Quantum Mechanics Formulae

From "*Introduction to Quantum Mechanics*"
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◈ Chapter 1: The Wave Function ◈

§1.1: The Schrödinger Equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi. \quad \text{(1.1—TDSE)}$$

§1.2: The Statistical Interpretation:

$$\int_a^b |\Psi(x,t)|^2\, dx = \left\{ \begin{array}{l} \text{probability of finding the particle} \\ \text{between } a \text{ and } b, \text{ at time } t. \end{array} \right\} \quad (1.3)$$

§1.3: Probability:

$$\langle f(j)\rangle = \sum_{j=0}^{\infty} f(j)P(j). \quad (1.9)$$

$$\sigma = \sqrt{\langle j^2\rangle - \langle j\rangle^2}. \quad (1.12)$$

$$\int_{-\infty}^{+\infty} \rho(x)\, dx = 1, \quad (1.16)$$

$$\langle x\rangle = \int_{-\infty}^{+\infty} x\rho(x)\, dx, \quad (1.17)$$

$$\langle f(x)\rangle = \int_{-\infty}^{+\infty} f(x)\,\rho(x)\, dx, \quad (1.18)$$

$$\sigma^2 \equiv \langle(\Delta x)^2\rangle = \langle x^2\rangle - \langle x\rangle^2. \quad (1.19)$$

§1.4: Normalization:

$$\int_{-\infty}^{+\infty} |\Psi(x,t)|^2\, dx = 1. \quad (1.20)$$

§1.5: Momentum:

$$\langle x \rangle = \int_{-\infty}^{+\infty} x \, |\Psi(x,t)|^2 \, dx. \quad (1.28)$$

$$\langle p \rangle = m\frac{d\langle x \rangle}{dt} = -i\hbar \int \left(\Psi^* \frac{\partial \Psi}{\partial x}\right) dx. \quad (1.33)$$

$$\langle x \rangle = \int \Psi^* [x] \Psi \, dx, \quad (1.34)$$

$$\langle p \rangle = \int \Psi^* [-i\hbar \, (\partial/\partial x)] \Psi \, dx. \quad (1.35)$$

$$\langle Q(x,p) \rangle = \int \Psi^* [Q(x, -i\hbar \partial/\partial x)] \Psi \, dx. \quad (1.36)$$

§1.6: The Uncertainty Principle:

$$\sigma_x \sigma_p \geq \frac{\hbar}{2}, \quad (1.40)$$

◈ Chapter 2: Time-Independent Schrödinger Equation [TISE] ◈

§2.1: Stationary States:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi. \quad \text{(2.5—TISE)}$$

$\varphi(t) = e^{-iEt/\hbar}$. (2.6)

$$\Psi(x,t) = \sum_{n=1}^{\infty} c_n \psi_n(x) e^{-iE_n t/\hbar} = \sum_{n=1}^{\infty} c_n \Psi_n(x,t). \quad (2.17)$$

$|c_n|^2$ is the *probability* that a measurement of the energy would return the value E_n. (2.19)

$$\sum_{n=1}^{\infty} |c_n|^2 = 1, \quad (2.20)$$

$$\langle H \rangle = \sum_{n=1}^{\infty} |c_n|^2 E_n. \quad (2.21)$$

§2.2: The Infinite Square Well:

$$V(x) = \begin{cases} 0, & 0 \le x \le a, \\ \infty, & \text{otherwise} \end{cases} \quad (2.22)$$

$$E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{n^2 \pi^2 \hbar^2}{2ma^2}. \quad (2.30)$$

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right). \quad (2.31)$$

$$\int \psi_m(x)^* \psi_n(x)\, dx = \delta_{mn}, \quad (2.33)$$

$$f(x) = \sum_{n=1}^{\infty} c_n \psi_n(x) = \sqrt{\frac{2}{a}} \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi}{a}x\right). \quad (2.35)$$

$$c_n = \int \psi_n(x)^* f(x)\, dx. \quad (2.37)$$

$$c_n = \sqrt{\frac{2}{a}} \int_0^a \sin\left(\frac{n\pi}{a}x\right) \Psi(x, 0)\, dx. \quad (2.40)$$

