Chapter 4:

thx

Q1) (a) $y = 1-(x+1)^3$

1.

$* \quad y = 1-(x+1)^3 \Rightarrow y' = -3(x+1)^2$

$y' = \text{zero} \Rightarrow \text{zero} = -3(x+1)^2 \Rightarrow \boxed{x = -1}$

2.

$\longleftarrow - - - - \underset{-1}{|} - - - - \longrightarrow \quad f'(x)$

- $f(x)$ decreasing on $(-\infty, \infty)$
- no local (maximum, minimum)

$f(x)$ behav

(x) behave

3.

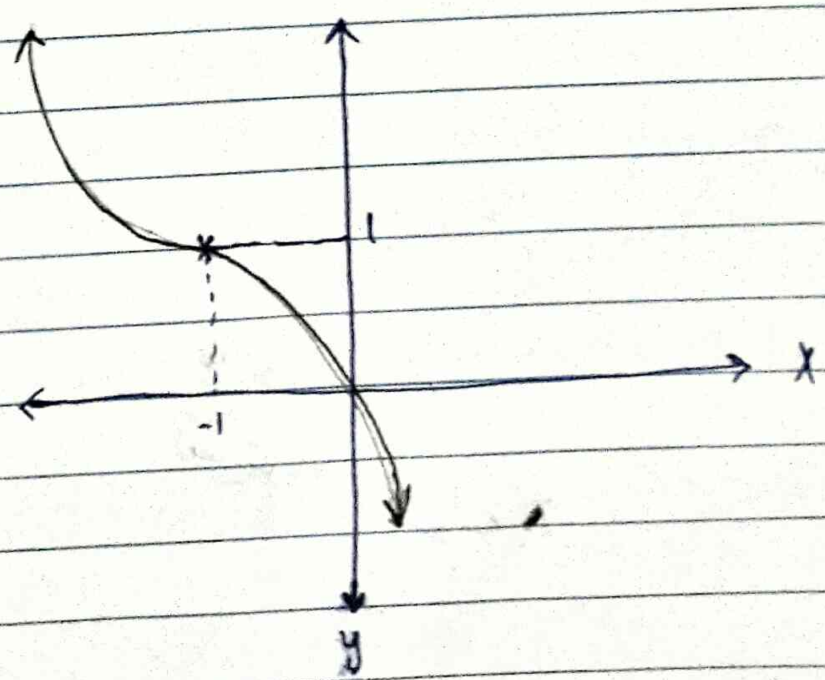
$* \quad y'' = -6(x+1) \Rightarrow y'' = \text{zero} \Rightarrow \text{zero} = -6(x+1)$

$\boxed{x = -1}$

- $f(x)$ concave up on $(-\infty, -1]$
- $f(x)$ concave down on $[-1, \infty)$
- In flection points $f(-1) = 1$.

$\longleftarrow + + + + + \overset{0}{\underset{-1}{|}} - - - - - \longrightarrow \quad f''(x)$

$\cup \quad \cap \quad f(x)$ behave



(b) $y = \frac{x^2+1}{x}$ $\qquad x \neq 0$

to help:
- → No. H.asy
- → $x = 0$ V. asy
- → $y = x$ Obliqu. asy

$y = x + \frac{1}{x}$

$y' = 1 - \frac{1}{x^2} \Rightarrow$ zero $= 1 - \frac{1}{x^2}$

$1 = \frac{1}{x^2}$

$x = -1, 1$

$f'(x)$: $- - - - - - - - \; 0 \; + + + + + + +$ (at -1, 0, 1)

f(x) behave

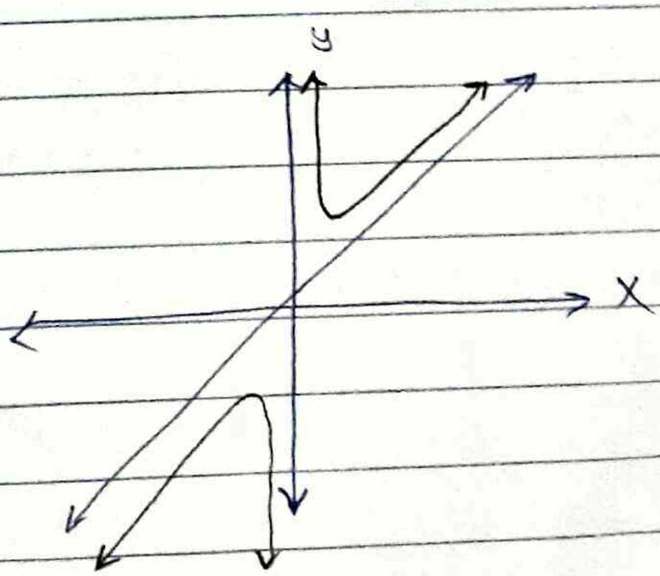
- f(x) increasing on $(-\infty, 0)$
- f(x) decreasing on $(0, \infty)$
- ~~f(x)~~ No extreme values.

