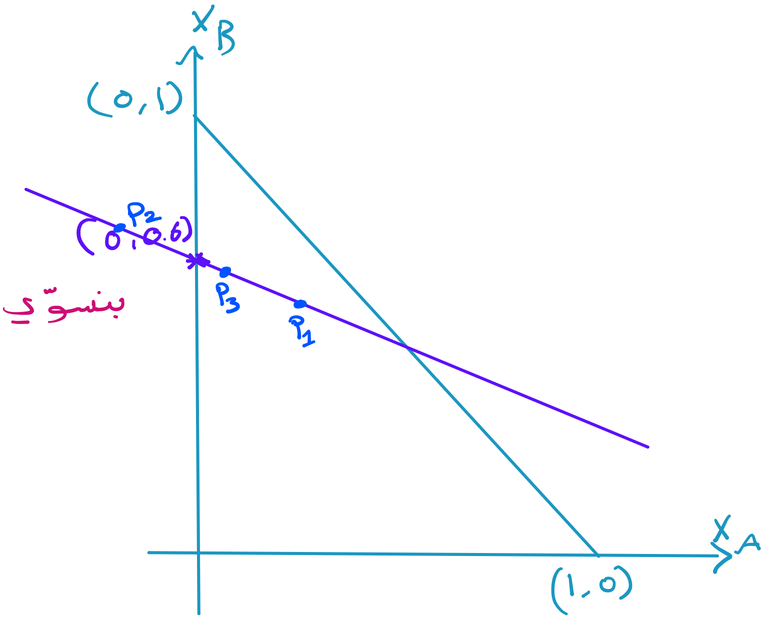
	x_A	x_B	x_C
P_1:	$(0.24 * 2{,}000)$	$(0.52 * 2{,}000)$	$(0.24 * 2{,}000)$
	\$480	\$1040	\$480
	+	+	+
P_2:	$(-0.36 * 1{,}000)$	$(0.72 * 1{,}000)$	$(0.64 * 1{,}000)$
	\$ −360	\$720	\$640
P_3:	\$120	\$1,760	\$1,120
Weights:	$\frac{120}{3{,}000}$	$\frac{1{,}760}{3000}$	$\frac{1{,}120}{3000}$

$W_A = 0.04 \qquad W_B = 0.59 \qquad W_C = 0.27$

$\boxed{b}$ معادلة الخط :

المقطع الصادي ← $Y = a + bx$ → slope

$x_B = \overset{?}{a} + \overset{?}{b} x_A$



بنسوّي معادلتين :

① .... $0.52 = a + b(0.24) \quad \times(-1)$

② .... $0.72 = a + b(-0.36)$

$-0.52 = \cancel{-a} - 0.24b$

$0.72 = \cancel{a} - 0.36b$

$0.2 = -0.6b$

$\therefore b = \frac{-1}{3} \qquad \therefore a = 0.6$

So, the line equation:

$x_B = 0.6 - \frac{1}{3} x_A$

نعوّض →

* To check if P_3 is on the critical line :

$W_B = 0.6 - \frac{1}{3}(0.04)$

$= 0.59$ ✓

$W_B =$ ✓

$\therefore P_3$ is on the critical line

$\boxed{c}$ $x_A = 0.5, \; x_B = ? \; , \; x_C = ?$

* if P_4 is on the min Variance set:

$\therefore x_B = 0.6 - \frac{1}{3}(\frac{1}{2}) = 0.433$

$x_C = 1 - x_A - x_B$

$= 1 - 0.5 - 0.43$

$= 0.07$

4. Suppose the expected returns on three stocks are as follows:

	X	Y	Z
$E(r)$	.07	.11	.16

a. Find the equation of the isoexpected return line that corresponds to a portfolio expected return of .15 for these three stocks. (The line is to be expressed in terms of the weights on X and Y.) $E(r_p) = 0.15$

b. If the weight on stock Y were restricted to zero, what weights for stocks X and Z would result in a portfolio expected return of .15?

