Thermal Fluid Engineering
ENMC4411
Chapter 8 Conduction Heat Transfer

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Outline

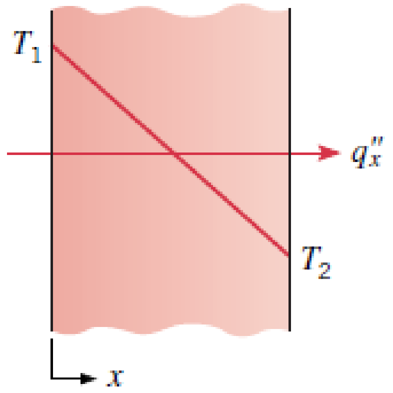
- Conduction heat transfer
- Heat diffusion equation
- SS one dimension no heat source
  - plane wall
  - Cylinder
  - Sphere
- SS one dimension with heat source
- Extended surfaces (fins)
- Transient conduction

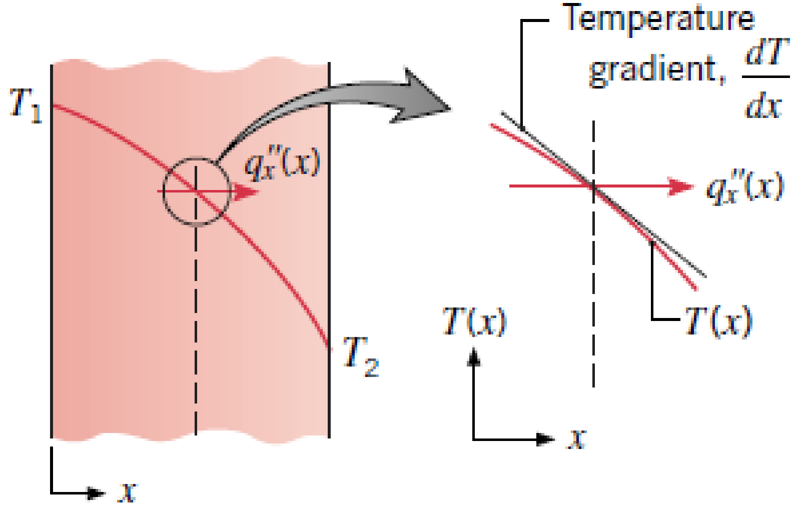
Conduction

- Heat flux is given by Fourier law

$$q''_x = -k\frac{dT}{dx} = q_x/A$$

q'' is normal to the cross section area “A” or direction of heat flow will be always normal to surface of constant temperature.(isothermal surface).

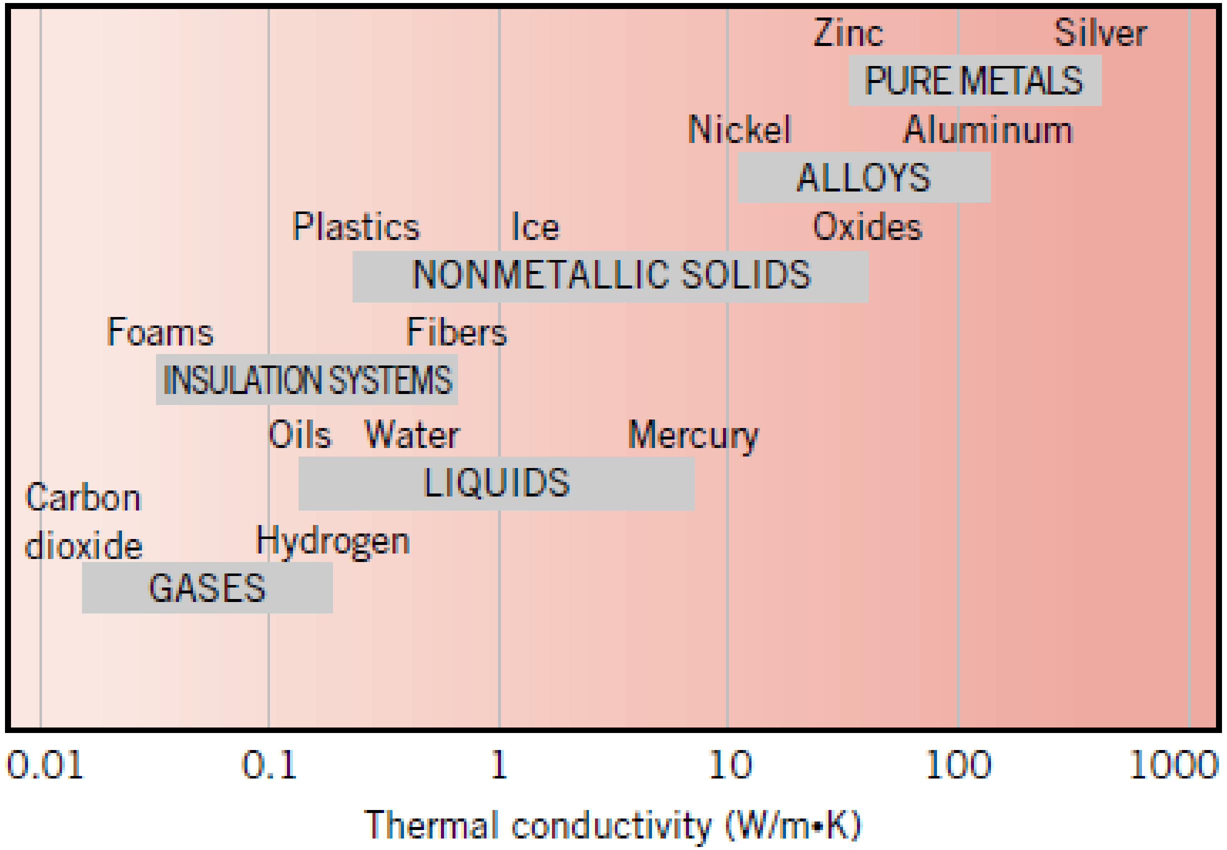




Thermal conductivity

$$k = -\frac{q''_x}{\partial T / \partial x};$$

Tabulated values of the thermophysical properties required for solution of heat transfer problems are provided in Appendix HT



thermal conductivity of solid is larger than that of liquid which is larger than that of gases. As shown in *figure 2.4* page 46 “k” of solids may be four orders of magnitude of that of gas.

$K_{solid} >> K_{liquid} >> K_{gas}$.

Insulation systems

- Low thermal conductivity materials such as; wood, plastic or combined in manner to achieve lower system conductivity are called insulators.
- For example: introducing voids (air or space voids) reducing bulk density as well. e.g. plastic foams. Different heat transfer modes do exist in the material conduction through solid, convection through voids or radiation through voids.

TABLES AND FIGURES FOR HEAT TRANSFER[1]

Table HT-1 Thermophysical Properties of Selected Technical Materials

Composition	Melting Point (K)	Properties at 300 K: ρ (kg/m^3)	c_p (J/kg · K)	k (W/m · K)	$\alpha \cdot 10^6$ (m^2/s)	Properties at Various Temperatures (K), k (W/m · K) / c_p (J/kg · K): 100	200	400	600	800	1000	1200	1500
Metallic Solids													
Aluminum													
Pure	933	2702	903	237	97.1	302	237	240	231	218			
						482	798	949	1033	1146			
Alloy 2024-T6	775	2770	875	177	73.0	65	163	186	186				
						473	787	925	1042				
Beryllium	1550	1850	1825	200	59.2	990	301	161	126	106	90.8	78.7	
						203	1114	2191	2604	2823	3018	3227	3519
Copper													
Pure	1358	8933	385	401	117	482	413	393	379	366	352	339	
						252	356	397	417	433	451	480	
Cartridge brass	1188	8530	380	110	33.9	75	95	137	149				
(70% Cu, 30% Zn)							360	395	425				
Germanium	1211	5360	322	59.9	34.7	232	96.8	43.2	27.3	19.8	17.4	17.4	
						190	290	337	348	357	375	395	
Gold	1336	19,300	129	317	127	327	323	311	298	284	270	255	
						109	124	131	135	140	145	155	
Iron													
Pure	1810	7870	447	80.2	23.1	134	94.0	69.5	54.7	43.3	32.8	28.3	32.1
						216	384	490	574	680	975	609	654
Plain carbon steel		7854	434	60.5	17.7			58.7	48.8	39.2	31.3		
								487	559	685	1168		
AISI 1010		7832	434	63.9	18.8			58.7	48.8	39.2	31.3		
								487	559	685	1168		
Stainless steel		8238	468	13.4	3.48			15.2	18.3	21.3	24.2		

Table HT-2 Thermophysical Properties of Selected Common Materials

Description/Composition	Typical Properties at 300 K		
	Density, ρ (kg/m^3)	Thermal Conductivity, k (W/m · K)	Specifi Heat, c_p (J/kg · K
Insulating Materials and Systems			
Blanket and Batt			
Glass fiber, paper faced	16	0.046	—
	28	0.038	—
	40	0.035	—
Board and Slab			
Cellular glass	145	0.058	1000
Glass fiber, organic bonded	105	0.036	795
Polystyrene, expanded			
Extruded (R-12)	55	0.027	1210
Molded beads	16	0.040	1210
Loose Fill			
Glass fiber, poured or blown	16	0.043	835
Vermiculite, flakes	80	0.068	835
	160	0.063	1000
Formed/Foamed-in-Place			
Polyvinyl acetate cork mastic; sprayed or troweled	—	0.100	—
Urethane, two-part mixture; rigid foam	70	0.026	1045

Table HT-2 Thermophysical Properties of Selected Common Materials

Description/Composition	Typical Properties at 300 K		
	Density, ρ (kg/m^3)	Thermal Conductivity, k $(W/m \cdot K)$	Specific Heat, c_p $(J/kg \cdot K)$
Structural Building Materials			
Building Boards			
Gypsum or plaster board	800	0.17	—
Hardboard, siding	640	0.094	1170
Particle board, low density	590	0.078	1300
Particle board, high density	1000	0.170	1300
Plywood	545	0.12	1215
Woods			
Hardwoods (oak, maple)	720	0.16	1255
Softwoods (fir, pine)	510	0.12	1380
Masonry Materials			
Brick, common	1920	0.72	835
Concrete (stone mix)	2300	1.4	880

General heat diffusion equation

$$\frac{\partial^{2T}}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\dot{q}\big/_{k} = 1\big/_{\alpha}\frac{\partial T}{\partial t} \qquad \rho C\big/_{k} = 1/\alpha$$

For one dimension this simplifies to

$$\frac{\partial^{2T}}{\partial x^2}+\dot{q}\big/_{k} = 1\big/_{\alpha}\frac{\partial T}{\partial t}$$

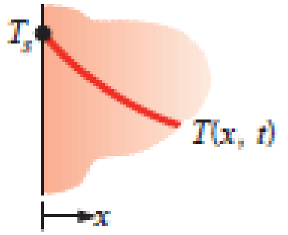
For steady state (no change with time) for steady state $\frac{\partial T}{\partial t}=0$

For no heat source it becomes $\frac{d^2 T}{\partial x^2} = 0$

TABLE 2.2 Boundary conditions for the heat diffusion equation at the surface ($x = 0$)

1. Constant surface temperature

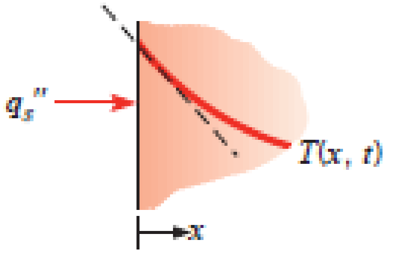
$$T(0, t) = T_s \tag{2.31}$$



2. Constant surface heat flux

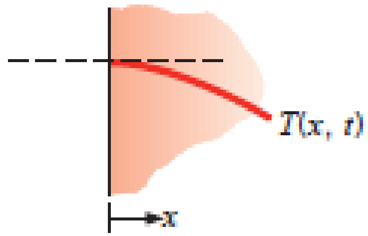
(a) Finite heat flux

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = q_s'' \tag{2.32}$$



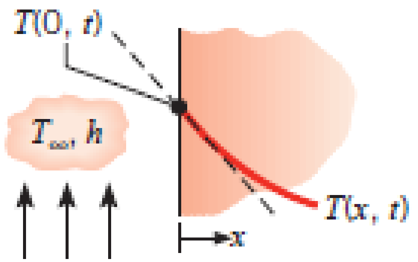
(b) Adiabatic or insulated surface

$$\frac{\partial T}{\partial x}\bigg|_{x=0} = 0 \tag{2.33}$$



3. Convection surface condition

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = h[T_\infty - T(0, t)] \tag{2.34}$$



