FUNICULAR STRUCTURES: CABLES AND ARCHES

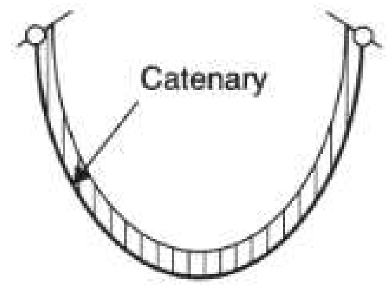
Section 6.2

Introduction

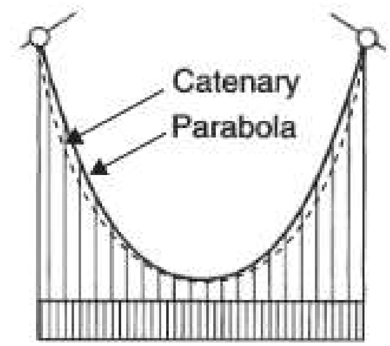
- Cable and arch, these two - apparently - different structures, share some fundamental characteristics that cause them to be more closely related than might initially appear, particularly in terms of their basic structural behavior.
- A cable subjected to external loads will deform in a way that is dependent on the magnitude and location of the external forces. The form acquired is often called the funicular shape of the cable. Only tension forces will be developed in the cable.
- Inverting the structural form obtained yields a new structure – Arch - that is analogous to the cable structure, except that compression rather than tension forces are developed.

General Principles of Funicular Shapes

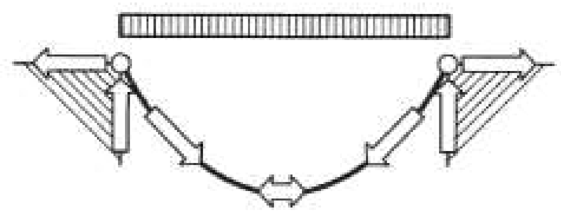
- The funicular shape is naturally assumed by a freely deforming cable subjected to loading.
- It is important to know what exact curve or series of straight-line segments defines the funicular shape for a given loading.



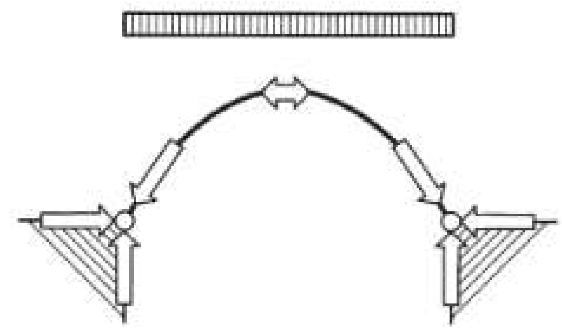
(a) Uniformly distributed load along length of cable (e.g., self-weight of cable)



(b) Uniformly distributed load on horizontal projection

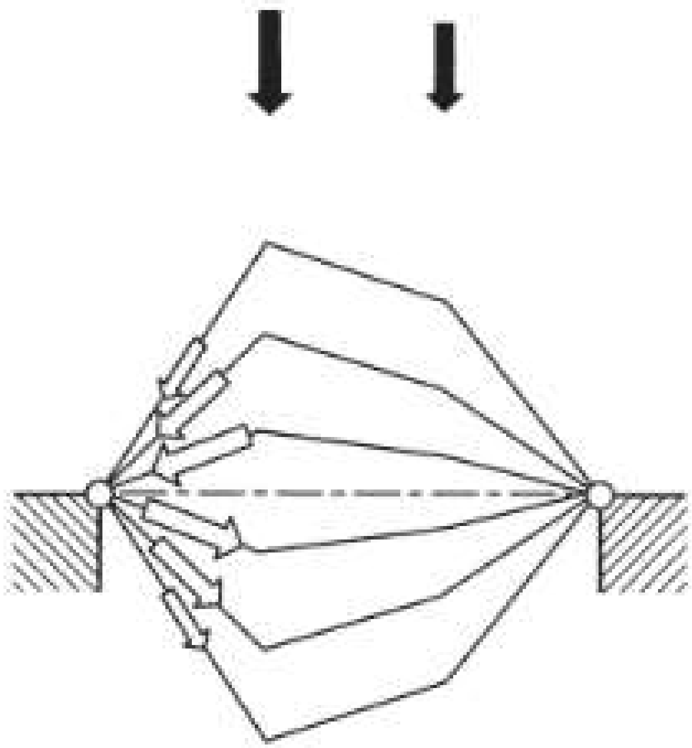


(c) Cable forces: Only tension forces are developed in a cable. Maximum forces occur at the reactions.

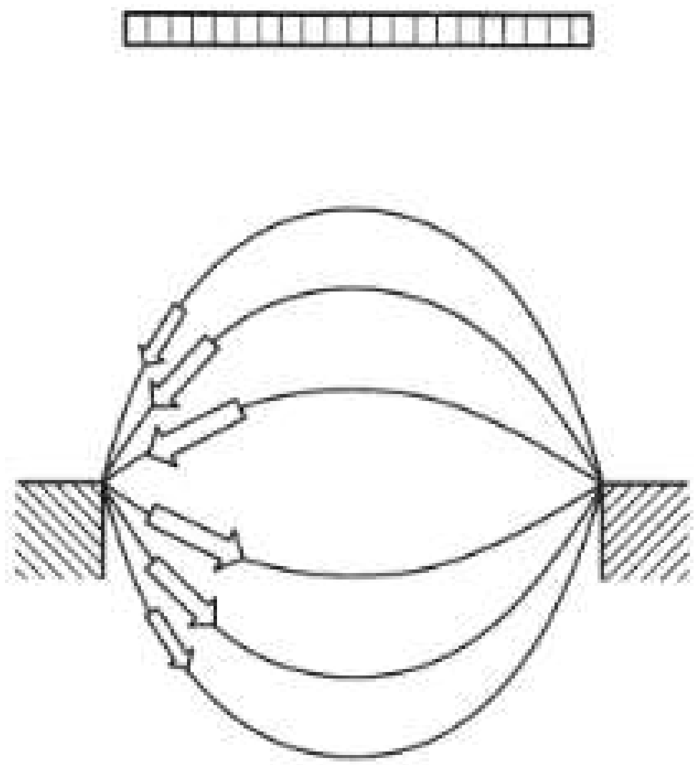


(d) Arch forces: Only compression forces are developed in an ideal arch. Maximum forces occur at reactions.

General Principles of Funicular Shapes



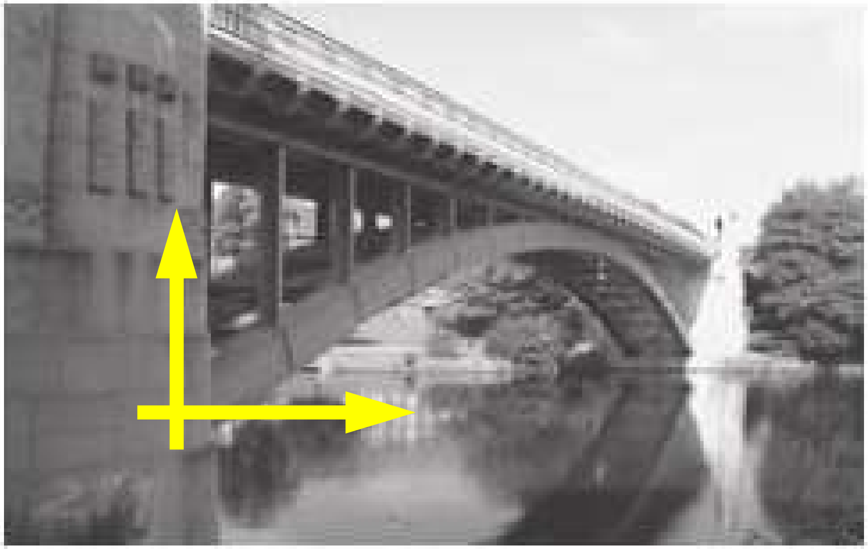
(e) Concentrated loads: Family of funicular shapes for a typical loading. As the structural depth of a funicular structure decreases, internal forces increase, and vice versa.

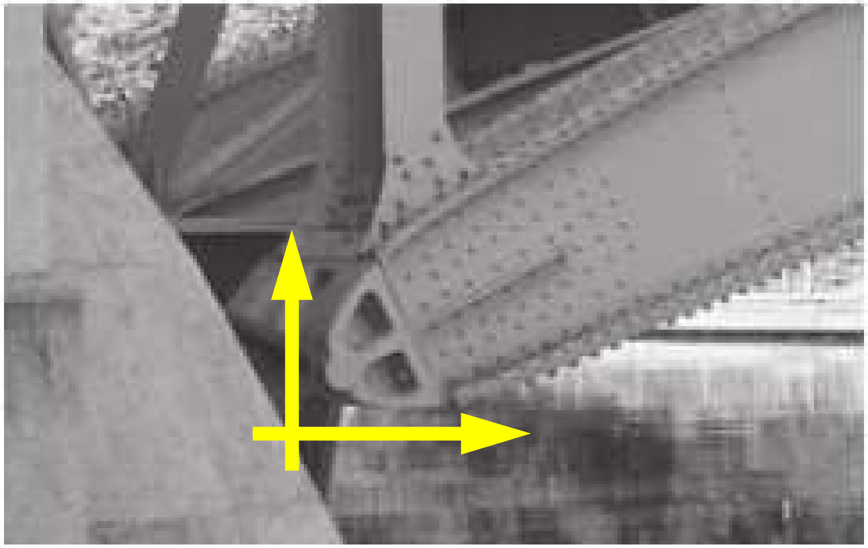


(f) Uniformly distributed loads: Family of funicular shapes for a horizontally distributed uniform loading

General Principles of Funicular Shapes

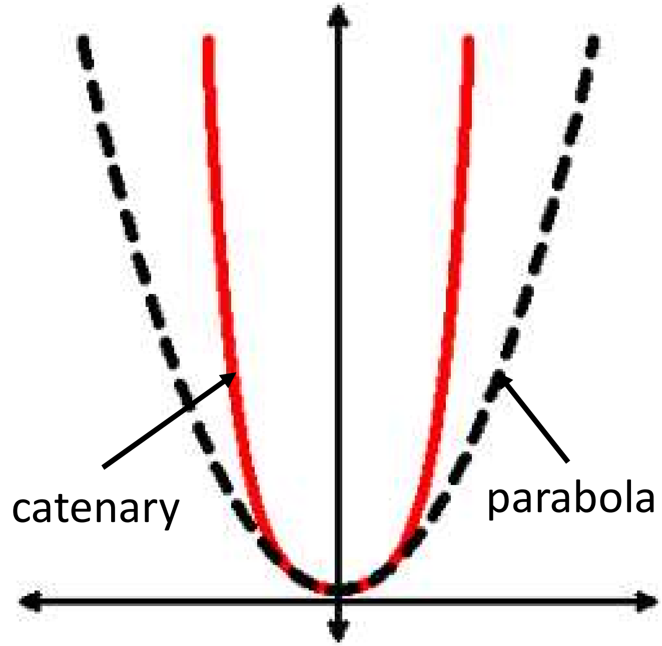
- Reactions developed at the arch or cable ends also depend on loads, and funicular shape.
- End reactions have vertical and horizontal thrusts that must be resisted by the foundations or some other element, such as a compressive strut or tie-rod.





Catenary & Parabolic arch

- For relatively small values of x the parabola and catenary appear quite similar, but as x increases in size, the difference is rather dramatic.

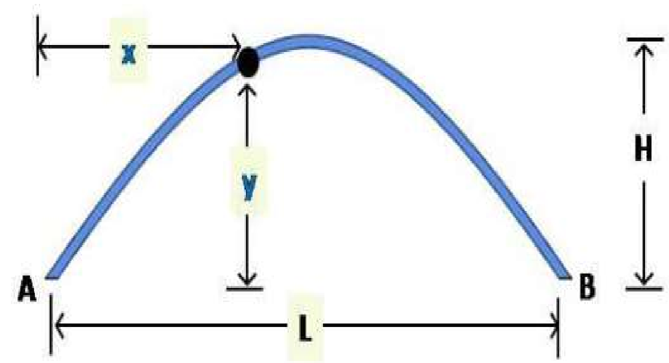


- Catenary function formula (red)

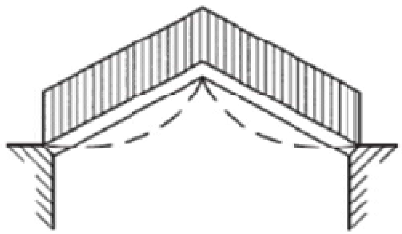
$$\frac{y}{c} = \cosh\left(\frac{x}{c}\right) - 1 \; or \; y = e^{\frac{x}{2}} + e^{\frac{-x}{2}}$$

- Parabolic arch formula

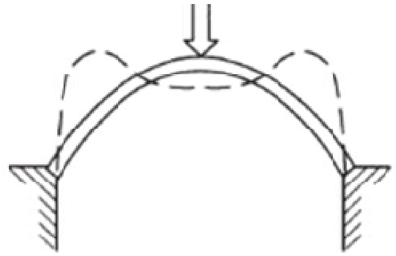
$$y = \frac{4H}{L^2}(L - x)x$$



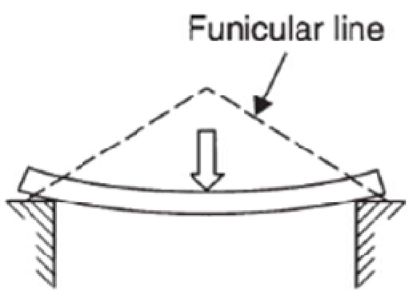
General Principles of Funicular Shapes



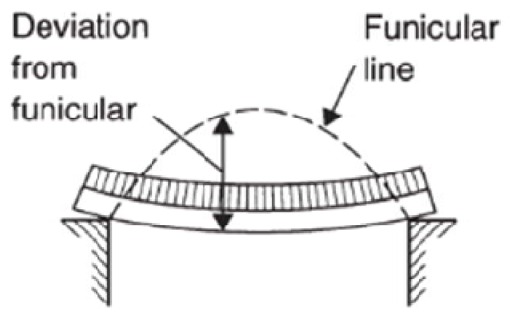
(e) Structure in bending:
A structure that is shaped to be funicular for a concentrated load at midspan becomes subject to bending when the loading changes.



(f) Structure in bending:
A funicular structure for a uniformly distributed load becomes subject to bending when the loading changes.



(g) Structure in bending:
When the shape of the structure does not correspond to the funicular shape for the loading, bending invariably develops.

