$f>0$, diff

$y = \ln f(x)$

$y' = \dfrac{f'(x)}{f(x)}$

$\ln e = 1$

14. $y = \ln(3\theta e^{-\theta}) = \ln 3 + \ln\theta + \ln e^{-\theta}$

$= \ln 3 + \ln\theta - \theta$

$y' = 0 + \dfrac{1}{\theta} - 1$

$= \dfrac{1}{\theta} - 1$

$y = x^2$

$y' = 2x$

$y = 2^x$

$y' = 2^x(\ln 2)(1)$

$y = a^{f(x)}$ $a>0$

$y' = a^{f(x)}(\ln a)\,f'(x)$

21. $y = e^{(\cos t + \ln t)}$

$y' = e^{\cos t + \ln t}\underbrace{(\ln e)}_{1}\left(-\sin t + \frac{1}{t}\right) = e^{\cos t}\, e^{\ln t}\left(\frac{1}{t} - \sin t\right)$

$= e^{\cos t}\, t\left(\frac{1}{t} - \sin t\right)$

$= e^{\cos t}(1 - t\sin t)$

$y = e^{\cos t}e^{\ln t} = t\, e^{\cos t}$

$y' = t\left[e^{\cos t}\ln e\,(-\sin t)\right] + e^{\cos t}(1)$

$= -t\, e^{\cos t}\sin t + e^{\cos t} = \boxed{e^{\cos t}(1 - t\sin t)}$

48. $\displaystyle\int_0^{\sqrt{\ln\pi}} 2x e^{x^2}\cos(e^{x^2})\,dx$

$u = e^{x^2} \Rightarrow du = e^{x^2}\underbrace{(\ln e)}_{1}(2x)\,dx$

$= 2x e^{x^2}\,dx$

$x = 0 \Rightarrow u = e^0 = 1$

$x = \sqrt{\ln\pi} = u = e^{\ln\pi} = \pi$

$\displaystyle\int_1^{\pi}\cos u\,du = \sin u\Big|_1^{\pi}$

$= \sin\pi - \sin 1$

$= 0 - \sin 1$

$= -\sin 1$

50. $\displaystyle\int \frac{dx}{1+e^x}$ $\boxed{\dfrac{e^{-x}}{e^{-x}}}1$

$u = 1 + e^{-x}$

$\int \frac{e^{-x}}{e^{-x}+1}\,dx$

$u = 1+e^{-x}$

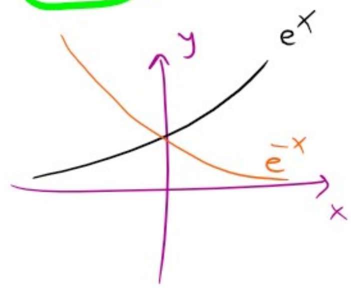
$du = e^{-x}\,\ln e\,(-1)\,dx$

$= -e^{-x}\,dx$

$$\int \frac{-du}{u} = -\ln|u|+c$$

$$= -\ln|1+e^{-x}|+c$$

$$= -\ln(1+e^{-x})+c$$



79. $y = 3^{\log_2 t} = 3^{\frac{\ln t}{\ln 2}}$

$a>0,\quad u(x)>0$

$$y = \log_a u(x) = \frac{\ln u(x)}{\ln a}$$

$$y' = 3^{\log_2 t}(\ln 3)\left(\frac{1}{\ln 2}\,\frac{1}{t}\right)$$

$$= 3^{\log_2 t}\,\frac{\ln 3}{\ln 2}\,\frac{1}{t}$$

$$\log_2 3 = \frac{\ln 3}{\ln 2}$$

$$y' = \frac{1}{\ln a}\,\frac{u'(x)}{u(x)}$$

$$= \frac{1}{t}\,3^{\log_2 t}\,\log_2 3$$

106. $\int \frac{dx}{x(\log_8 x)^2} = \int \frac{dx}{x\left(\frac{\ln x}{\ln 8}\right)^2}$

$$\log_8 x = \frac{\ln x}{\ln 8}$$

$$= (\ln 8)^2 \int \frac{dx}{x\,(\ln x)^2}$$

$u = \ln x$ | $u = \log_8 x$

$du = \frac{dx}{x}$ | $du = \frac{1}{\ln 8}\,\frac{1}{x}\,dx$

$$\ldots \frac{\ldots}{x (\ln x)^2}$$

$$du = \frac{dx}{x}$$

$$du = \frac{1}{\ln 8} \frac{1}{x} dx$$

$$\int \frac{\ln 8 \, du}{u^2}$$

$$\ln 8 \frac{u^{-1}}{-1} + c$$

$$\frac{-\ln 8}{\log_8 x} + c$$

$$\frac{-\ln 8}{\frac{\ln x}{\ln 8}} + c$$

$$= (\ln 8)^2 \int \frac{du}{u^2} = (\ln 8)^2 \int u^{-2} du$$

$$= (\ln 8)^2 \frac{u^{-1}}{-1} + c = -(\ln 8)^2 \frac{1}{u} + c$$

$$= \frac{-(\ln 8)^2}{\ln x} + C$$

108. $\int_1^{e^x} \frac{1}{t} dt$

$$= \ln|t| \Big|_1^{e^x} = \ln|e^x| - \ln|1|$$

$$= \ln e^x - \ln 1$$

$$= x \underbrace{(\ln e)}_{1} - 0$$

$$= x$$