Outline

- Heat diffusion equation
- SS one dimension no heat source
  - plane wall
  - **Cylinder**
  - Sphere
- SS one dimension with heat source
- Extended surfaces (fins)
- Transient conduction

Plane wall

$$\frac{d\left(k\,dT/dx\right)}{dx}=0 \quad \text{if k = constant} \quad \frac{d^2T}{dx^2}=0$$

integrating twice $\frac{dT}{dx}=c_1$

$T(x) = c_1 X + c_2$

Apply B.c to obtain C_1 & C_2

$$T(x=0)=T_1 \Rightarrow c_2=T_1$$

$$T(x=L)=T_2 \Rightarrow T_2=c_1L+T_1$$

$$c_1=\frac{T_2-T_1}{L}$$

Then

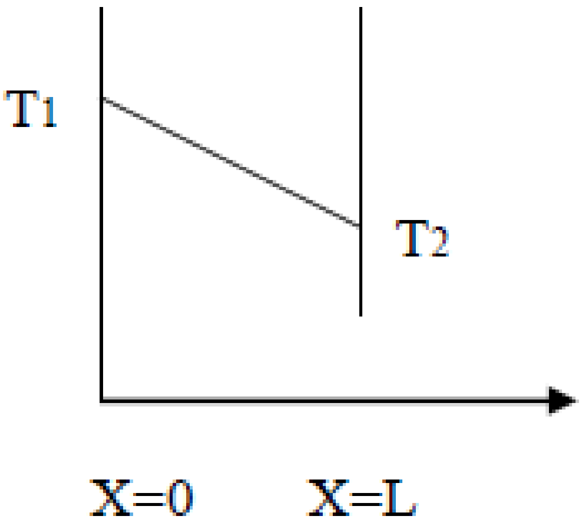
$$T(x)=(T_2-T_1)\frac{x}{L}+T_1$$

Heat transfer: $q_x=-kA\frac{dT}{dx}$

$$\frac{dT}{dx}=\frac{T_2-T_1}{L}=c_1 \quad \text{Constant.}$$

$$q_x=-kA\frac{(T_2-T_1)}{L}=kA\frac{(T_1-T_2)}{L}$$

Flux $q''=\frac{q_x}{A}=\frac{k}{L}(T_1-T_2)$



Alternative conduction Analysis

Starting with Fourier law for 1-dimension

$$q_x = -kA\frac{dT}{dx}$$

Since q_x = constant, integrating the above eq.

$$\int q_x dx = \int -kA dT$$

If k= constant

$$\int_{x_1}^{x_2} q_x dx = -kA\int_{T_1}^{T_2} dT$$

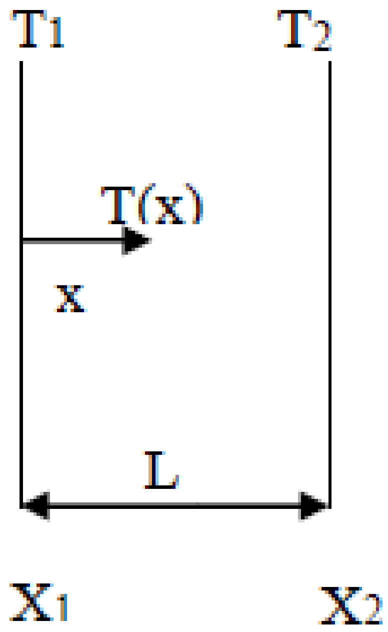
If limits are know T1= T(x=x1) & T2= T(x=x2)

Then

$q_x (x_2-x_1) = -kA (T_2-T_1)$

$q_x = -kA (T_2-T_1)/(x_2-x_1)$

$$q_x = kA\frac{(T_1 - T_2)}{(x_2 - x_1)} = kA\frac{(T_1 - T_2)}{L} \quad T_1>T_2$$



Electrical Analogy - conduction

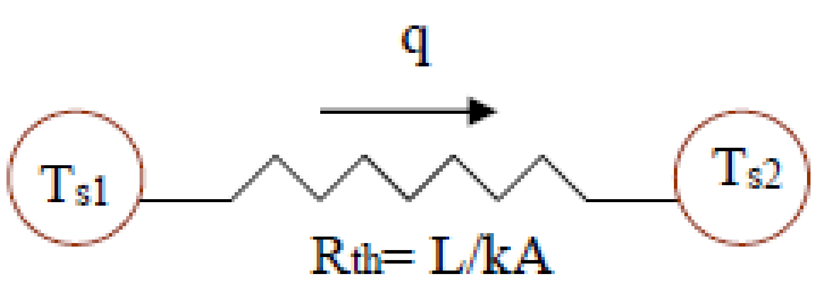
Heat transfer $q = kA\dfrac{(T_{s1} - T_{s2})}{L}$

Current $I = \Delta V/R$.

q analogous to I

ΔV analogous to $T_{s1}-T_{s2} = \Delta T$, driving force.
Then $R = L/kA$

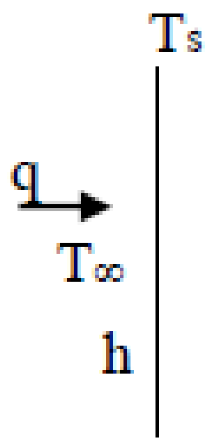
$R_{th} = L/kA$
hen
$q_x = \Delta T/R_{th}$ *known as thermal resistance.*

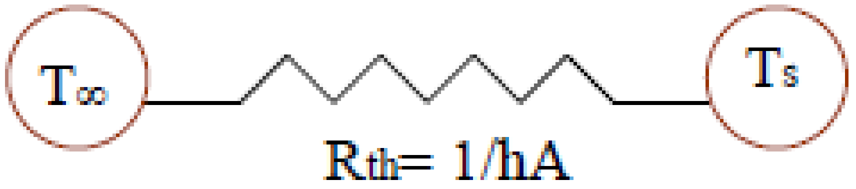


Electrical Analogy - convection

$$q_{conv} = hA(Ts - T\infty) = \frac{(T_s - T\infty)}{\frac{1}{hA}}$$

$$q_{conv} = \frac{\Delta T}{R_{th}} \quad \text{then } R_{th} = 1/hA$$





Convection – conduction- convection

At steady state:

$q_{conv1} = q_{cond} = q_{conv2} = q$

$q_{conv1} = h_1 A_1 (T_{\infty 1} - T_{s1})$

$q_{cond} = kA(T_{s1} - T_{s2})/L$

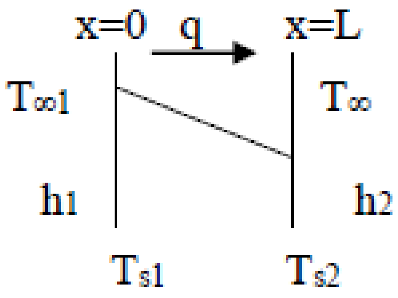
$q_{conv2} = h_2 A_2 (T_{s2} - T_{\infty 2})$

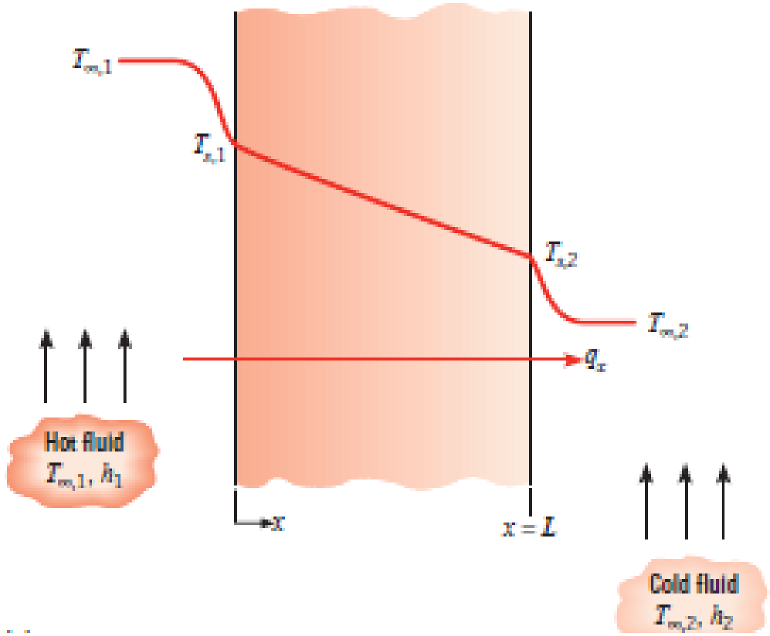
Then:

$T_{\infty 1} - T_{s1} = q / h_1 A_1$

$T_{s1} - T_{s2} = q / kA/L$

$T_{s2} - T_{\infty 2} = q / h_2 A_2$



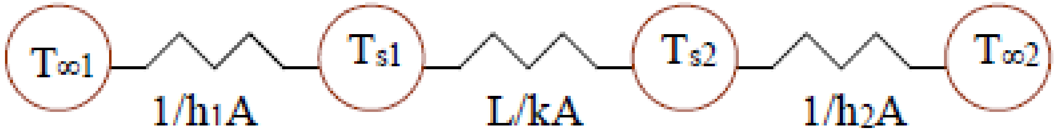


Add above eqs:

$$T_{\infty 1} - T_{\infty 2} = q\left[\frac{1}{h_1 A_1} + \frac{L}{KA} + \frac{1}{h_2 A_2}\right]$$

Convection – conduction- convection

$$q = \left(T_{\infty 1} - T_{\infty 2}\right) \Big/ \left[\frac{1}{h_1 A_1} + \frac{L}{KA} + \frac{1}{h_2 A_2}\right]$$



$$\left(T_{\infty 1} - T_{\infty 2}\right) = \Delta\text{T overall}$$

$$R_{total} = \sum R_{total} = R_1 + R_2 + R_3 + \ldots\ldots + R_n$$

$$R_{th,conv1} = \frac{1}{h_1 A_1}, \text{ since } q_{conv_1} = \frac{T_{\infty_1} - T_{S_1}}{R_{th}} = \frac{T_{\infty_1} - T_{S_1}}{1/h_1 \, A}$$

$$R_{total} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}.$$

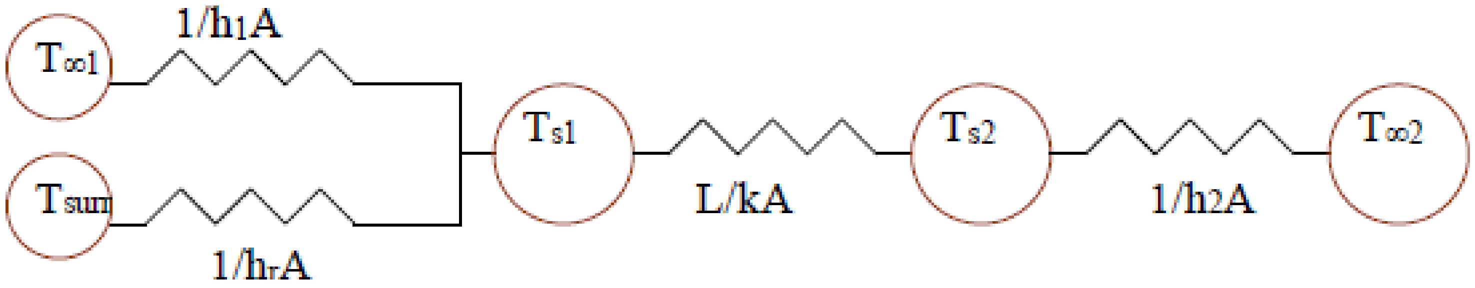
$$R_{th,cond} = \frac{L}{kA}, \text{ since } q_{cond} = \frac{T_{S1} - T_{S2}}{L/kA} = \frac{\Delta T}{R_{th}}$$

$$\text{Hence; } \quad q = \frac{\Delta T_{overall}}{\sum R_{th}} = \frac{\Delta T}{R_{total}}$$

$$R_{th,conv2} = \frac{1}{h_2 A_2}, \text{ since } q_{conv2} = \frac{T_{S2} - T_{\infty 2}}{1/h_2 \, A} = \frac{\Delta T}{R_{th}}$$