§2.3: The Harmonic Oscillator:

$$\hat{a}_\pm \equiv \frac{1}{\sqrt{2\hbar m\omega}}\left(\mp i\hat{p} + m\omega x\right) \quad (2.48)$$

$$\boxed{[x, \hat{p}] = i\hbar.} \quad (2.52)$$

$$\boxed{\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2}.} \quad (2.60)$$

$$\boxed{\psi_n(x) = A_n\left(\hat{a}_+\right)^n \psi_0(x), \quad \text{with} \quad E_n = \left(n + \frac{1}{2}\right)\hbar\omega,} \quad (2.62)$$

$$\boxed{\hat{a}_+\psi_n = \sqrt{n+1}\,\psi_{n+1}, \quad \hat{a}_-\psi_n = \sqrt{n}\,\psi_{n-1}.} \quad (2.67)$$

$$\boxed{\psi_n = \frac{1}{\sqrt{n!}}\left(\hat{a}_+\right)^n \psi_0,} \quad (2.68)$$

$$\boxed{x = \sqrt{\frac{\hbar}{2m\omega}}\left(\hat{a}_+ + \hat{a}_-\right); \quad \hat{p} = i\sqrt{\frac{\hbar m\omega}{2}}\left(\hat{a}_+ - \hat{a}_-\right).} \quad (2.70)$$

$$H_n(\xi) = (-1)^n e^{\xi^2}\left(\frac{d}{d\xi}\right)^n e^{-\xi^2}.$$ (2.87—Rodrigues formula for Hermite operators)

$$H_{n+1}(\xi) = 2\xi H_n(\xi) - 2n H_{n-1}(\xi). \quad (2.88)$$

§2.4: The Free Particle:

$$\boxed{\Psi(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} \phi(k) e^{i\left(kx - \frac{\hbar k^2}{2m}t\right)}\, dk.} \quad (2.101)$$

$$\boxed{f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} F(k)\, e^{ikx}\, dk \iff F(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)\, e^{-ikx}\, dx.} \quad (2.103)$$

$$\boxed{\phi(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \Psi(x,0)\, e^{-ikx}\, dx.}$$ (2.104)

$$v_{\text{phase}} = \frac{\omega}{k},$$ (2.108)

$$v_{\text{group}} = \frac{d\omega}{dk}$$ (2.109)

§2.5: The Delta-Function Potential:

$$\begin{cases} E < V(-\infty) \text{ and } V(+\infty) \Rightarrow & \text{bound state,} \\ E > V(-\infty) \text{ or } V(+\infty) \Rightarrow & \text{scattering state.} \end{cases}$$ (2.112)

$$\begin{cases} E < 0 \Rightarrow & \text{bound state,} \\ E > 0 \Rightarrow & \text{scattering state.} \end{cases}$$ (2.113—Simplification of 2.112)

$$\delta(x) \equiv \left\{ \begin{matrix} 0, & \text{if } x \neq 0 \\ \infty, & \text{if } x = 0 \end{matrix} \right\}, \text{ with } \int_{-\infty}^{+\infty} \delta(x)\, dx = 1.$$ (2.114)

$$V(x) = -\alpha\delta(x),$$ (2.117)

$$\boxed{\begin{cases} 1.\ \psi & \text{is always continuous;} \\ 2.\ d\psi/dx & \text{is continuous except at points where the potential is infinite.} \end{cases}}$$ (2.124—The standard boundary conditions for ψ)

$$\boxed{\Delta\left(\frac{d\psi}{dx}\right) = -\frac{2m\alpha}{\hbar^2}\psi(0)}$$ (2.128—Discontinuity in $d\psi/dx$ when V(0) = ∞)

$$\boxed{\psi(x) = \frac{\sqrt{m\alpha}}{\hbar} e^{-m\alpha|x|/\hbar^2}; \quad E = -\frac{m\alpha^2}{2\hbar^2}.}$$ (2.132—The only bound state)

$$\boxed{R = \frac{1}{1 + (2\hbar^2 E/m\alpha^2)}, \quad T = \frac{1}{1 + (m\alpha^2/2\hbar^2 E)}.}$$ (2.144—Reflection coefficient and Transmission coefficient for scattering states)

§2.6: The Finite Square Well:

$$V(x) = \begin{cases} -V_0, & -a \le x \le a, \\ 0, & |x| > a, \end{cases} \quad (2.148)$$