$\boxed{a}$

$$\rightarrow (1-w_x-w_y)$$

$$E(r_p) = w_x E(r_x) + w_y E(r_y) + W_z E(r_z)$$

$$0.15 = 0.07\,w_x + 0.11\,w_y + 0.16 - 0.16\,w_x - 0.16\,w_y$$

$$0.15 = -0.09\,w_x - 0.05\,w_y + 0.16$$

$$\frac{0.05\,w_y}{0.05} = \frac{0.01 - 0.09\,w_x}{0.05}$$

$$w_y = \frac{1}{5} - \frac{9}{5}\,w_x$$ → equation for the iso-expected return line

$\boxed{b}$

$$w_x + 0 + w_z = 1 \longrightarrow w_x = 1 - w_z$$

$$0.15 = 0.07\,w_x + 0.16\,w_z$$

$$0.15 = 0.07 - 0.07\,w_z + 0.16\,w_z$$

$$0.15 = 0.07 + 0.09\,w_z$$

$$\therefore w_z = 0.\overline{8} \quad \text{and} \quad w_x = 0.\overline{1}$$

Chapter (6): Factor Model

* Efficient Set ⟹ Estimates

E(r) covariance

طرق لإيجاد ال efficient Set

1. Sampling (Past Prices) ⟹ unreliable estimate
   - ↓ Returns
   - ↓ covariance

1980 – 2022 → long
↓
Trade off
- reliable
- irrelevent

* There is a % of error in sampling

2015 – 2022 → Short
↓
Trade off
- unreliable
- relevant

** 2. Factor Models

مطلوب للكبر

- ↳ Risk Factors ⟹ variables (inflation, interest, growth)
  - ↳ affect stock prices / returns
- ↳ E(r) Factors ⟹ Firm charachteristics خصائص من الشركة

* Risk Factor Models to Estimate Volatity (σ^2) of Returns

* Single Factor Model
  - ↳ assumption: Security returns are correlated with one single factor