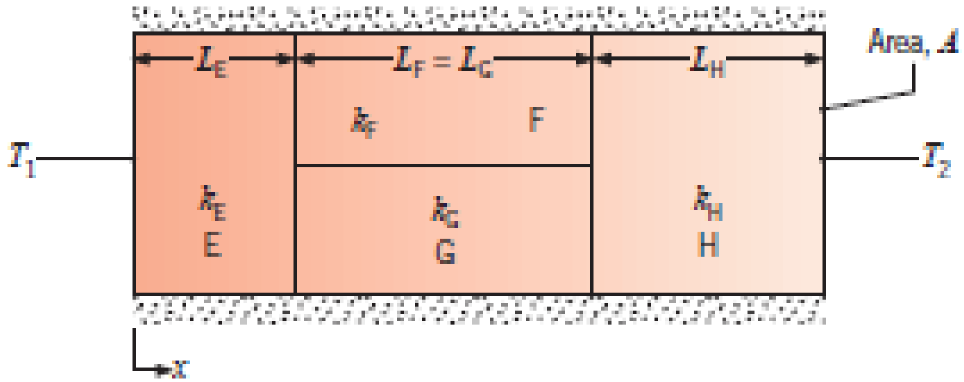
Combined heat transfer

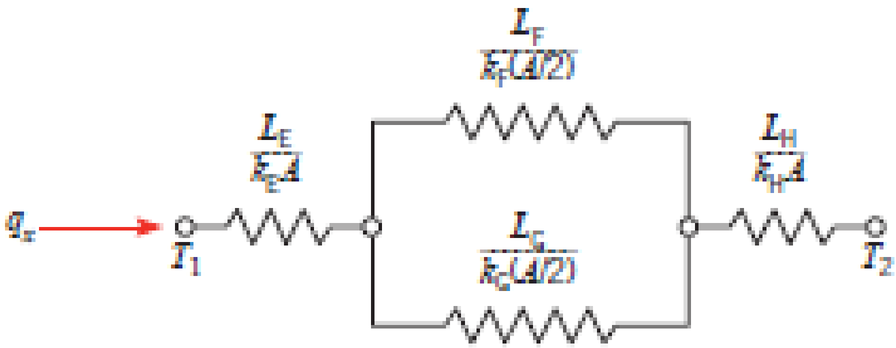
- Derive expression for qx if convection or radiation exist at surface 1



- For parallel resistant

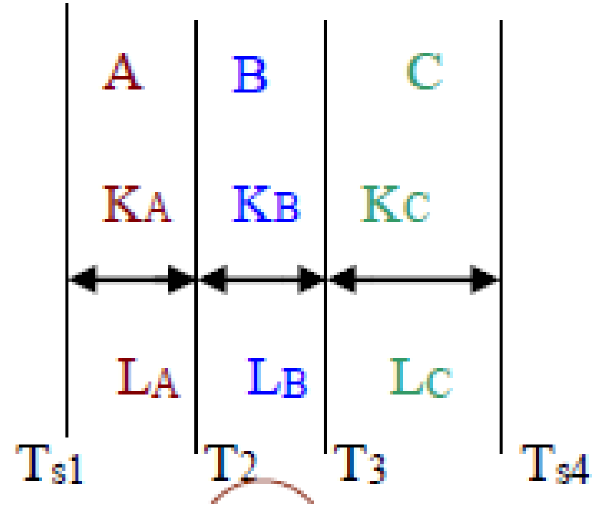
$$\frac{1}{Rtotal} = \frac{1}{R1} + \frac{1}{R2} \ldots \ldots + \frac{1}{Rn}$$



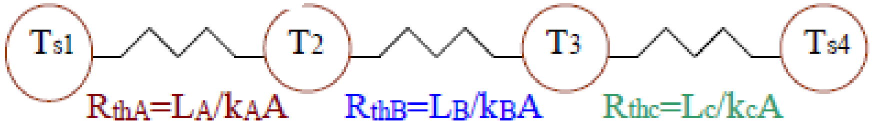


Composite walls

- Consider composite multilayer wall: A+B+C
- By electrical analogy

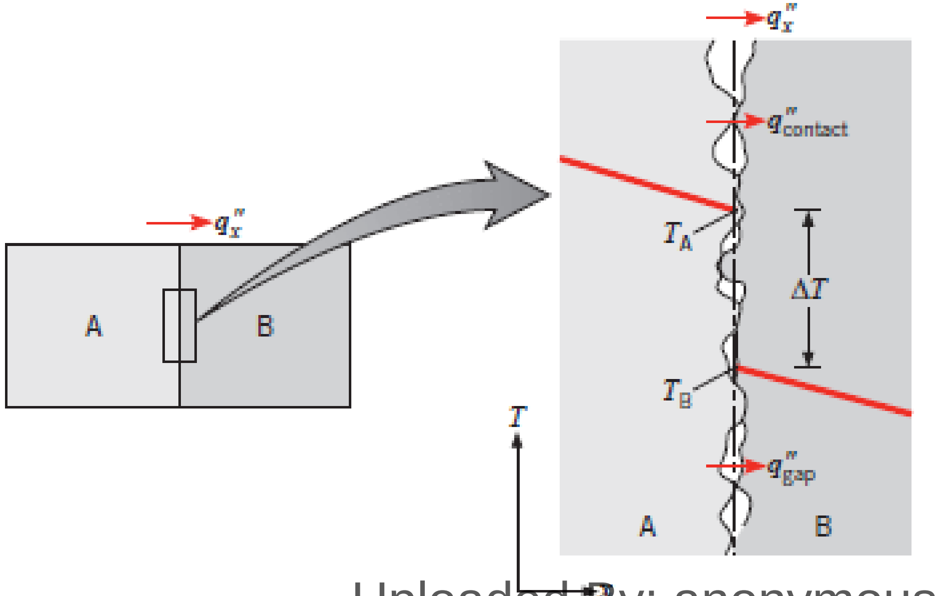


$$q = \frac{\Delta T_{overall}}{R_{total}} = \frac{(T_{S1} - T_{S4})}{\left[\frac{L_A}{k_A A} + \frac{L_B}{k_B A} + \frac{L_C}{k_C A}\right]}$$

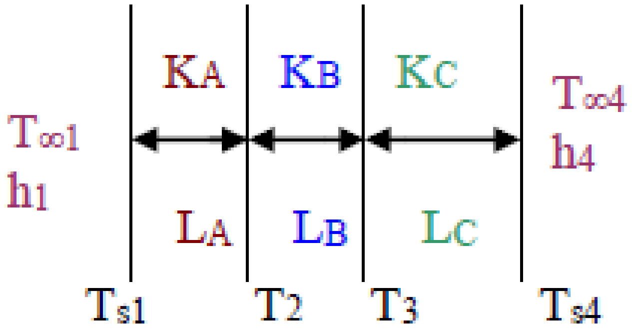


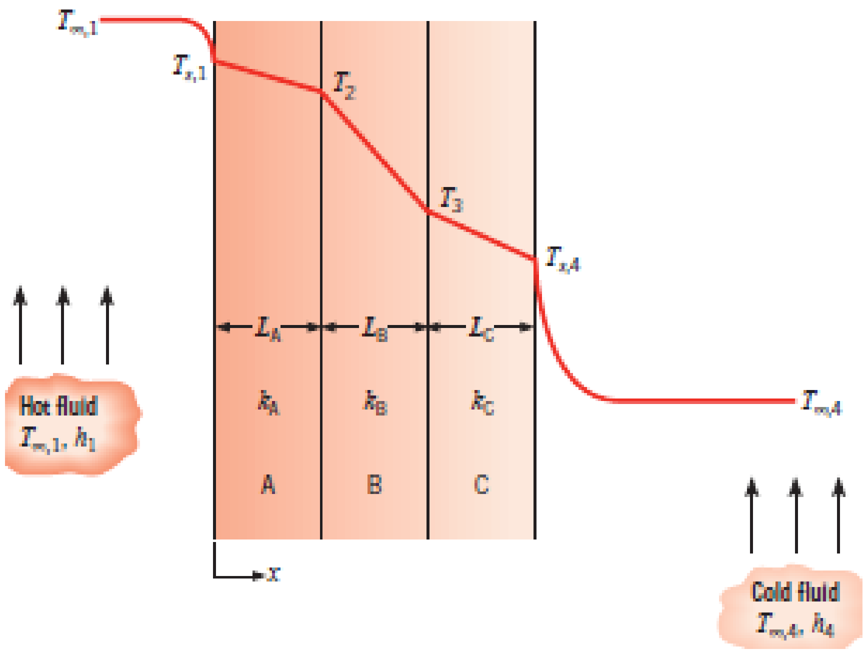
- Contact resistance

The temperature drop across the interface between materials may be appreciable. This temperature change is attributed to what is known as the thermal contact resistance, Rt,c. Usually this resistance is neglected

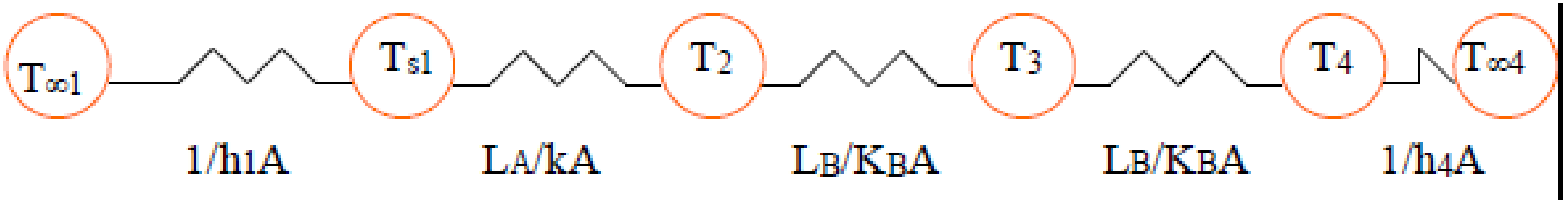


Composite wall with convection





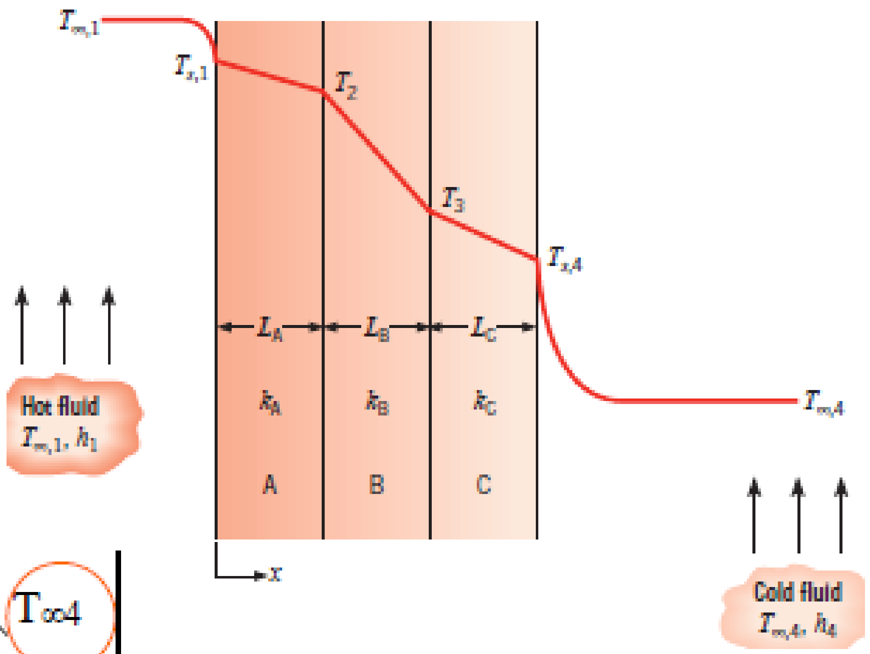
Equivalent circuit



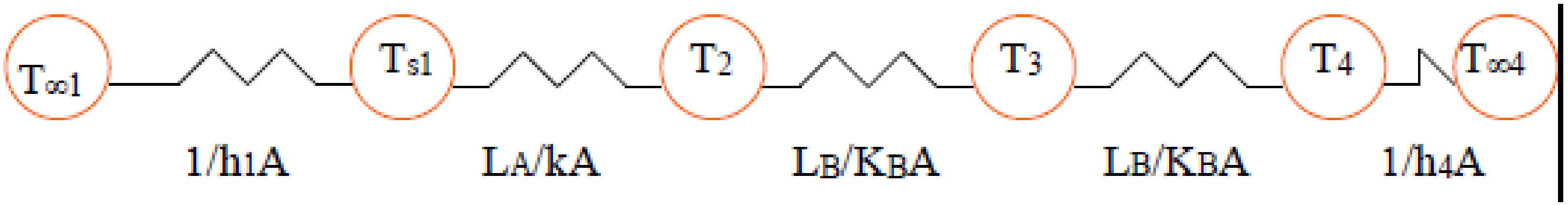
$$q = \frac{\Delta T_{overall}}{R_{th}} \qquad q = (T_{\infty 1} - T_{\infty 4}) \Big/ \left[\frac{1}{h_1 A} + \frac{L_A}{K_A A} + \frac{L_B}{K_B A} + \frac{L_C}{K_C A} + \frac{1}{h_4 A}\right]$$