In the region $x < -a$ the potential is zero, so the Schrödinger equation reads

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi, \quad \text{or} \quad \frac{d^2\psi}{dx^2} = \kappa^2\psi,$$

$$\kappa \equiv \frac{\sqrt{-2mE}}{\hbar} \quad (2.149)$$

In the region $-a < x < a$, $V(x) = -V_0$, and the Schrödinger equation reads

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} - V_0\psi = E\psi, \quad \text{or} \quad \frac{d^2\psi}{dx^2} = -l^2\psi,$$

$$l \equiv \frac{\sqrt{2m(E+V_0)}}{\hbar}. \quad (2.151)$$

$$\psi(x) = \begin{cases} Fe^{-\kappa x}, & (x > a), \\ D\cos(lx), & (0 < x < a), \\ \psi(-x), & (x < 0). \end{cases} \quad (2.154)$$

$$T^{-1} = 1 + \frac{V_0^2}{4E(E+V_0)}\sin^2\left(\frac{2a}{\hbar}\sqrt{2m(E+V_0)}\right). \quad (2.172)$$

◈ Chapter 3: Formalism ◈

§3.1: Hilbert Space:

Wave functions live in Hilbert space. (3.5)

$$\langle f|g\rangle \equiv \int_a^b f(x)^* g(x)\,dx. \quad (3.6)$$

$\langle f_m|f_n\rangle = \delta_{mn}$. (3.10—The set $\{f_n\}$ is orthonormal)

$$f(x) = \sum_{n=1}^{\infty} c_n f_n(x).$$; $c_n = \langle f_n | f \rangle$, (3.11—Completeness)

§3.2: Observables:

$$\langle f | \hat{Q} f \rangle = \langle \hat{Q} f | f \rangle \quad \text{for all } f(x).$$ (3.15—Hermitian operator definition)

Observables are represented by hermitian operators. (3.18)

Determinate states of Q are eigenfunctions of $\hat{Q}$. (3.23)

§3.3: Eigenfunctions of a Hermitian Operator:

> **Axiom:** The eigenfunctions of an observable operator are *complete*: Any function (in Hilbert space) can be expressed as a linear combination of them.[13]

$$f_p(x) = \frac{1}{\sqrt{2\pi\hbar}} e^{ipx/\hbar},$$ (3.32)

§3.4: Generalized Statistical Interpretation:

Generalized statistical interpretation: If you measure an observable $Q(x, p)$ on a particle in the state $\Psi(x, t)$, you are certain to get *one of the eigenvalues* of the hermitian operator $\hat{Q}(x, -i\hbar d/dx)$.[15] If the spectrum of $\hat{Q}$ is discrete, the probability of getting the particular eigenvalue q_n associated with the (orthonormalized) eigenfunction $f_n(x)$ is

$$|c_n|^2, \quad \text{where} \quad c_n = \langle f_n | \Psi \rangle. \tag{3.43}$$

If the spectrum is continuous, with real eigenvalues $q(z)$ and associated (Dirac-orthonormalized) eigenfunctions $f_z(x)$, the probability of getting a result in the range dz is

$$|c(z)|^2 dz \quad \text{where} \quad c(z) = \langle f_z | \Psi \rangle. \tag{3.44}$$

Upon measurement, the wave function "collapses" to the corresponding eigenstate.[16]

$$\Phi(p, t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} e^{-ipx/\hbar} \Psi(x, t) dx;$$ (3.54)

$$\Psi(x, t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} e^{ipx/\hbar} \Phi(p, t) dp.$$ (3.55)

§3.5: The Uncertainty Principle:

$$\sigma_A^2\sigma_B^2 \geq \left(\frac{1}{2i}\left\langle\left[\hat{A},\hat{B}\right]\right\rangle\right)^2.$$ (3.62—Generalized uncertainty principle)

$$\left[\hat{A}+\hat{B},\hat{C}\right]=\left[\hat{A},\hat{C}\right]+\left[\hat{B},\hat{C}\right],$$ (3.64)

$$\left[\hat{A}\hat{B},\hat{C}\right]=\hat{A}\left[\hat{B},\hat{C}\right]+\left[\hat{A},\hat{C}\right]\hat{B}.$$ (3.65)

$$\frac{d}{dt}\langle Q\rangle=\frac{i}{\hbar}\left\langle\left[\hat{H},\hat{Q}\right]\right\rangle+\left\langle\frac{\partial\hat{Q}}{\partial t}\right\rangle.$$ (3.73—"Generalized Ehrenfest theorem")