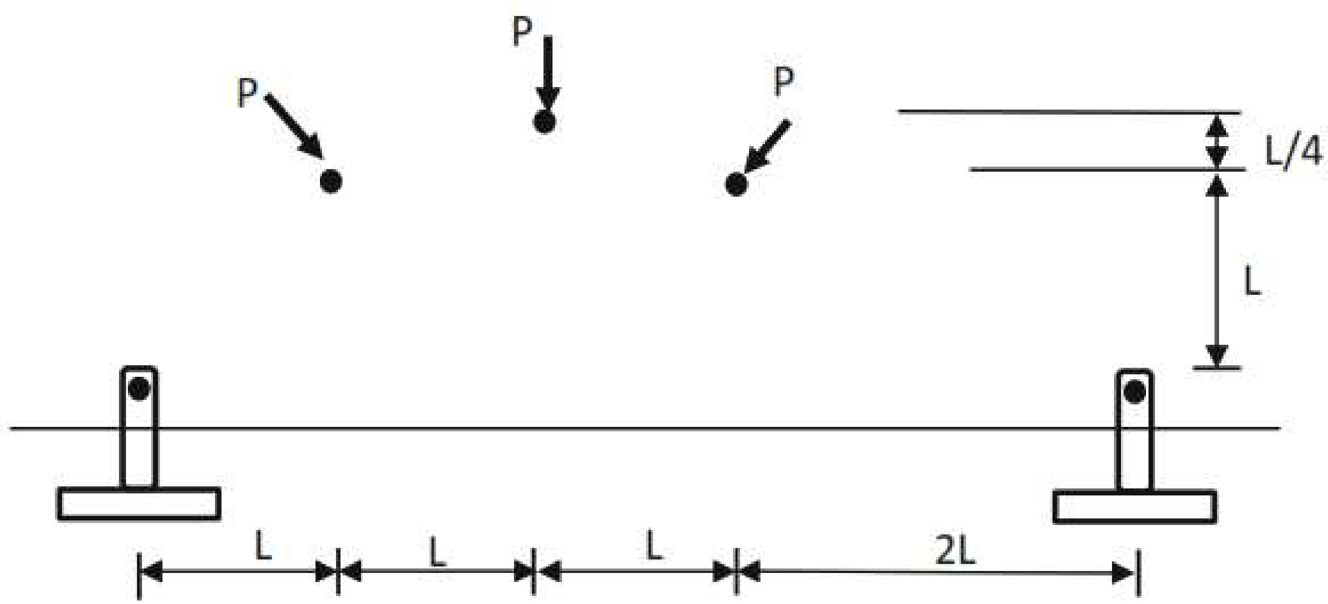


(h) Structure in bending:
The deviation at any point between the structure and the funicular line for the loading generally reflects the amount of bending present in the structure at that point.

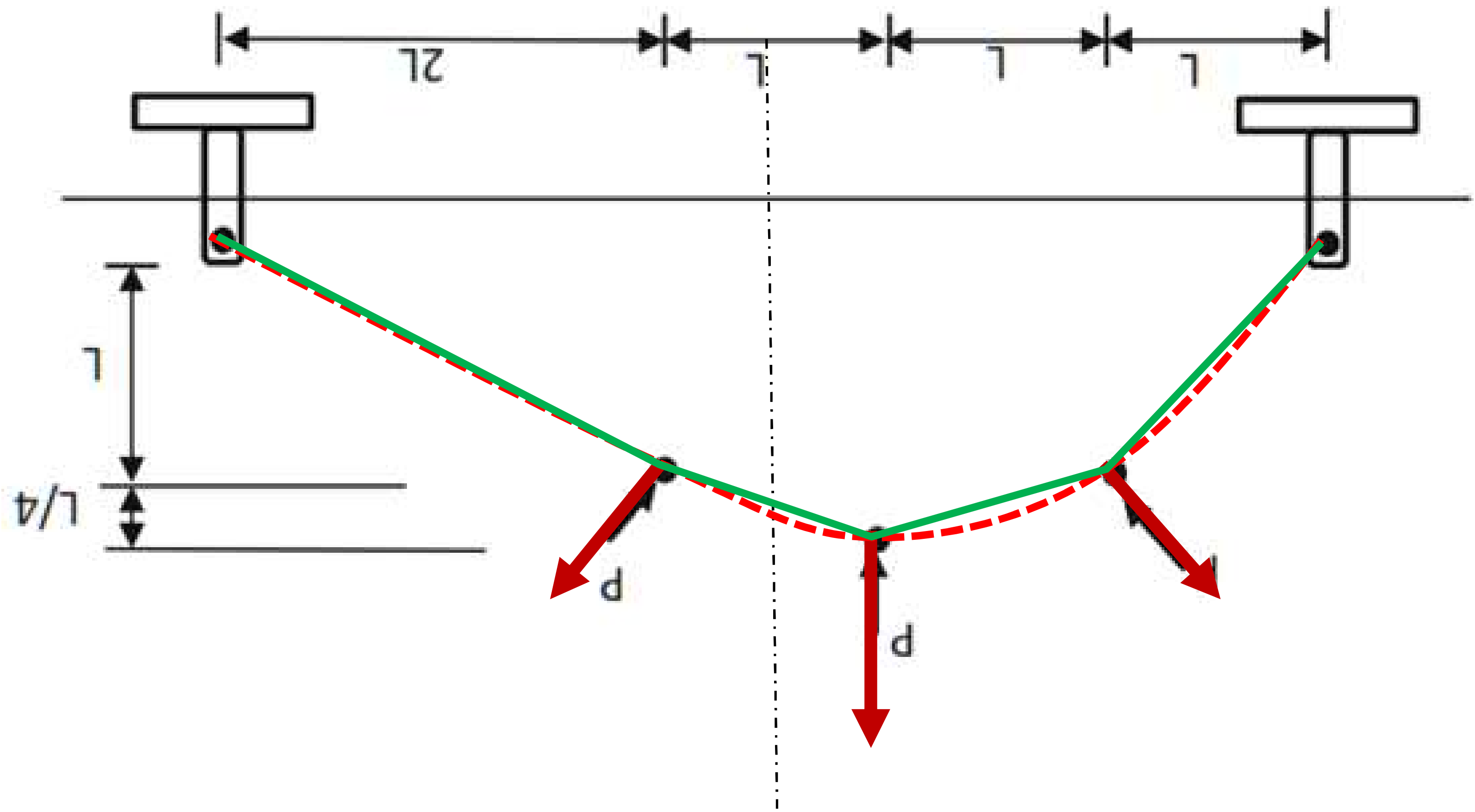
Note that if the shape of a funicular structure is imagined and superimposed on the actual structure considered, the amount of deviation of the actual structure from the funicular shape generally reflects the severity of the bending present in the structure.

Finding Funicular Shape

Provide a structure of curved or linear members for the loads given below. Two foundations are given to support the structure. Draw a pure compression structure to support the given loads, using the hanging chain analogy. Firstly, assume that the self-weight of the support structure is negligible compared to the forces (P). Secondly, assume that the self-weight is significant, and must be considered in the configuration of the support structure.

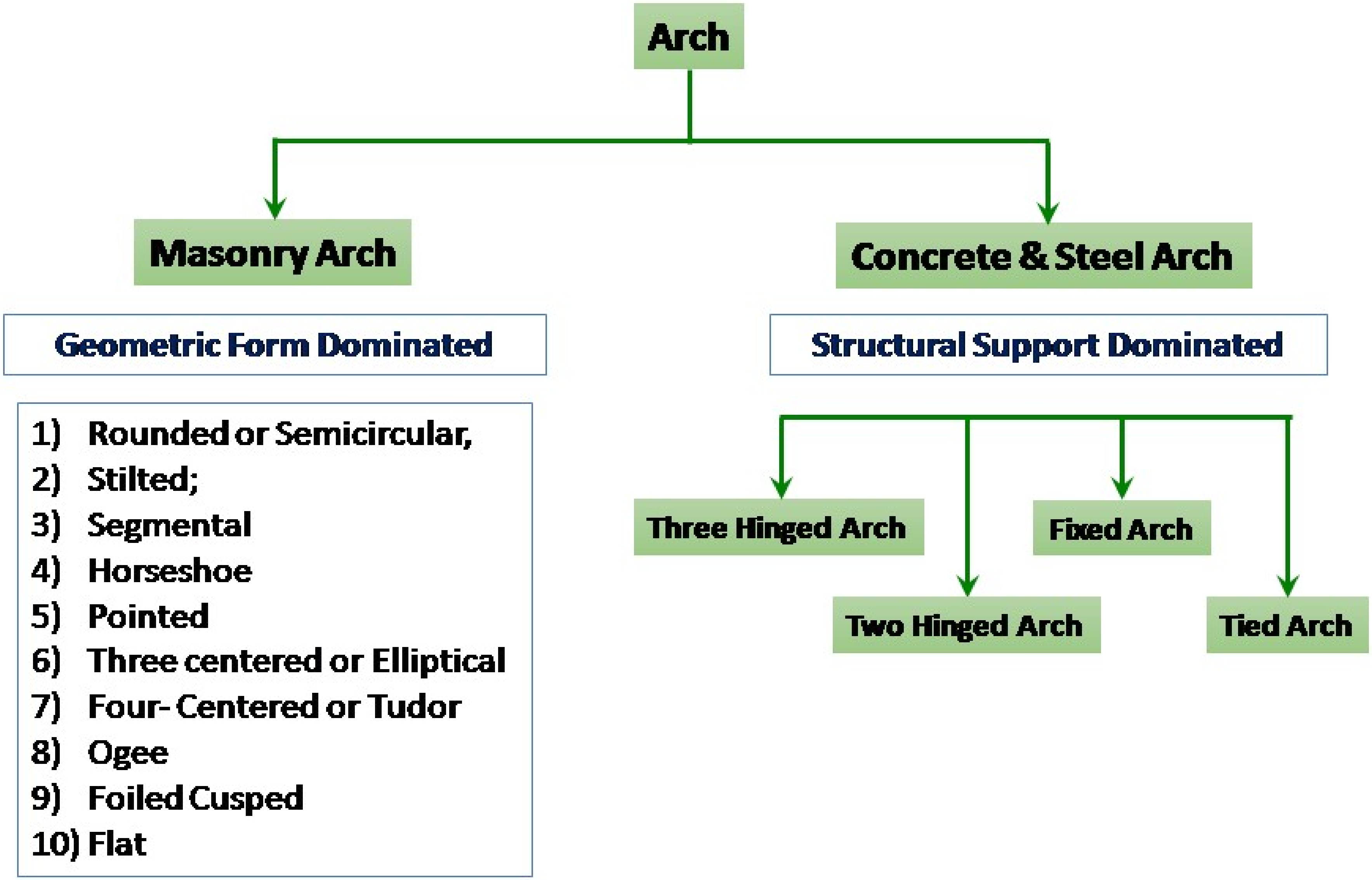


Finding shape



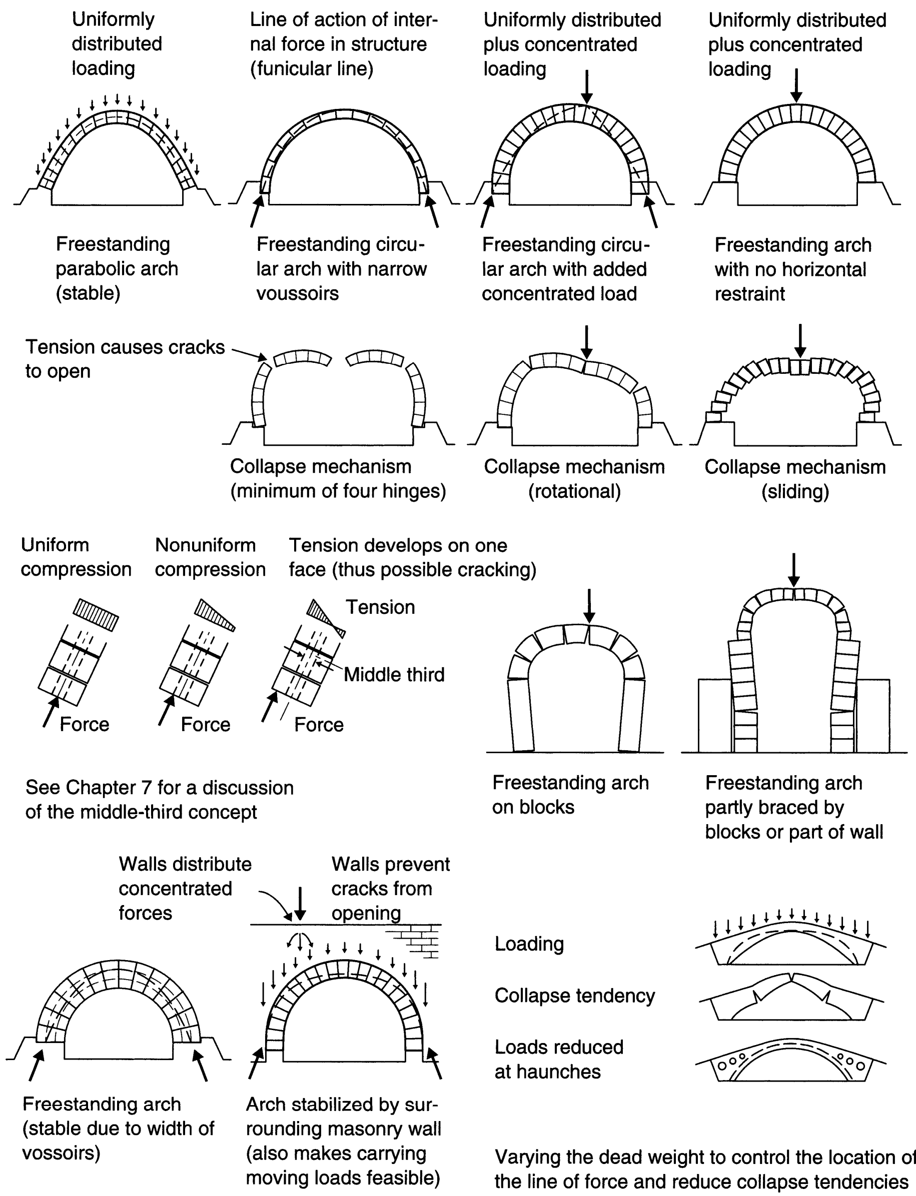
Arches

Arches Types



Masonry Arches

- Masonry arches derive their load-carrying capability from their curved geometry, which causes only compressive forces. However, the shapes of masonry arches rarely follow the funicular shaping. So some bending – tension can develop.
- Such issue is typically not problematic because of the large dimensions of the blocks in common masonry arches which enabled the funicular line to be contained within the middle third of the masonry blocks.