Interface temperatures

- At Steady state same heat transfer rate in three layers and same as convection.
- May use this relation to find temperate at interface of wall layers once q is calculated.



$$q_x = \frac{T_{\infty,1} - T_{s,1}}{(1/h_1 A)} = \frac{T_{s,1} - T_2}{(L_A/k_A A)} = \frac{T_2 - T_3}{(L_B/k_B A)} = \cdot$$

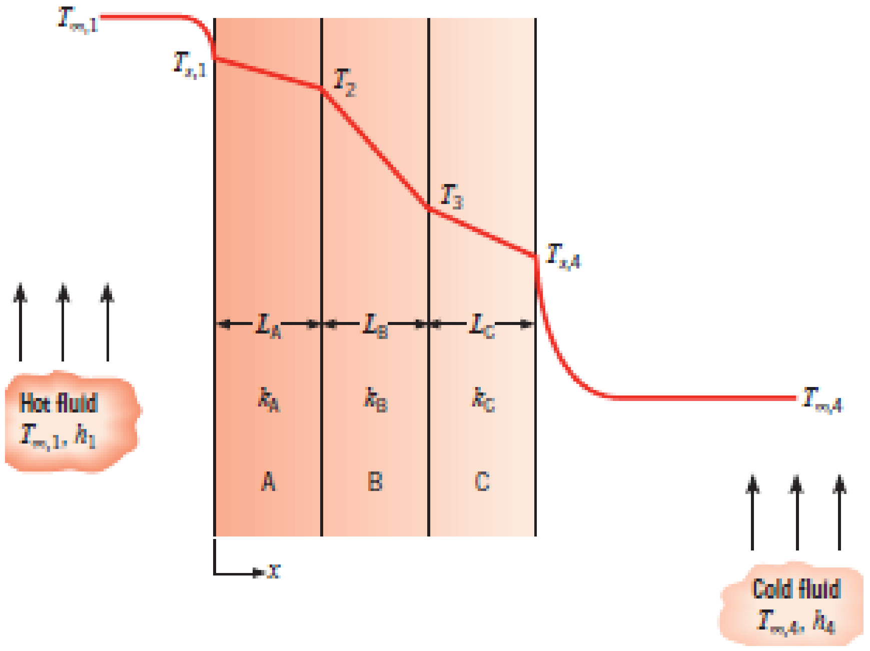


Overall heat transfer coefficient

- For above composite wall

$$q = \frac{\Delta T_{overall}}{R_{th}}$$

$$q = \left(T_{\infty 1} - T_{\infty 4}\right) \Big/ \left[\frac{1}{h_1 A} + \frac{L_A}{K_A A} + \frac{L_B}{K_B A} + \frac{L_C}{K_C A} + \frac{1}{h_4 A}\right]$$



- If written as $q = UA\,\Delta T$

- Then $$U = \frac{1}{R_{total} A} = 1 \Big/ \left[\frac{1}{h_1} + \frac{L_A}{K_A} + \frac{L_B}{K_B} + \frac{L_C}{K_C} + \frac{1}{h_4}\right]$$

In general
Rtotal = 1/ UA

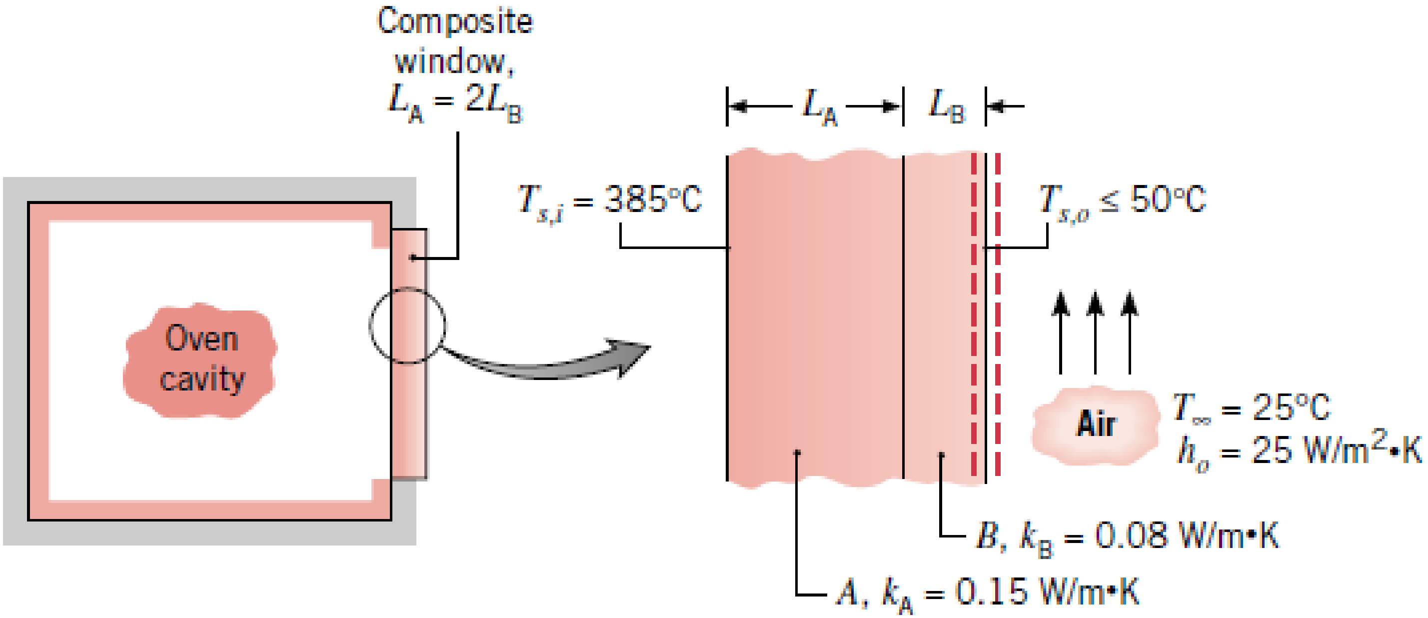
$$\boxed{R_{tot} = \sum R_t = \frac{\Delta T}{q} = \frac{1}{UA}}$$

Example 16.1 Thermal Circuit Analysis–the Plane Wall

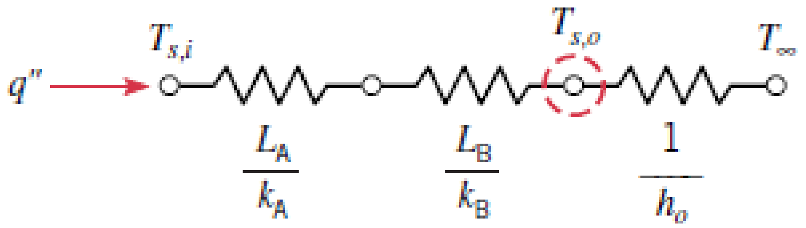
A manufacturer of household appliances is proposing a self-cleaning oven design that involves use of a composite window separating the oven cavity from the room air. The composite is to consist of two high-temperature plastics (A and B) of thicknesses $L_A = 2L_B$ and thermal conductivities $k_A = 0.15$ and $k_B = 0.08$W/m.k. During the self-cleaning process, the inside window temperature *Ts,i* is 385C, while the room air temperature *T* is 25C and the outside convection coefficient is 25W/m2.K . What is the minimum window thickness, $L = L_A + L_B$, needed to ensure a temperature that is 50 C or less at the outer surface of the window during steady-state operation? This temperature must not be exceeded for safety reasons

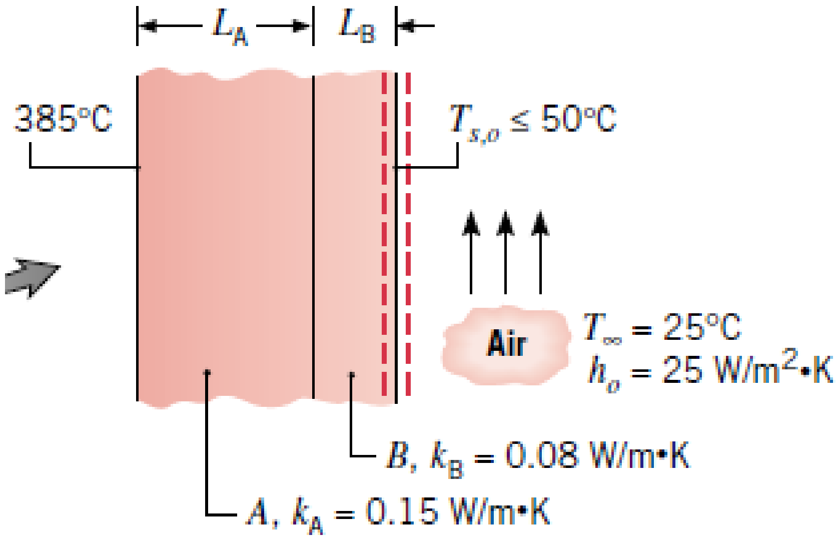
Assumptions:

1. Steady-state conditions exist.
2. Conduction through the window is one-dimensional.
3. Contact resistance between the plastics is negligible.
4. Radiation transfer through the window is negligible.
5. Plastics are homogenous with constant properties.



Surface energy balance, hence, the heat flux into the node (surface) is equal to the heat flux out of the node (surface). As such, the heat rate can be expressed as





$$q'' = \frac{T_{s,i} - T_{s,o}}{L_A/k_A + L_B/k_B} = \frac{T_{s,o} - T_\infty}{1/h_o}$$

With $L_B = L_A/2$, and substituting numerical values, find L_A

$$\frac{(385 - 50)°C}{(L_A/0.15 + 0.5L_A/0.08)\,m \cdot K/W} = \frac{(50 - 25)°C}{(1/25)\,m^2 \cdot K/W}$$

$$L_A = 0.0415\ m$$

Hence, the required thickness for the composite window is

$$L = L_A + L_B = (0.0415 + 0.5 \times 0.0415)\,m = 0.0622\ m = 62.2\ mm \quad \triangleleft$$

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- Heat diffusion equation
- SS one dimension no heat source
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Cylindrical System