§3.6: Vectors and Operators:

$$\Psi(x,t)=\langle x|\mathcal{S}(t)\rangle,$$ (3.77)

$$\Phi(p,t)=\langle p|\mathcal{S}(t)\rangle$$ (3.78)

$$c_n(t)=\langle n|\mathcal{S}(t)\rangle$$ (3.79)

$$\left\langle e_m\middle|\hat{Q}\middle|e_n\right\rangle\equiv Q_{mn}.$$ (3.83)

$$\hat{x}\text{ (the position operator)}\rightarrow\begin{cases}x & \text{(in position space)},\\ i\hbar\partial/\partial p & \text{(in momentum space)};\end{cases}$$

$$\hat{p}\text{ (the momentum operator)}\rightarrow\begin{cases}-i\hbar\partial/\partial x & \text{(in position space)},\\ p & \text{(in momentum space)}.\end{cases}$$

$\hat{P}\equiv|\alpha\rangle\langle\alpha|$ (3.91—Projection operator)

$$\sum_n |e_n\rangle\langle e_n| = 1$$ (3.93—Identity operator for *discrete* spectra)

$$\int |e_z\rangle\langle e_z|\,dz=1.$$ (3.96—Identity operator for *continuous* spectra)

$$e^{\hat{Q}} \equiv 1 + \hat{Q} + \frac{1}{2}\hat{Q}^2 + \frac{1}{3!}\hat{Q}^3 + \cdots \quad (3.100)$$

$$\frac{1}{1-\hat{Q}} \equiv 1 + \hat{Q} + \hat{Q}^2 + \hat{Q}^3 + \hat{Q}^4 + \cdots \quad (3.101)$$

$$\ln\left(1+\hat{Q}\right) \equiv \hat{Q} - \frac{1}{2}\hat{Q}^2 + \frac{1}{3}\hat{Q}^3 - \frac{1}{4}\hat{Q}^4 + \cdots \quad (3.102)$$

$$1 = \int dx\, |x\rangle \langle x|,$$

$$1 = \int dp\, |p\rangle \langle p|,$$

$$1 = \sum |n\rangle \langle n|. \quad (3.106)$$

◈ Chapter 4: Quantum Mechanics in Three Dimensions ◈

§4.1: The Schrödinger Equation:

$$\boxed{\mathbf{p} \to -i\hbar\nabla,} \quad (4.3)$$

$$\boxed{i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi,} \quad (4.4\text{—TDSE in 3D})$$

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}, \quad (4.5\text{—Laplacian})$$

$$\boxed{-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi.} \quad (4.8\text{—TISE in 3D})$$

$$\psi(r,\theta,\phi) = R(r)\,Y(\theta,\phi). \quad (4.15)$$

$$P_\ell^m(x) \equiv (-1)^m\left(1-x^2\right)^{m/2}\left(\frac{d}{dx}\right)^m P_\ell(x), \quad (4.27\text{—Associated Legendre polynomial})$$

$$P_\ell(x) \equiv \frac{1}{2^\ell \ell!}\left(\frac{d}{dx}\right)^\ell \left(x^2-1\right)^\ell .$$ (4.28)

$$\boxed{Y_\ell^m(\theta,\phi) = \sqrt{\frac{(2\ell+1)}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}}\, e^{im\phi}\, P_\ell^m(\cos\theta)\,,}$$ (4.32—Spherical harmonics)

$$u(r) \equiv r\,R(r)$$ (4.36)

$$\boxed{-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + \left[V + \frac{\hbar^2}{2m}\frac{\ell(\ell+1)}{r^2}\right]u = Eu.}$$ (4.37—Radial equation)

$$V_{\text{eff}} = V + \frac{\hbar^2}{2m}\frac{\ell(\ell+1)}{r^2},$$ (4.38)

§4.2: The Hydrogen Atom:

$$\boxed{E_n = -\left[\frac{m_e}{2\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2\right]\frac{1}{n^2} = \frac{E_1}{n^2}, \quad n = 1, 2, 3, \ldots .}$$ (4.70—Bohr formula)

$$R_{n\ell}(r) = \frac{1}{r}\rho^{\ell+1}e^{-\rho}v(\rho)$$ (4.75)

$$\boxed{\psi_{100}(r,\theta,\phi) = \frac{1}{\sqrt{\pi a^3}}e^{-r/a}.}$$ (4.80)

$$L_q^p(x) \equiv (-1)^p\left(\frac{d}{dx}\right)^p L_{p+q}(x)$$ (4.87—Associated Laguerre polynomial)

$$L_q(x) \equiv \frac{e^x}{q!}\left(\frac{d}{dx}\right)^q \left(e^{-x}x^q\right)$$ (4.88—Laguerre polynomial)

$$\boxed{\psi_{n\ell m} = \sqrt{\left(\frac{2}{na}\right)^3\frac{(n-\ell-1)!}{2n(n+\ell)!}}\, e^{-r/na}\left(\frac{2r}{na}\right)^\ell \left[L_{n-\ell-1}^{2\ell+1}(2r/na)\right] Y_\ell^m(\theta,\phi).}$$ (4.89)