Design of Arch Structures

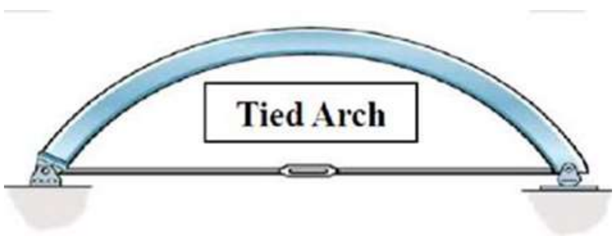
- Rigid Arches. With the advent of steel and reinforced concrete, the member rigidity made possible with the new materials gave the arch a more versatile form because that rigidity allowed the arch to maintain its basic shape and prevented collapse when unanticipated loads occurred.
- A modern rigid arch is often shaped in response to a primary loading condition and carries loads in axial compression when such loads are present. However, it is also designed to have sufficient bending resistance to carry load variations.

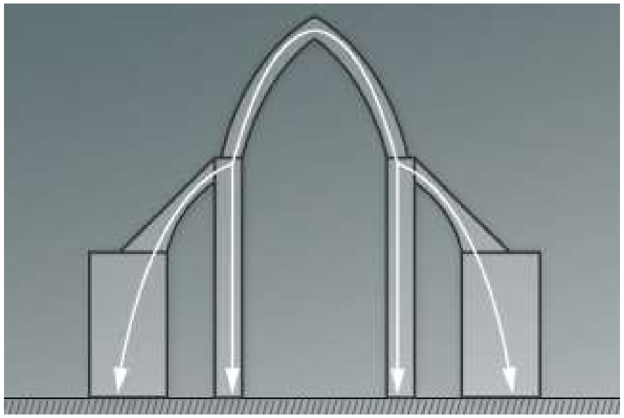
Padre Pio Pilgrimage Church , S. Giovanni Rotondo (Foggia), Italy

Design of Arch Structures

Support elements

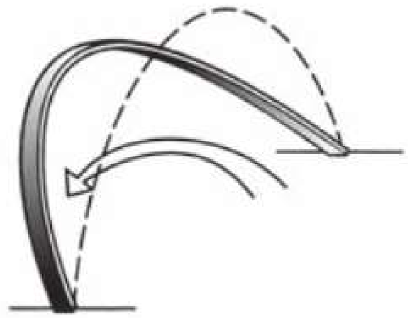
- The arch foundation shall be designed to resist both the horizontal and the vertical component of the arch reaction.
- A Tie-Rod can be used to absorb horizontal thrusts in arches efficiently through tension. In this case, footing will be designed for vertical loads only, and the arch is called tied arch.
- If arches must be located on top of vertical elements, using buttressing elements is preferable to attempting to design the verticals to act as vertical beams that carry the horizontal thrusts by bending.



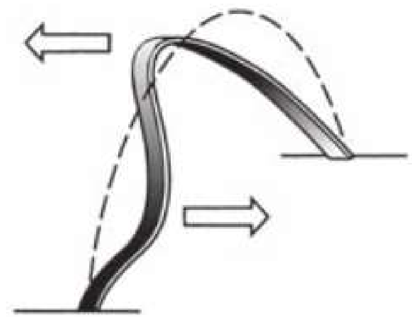


Design of Arch Structures

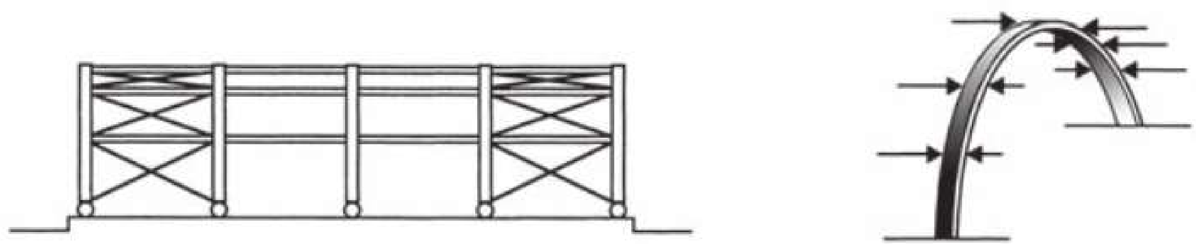
- Fixed arch with huge foundations may be needed.
- Alternative: Lateral bracing as in c.



(a) Lateral stability can be a problem.



(b) Lateral buckling due to high internal forces can be a problem.



(c) The lateral buckling problem can be solved by laterally bracing arches with other elements, such as those from the roof structure. Cross bracing or some other mechanism is needed to assure lateral stability.

Arches lateral Stability

Primary Types Of Arches

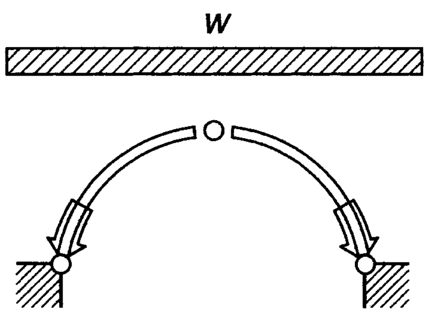
Three primary types of arches are normally described in terms of their end conditions:

- the three-hinged arch,
- the two-hinged arch, and
- the fixed-ended arch.

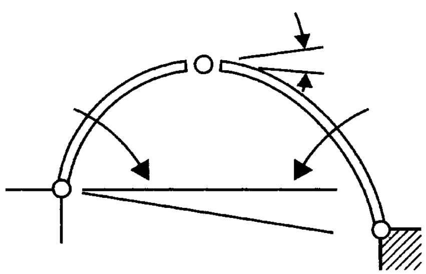
- If funicularly shaped, the primary internal forces in each type will be similar.

Primary Types Of Arches

Three-hinged arch



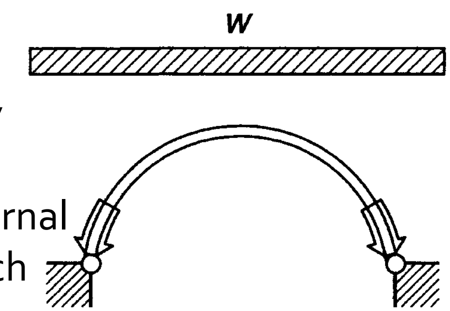
Primary forces



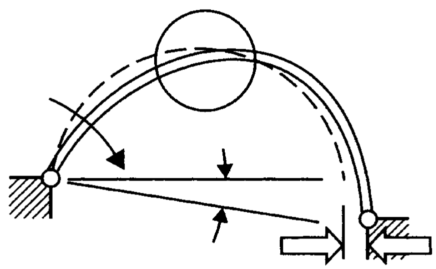
The three-hinged arch is relatively unaffected by support settlements since the two arch segments merely rotate with respect to one another (the hinges allow the structure to flex freely).

If funicularly shaped, the primary internal forces in each type will be similar.

Two-hinged arch

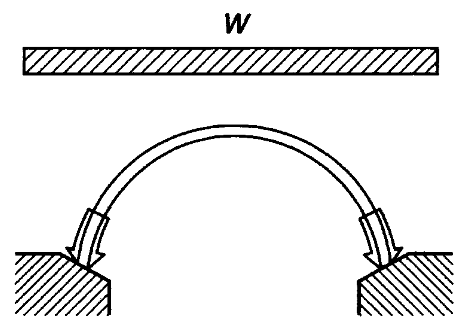


Primary forces

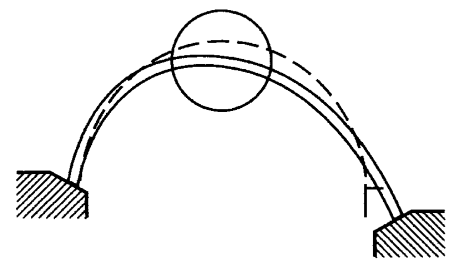


The two-hinged arch is relatively unaffected by vertical settlements since the hinges allow the structure to simply rotate as a unit. Horizontal spreading of the foundations, however, induces destructive bending at the crown of the arch.

Fixed-ended arch



Primary forces

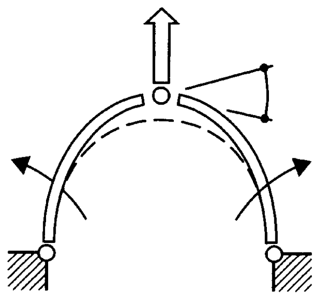


The fixed-ended arch is severely affected by any type of foundation settlement. The absence of hinges does not allow the structure to flex freely, and destructive bending moments are consequently induced in the structure.

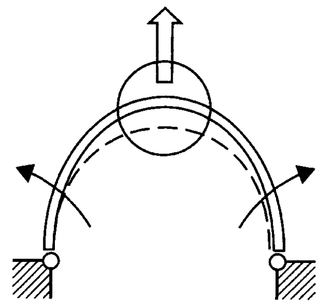
Support settlements. The three-hinged arch is least affected by support settlements, whereas the fixed-end arch is most affected.

Primary Types Of Arches

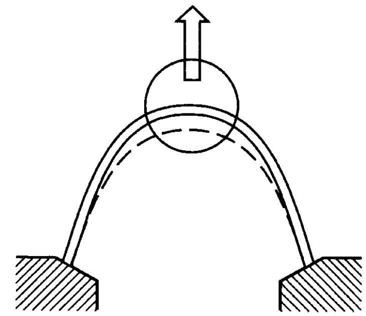
Expansions and contractions effect



The three hinges allow relative rotations to occur between members, which reduces the stresses associated with temperature expansions and contractions.

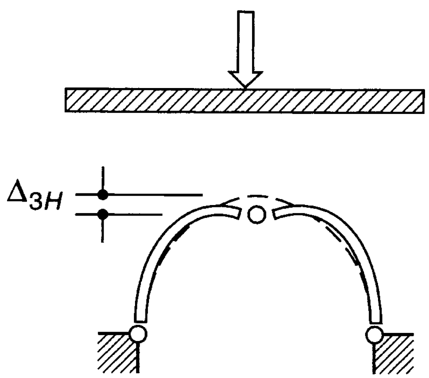


While unable to flex as freely as the three-hinged arch, the pins present in the two-hinged arch allow some flexing to occur, which reduces the effects of temperature movements. Some undesirable bending moments, however, are still generated in the structure.

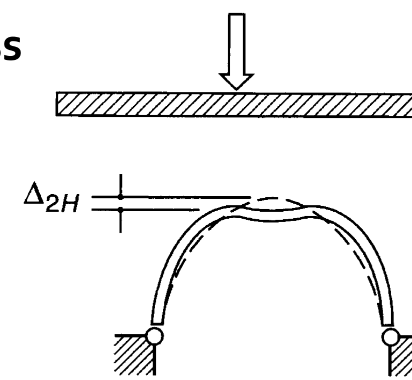


The fixed-ended arch is severely affected by temperature expansions and contractions since no relief mechanism is present. Undesirable bending moments are created.

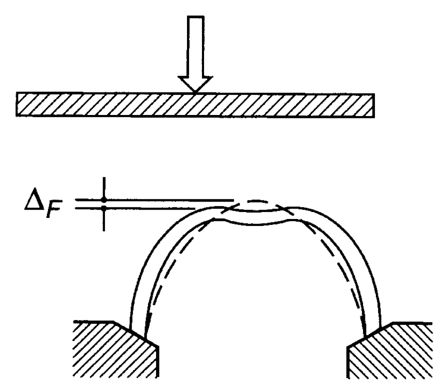
Stiffness



The presence of the hinges reduces the overall stiffness of the three-hinged arch and increases its sensitivity to deflections.



The two-hinged arch is stiffer than the three-hinged arch but less rigid than the fixed-ended arch.

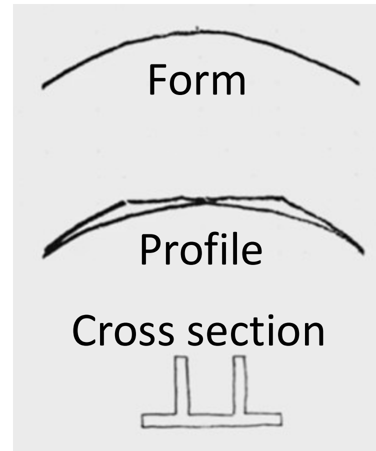


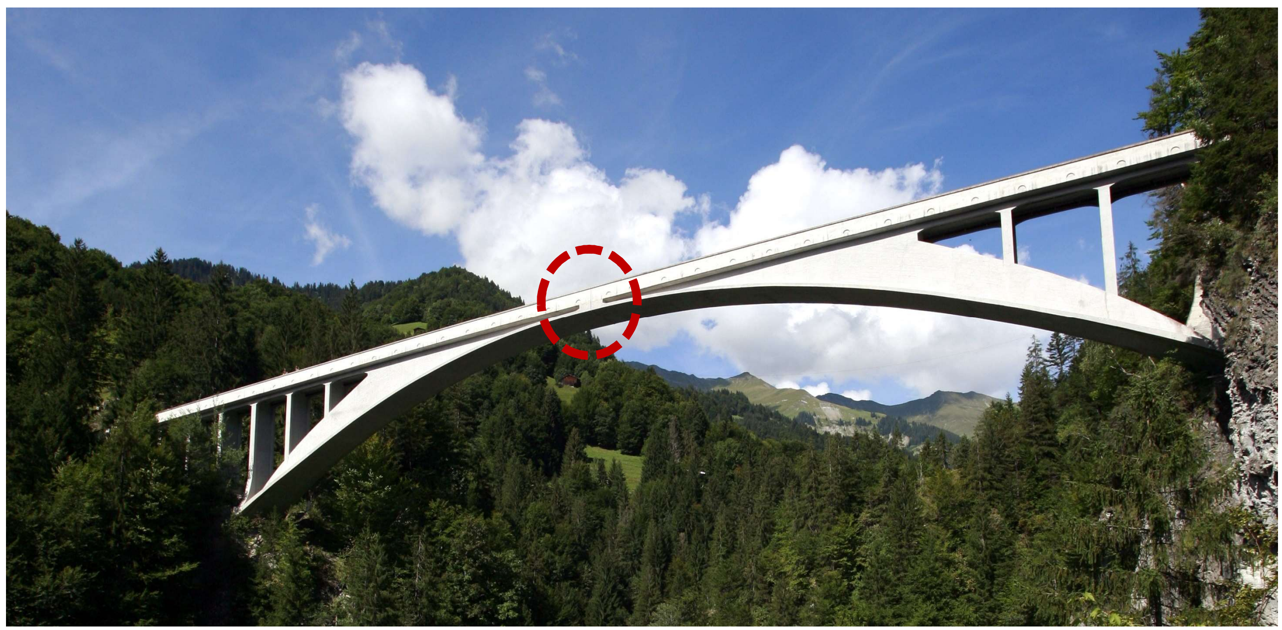
The fixed-ended arch is very stiff and highly resistant to deflections.