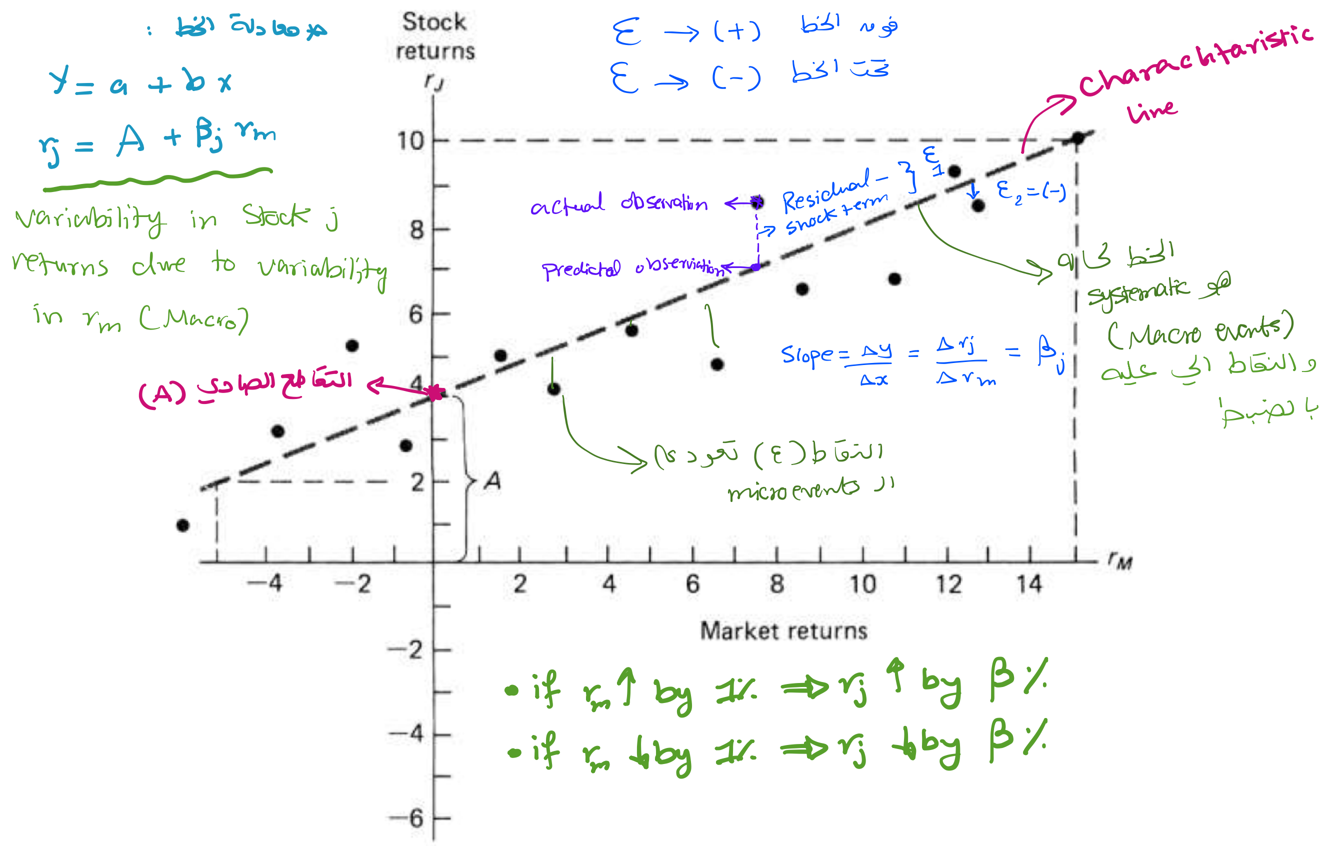
↓

افتراضي

(M) market Portfolio: portfolio (hypothetical) of all risky assets

Variability of the market portfolio accounts for all the comovement between the stocks

FIGURE 6.1 RELATIONSHIP BETWEEN THE RETURNS ON AN INDIVIDUAL INVESTMENT AND THE RETURNS ON THE MARKET PORTFOLIO.



* Single Factor Model assumes two types of events produce variability in Stock Returns

① Macro events : • unexpected changes (i)
• unexpected changes inflation
• COVID

② Micro events : • new product
↳ have an impact on individual stocks not on all stocks

* Residuals for difference firms are UNCORRELATED with one another

↳ $Cov(r_i, r_j) = \beta_i \beta_j \sigma_M^2$

σ_M^2 → magnitude of the market movement "Strength of the pull"

$\beta_i \beta_j$ ↳ respond of i, j to the market pull

$I \xrightarrow{\beta_i} M$, $J \xrightarrow{\beta_j} M$

* You can split the variance of returns on a security (portfolio) into 2 parts

$$\sigma_{r_j}^2 = \beta^2 \sigma_M^2 + \sigma_\varepsilon^2 \rightarrow \text{residual variance}$$

لعاليه بتغيرا عوائد j ؟ → بسبب The market + micro events

Total risk = Systematic risk + unsystematic risk } → "diversifiable"

Systematic risk: You will always be exposed to it "undiversifiable" → ما بتقدر تهرب منها أو تتجنبها

* $\beta_p = w_i \beta_i + w_j \beta_j + \dots$

* $\sigma_{\varepsilon p}^2 \text{(portfolio)} = \sum_{i=1}^{n} w_i^2 \sigma_{\varepsilon_i}^2$

AN EXAMPLE WHERE THE SINGLE-FACTOR MODEL WORKS

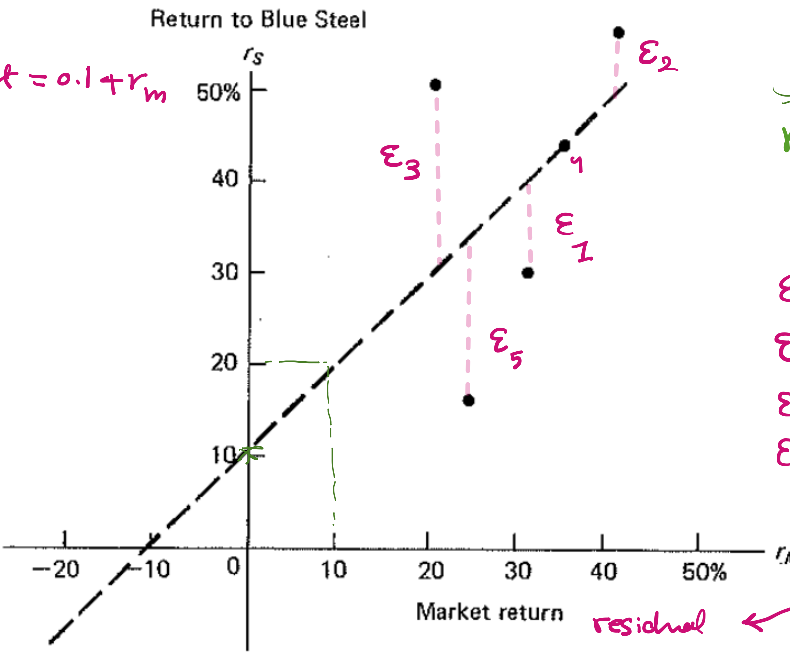
Consider two hypothetical stocks, Blue Steel and Black Rubber. In Table 6.1 are the rates of return for these companies, for the market portfolio, and for an equally weighted (→ 50%) portfolio of the two stocks for five periods of time. The two-stock portfolio is assumed to be rebalanced to equal weights at the beginning of each period. Given this, the return for the portfolio is a simple average of the returns to the stocks in each period.