- Fourier law for conduction
- q_r = kA dT/dr
- Consider a cylinder with T1, at r1, ; T2 at r2 length L.
- Area of heat transfer is A = 2πr L
- qr= -k 2πr L dT/dr = -2 π Lk rdT/dr
- q= constant; rearrange equation

$$\int q_r \frac{dr}{r} = \int -2\Pi\,\mathrm{Lk}\ \ \mathrm{dT}$$

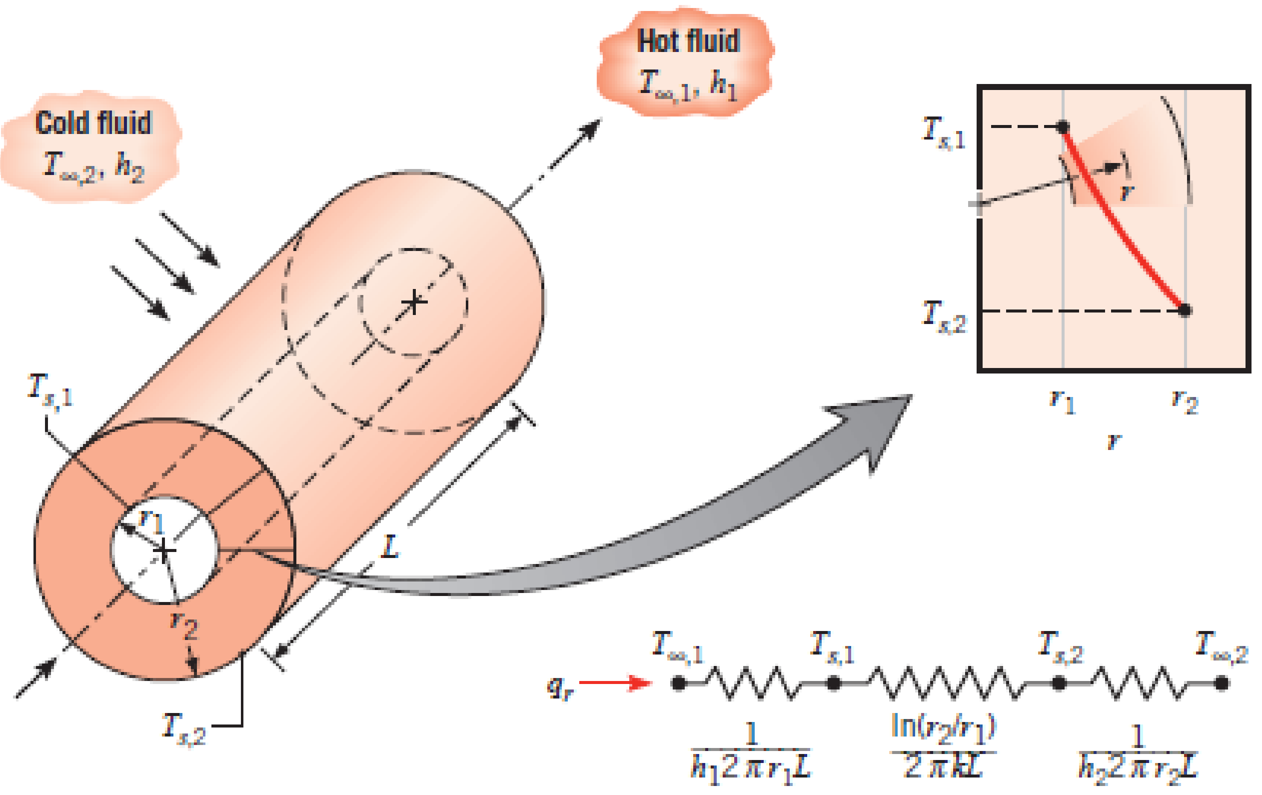
- Integrating
- $q_r \ln(\frac{r2}{r1}) = 2\pi Lk(T2 - T1)$

Cylindrical System

$$q_r = -2\Pi lk\frac{(T_2 - T_1)}{\ln\left(\frac{r_2}{r_1}\right)} = 2\Pi lk\frac{(T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)}$$

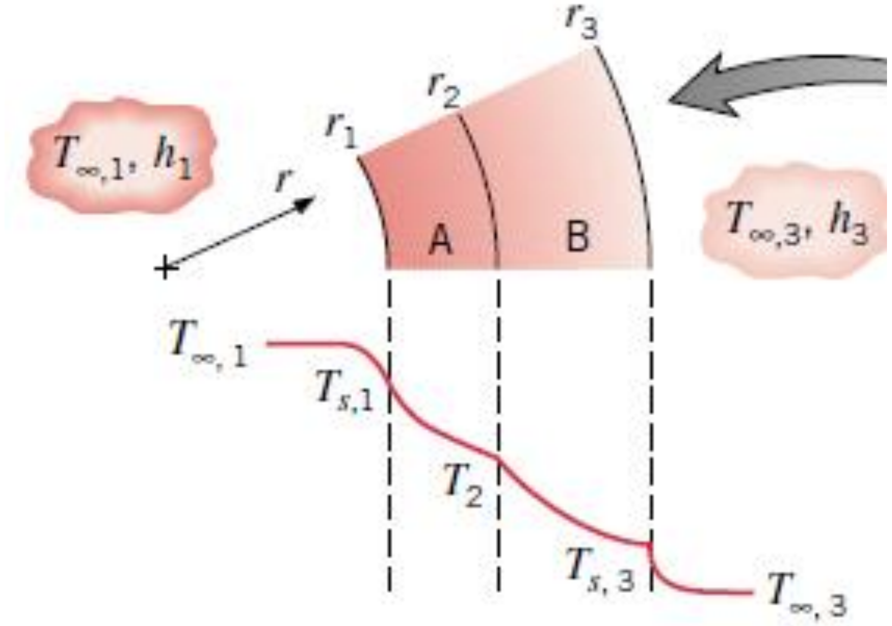
$$q = \frac{\Delta T}{R_{TH}}$$

$$R_{th} = \ln[r2/r1]/2\Pi kl$$



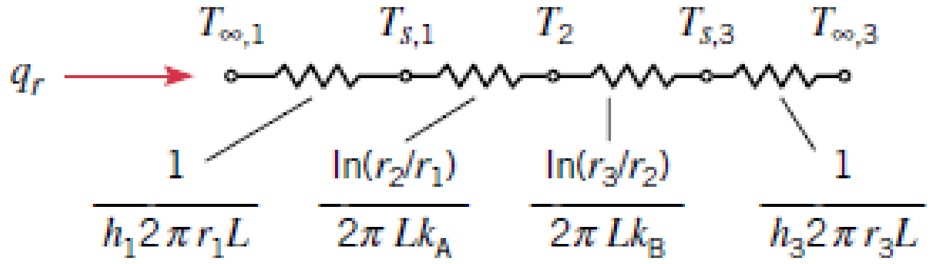
Composite system

$$q_r = \frac{T_{\infty,1} - T_{\infty,3}}{\dfrac{1}{2\pi r_1 L h_1} + \dfrac{\ln(r_2/r_1)}{2\pi L k_A} + \dfrac{\ln(r_3/r_2)}{2\pi L k_B} + \dfrac{1}{2\pi r_3 L h_3}}$$



$$q_r = \frac{T_{\infty,1} - T_{\infty,3}}{R_{tot}} = UA(T_{\infty,1} - T_{\infty,3})$$

If U is defined in terms of the inside area, $A1 = 2\pi r1L$



$$U_1 = \frac{1}{\dfrac{1}{h_1} + \dfrac{r_1}{k_A}\ln\dfrac{r_2}{r_1} + \dfrac{r_1}{k_B}\ln\dfrac{r_3}{r_2} + \dfrac{r_1}{r_3}\dfrac{1}{h_3}}$$

$$U_1 A_1 = U_2 A_2 = U_3 A_3 = R_{tot}^{-1}$$

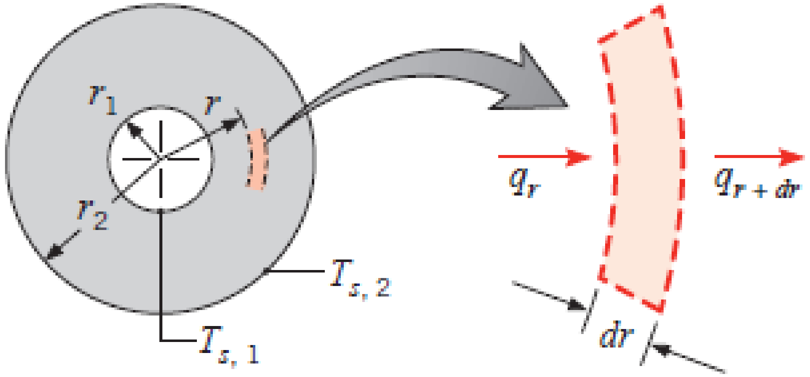
Outline

- Heat diffusion equation
- SS one dimension no heat source
  - plane wall
  - Cylinder
  - Sphere
- SS one dimension with heat source
- Extended surfaces (fins)
- Transient conduction

Spherical systems

- Consider a sphere with B.C.: T(r = r1)=T1, T(r = r2)= T2.
- Fourier Law:

for sphere A=4л r^2; q_r= constant



$$q = -kA\frac{dT}{dr} \qquad q_r = -k4\pi r^2\frac{dT}{dr}$$

Assume k= constant and integrating ;

$$\frac{-q_r}{4\pi}\left(\frac{1}{r_2}-\frac{1}{r_1}\right) = -k(T_2 - T_1)$$

$$q_r = 4\pi k\frac{(T_1 - T_2)}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)} = \frac{\Delta T}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\Big/4\pi k} = \frac{\Delta T}{R_{th}}$$

$$R_{th} = \left(\frac{1}{r_1}-\frac{1}{r_2}\right)\frac{1}{4\pi k}$$

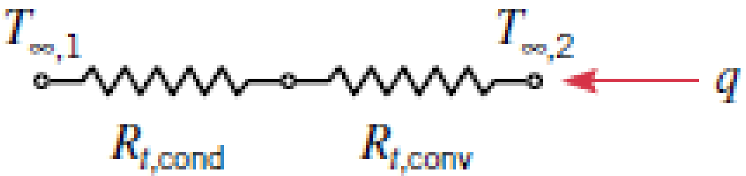
Example 16.3 Thermal Circuit Analysis–Spherical System

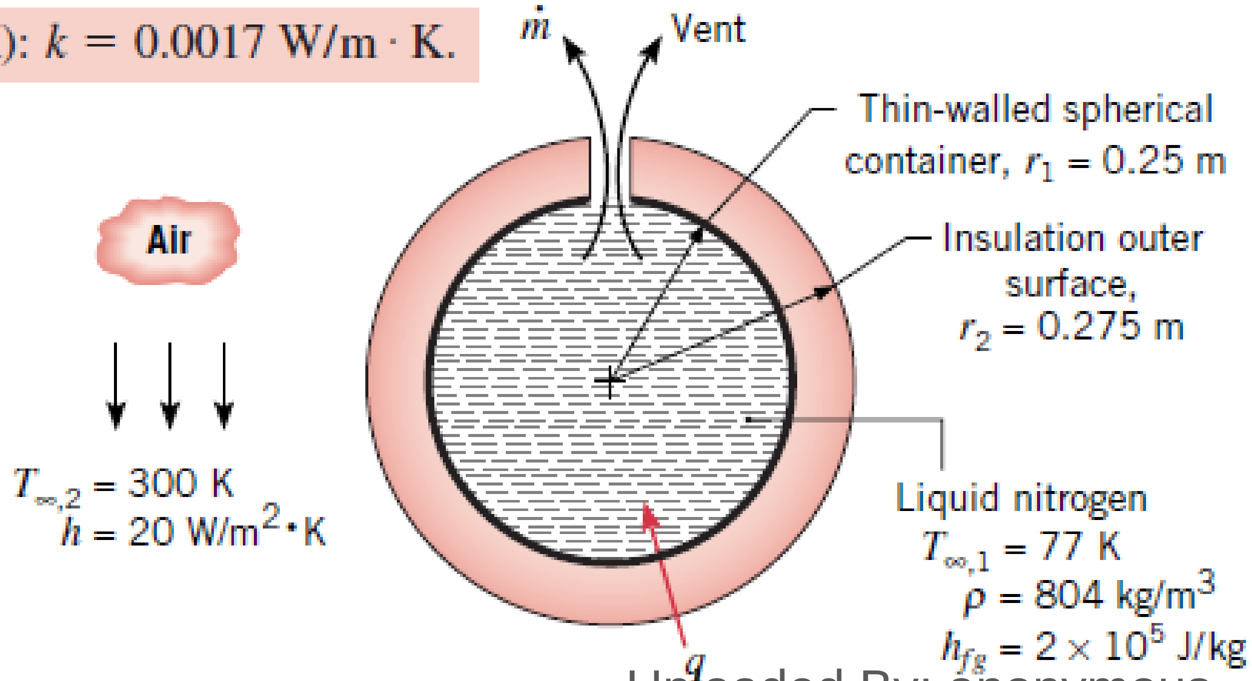
A spherical, thin-walled metallic container is used to store liquid nitrogen at 77 K. The container has a diameter of 0.5 m and is covered with an evacuated, reflective insulation composed of silica powder. The insulation is 25 mm thick, and its outer surface is exposed to ambient air at 300 K. The convection coefficient is known to be 20 W/m2.K.The heat of vaporization and the density of liquid nitrogen are 2X 10^5 J/kg and 804 kg/m^3, respectively.

(a) What is the rate of heat transfer to the liquid nitrogen?

(b) What is the rate of liquid boil-off (liters/day)?

Properties: Table HT-2, evacuated silica powder (300 K): $k = 0.0017$ W/m · K.