The Salginatobel Bridge

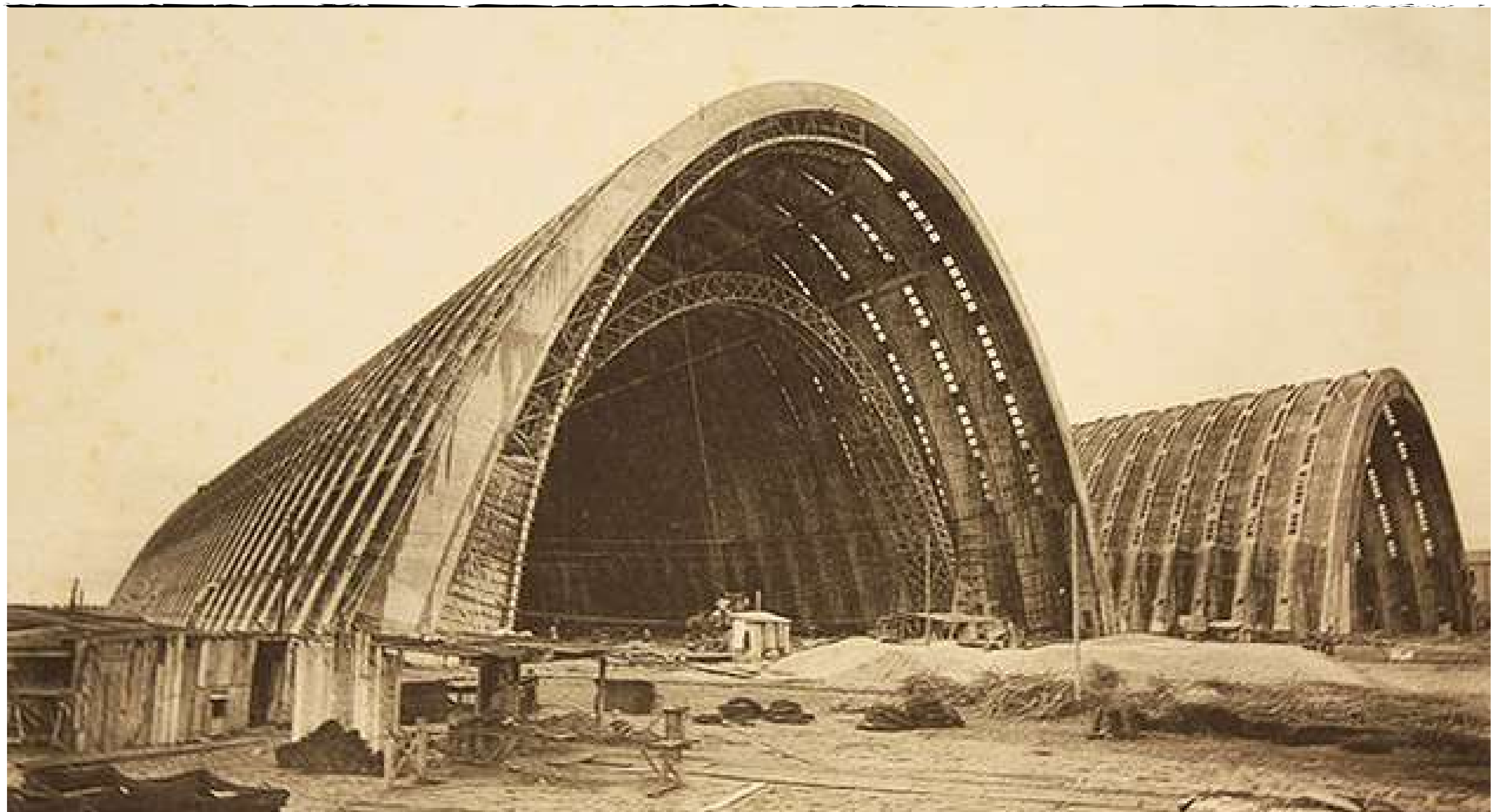
- **<u>Three-Hinged Arch Example. S</u>**panning the Salgina Valley ravine, is the earliest surviving three-hinged, hollow box arch bridge designed by Robert Maillart. The design provided the lowest cost of 19 designs submitted for the bridge's original design competition.

Efficiency Factors



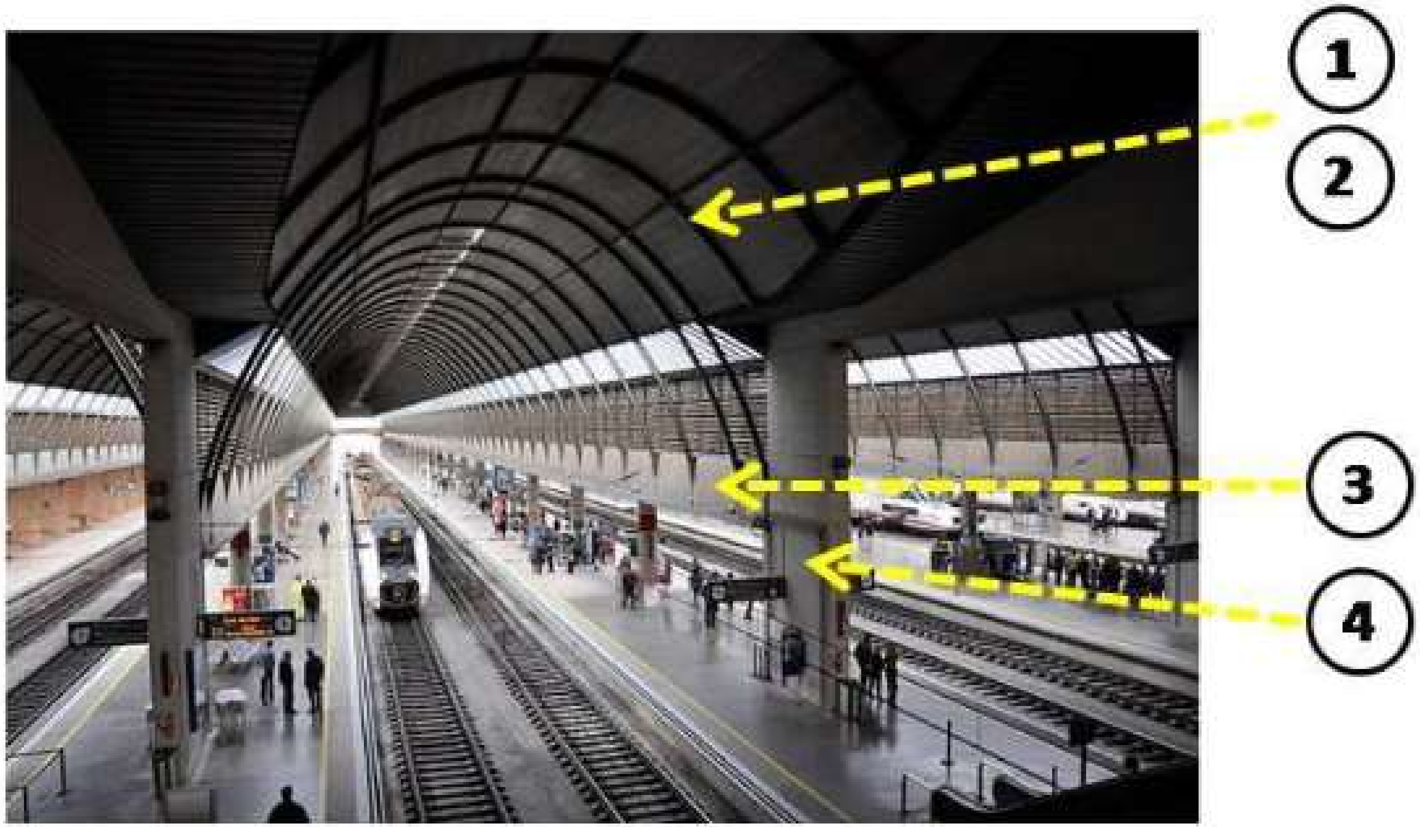


Airship hangars at Paris-Orly airport



The structure was formed from a series of parabolic fixed arches around 90 m span and 60 m high. It is the first folded reinforced concrete building, built in 1923.

Santa Justa Train Station, Seville, Spain



1. Parabolic arch profile is subjected to uniform load. 2. A funicular form minimizes the bending action. 3. Arches are supported by a deep side beam. 4. Beams are supported by circular mega columns.

Cable Structures

Historical Background

- Originally, the cable structures were developed to provide a bridging solution. The flexible suspension bridge, originating in ancient China, India, and South America, has a long history, with early bridges made from ropes, often bamboo, and later chains in China as early as the first century BCE.
- In addition to the varieties of tents that used ropes, a rope cable structure was used in about AD 70 to roof a Roman amphitheater.
- In the early 19th century, James Findley revolutionized suspension bridge design in America by introducing a stiffened deck to carry vehicular traffic, which prevented changes in the cable shape by distributing concentrated vehicular loads over a larger portion of the cable.

Historical Background

- Cable structures in buildings advanced more slowly, with early examples like the 1853 Crystal Palace proposal and later developments.
- The longest modern suspension bridge, the Akashi Kaikyo Bridge in Japan, spans 1991 m.

The Bridge is 3,911 m long and has three spans. The central span is 1,991 m long, and each of the two side spans measures 960 m. The two main supporting towers stand 297 m above the strait's surface, making it one of the tallest bridges in the world.

Modern Cable Structures

(1) suspension bridge, Highway Bridge Khor al Batah, Oman; (2) cable-stayed bridge, Evripos Bridge, Greece; (3) cable truss roof, Annex Lutherhaus Roof, Germany; (4) cable suspension roof, Glass Canopy of Station Plaza Heilbronn, Germany; (5) cable net facade, Façade Airport Málaga, Spain; (6) saddle-shaped cable net, Canopy in Autostadt Wolfsburg, Germany; (7) cable net tower, Dry Cooling Tower in Schmehausen near Hamm, Germany; (8) arch tower cable supported roof, Moses Mabhida Stadium, South Africa; and (9) spoked wheel cable roof, Bay Arena Leverkusen, Germany

Cable and cable structure

- **Cable** means any structural element whose diameter is very small in relation to its length. Cables make use of the tensile strength of material and can support incredible forces for their size. However, they have no strength in compression and are used almost exclusively in suspension structures.
- **A cable structure** can be defined as a structure in which a cable or cable system is used as a structural element to withstand the initial load. They are some of the most beautiful and recognized structures in engineering.
- Cable structures are commonly used for relatively long spans and in situations that allow the structure to be shaped with a significant depth. The ability to design **large spans** with **little self-weight** and high **load capacities** and explore **new architectural expressions** made cable structures very popular.

Cable Characteristics

- The structural behavior of the cable is unique and unconventional compared to common structures. Unlike the common design, where the strength of a member is derived from its section and material properties, the cable has an additional variable that influences its capacity: its geometry.
- Because a cable has negligible bending strength, its deformation from loads follow its funicular curve. The cable always adopt its shape so that it is in pure tension under the action of the applied load.
- The above implies that when the load configuration changes, the shape of the cable must change. In addition, the strain of the cable, as well as movements at its support, influence its shape and thus its load carrying capacity significantly.

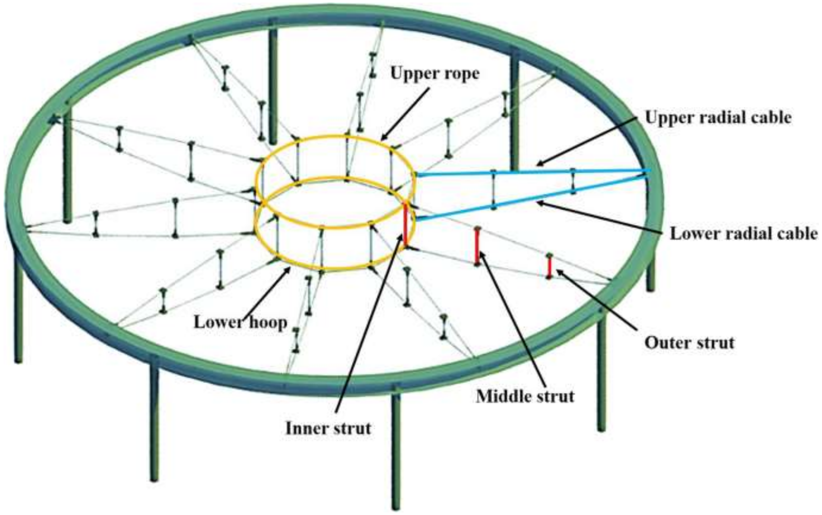
The System Supporting Elements

- A cable structure's overall behavior, appearance, and economy may depend as much on its supporting elements as on the cable system itself.
- A key concern is the reaction forces at the points where the cables connect to the supporting structure, particularly the horizontal components of these reactions.
- These forces can be managed internally within the structural system or through properly designed tiebacks and large foundations.
- Resolving horizontal forces within the structure often involves the use of compression struts or rings. As with all components subject to compression, these elements must be designed to prevent buckling.