The returns for each stock and for the portfolio are plotted against the returns for the market in Figures 6.5, 6.6, and 6.7. Note that the beta factor for Blue Steel is

TABLE 6.1 Rates of Return to the Market, Two Stocks, and a Portfolio

Period	*Market Portfolio* r_M	*Blue Steel* r_S (actual)	*Black Rubber* r_R	*Two-Stock Portfolio* $r_P = w_S r_S + w_R r_R$
1	30%	30%	55%	42.5%
2	40	60	40	50
3	20	50	30	40
4	35	45	27.5	36.25
5	25	15	22.5	18.75

Predict $= 0.1 + r_m$



$\beta = \frac{20-10}{10-0} = \frac{10}{10} = 1$

المقطع الصادي

$r_s = \underline{0.1} + \underline{\beta}\, r_m$

$r_s = 0.1 + r_m$

$\varepsilon_1 = 0.3 - (0.1 + 0.3) = (\quad)^2$

$\varepsilon_2 = 0.6 - (0.1 + 0.4) = (\quad)^2$

$\varepsilon_3 = 0.5 - (0.1 + 0.2) = (\quad)^2$

$\varepsilon_5 = 0.15 - (0.1 + 0.25) = (\quad)^2$

المجموع = 0.1

$\sigma^2_{\varepsilon_S} = \sum_{i=1}^{n} \varepsilon_i^2 \div (n-2)$

residual variance

$= 1 \div (5-2) = 0.0333$

FIGURE 6.5 BLUE STEEL.