Assumptions:

1. Steady-state heat transfer.
2. One-dimensional transfer in the radial direction.
3. Negligible resistance to heat transfer through the container wall and from the container to the nitrogen.
4. Negligible radiation exchange between outer surface of insulation and surroundings.
5. Constant properties.

Analysis:

(a) By assumption 3, the only elements in the thermal circuit as shown above are the resistances due to conduction through the insulation and convection from the outer surface where, from Table 16.3

$$R_{t,\text{cond}} = \frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad R_{t,\text{conv}} = \frac{1}{h4\pi r_2^2}$$

The rate of heat transfer to the liquid nitrogen is then

$$q = \frac{T_{\infty,2} - T_{\infty,1}}{(1/4\pi k)[(1/r_1) - (1/r_2)] + (1/h4\pi r_2^2)}$$

$$q = \frac{(300 - 77)\text{ K}}{\left[\frac{1}{4\pi(0.0017\text{ W/m}\cdot\text{K})}\left(\frac{1}{0.25\text{ m}} - \frac{1}{0.275\text{ m}}\right) + \frac{1}{(20\text{ W/m}^2\cdot\text{K})4\pi(0.275\text{ m})^2}\right]}$$

$$q = \frac{223}{17.02 + 0.05}\text{ W} = 13.06\text{ W} \quad \triangleleft$$

(b) The heat transfer to the liquid nitrogen provides energy to vaporize the liquid nitrogen by boiling

$$q = \dot{m} h_{fg}$$

and the mass rate of nitrogen boil-off is

$$\dot{m} = \frac{q}{h_{fg}} = \frac{13.06 \text{ J/s}}{2 \times 10^5 \text{ J/kg}} = 6.53 \times 10^{-5} \text{ kg/s}$$

The mass rate per day is

$$\dot{m} = 6.53 \times 10^{-5} \text{ kg/s} \left|\frac{3600 \text{ s}}{\text{h}}\right| \left|\frac{24 \text{ h}}{\text{day}}\right| = 5.64 \text{ kg/day}$$

or on a volumetric flow rate basis

$$\frac{\dot{m}}{\rho} = \frac{5.64 \text{ kg/day}}{804 \text{ kg/m}^3} = 0.007 \text{ m}^3\text{/day} \left|\frac{10^3 \text{ liters}}{\text{m}^3}\right| = 7 \text{ liters/day} \quad \triangleleft$$

Comments:

1. Since $R_{t,conv} \ll R_{t,cond}$, the dominant contribution to the total thermal resistance is that due to conduction in the insulation. Even if the convection coefficient were reduced by a factor of 10, thereby increasing the convection resistance by the same proportion, the effect on the boil-off rate would be small.
2. With a container volume of $(4/3)(\pi r_1^3) = 0.065 \text{ m}^3 = 65$ liters, the daily evaporation rate amounts to (7 liters/65 liters) 100% = 10.8% of capacity.

TABLE 3.3 One-dimensional, steady-state solutions to the heat equation with no generation

	Plane Wall	Cylindrical Wall[a]	Spherical Wall[a]
Heat equation	$\frac{d^2T}{dx^2} = 0$	$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$	$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$
Temperature distribution	$T_{s,1} - \Delta T\frac{x}{L}$	$T_{s,2} + \Delta T\frac{\ln(r/r_2)}{\ln(r_1/r_2)}$	$T_{s,1} - \Delta T\left[\frac{1-(r_1/r)}{1-(r_1/r_2)}\right]$
Heat flux (q'')	$k\frac{\Delta T}{L}$	$\frac{k\,\Delta T}{r\ln(r_2/r_1)}$	$\frac{k\,\Delta T}{r^2[(1/r_1)-(1/r_2)]}$
Heat rate (q)	$kA\frac{\Delta T}{L}$	$\frac{2\pi Lk\,\Delta T}{\ln(r_2/r_1)}$	$\frac{4\pi k\,\Delta T}{(1/r_1)-(1/r_2)}$
Thermal resistance ($R_{t,\text{cond}}$)	$\frac{L}{kA}$	$\frac{\ln(r_2/r_1)}{2\pi Lk}$	$\frac{(1/r_1)-(1/r_2)}{4\pi k}$

[a]The critical radius of insulation is $r_{cr} = k/h$ for the cylinder and $r_{cr} = 2k/h$ for the sphere.

Outline

- Heat diffusion equation
- SS one dimension no heat source
  - plane wall
  - Cylinder
  - Sphere
- SS one dimension with heat source
- Extended surfaces (fins)
- Transient conduction

Conduction with Energy Generation

Consider the plane wall of Fig. 16.11*a*, in which there is *uniform* energy generation per unit volume (is constant) and the surfaces are maintained at *Ts*,1 and *Ts*,2. For constant thermal conductivity *k*, the appropriate form of *the heat equation*

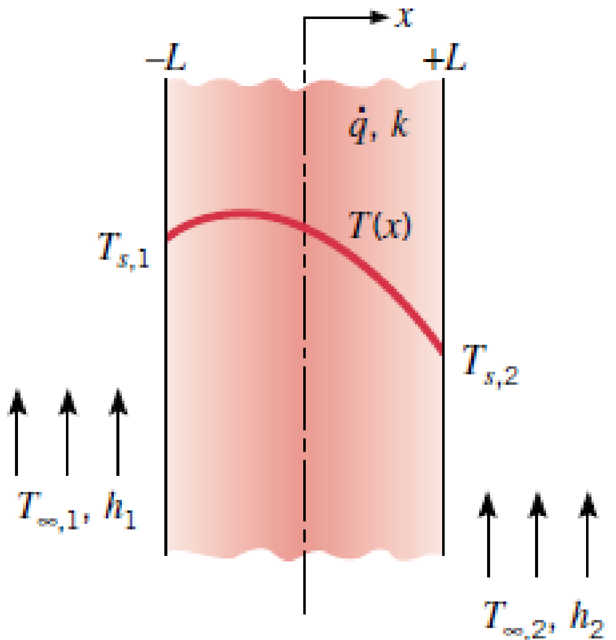
$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0$$

$$T = -\frac{q}{2k}x^2 + C_1 x + C_2$$

$T(-L) = T_{s,1}$ and $T(L) = T_{s,2}$

$$C_1 = \frac{T_{s,2} - T_{s,1}}{2L} \quad \text{and} \quad C_2 = \frac{\dot{q}}{2k}L^2 + \frac{T_{s,1} + T_{s,2}}{2}$$

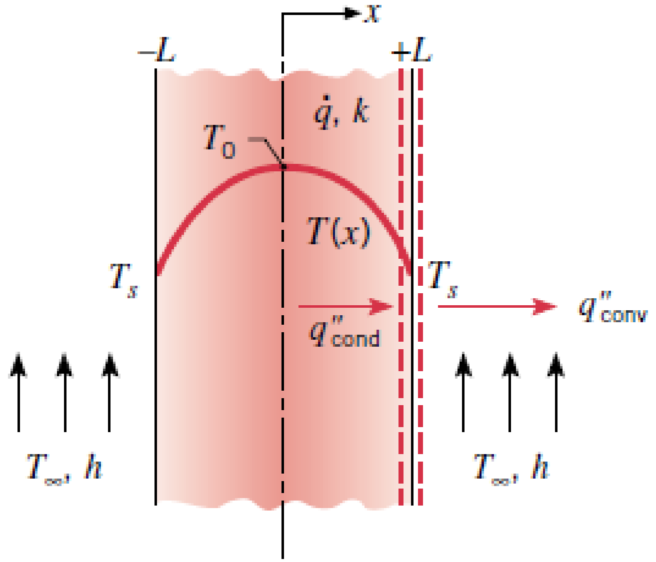
$$T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + \frac{T_{s,2} - T_{s,1}}{2}\frac{x}{L} + \frac{T_{s,1} + T_{s,2}}{2}$$



(a)

Conduction with Energy Generation

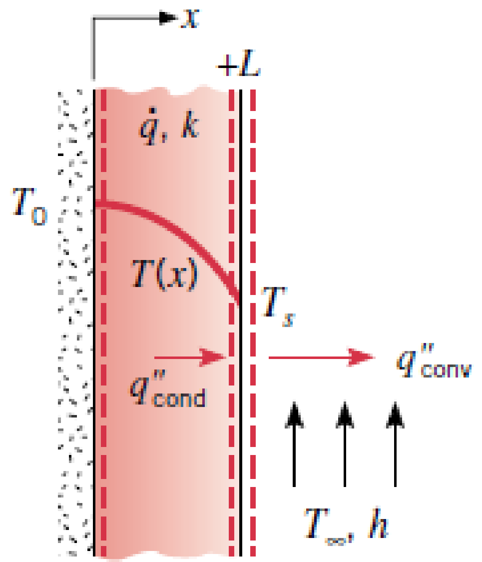
The preceding result simplifies when both surfaces are maintained at a common temperature, *Ts*,1=*Ts*,2 = *Ts* as shown in Fig. 16.11*b*.



$$T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + T_s$$

The maximum temperature exists at the midplane, $x = 0$

$$T(0) \equiv T_0 = \frac{\dot{q}L^2}{2k} + T_s$$



The surface energy balance, has the form *q*cond =*q*conv,=Generation or using surface balance gives;

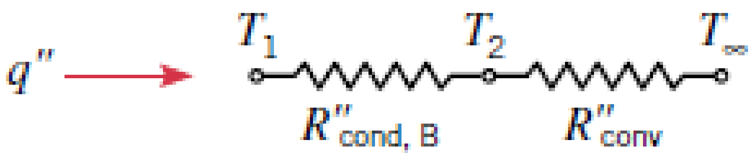
$$-k\frac{dT}{dx}\bigg|_{x=L} = h(T_s - T_\infty) \qquad T_s = T_\infty + \frac{\dot{q}L}{h}$$

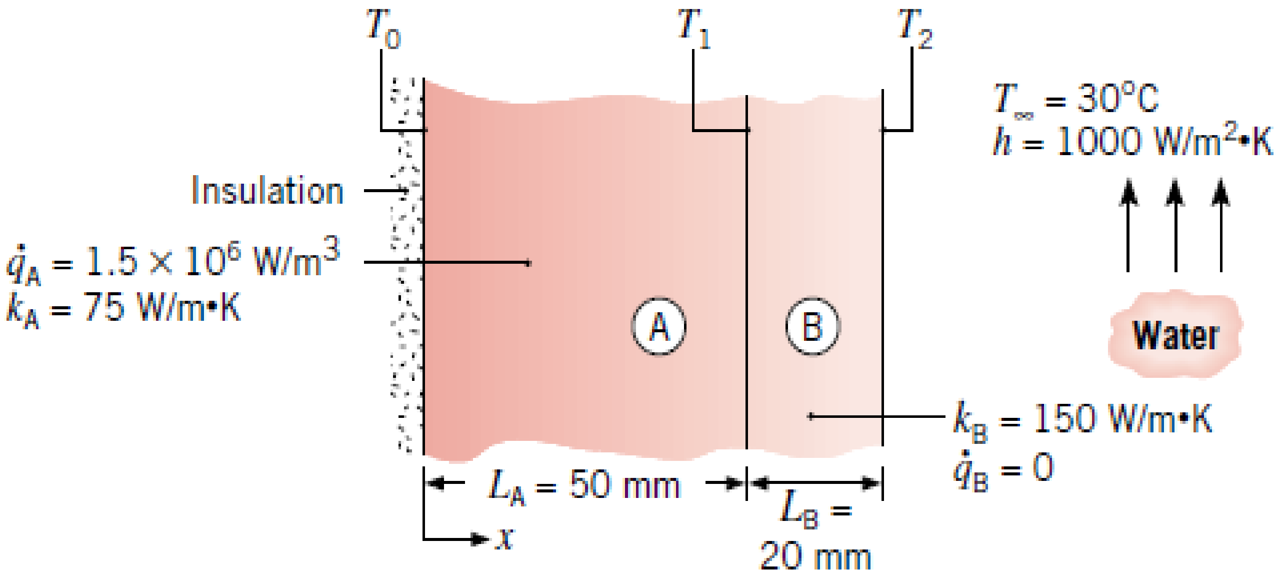
Example 16.4 Energy Generation in a Plane Wall

A plane wall is a composite of two materials, A and B. The wall of material A has uniform energy generation W/m3, *k*A =75 W/m.K, and thickness *L*A =50 mm. The wall of material B has no generation, with *k*B =150 and thickness *L*B =20 mm. The inner surface of material A is well insulated, while the outer surface of material B is cooled by a water stream with *T* =30C and *h* = 1000W/m2.K

(a) Determine the temperature *T*0 of the insulated surface and the temperature *T*2 of the cooled surface.
(b) Sketch the temperature distribution that exists in the composite under steady-state conditions.