Types of cable structure

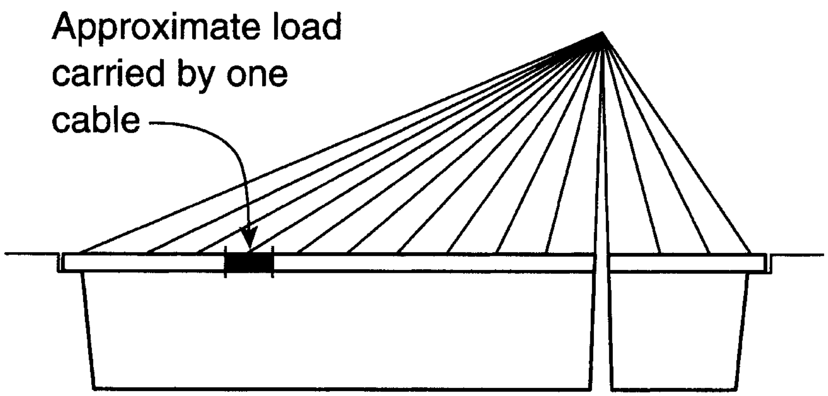
Cable structures are categorized into:

- Cable-Stayed Structures.
- Suspended cables and systems.
  - Singly-Curved Structures
  - Double-Curvature Structures

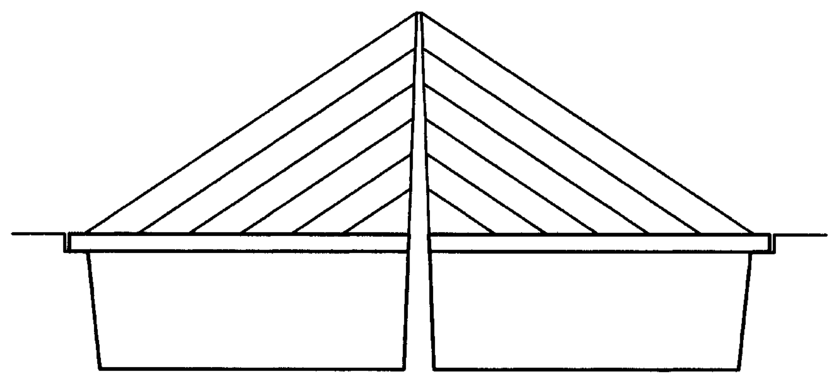


Cable-Stayed Structures

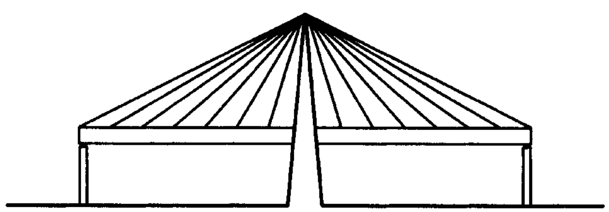
- Cable-stayed structures consist of towers or **masts** from which **cables extend to support horizontally spanning members**.



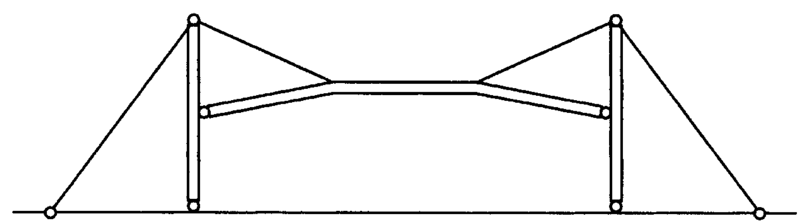
(a) Cable-stayed bridge with asymmetrical mast and radiating cables



(b) Cable-stayed bridge with central mast and parallel cables



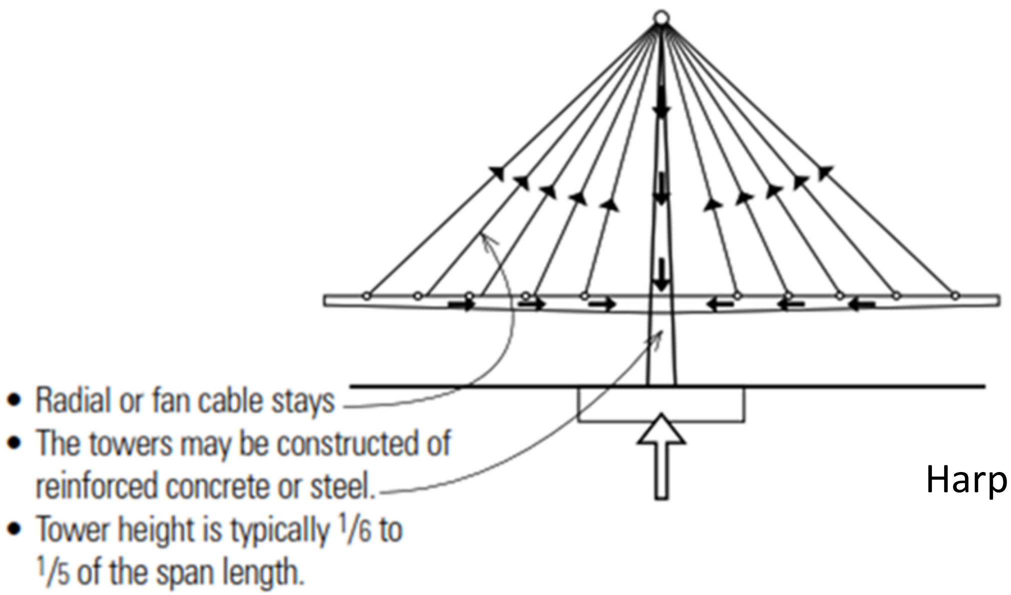
(c) Cable-stayed roof structure with central mast and non-load-bearing side enclosure walls

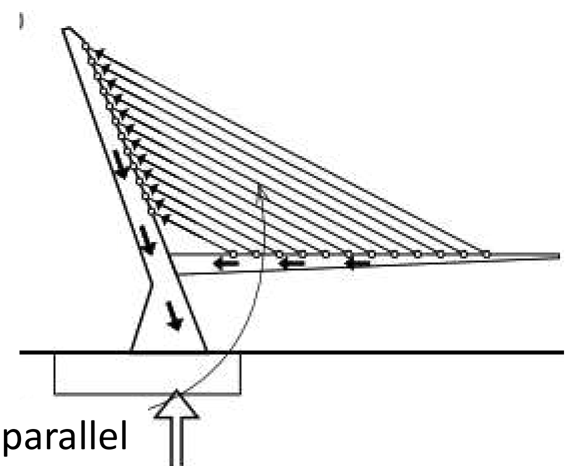


(d) Cable-stayed rigid roof with stabilizing tieback cables

Cable-Stayed Structures

- There are two primary cable configurations: **radial or fan** patterns and **parallel or harp systems**. Radial systems attach the upper ends of the cable stays to a single point at the top of the tower, while parallel systems secure the upper ends of the cable to the mast at different heights.





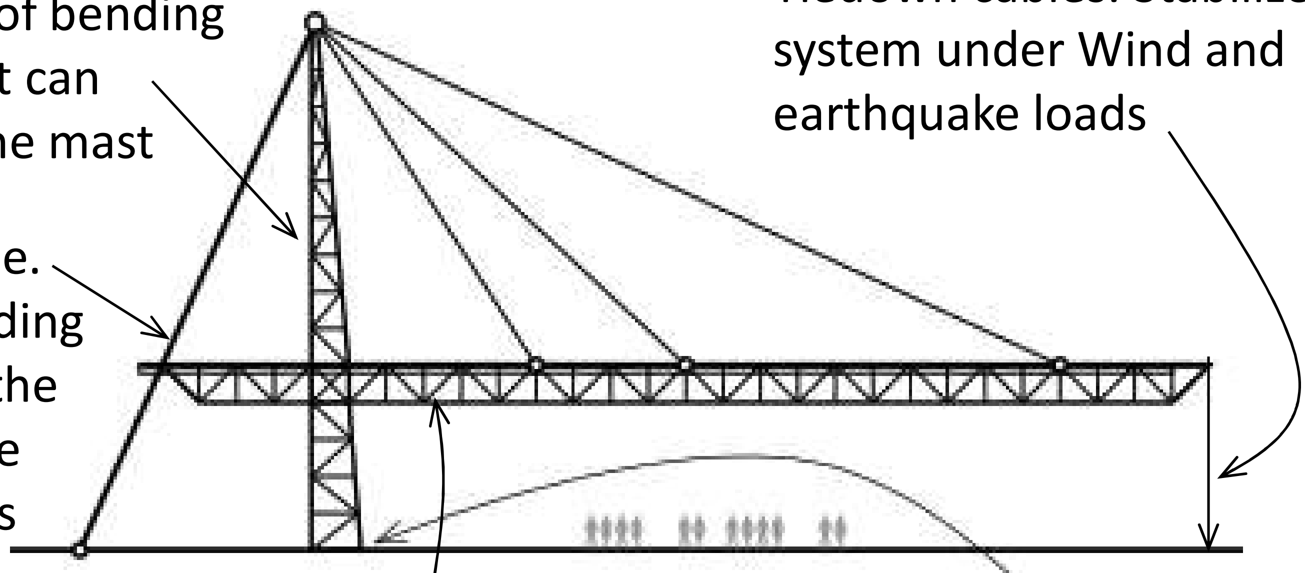
Harp system of parallel cable stays

System's Elements Functions

Inclined mast. Reduce the amount of bending moment that can develop in the mast

Tiedown cables. Stabilize the system under Wind and earthquake loads

Tieback cable. Reduce bending moment in the mast and the footing loads

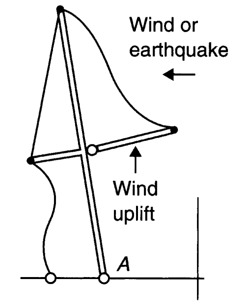


Footing. Large gravity loads and possible overturning moments may require a substantial foundation.

Primary spanning element. External loading is shared between the cable system and the element. The number of cables depends on the size and stiffness of the spanning element.

Cable-Stayed Design

- The system shall be designed to have the necessary stability under all anticipated types of loading conditions. In this respect wind and earthquake loads may cause instability. Tiedown cables at the ends could remedy the latter problem but are spatially awkward.

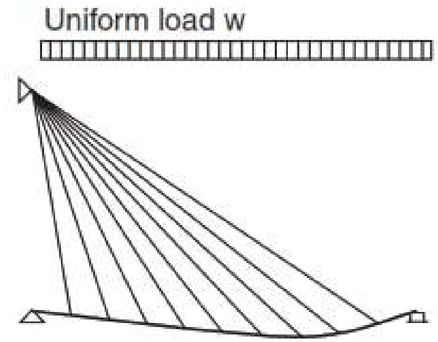


Possible instabilities due to wind or earthquake forces

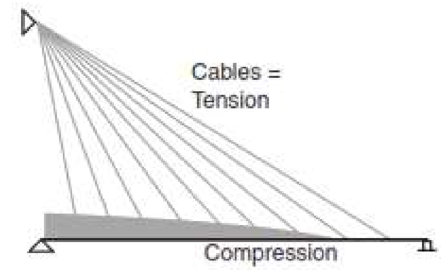
- Cables are invariably in tension. Determining cable forces can be simple or complex, depending on the number of cables in the system and the relative stiffness of the beam or truss. A useful first approximation for designing the cable-and-mast system is to ignore the stiffness of the beam or truss and to assume that the cable system carries all the loads. Accordingly, cable load can be estimated based on its tributary area.
- Forces in masts and tiebacks that stabilize masts may be determined readily through basic equilibrium considerations. Masts can become quite tall in cable-stayed structures and pose their design difficulties due to buckling phenomena.

Cable-Stayed Design

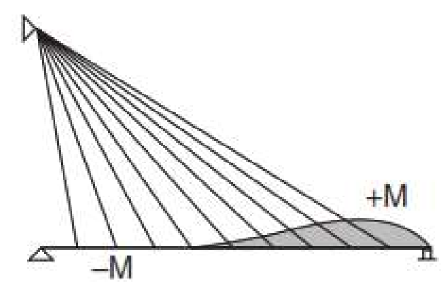
- The angles formed by the supporting cable to the suspended load are critical. Avoid shallow angles because attached cables would not provide the stiff support necessary to the supported beam, and forces developed in the cables would become extremely high.
- Supported beams or trusses are designed like continuous beams over several supports (in this case, the cable attachments). It must be sufficiently stiff to transfer or resist the lateral and torsional stresses induced by wind, unbalanced live loads, and the normal force created by the upward pull of the stays.



(a) Deflections (shown exaggerated): Cables that attach to the deck at acute angles provide little support.



(b) Axial forces: Compression builds up in the deck toward the support on the left.



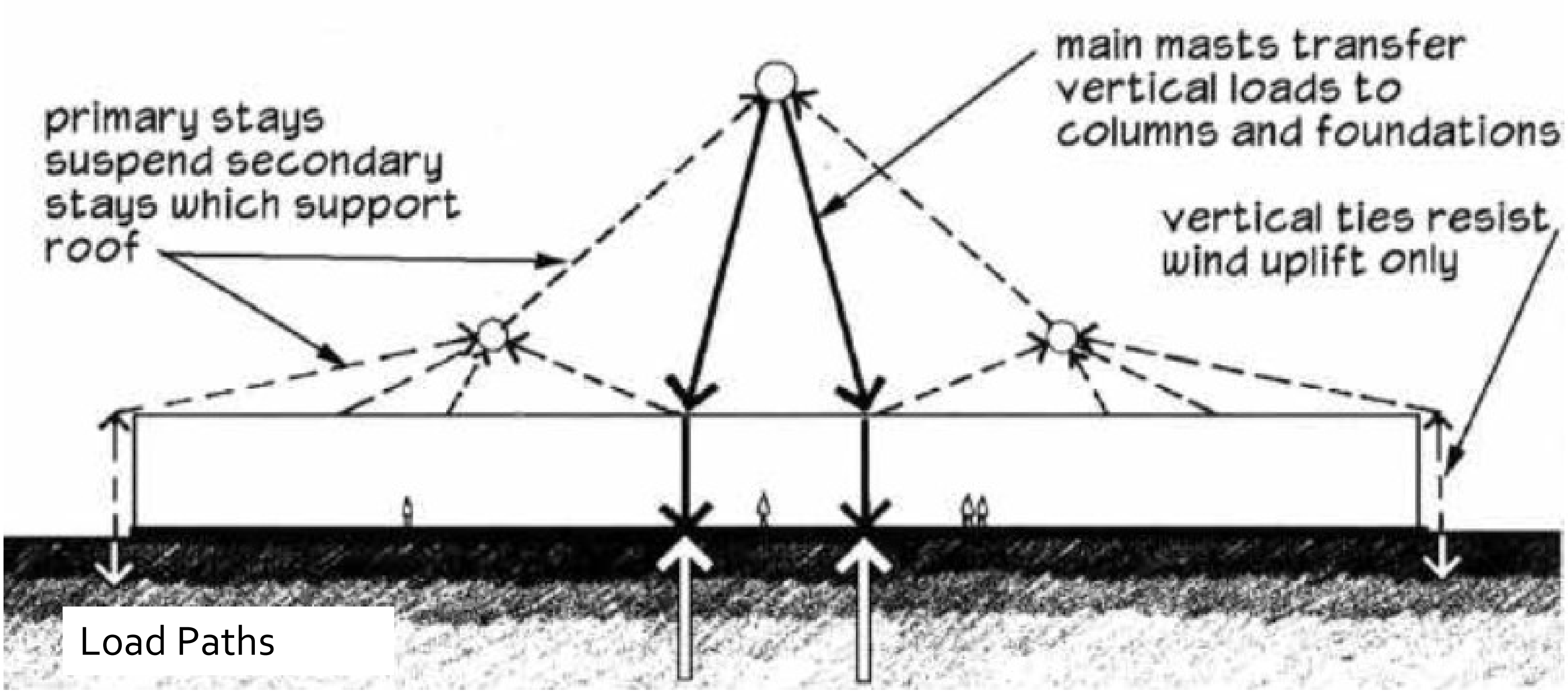
(c) Bending moments: Bending moments in the deck are greatest where curvatures induced through large deflections are present.