$f''(x) = \frac{1}{x^4}$ $\qquad f'' \neq 0$

- f(x) concave up on $(\infty, -\infty)$ $\qquad$ $+ + + + \; 0 \; + + +$ (at 0) $\qquad f''(x)$

$\lim_{x \to 0^-} f(x) = -\infty$

$\lim_{x \to 0^+} f(x) = \infty$



c) $y = x^4 - 2x^2$

$\Rightarrow y' = 4x^3 - 4x \Rightarrow y' = \text{zero}$

$\text{zero} = 4x^3 - 4x$

$\Rightarrow x^3 - x = 0$

$x(x^2 - 1) = 0 \Rightarrow x = 0, 1, -1$

- $f(x)$ decreasing on $(-\infty, -1] \cup [0, 1]$ $\leftarrow - - - 0 + + + 0 - - - 0 + + + \rightarrow f'(x)$ $-1 \quad 0 \quad 1$
- $f(x)$ increasing on $[-1, 0] \cup [1, \infty)$
- $f(-1) = -1$ local minimum values
- $f(0) = 0$ local maximum values & $f(1) = -1$ local minimum value

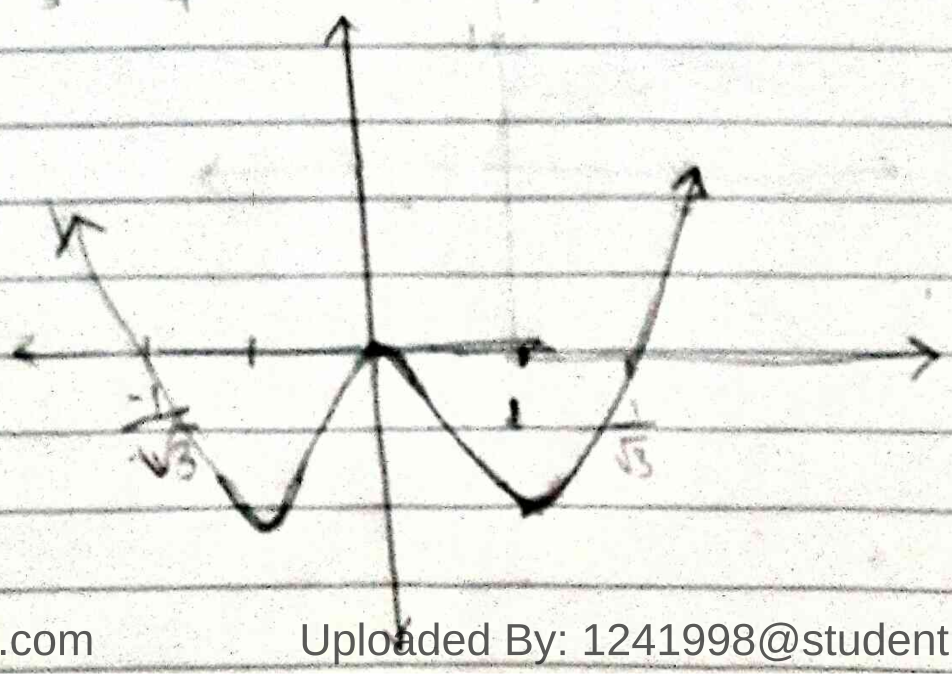
$\Rightarrow y'' = 12x^2 - 4 \Rightarrow y'' = 0$

$0 = 12x^2 - 4 \Rightarrow 0 = 3x^2 - 1$

$\Rightarrow 3x^2 = 1 \Rightarrow x^2 = \frac{1}{3}$

$\Rightarrow x = \pm\sqrt{\frac{1}{3}}$

- $f(x)$ concave up on $(-\infty, -\frac{1}{\sqrt{3}}] \cup [\frac{1}{\sqrt{3}}, \infty)$ $\leftarrow + + + 0 - - - - - 0 + + + \rightarrow f''(x)$ $-\sqrt{\frac{1}{3}} \quad \sqrt{\frac{1}{3}}$
- $f(x)$ concave down on $[-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}]$
- $f(-\frac{1}{\sqrt{3}}) = \frac{1}{9} - \frac{2}{3} = \frac{-5}{9}$ inflection points
- $f(\frac{1}{\sqrt{3}}) = \frac{1}{9} - \frac{2}{3} = \frac{-5}{9}$ inflection points