Assumptions:
1. Steady-state conditions.
2. One-dimensional conduction in *x*-direction.
3. Negligible contact resistance between walls.
4. Inner surface of A is adiabatic.
5. Constant properties for materials A and B.





Analysis:

(a) The outer surface temperature T_2 can be obtained by performing an energy balance on a system about material B (Fig. E16.4*b*). Since there is no generation in this material, it follows that, for steady-state conditions and a unit surface area, the heat flux into the material at $x = L_A$ must equal the heat flux from the material due to convection at $x = L_A + L_B$. Hence

$$q'' = h(T_2 - T_\infty) \tag{1}$$

The heat flux q'' can be determined by performing a second energy balance about material A. In particular, since the surface at $x = 0$ is adiabatic, there is no inflow and the rate at which energy is generated must equal the outflow. Accordingly, for a unit surface area

$$\dot{q}L_A = q'' \tag{2}$$

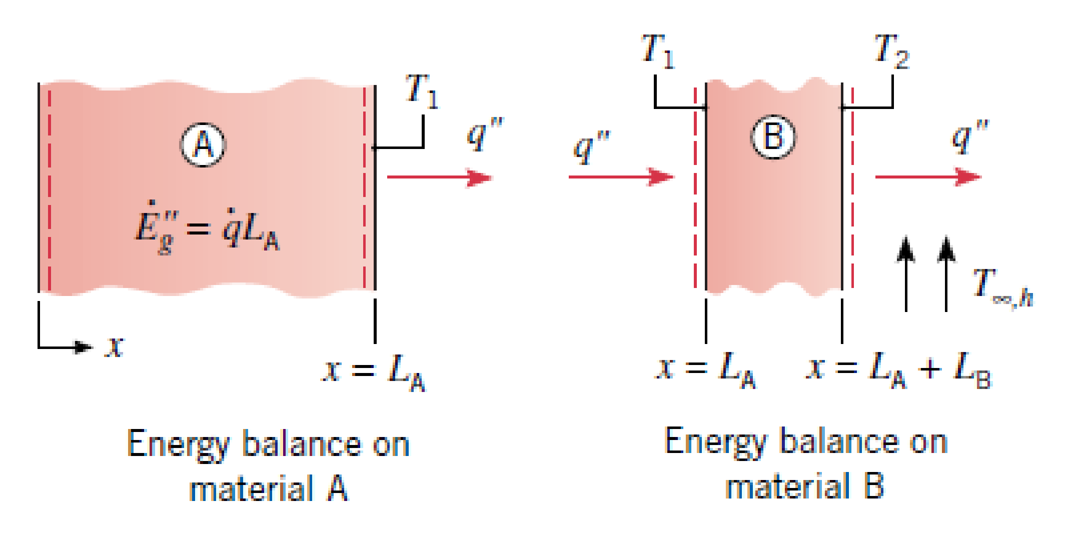


Figure E16.4b

Combining Eqs. 1 and 2, the outer surface temperature is

$$T_2 = T_\infty + \frac{\dot{q}L_A}{h} = 30°\text{C} + \frac{1.5 \times 10^6 \text{ W/m}^3 \times 0.05 \text{ m}}{1000 \text{ W/m}^2 \cdot \text{K}} = 105°\text{C} \triangleleft$$

$$T_0 = \frac{\dot{q}L_A^2}{2k_A} + T_1 \tag{3}$$

where T_1 may be obtained from the thermal circuit shown in Fig. E16.4*a* representing the wall B conduction and convection processes. That is,

$$T_1 = T_\infty + (R''_{\text{cond,B}} + R''_{\text{conv}})\, q''$$

where the resistances for a unit surface area are

$$R''_{\text{cond, B}} = \frac{L_B}{k_B} \qquad R''_{\text{conv}} = \frac{1}{h}$$

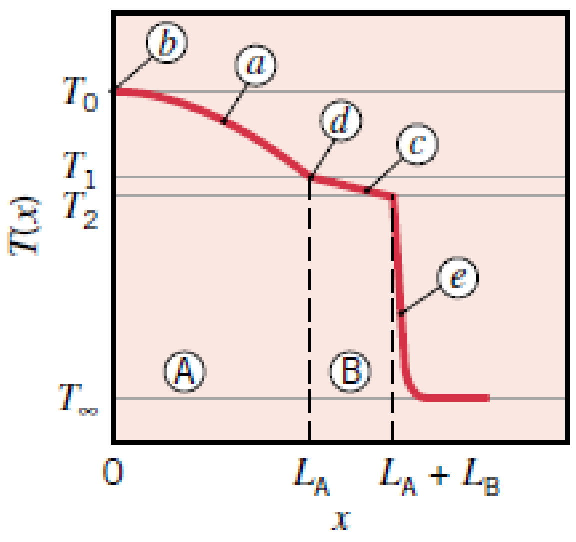
Hence, the temperature at the composite interface is

$$T_1 = 30°\text{C} + \left(\frac{0.02\ \text{m}}{150\ \text{W/m}\cdot\text{K}} + \frac{1}{1000\ \text{W/m}^2\cdot\text{K}}\right)(1.5 \times 10^6\ \text{W/m}^3)\ 0.05\ \text{m}$$

$$T_1 = 30°\text{C} + 85°\text{C} = 115°\text{C}$$

Substituting into Eq. 3, the inner surface temperature of the composite is

$$T_0 = \frac{1.5 \times 10^6\ \text{W/m}^3\ (0.05\ \text{m})^2}{2 \times 75\ \text{W/m}\cdot\text{K}} + 115°\text{C} = 25°\text{C} + 115°\text{C} = 140°\text{C} \quad \triangleleft$$



(a) Parabolic in material A.
(b) Zero slope at insulated boundary.
(c) Linear in material B.
(d) Slope change $= k_B/k_A$ at interface.

The temperature distribution in the *water* is characterized by large gradients near the surface (e).

Figure E16.4c

Comments:

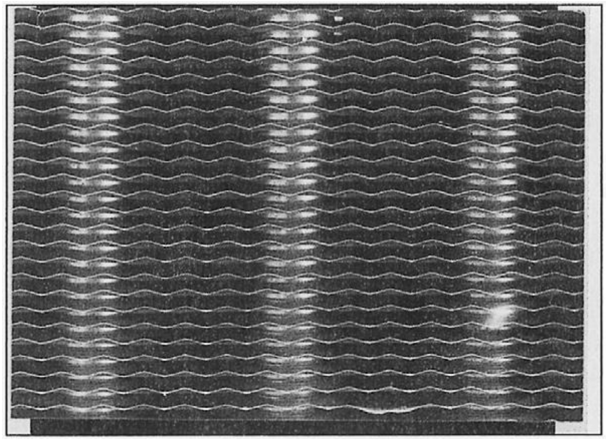
1. Material A, having energy generation, cannot be represented by a thermal circuit element.
2. Since the resistance to heat transfer by convection is significantly larger than that due to conduction in material B, $R''_{conv}/R''_{cond} = 7.5$, the surface-to-fluid temperature difference is much larger than the temperature drop across material B, $(T_2 - T_\infty)/(T_1 - T_2) = 7.5$. This result is consistent with the temperature distribution plotted in Fig. E16.4*c*.

Outline

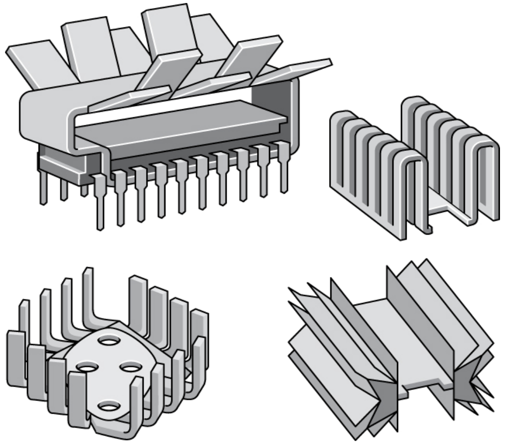
- Heat diffusion equation
- SS one dimension no heat source
  - plane wall
  - Cylinder
  - Sphere
- SS one dimension with heat source
- Extended surfaces (fins)
- Transient conduction

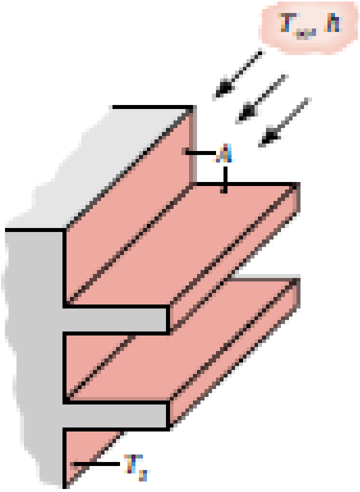
Conduction – convection systems

- Extended surfaces are used to enhance rate of heat transfer.
- Conduction transfer in the solid is followed by convection from the surface to the fluid.



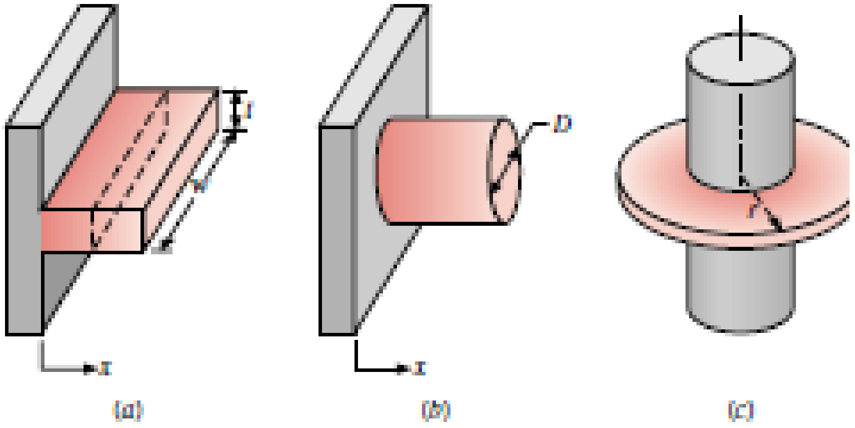
Thin plate fins of a car radiator and innovative fin designs to increase rate if heat transfer.

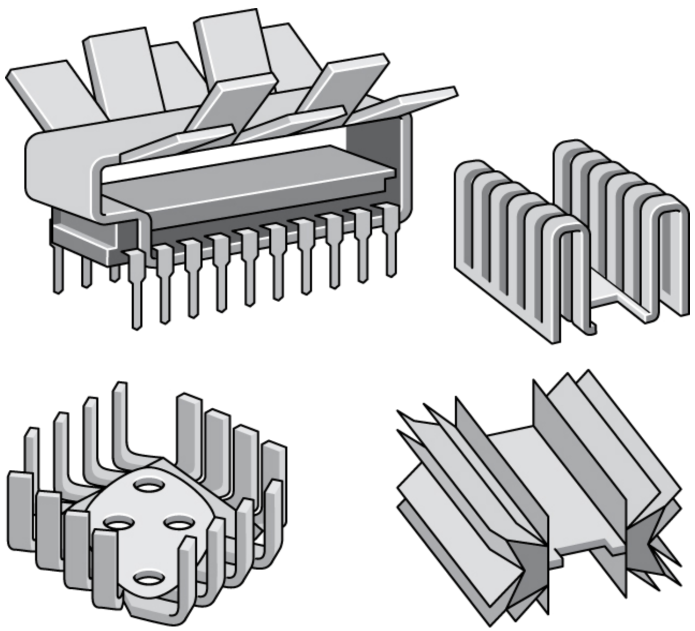


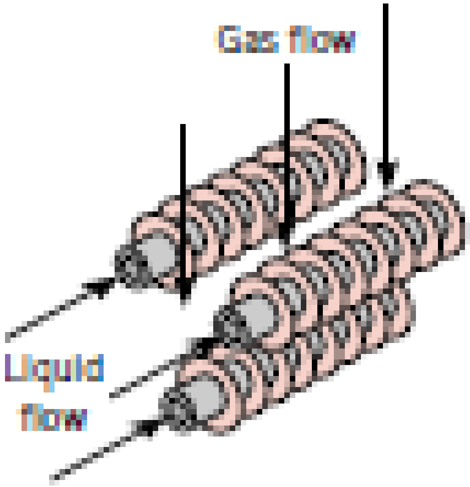


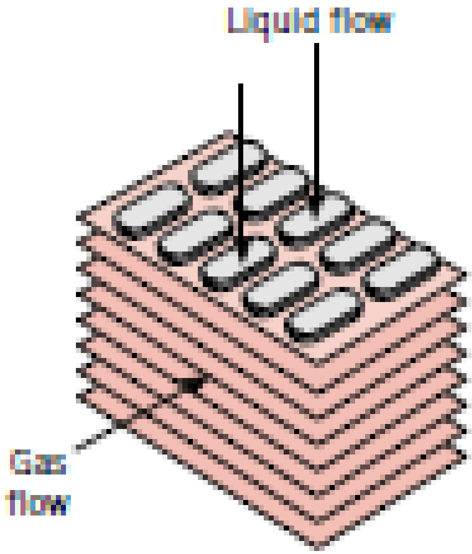
Electronic components innovative fin designs to increase rate if heat transfer.