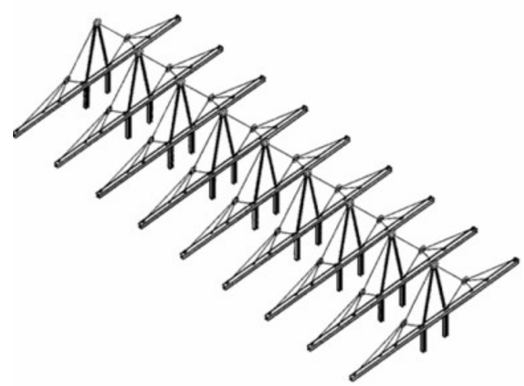
Cable-Stayed Design

- The cable stays are usually attached symmetrically to a single tower or mast with an equal number of stays on both sides so that the horizontal force component of the inclined cables will cancel each other out and minimize the moment at the top of the tower or mast.

- Guadiana International Bridge
- Cable-stayed bridge with semi-fan system
- Material: Prestressed concrete
- Main span = 324 m
- Total length = 666.00 m
- Height above water = 20 m
- Pylon height = 95-96 m
- Deck depth = 2.50 m
- Deck width = 18.04 m

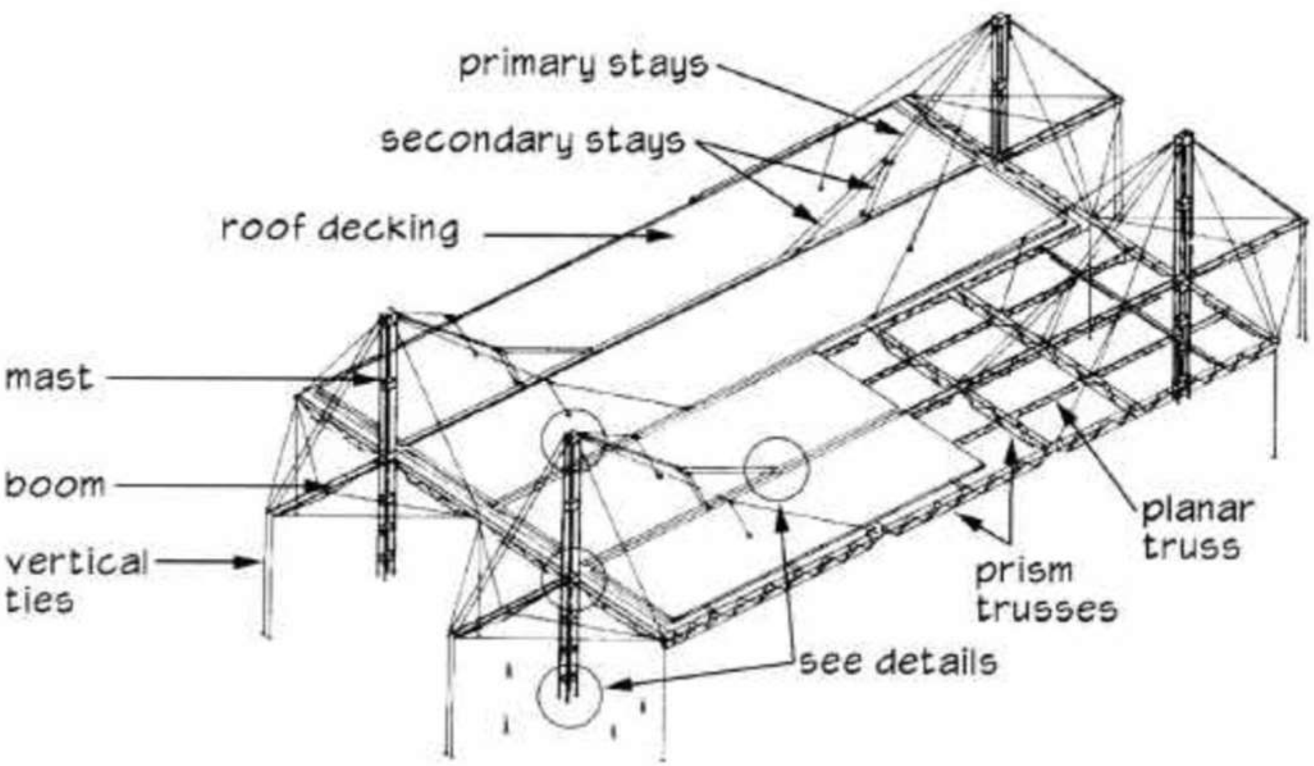
Patscenter, Princeton, NJ, USA

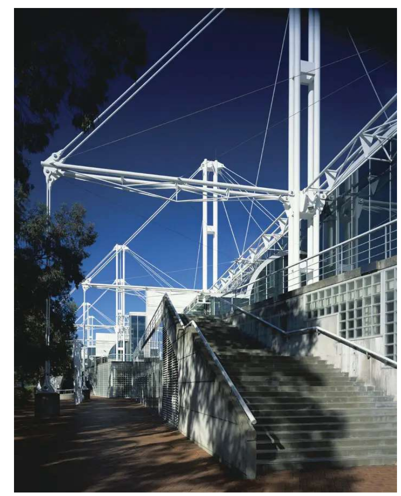




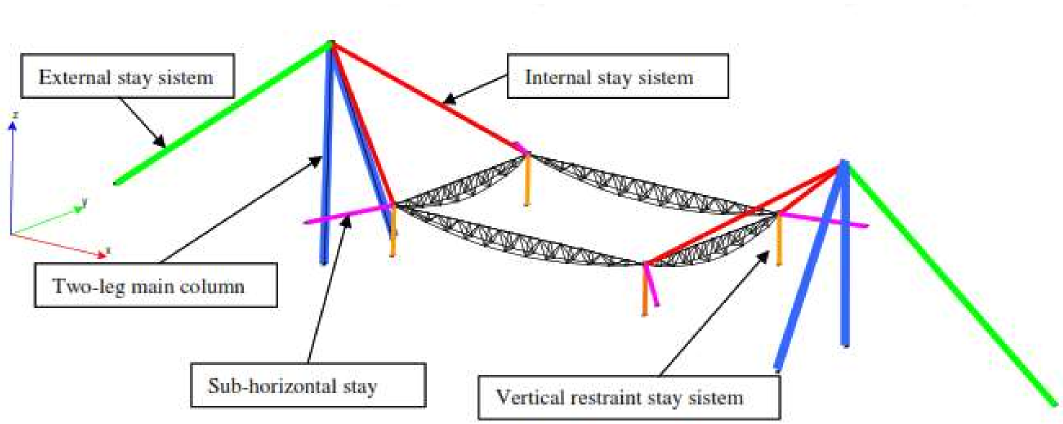
Darling Harbor Exhibition Centre

Darling Harbor Exhibition Centre 100-meter span



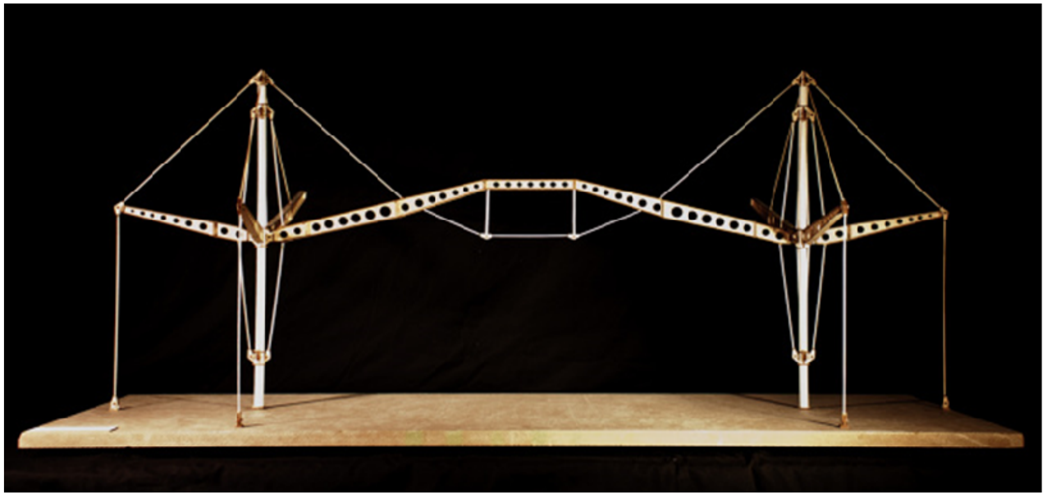


Juventus Stadium



playing field	width	68 m
	length	105 m

Cable-Stayed Structures - Examples



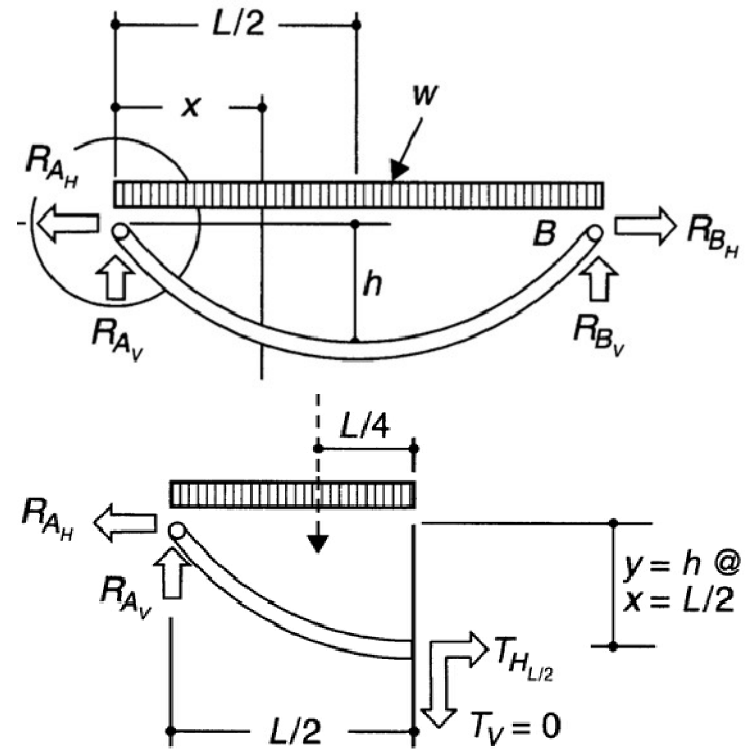

Simple Suspension cables

- **Cable Sags.** The primary design challenge for cable structures is determining the appropriate sag-to-span ratio, as this affects the cable's forces, length, and diameter. This ratio also influences the forces in supporting masts or tieback cables, impacting the size of these structural elements.

- The horizontal component of the force in a uniformly loaded level-ended cable is given by

  $T_H = R_{A_H} = {wL^2}/{8h_{max}}$, where h_{max} is the maximum sag.

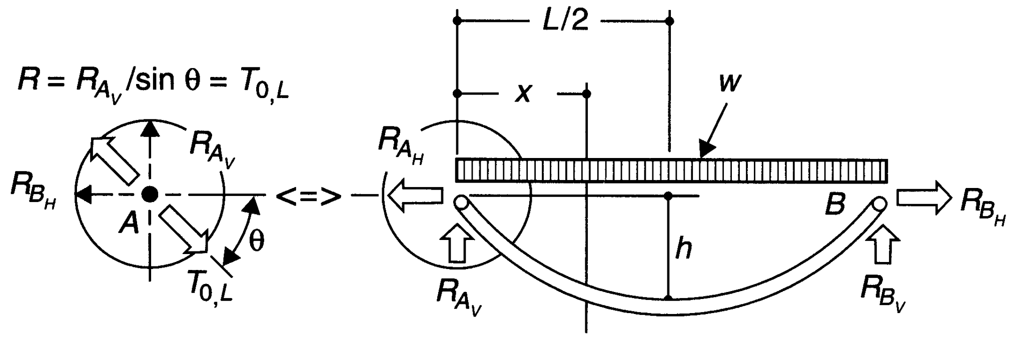
- Doubling the maximum sag decreases T_H by a factor of 2. In general, as the sag h_{max} is decrease, the cable force increase.



Simple Suspension cables

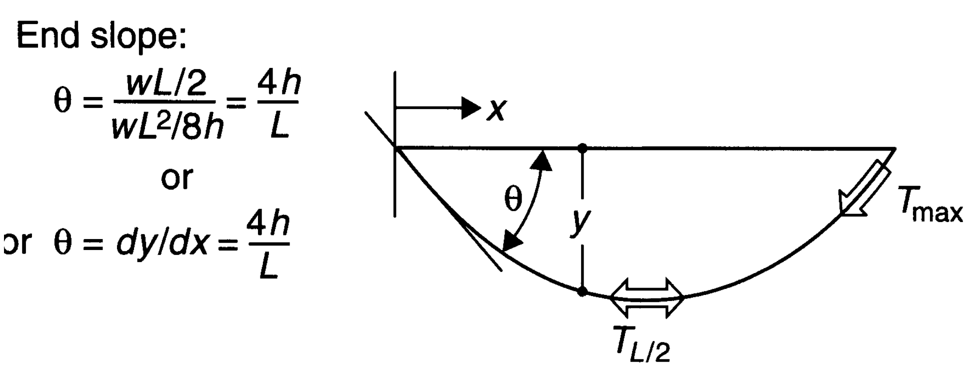
- Shape of cable:

$$y = 4h(Lx - x^2)/L^2$$



- Cable forces: maximum force in the cable occur at the support ($T_{0,L}$).

$$T_{0.L} = \left({wL^2}/{8h_{max}} \right) \sqrt{1 + {16h_{max}^2}/{L^2}}$$



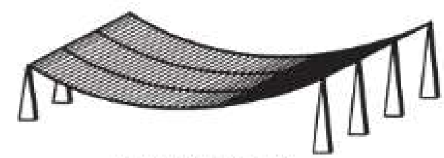
Simple Suspension cables

- **Typical sag to span ratio**. The ideal sag-to-span ratio for uniformly loaded level-ended cable is 1:3. The exact sag or rise chosen, however, invariably depends on the overall context in which the cable is used (including the design of supporting masts). Most cable structures used in buildings have sag-span ratios between 1:8 and 1:10.

