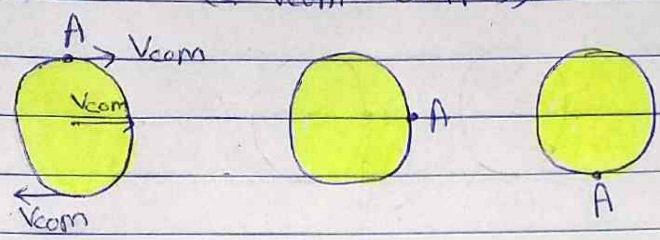
Rolling

→ When a wheel of Radius R rolling then

$$\ll V_{com} = \omega R \gg$$



→ Forces and Kinetic energy of rolling

$$K = \tfrac{1}{2} I\omega^2 + \tfrac{1}{2} mv^2$$

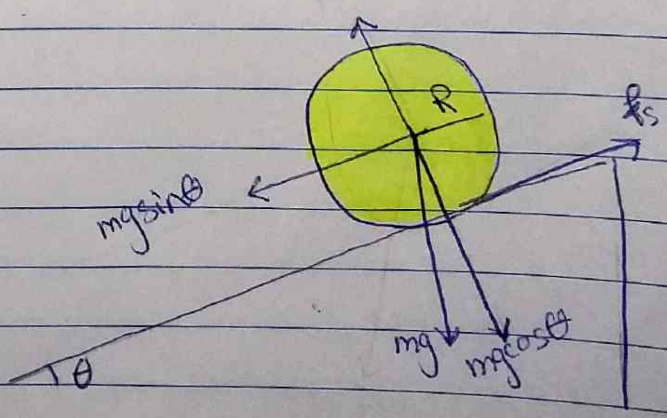
$\frac{1}{2} I\omega^2$ ↓ from the rotation motion

$\frac{1}{2} mv^2$ ↓ from ther linear motion

→ $a_{com} = \alpha R$

• If the wheel rolls smoothly down a ramp of angle θ its acceleration along an X-axis extending up the ramp is

→ $$a_{com,x} = \frac{-g\sin\theta}{1+\left(\frac{I_{com}}{MR^2}\right)}$$

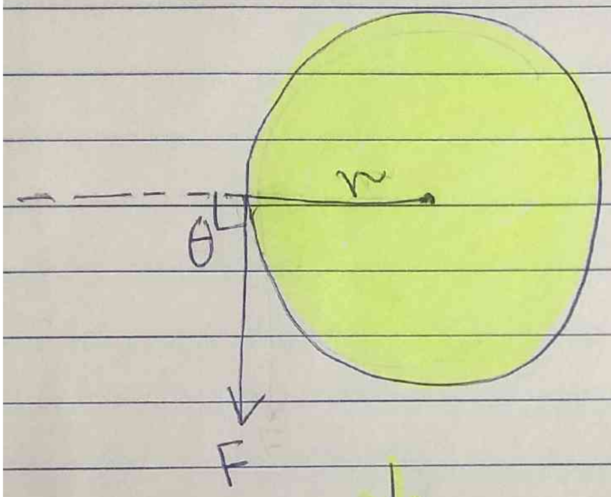


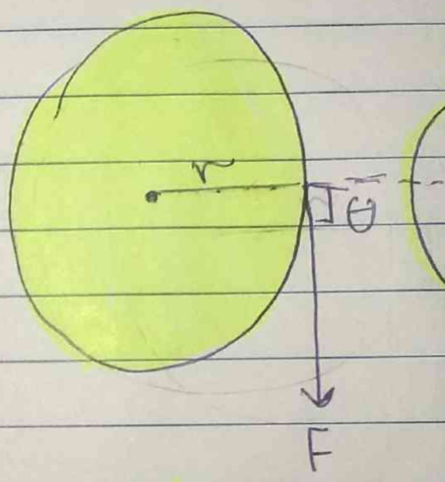
→ $$f_s = \frac{-I_{com}\, a_{com,x}}{R^2}$$

→ Torque revisited:-

$$(\vec{\tau}) = \vec{F} \times \vec{r} \rightarrow |\tau| = |F||r| \sin\theta$$

- The direction of (τ) is given By the right-hand rule for cross product.





the fingers are in the direction of r

(τ) is $(+\hat{k})$
(counter clockwise)
(out of the page)

(τ) is $(-\hat{k})$
(with clock wise)
(on the page)

→ Angular momentum:-

$$\vec{l} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$$

$$l = m r v \sin\theta \qquad \theta: \text{between } r \text{ and } v$$

- The direction of (l) is given By the right-hand rule for cross product.

the fingers are in the direction of r

$\rightarrow \vec{\ell} \quad I \, \vec{\omega}$

→ Newton's second law in angular form:-

$$\vec{\tau}_{net} = \frac{d\vec{l}}{dt}$$, lookLike $\vec{F}_{net} = \frac{d\vec{p}}{dt}$

Note:

$$\frac{d}{dt}(\vec{r}) = \vec{V}$$

$$m\vec{r} \times \vec{V} = \vec{l}$$

$$\frac{d\vec{l}}{dt} = \vec{\tau}$$

→ look to the sample 11.04

→ Conservation of angular momentum:-

- The angular momentum $\vec{l}$ of a system remains constant if the external torque acting on the system is zero:

$\vec{l}$ = a constant (isolated system)

$\vec{l}_i = \vec{l}_f$