$x = 2$ V. asy

$y = x + 2$ oblique.asy

d) $y = \dfrac{x^2 - 3}{x - 2}$, $x \neq 2$

$y' = \dfrac{x^2 - 4x + 3}{(x-2)^2}$ $y'' = \dfrac{2}{(x-2)^3}$

$y' = 0 \Rightarrow x = 1, 3 \rightarrow$ critical values 1, 3

y' DNE $\Rightarrow x = 2$ $2 \notin D$

y' + + + | − − − | − − − − | + +
1 2 3

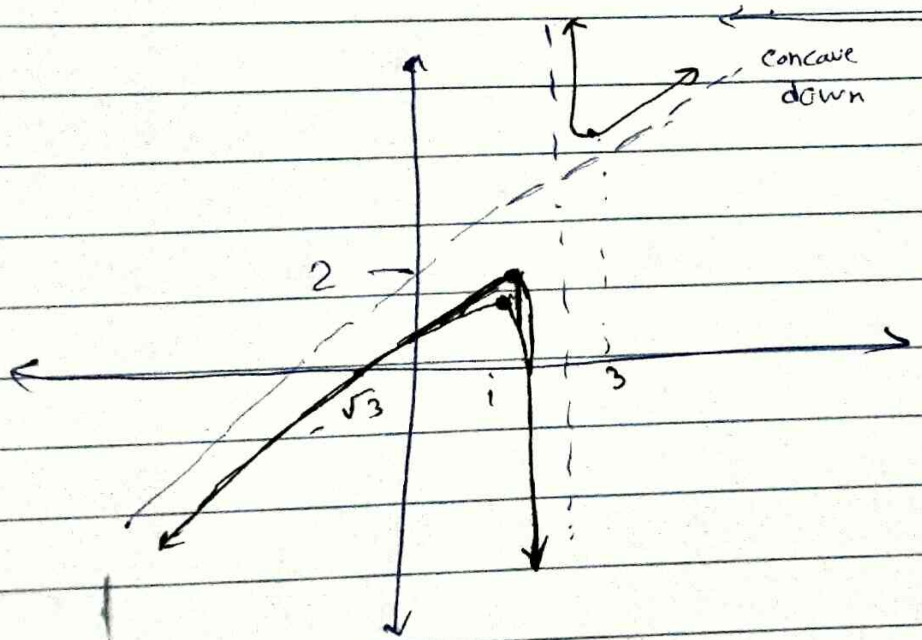
- $f(x)$ increasing on $(-\infty, 1]$ $\cup [3, \infty)$
- $f(x)$ decreasing on $[1, 3]$
- $(1, 2)$ local max
- $(3, 6)$ local min

$y'' = 0 \Rightarrow$ No values

y'' DNE $\Rightarrow x = 2$ $2 \notin D$

− − − − | + + +
2

concave down concave up



$x \neq 1, 1$

$y = 0$ H.Asy

$x = 1$, $x = -1$ V.Asy

f) $y = \frac{x}{x^2 - 1}$

$$y' = \frac{(x^2-1) - 2x^2}{(x^2-1)^2}$$

$y' =$ zero $\Rightarrow x^2 - 1 = 2x^2 \Rightarrow x^2 \neq -1$

. $f'(x)$ decreasing on $(-\infty, \infty)$

--- -1 --- 1 --- $f'(x)$

$$y' = \frac{1}{x^2-1} - \frac{2x^2}{(x^2-1)^2}$$

$$y'' = \frac{-2x}{(x^2-1)^2} - \left(\frac{(x^2-1)^2 4x - 2x^2 * 2x * 2(x^2-1)^2}{(x^2-1)^4} \right)$$

$0 = -2x - 4x + 8x^3$

$-6x + 8x^3 = 0 \Rightarrow 2x(-3 + 4x^2) = 0$

$x = 0$, $-\frac{\sqrt{3}}{2}$, $\frac{\sqrt{3}}{2}$

-- $-\frac{\sqrt{3}}{2}$ ++++ 0 ----- $\frac{\sqrt{3}}{2}$ +++

. $f(x)$ concave up on $[-\frac{\sqrt{3}}{2}, 0] \cup [\frac{\sqrt{3}}{2}, \infty)$