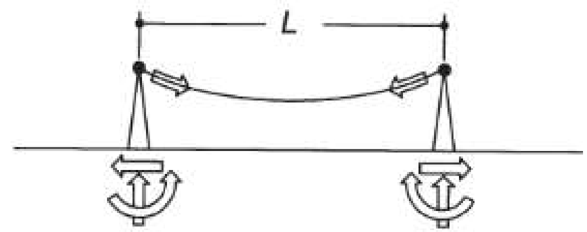
Simple Suspension cables

Supporting elements. Several common supporting elements can be used in cable structures, and the chosen cable sag directly affects the length and forces on piers or masts. A deeper sag requires more substantial support members, as longer columns are more prone to buckling and need stronger control.

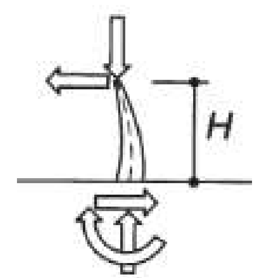
a) Pier Supports. The vertical piers supporting the ends of the cables carry the vertical components of the cable reactions by axial compression and the horizontal component by bending. This system is good for cables of relatively short span. Note that the foundation must prevent the piers from overturning.



Pier supports



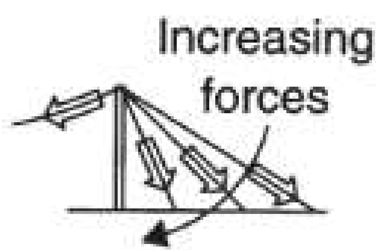
The greater the span L or the height H, the larger must be the supporting piers.

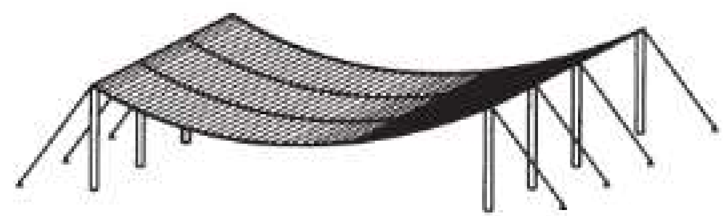


Simple Suspension cables

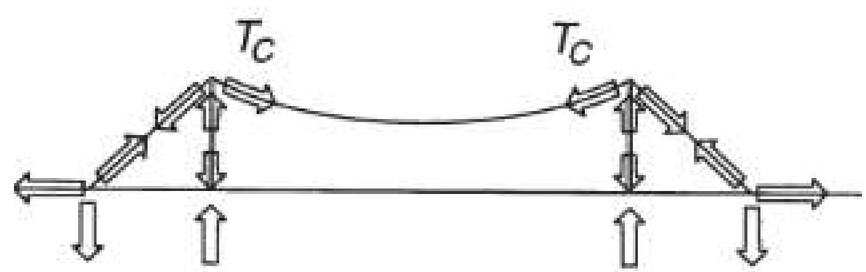
Supporting elements

b) Guyed mast Supports. The horizontal components of the cable thrusts are absorbed by diagonal guy cables and transferred to the ground. The vertical masts act in axial compression. This system is good for relatively long-span cables. The masts carry compression forces and the Guy cable foundation must prevent guy uplift and sliding.





Guyed mast supports

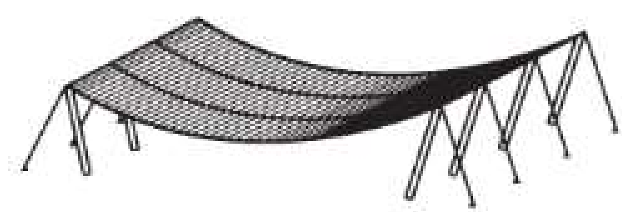


As the guy cable angle changes as shown, the forces in the guy cable tend to increase. Mast forces also increase accordingly.

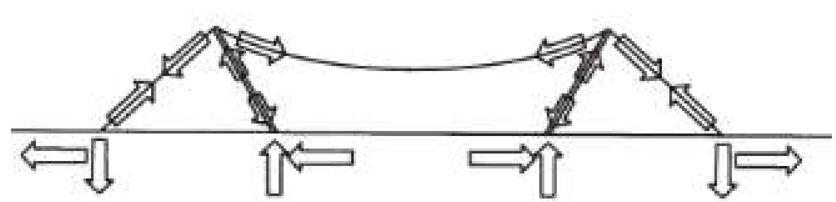
Simple Suspension cables

Supporting elements

c) Inclined Guyed mast. Inclining the masts causes them to pick up some of the horizontal cable thrusts, thus reducing the forces in the diagonal guy cables. This system is good for long-span cables. The masts carry compression forces and the Guy cable foundation must prevent guy uplift and sliding.



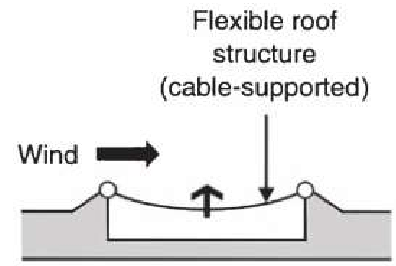
Inclined guyed masts



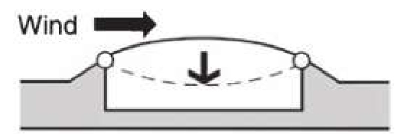
Simple Suspension cables

Wind effects

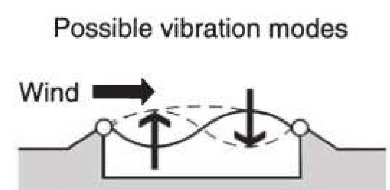
- Simply suspended cable systems are particularly sensitive to vibratory wind effects which have destroyed major cable structures in the past.
- Special precautions must be taken to prevent wind-induced instability. For example, cable tensions, which affect a cable's natural period of vibration and hence its susceptibility to vibratory phenomena, can be controlled. Various tiedowns or stiffening devices can be used as well.



(a) Winds blowing over roof surface in its naturally deflected shape cause suction forces to develop. These suction forces cause the flexible roof to begin rising.



(b) As the roof changes shape due to the suction forces, the effect of the wind on the new shape becomes one of pressure rather than suction. This causes the roof to move downward again.

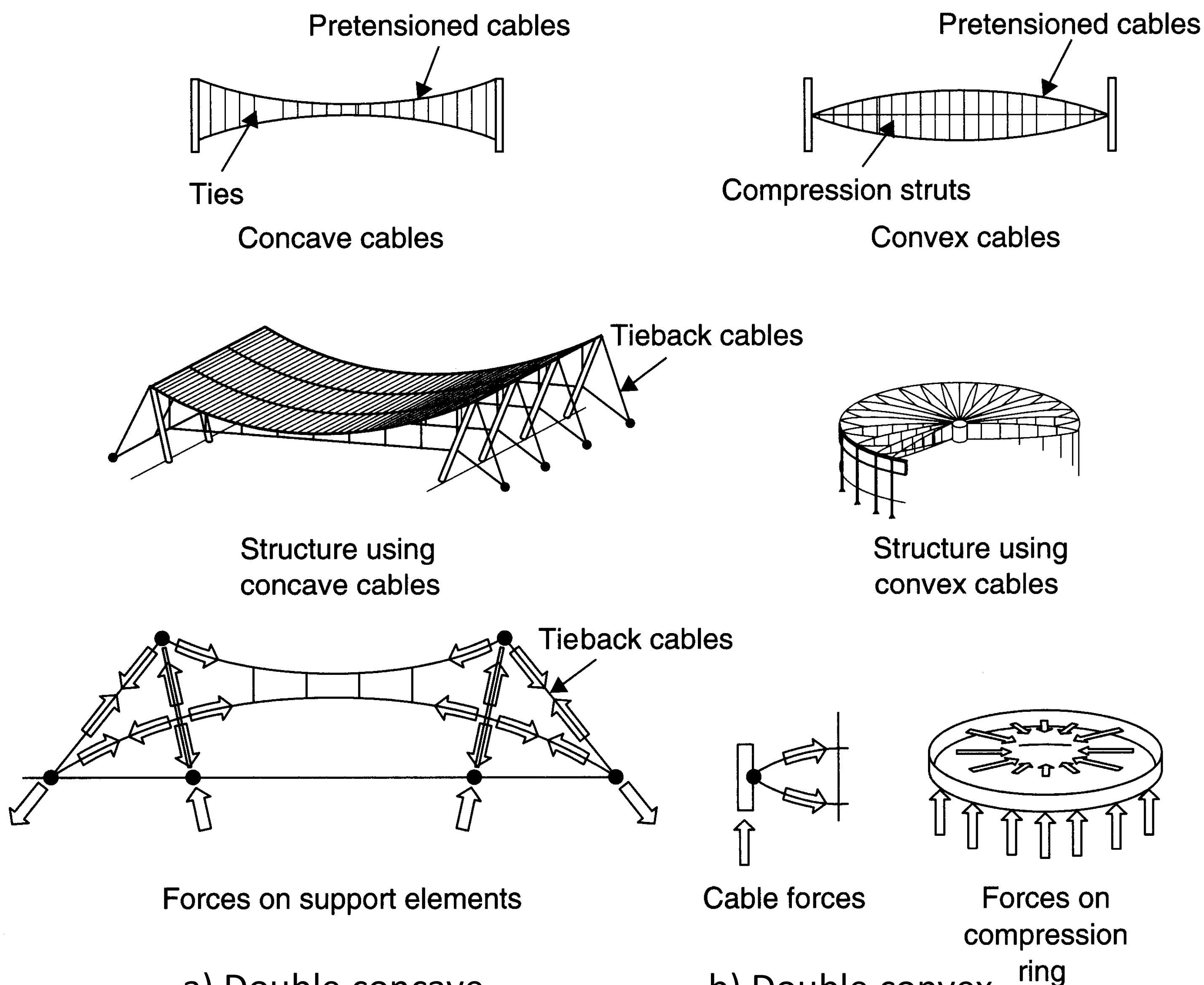


(c) As the roof moves up and down, the effect of the wind alternately produces suctions and pressures causing further movements. A constant fluttering of the roof results.

Double-Cable Systems

- A double-cable structure typically consists of two interconnected, pretensioned cables, along with associated struts or ties, that work together to bear external forces.
- There are two common types of this system double- concave and double convex system as illustrated in the Figure (next slide).
- In the double-concave system, pre-tensioning is achieved by stretching the tieback cables. In the convex system, the upper and lower cables are internally pre-tensioned.
- Double-cable systems provide an effective design solution for addressing the challenge of wind-induced vibration flutter, a common issue with simple suspended cable systems.
- In both structures, the external loads and pre-tensioning forces generate significant horizontal forces on the supporting elements. A series of tieback cables is used to manage these forces. In the structure shown in Figure (b), a large continuous compression ring, typically made of steel, absorbs the horizontal forces, allowing the columns to carry only the vertical loads.