$\beta = \frac{30-20}{20-0} = \frac{10}{20} = 0.5$

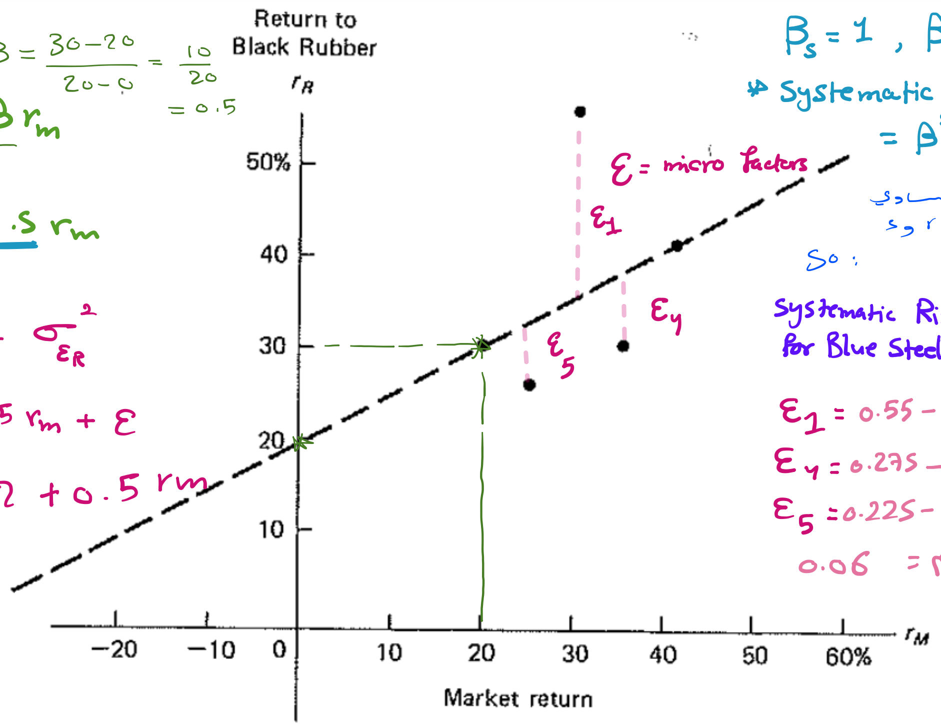
$r_R = 0.2 + \underline{\beta}\, r_m$

$r_R = 0.2 + \underline{0.5}\, r_m$

$\sigma_R^2 = \beta_R^2 \sigma_M^2 + \sigma_{\varepsilon_R}^2$

$= 0.2 + 0.5\, r_m + \varepsilon$

Predict $= 0.2 + 0.5\, r_m$



$\beta_s = 1 , \beta_r = 0.5$

* Systematic Risk:

$= \beta^2 \sigma_M^2$

لـ متساوي
عند دو

So:

Systematic Risk for Blue Steel > Systematic Risk for Black Rubber

$\varepsilon_1 = 0.55 - (0.2 + 0.5 * 0.3)$

$\varepsilon_4 = 0.275 - (0.2 + 0.5 * 0.35)$

$\varepsilon_5 = 0.225 - (0.2 + 0.5 * 0.25)$

0.06 = مجموع تربيعهم

$\sigma^2_{\varepsilon_R} = \frac{0.06}{5-2}$

$= 0.02$

FIGURE 6.6 BLACK RUBBER.

$\sum_{i=1}^{n} w_i \beta_i = \beta_p = \frac{30-15}{20-0} = 0.75$

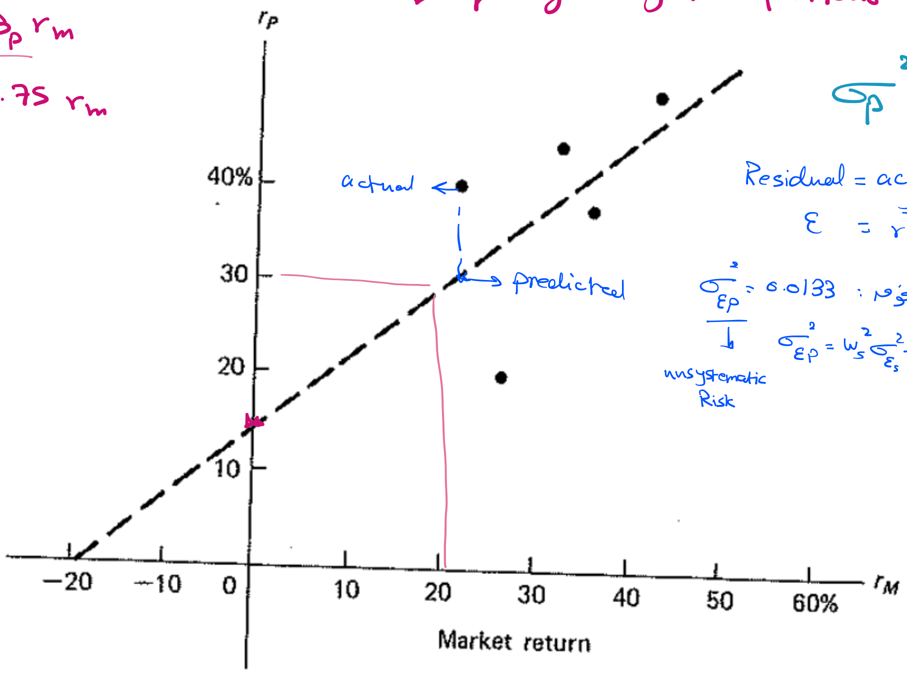
$r_p = 0.15 + \beta_p r_m$

$r_p = 0.15 + 0.75\, r_m$

$0.5 * 0.2 + 0.5 * 0.1 = 0.15$

في حال ما عنا رسمة

Return to portfolio ⇒ equally weighted portfolio



$\sigma_p^2 = \beta_p^2 \sigma_M^2 + \sigma_{\varepsilon p}^2$

Residual = actual − predicted

$\varepsilon = r - (\underset{\text{القاطع العمودي}}{\alpha} + \beta r_m)$

$\sigma_{\varepsilon p}^2 = 0.0133$: نفس الطريقة اي فورم

unsystematic Risk

أو

$\sigma_{\varepsilon p}^2 = w_s^2 \sigma_{\varepsilon_s}^2 + w_r^2 \sigma_{\varepsilon_r}^2$

FIGURE 6.7 TWO-STOCK PORTFOLIO.