§4.3: Angular Momentum:

$$\boxed{[L_x, L_y] = i\hbar L_z; \quad [L_y, L_z] = i\hbar L_x; \quad [L_z, L_x] = i\hbar L_y.} \quad (4.99)$$

$L_\pm \equiv L_x \pm i L_y.$ (4.105)

$$\boxed{L^2 f_\ell^m = \hbar^2 \ell(\ell+1) f_\ell^m; \quad L_z f_\ell^m = \hbar m f_\ell^m,} \quad (4.118)$$

$\ell = 0,\ 1/2,\ 1,\ 3/2, \ldots; \quad m = -\ell,\ -\ell+1,\ \ldots,\ \ell-1,\ \ell.$

$$\boxed{L_z = -i\hbar \frac{\partial}{\partial \phi}.} \quad (4.129)$$

$$\boxed{L^2 = -\hbar^2 \left[\frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial}{\partial\theta} \right) + \frac{1}{\sin^2\theta} \frac{\partial^2}{\partial\phi^2} \right].} \quad (4.132)$$

§4.4: Spin:

$\mathbf{L} = \mathbf{r} \times \mathbf{p}$ (Orbital angular momentum)

$\mathbf{S} = I\boldsymbol{\omega}$ (Spin angular momentum)

$\chi = \begin{pmatrix} a \\ b \end{pmatrix} = a\chi_+ + b\chi_-,$ (4.139—Spinor)

$$\boxed{\sigma_x \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma_y \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \sigma_z \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.} \quad \text{(4.148—Pauli spin matrices)}$$

$\boldsymbol{\mu} = \gamma \mathbf{S};$ (4.156)

$$\boxed{s = (s_1 + s_2),\ (s_1 + s_2 - 1),\ (s_1 + s_2 - 2),\ \ldots,\ |s_1 - s_2|.} \quad (4.182)$$

$$|sm\rangle = \sum_{m_1 + m_2 = m} C^{s_1 s_2 s}_{m_1 m_2 m} |s_1 s_2 m_1 m_2\rangle \quad (4.183)$$

§4.5: Electromagnetic Interactions:

$$i\hbar\frac{\partial\Psi}{\partial t} = \left[\frac{1}{2m}(-i\hbar\nabla - q\mathbf{A})^2 + q\varphi\right]\Psi.$$ (4.191—Minimal coupling rule)

◈ Aharonov and Bohm showed that the **vector** potential *can* affect the quantum behavior of a charged particle, *even when the particle is confined to a region where the field itself is zero*.

◈ Chapter 5: Identical Particles ◈

§5.1: Two-Particle Systems:

$$\int |\Psi(\mathbf{r}_1, \mathbf{r}_2, t)|^2 \, d^3\mathbf{r}_1 d^3\mathbf{r}_2 = 1.$$ (5.5)

$$\psi(\mathbf{r}_1, \mathbf{r}_2) = \psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2).$$ (5.9—Noninteracting particles)

$$\psi_\pm(\mathbf{r}_1, \mathbf{r}_2) = A\left[\psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2) \pm \psi_b(\mathbf{r}_1)\psi_a(\mathbf{r}_2)\right];$$ (5.17)

$$|(1, 2, \ldots, i, \ldots, j, \ldots, n)\rangle = \pm|(1, 2, \ldots, j, \ldots, i, \ldots, n)\rangle,$$ (5.34)

$$\Psi(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N) = \frac{1}{\sqrt{N!}}\begin{vmatrix} \chi_1(\mathbf{x}_1) & \chi_2(\mathbf{x}_1) & \cdots & \chi_N(\mathbf{x}_1) \\ \chi_1(\mathbf{x}_2) & \chi_2(\mathbf{x}_2) & \cdots & \chi_N(\mathbf{x}_2) \\ \vdots & \vdots & & \vdots \\ \chi_1(\mathbf{x}_N) & \chi_2(\mathbf{x}_N) & \cdots & \chi_N(\mathbf{x}_N) \end{vmatrix}.$$ (Slater determinant for fermionic systems)