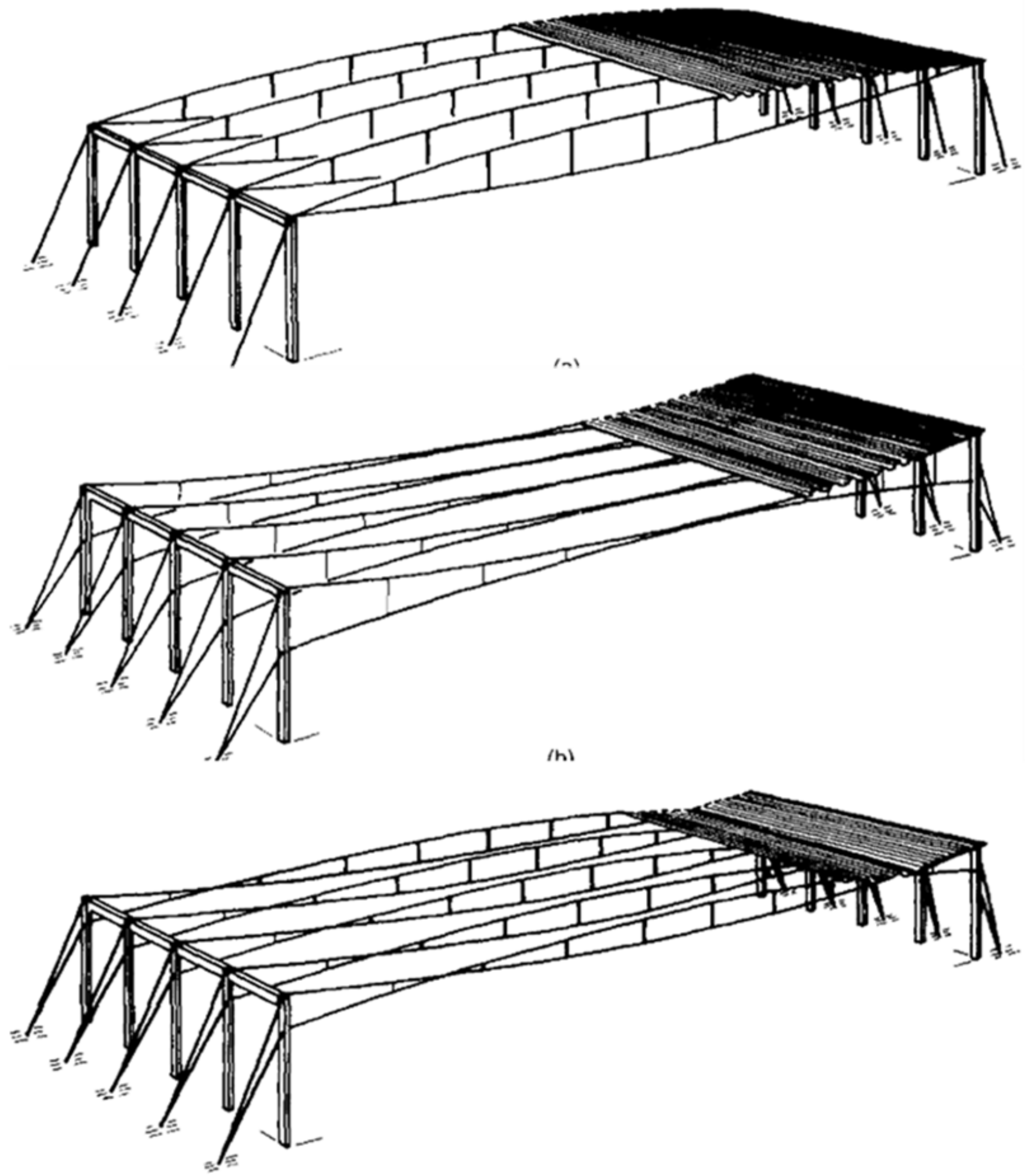
Double-Cable Systems



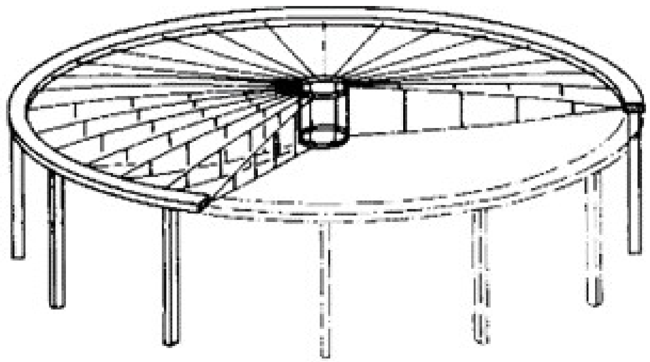
a) Double concave b) Double convex

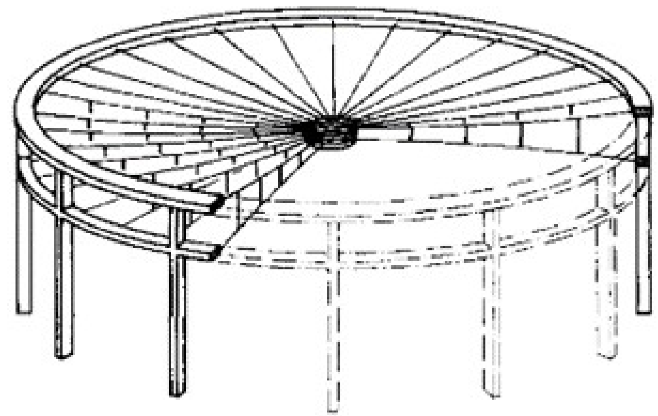
Double-Cable Systems

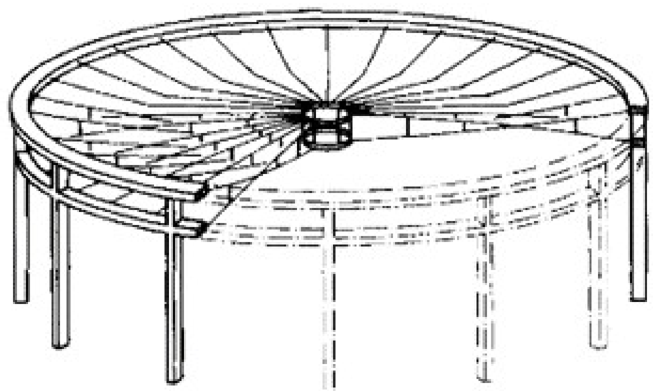
When the cables are tensioned to a level that ensures that both cables remain in tension under any combination of applied loading, the resulting systems or cable beams will be pretty stiff and stable.



Double-Cable Systems


(a)

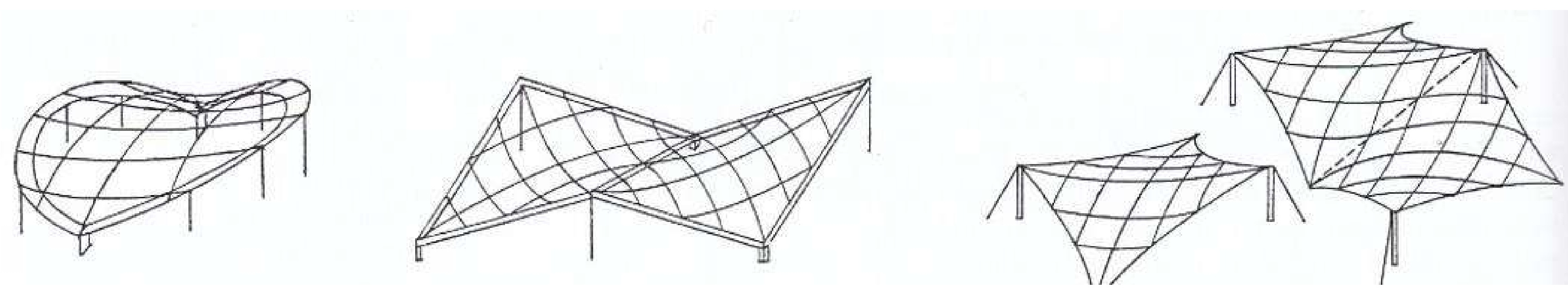

(b)


(c)

Radial convex cable beam structure with inner tension rings and an outer compression ring

Pretensioned cable net structures

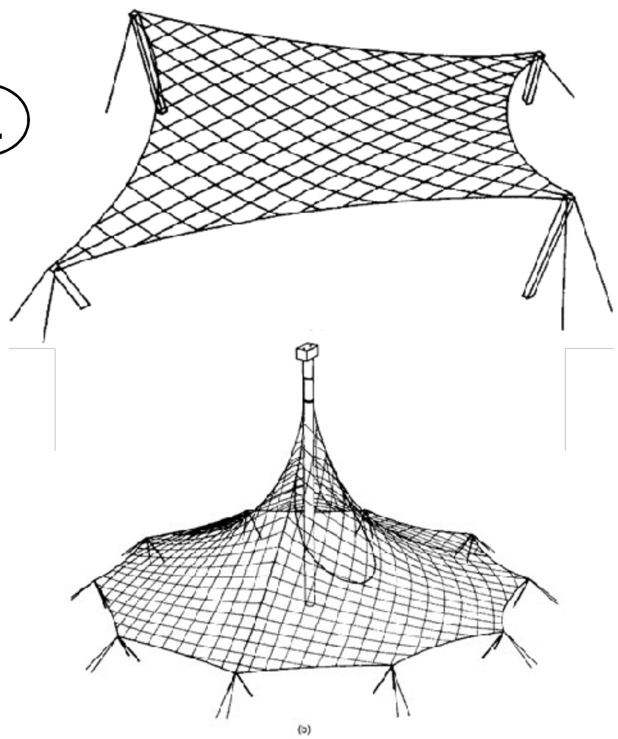
The suspension and pretension cables all lie in one surface and form a large net. As with cable beams, for a net to be stiff the cables must be in tension and it follows that the geometry of a net must be such that all the surface is anticlastic or saddle-shaped.



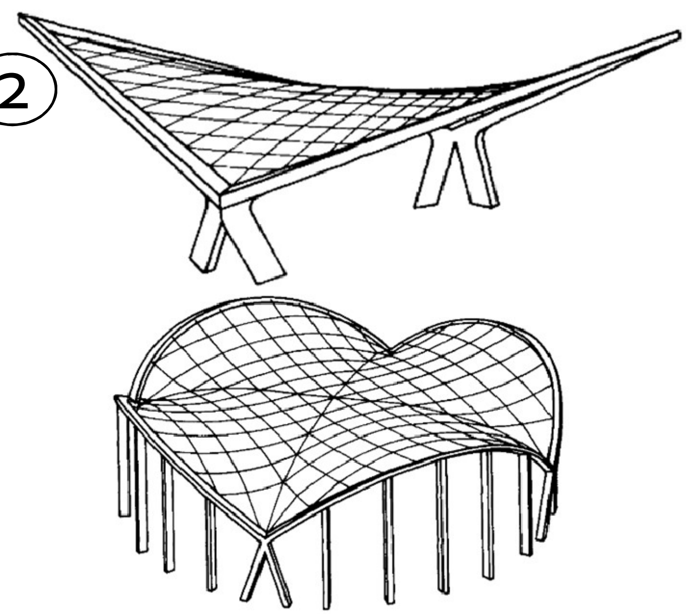
Pretensioned cable net structures

The geometries of cable nets are functions of their points of support and the tensions in the cables. They may be designed to resemble tent-like structures with masts and edge cables as shown in 1, or with stiff boundary members such as beams, arches and space rings as in 2.

1



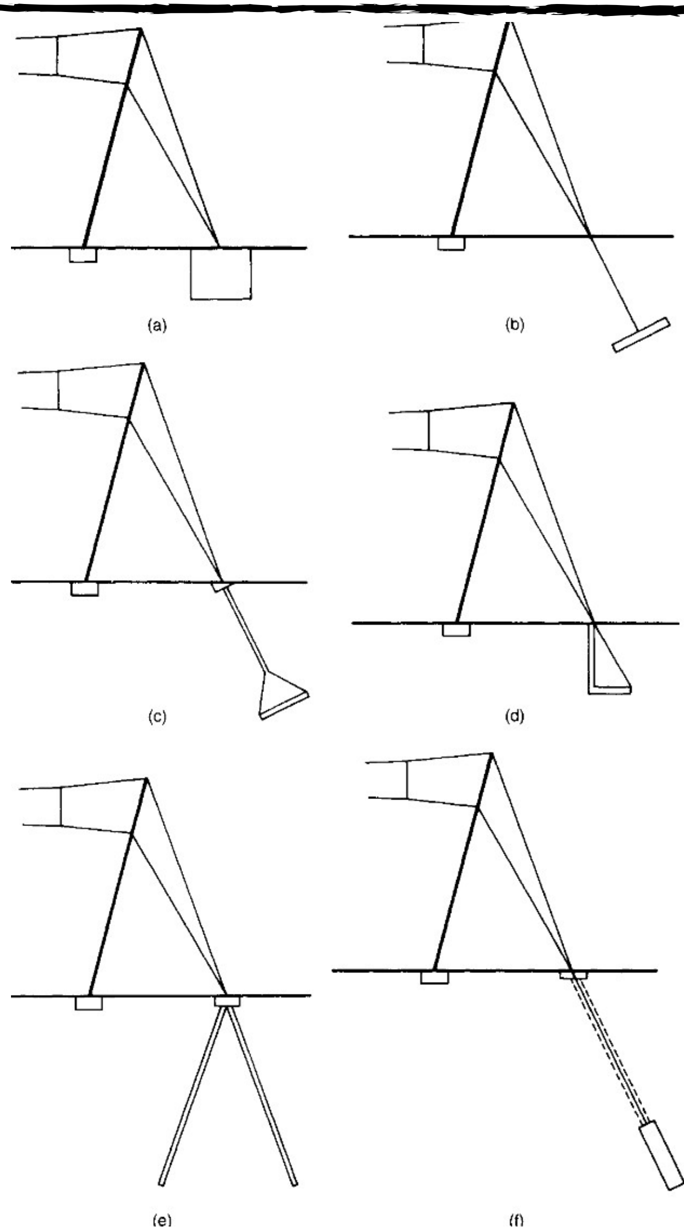
2



Cables Types

- Spiral strands (up): single-strand ropes which are composed of layers of round wires built up around a central king wire until the required strength is achieved.
- Locked coil type (down) which employs one or more layers of interlocking wires on the outside of the rope to produce a smooth external finish.
- Both these types of rope are capable of operating satisfactorily in structures, and the choice between the two is largely a matter of the personal preference of the designer.
- Cables mostly are made of high strength steel > 500 N/mm^2

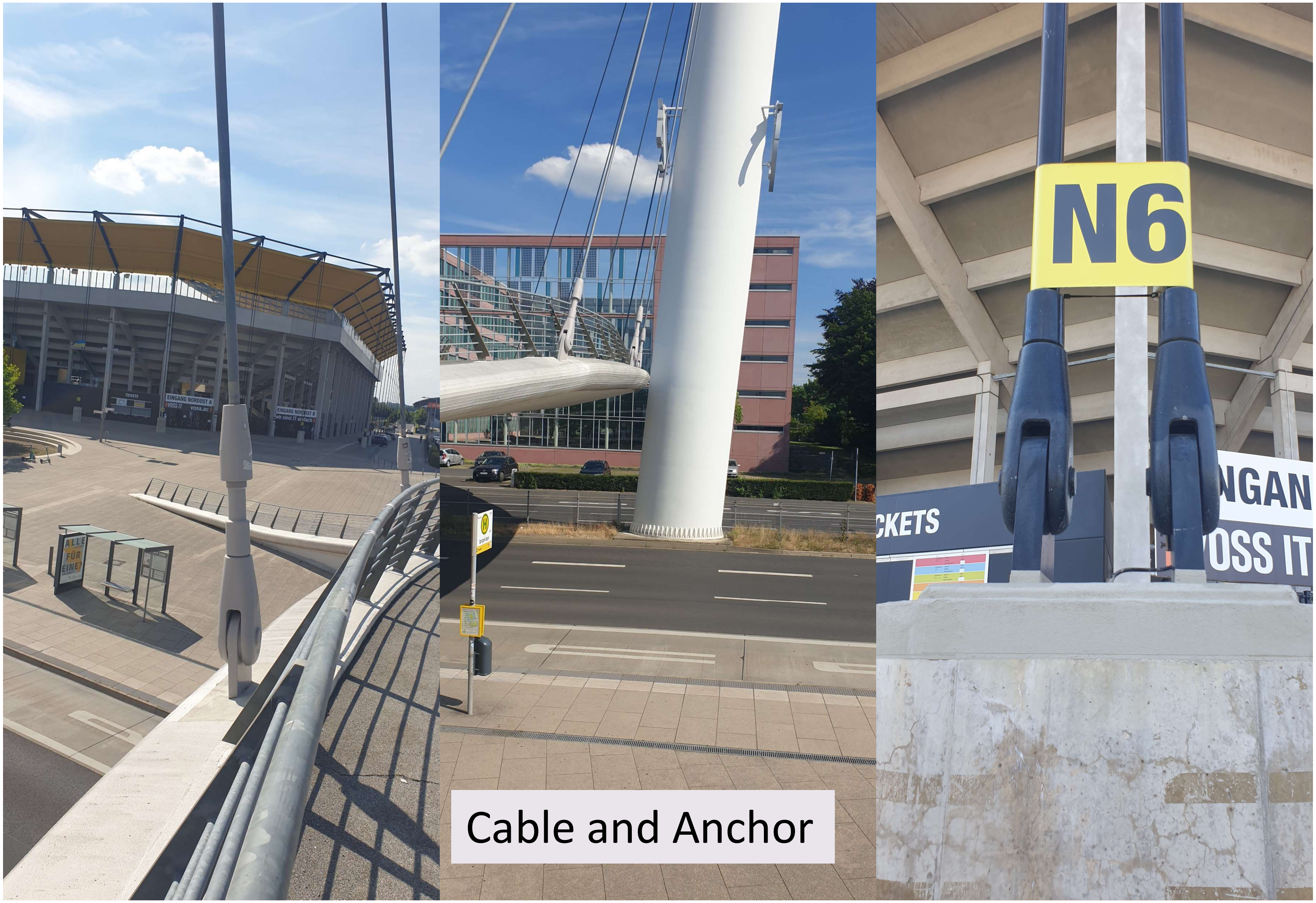
Cable Anchors



Types of tension anchors Examples of different anchors are shown in the figure.

(a) Gravity anchor; (b) plate anchor; (c) mushroom anchor; (d) retaining wall anchor; (e) tension piles; (f) ground anchor

Cable Anchors



Cable and Anchor