* Chapter (6) questions *

Refer to the following table for Problems 2 through 7.

Stocks	Portfolio Weight	Beta	Expected Return	$\sigma_2(r)$
A	.25	.50	.40	.07
B	.25	.50	.25	.05
C	.50	1.00	.21	.07

$$\sigma^2(r_M) = .06$$

2. Given the assumption of the single-factor model, what is the residual variance of each of the foregoing stocks?
3. What is the beta factor of the three-stock portfolio?
4. What is the variance of the portfolio?
5. What is the expected return on the portfolio?
6. Given the actual (Markowitz) covariance between the stocks' returns, what is the actual portfolio variance?

$$\mathrm{Cov}(r_A, r_B) = .020$$
$$\mathrm{Cov}(r_A, r_C) = .035$$
$$\mathrm{Cov}(r_B, r_C) = .035$$

7. Why might the actual covariance differ from those found using the single-factor model formula?

(2) $\sigma_A^2 = \beta_A^2 \sigma_M^2 + \sigma_{\varepsilon A}^2$

$0.07 = (0.5)^2 * 0.06 + \sigma_{\varepsilon A}^2 \Rightarrow 0.055$

$0.05 = (0.5)^2 * 0.06 + \sigma_{\varepsilon B}^2 \Rightarrow 0.035$

$0.07 = (1)^2 * 0.06 + \sigma_{\varepsilon c}^2 \Rightarrow 0.01$

(3) $\beta_P = w_A \beta_A + w_B \beta_B + w_c \beta_c$

$= 0.25(0.5) + 0.25(0.5) + 0.5(1)$

$= 0.75$

(4) $\sigma_P^2 = \beta_P^2 \sigma_M^2 + \sigma_{\varepsilon P}^2$ } unsystematic Risk Based on the single factor models

$\sigma_{\varepsilon P}^2 = w_A^2 \sigma_{\varepsilon A}^2 + w_B^2 \sigma_{\varepsilon B}^2 + w_c^2 \sigma_{\varepsilon c}^2$

$= 0.25^2 * 0.055 + 0.25^2 * 0.035 + 0.5^2 * 0.01 = 0.0081$

Systamatic risk (p) $= \beta_P^2 \sigma_M^2 = 0.75^2 * 0.06 = 0.0338$

$\therefore \sigma_P^2 = 0.0338 + 0.0081 = 0.049$

(5) $E(r_P) = w_A E(r_A) + w_B E(r_B) + w_c E(r_c)$

$= 0.25(0.4) + 0.25 * 0.25 + 0.5(0.21)$

$= 26.75\%$

6 For (Markowitz) :

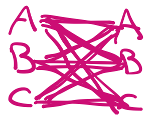
$$\sigma_p^2 = W_A^2 \sigma_A^2 + W_B^2 \sigma_B^2 + W_C^2 \sigma_C^2 + 2 W_A W_B \, Cov_{AB} + 2 W_A W_C \, Cov_{AC} + 2 W_B W_C \, Cov_{BC}$$

$$= (0.25)^2(0.07) + (0.25)^2(0.05) + (0.5)^2(0.07) + 2(0.25)(0.25)(0.02) + 2(0.25)(0.5)(0.035) + 2(0.25)(0.5)(0.035)$$

$$= 0.045$$

7 Markowitz take the full σ total Risk matrix (أدق)

$\hookrightarrow Cov_{AB} = \sigma_A \sigma_B \rho_{A,B}$



Single Factor Model assume that A and B are not correlated

$\hookrightarrow Cov_{AB} = \beta_A \beta_B \sigma_M^2$