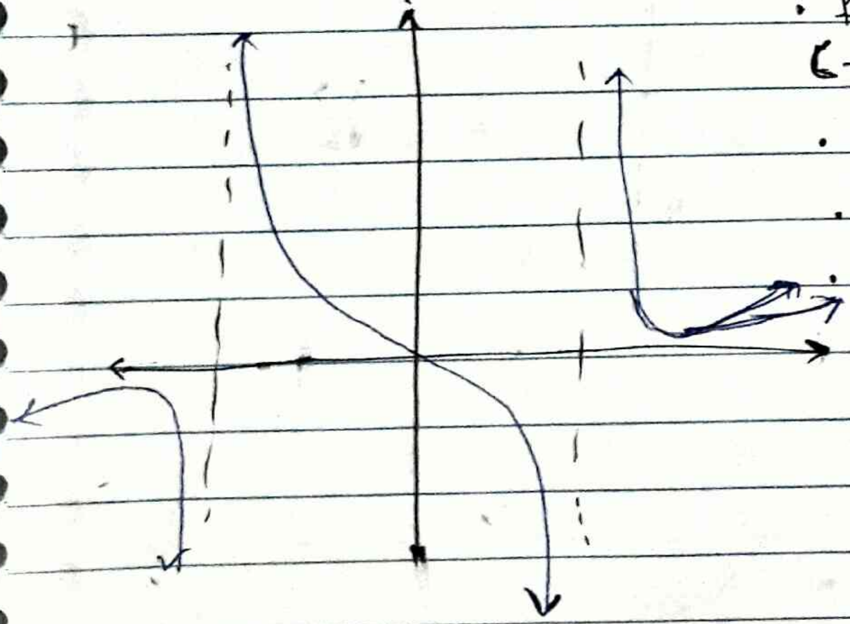
. $f(x)$ concave down on $(-\infty, -\frac{\sqrt{3}}{2}] \cup [0, \frac{\sqrt{3}}{2}]$

. $(f(\frac{\sqrt{3}}{2}), \frac{\sqrt{3}}{2}) = (-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2})$

. $(f(0), 0) = (0, 0)$

. $(f(-\frac{\sqrt{3}}{2}), -\frac{\sqrt{3}}{2}) = (\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2})$

inflection points



9) $y = x\sqrt{8 - x^2}$

$D \Rightarrow \quad 8 - x^2 \geq 0$

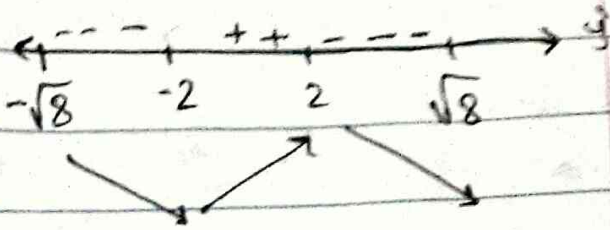
$\sqrt{8} \geq x \geq -\sqrt{8}$

$y' = \sqrt{8-x^2} + x \dfrac{-2x}{2\sqrt{8-x^2}}$

$y' = \dfrac{8 - x^2 - x^2}{\sqrt{8-x^2}} = \dfrac{8 - 2x^2}{\sqrt{8-x^2}}$

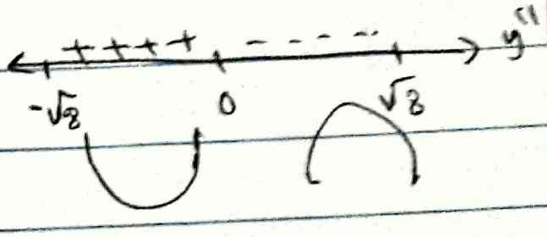
$y' = 0 \Rightarrow \quad x = 2, -2$

- y increasing on $[-2, 2]$
- y decreasing on $[-\sqrt{8}, -2] \cup [2, \sqrt{8}]$



- $y(-2) = -4$ abs. min
- $y(2) = 4$ abs. max

$y'' = \dfrac{2x(x^2 - 12)}{(8 - x^2)^{3/2}} \Rightarrow y'' = 0 \Rightarrow x = 0 \rightarrow y = 0$



- y concave up $[-\sqrt{8}, 0]$
- y concave down $[0, \sqrt{8}]$
- $(0,0)$ inflection point