§5.2: Atoms:

$$\hat{H} = \left\{-\frac{\hbar^2}{2m}\nabla_1^2 - \frac{1}{4\pi\epsilon_0}\frac{2e^2}{r_1}\right\} + \left\{-\frac{\hbar^2}{2m}\nabla_2^2 - \frac{1}{4\pi\epsilon_0}\frac{2e^2}{r_2}\right\} + \frac{1}{4\pi\epsilon_0}\frac{e^2}{|\mathbf{r}_1 - \mathbf{r}_2|},$$ (5.38)

$${}^{2S+1}L_J,$$ (5.45)

§5.3: Solids:

$$\psi_{n_x n_y n_z} = \sqrt{\frac{8}{l_x l_y l_z}} \sin\left(\frac{n_x\pi}{l_x}x\right) \sin\left(\frac{n_y\pi}{l_y}y\right) \sin\left(\frac{n_z\pi}{l_z}z\right),$$ (5.49)

$$E_{n_x n_y n_z} = \frac{\hbar^2 \pi^2}{2m}\left(\frac{n_x^2}{l_x^2} + \frac{n_y^2}{l_y^2} + \frac{n_z^2}{l_z^2}\right) = \frac{\hbar^2 k^2}{2m},$$ (5.50)

$$k_F = \left(3\rho\pi^2\right)^{1/3},$$ (5.52)

$$E_F = \frac{\hbar^2}{2m}\left(3\rho\pi^2\right)^{2/3}.$$ (5.54—Fermi energy)

$$E_{\text{tot}} = \frac{\hbar^2 V}{2\pi^2 m}\int_0^{k_F} k^4 dk = \frac{\hbar^2 k_F^5 V}{10\pi^2 m} = \frac{\hbar^2\left(3\pi^2 Nd\right)^{5/3}}{10\pi^2 m} V^{-2/3}.$$ (5.56)

$$P = \frac{2}{3}\frac{E_{\text{tot}}}{V} = \frac{2}{3}\frac{\hbar^2 k_F^5}{10\pi^2 m} = \frac{\left(3\pi^2\right)^{2/3}\hbar^2}{5m}\rho^{5/3}.$$ (5.57)

$\psi(x+a) = e^{iqa}\psi(x),$ (5.60—Bloch's theorem)

$$V(x) = \alpha\sum_{j=0}^{N-1}\delta(x - ja).$$ (5.64—Dirac comb)

◈ Chapter 6: Symmetries & Conservation Laws ◈

$$\boxed{\hat{T}(a) = \exp\left[-\frac{ia}{\hbar}\hat{p}\right].}$$ (6.3)

$\left\langle\psi'\middle|\hat{Q}\middle|\psi'\right\rangle = \left\langle\psi\middle|\hat{Q}'\middle|\psi\right\rangle.$ (Transformed operator)

$$\boxed{\hat{Q}' = \hat{T}^\dagger\hat{Q}\hat{T}.}$$ (6.6)

$$\boxed{\psi(x) = e^{iqx}u(x)}$$ (6.12—$u(x)$ is a periodic function of x)

$\left\{\hat{A}, \hat{B}\right\} \equiv \hat{A}\hat{B} + \hat{B}\hat{A}$ (Anti-Commutator)

$$\hat{R}_{\mathbf{n}}(\varphi) = \exp\left[-\frac{i\varphi}{\hbar}\mathbf{n}\cdot\hat{\mathbf{L}}\right]$$ (6.32)

$$\left[\hat{L}_i, \hat{V}_j\right] = i\hbar\epsilon_{ijk}\hat{V}_k,$$ (6.33)

$$\left[\hat{L}_i, \hat{f}\right] = 0.$$ (6.34)

$$\left\langle n'\,\ell'\,m' \,\middle|\, \hat{f} \,\middle|\, n\,\ell\,m\right\rangle = \delta_{\ell\ell'}\,\delta_{mm'}\,\langle n'\,\ell \,\|\, f \,\|\, n\,\ell\rangle.$$ (6.47)

$$\hat{U}(t) = \exp\left[-\frac{i\,t}{\hbar}\,\hat{H}\right].$$ (6.71)

$$\hat{Q}_H(t) = \hat{U}^\dagger(t)\,\hat{Q}\hat{U}(t).$$ (6.72—Heisenberg-picture operator)