Fin types



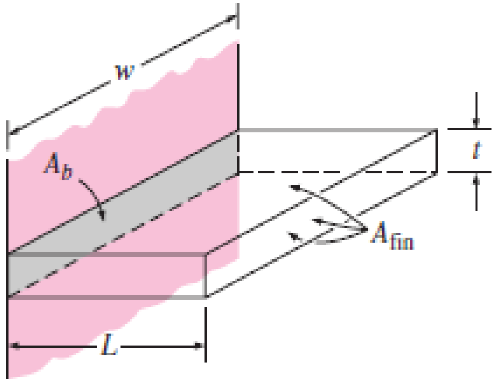




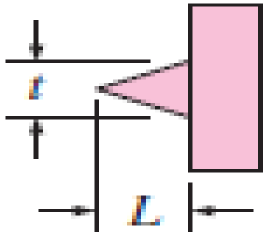


Fin types

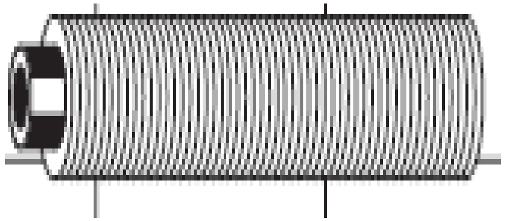
- Straight fin with uniform cross section.



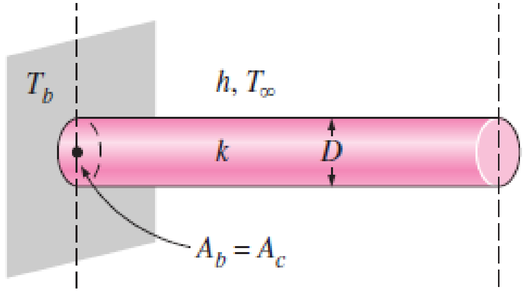
- Straight fin with non-uniform cross section (triangle profile).



- Annular (circumferential) fin



- Pin fin (cross section).



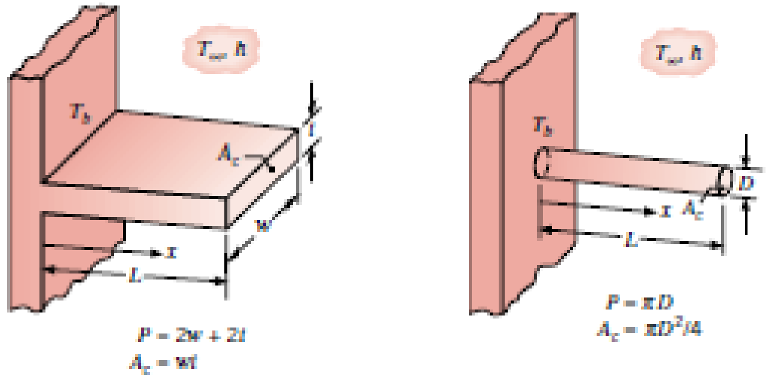
($p = \pi D$, $A_c = \pi D^2/4$ for a cylindrical fin)

Fin Model uniform cross section

- Rectangular or pin (circular) fins have constant cross section area A_c=const=A
- Base temperature T_b=T(x=0)
- Fluid temperature T_∞
- Surface area $A_s = PX$ $dA_s = Pdx$

Where P: perimeter, then

$$\frac{dA_c}{dx} = 0 \qquad \frac{dA_s}{dx} = P$$



- Simplifying fin equation $\dfrac{d^2T}{dx^2} - \dfrac{h}{kA_c} P(T - T\infty) = 0$

$$\theta = T - T\infty = T(x) - T\infty \rightarrow \frac{d\theta}{dx} = \frac{dT}{dx} \qquad m^2 = \frac{hP}{A_c K} \qquad \frac{d^2\theta}{dx^2} - m^2\theta = 0$$

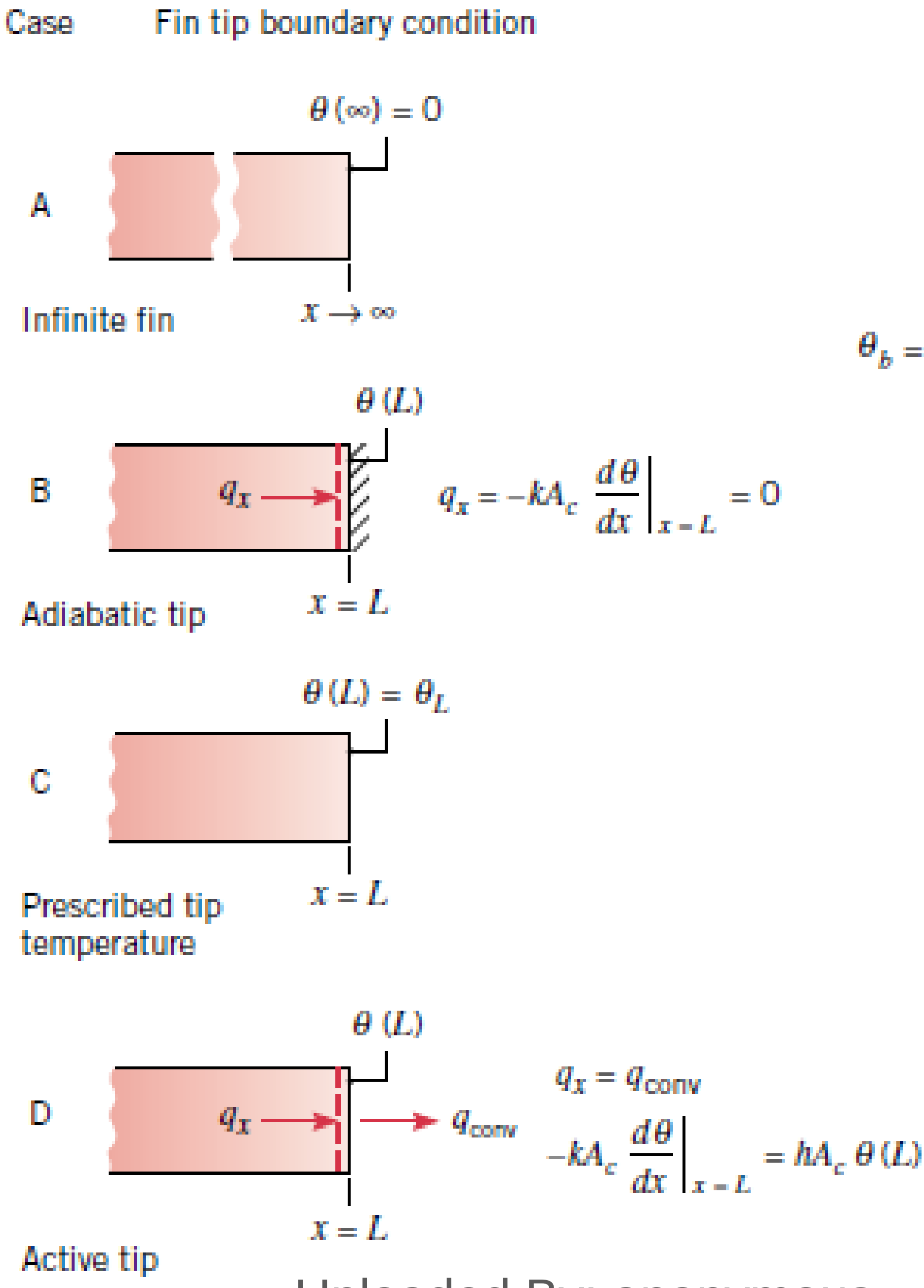
- General solution $\theta(x) = c_1 e^{mx} + c_2 e^{-mx}$ constants c_1 and c_2 evaluated from first B.C

B.C

- first B.C at the base (x=0) where T=T_b

$$T(x=0)=T_b \qquad \theta(x=0)=T_b - T_\infty = \theta_b$$
$$\theta_b = c_1 + c_2$$

- second B.C depends on physical condition at other end
  - Case A: infinite fin (very long fin)
  - Case B: Negligible convection (insulated)or adiabatic tip
  - Case C: Prescribed or finite temperature at tip
  - Case D: active tip (convection at the tip (x=L))



Solutions Case A infinite fin (very long fin)

- Temperature distribution in fin

$$\theta_l = 0 = c_1 e^{-\infty} + c_2 e^{\infty} \rightarrow c_2 = 0 \qquad c_1 + c_2 = \theta_b \Rightarrow c_1 = \theta_b \qquad \theta = \theta_b e^{-mx}$$

- Total heat transfer from fin
    - Energy balance over entire fin

$$q_f = q_{conductionbase}$$

$$q_f = -kA_c \left.\frac{dT}{dx}\right|_{x=0} = -kA_c \left.\frac{d\theta}{dx}\right|_{x=0}$$

$$q_f = -kA_c\left[-m\theta_b e^{-m(0)}\right] = kA_c m\theta_b = kA_c\sqrt{\frac{hP}{kA_c}}\theta_b$$

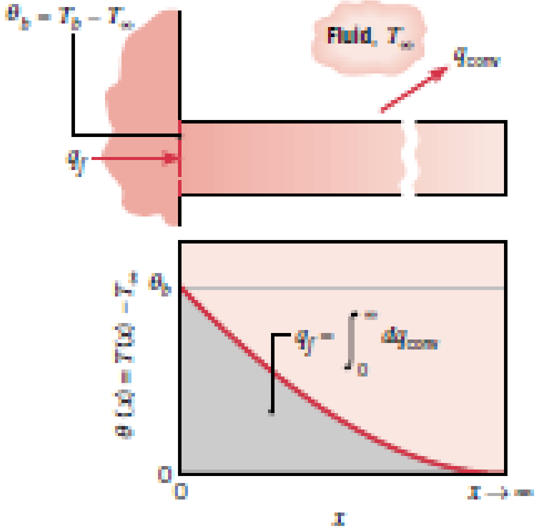
$$q_f = \sqrt{hPkA_c}\,\theta_b$$

- Convection for fin surface

$$q_f = q_{conv} = \int_0^\infty dq_{conv}$$

$$q_f = \int_0^\infty h[T(x) - T_\infty]Pdx = \int_0^\infty h\theta(x)Pdx = hP\theta_b \int_0^\infty e^{-mx}\,dx = hPm^{-1}\theta_b$$

$$q_f = \sqrt{hPkA_c}\,\theta_b$$



Fin temperature and heat loss

Table 16.4 Temperature Distribution and Heat Rate for Fins of Uniform Cross Section

Case	Tip Condition[a] $(x = L)$	Temperature Distribution θ/θ_b[b]		Fin Heat Transfer Rate q_f[c]	
A	Infinite fin $(L \rightarrow \infty)$: $\theta(L) = 0$	e^{-mx}	(16.67)	$M = \sqrt{hPkA_c}\,\theta_b$	(16.68)
		$m = \sqrt{\dfrac{hP}{kA_c}}$	(16.65)		
B	Adiabatic: $d\theta/dx\vert_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$	(16.69)[d]	$M \tanh mL$	(16.70)[d]
C	Prescribed temperature: $\theta(L) = \theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$	(16.71)	$M\dfrac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$	(16.72)
D	Active, convection heat transfer: $h\theta(L) = -kd\theta/dx\vert_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	(16.73)	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$	(16.74)[e]

Example 16.6 p. 381

Example 16.6 The Infinite Fin

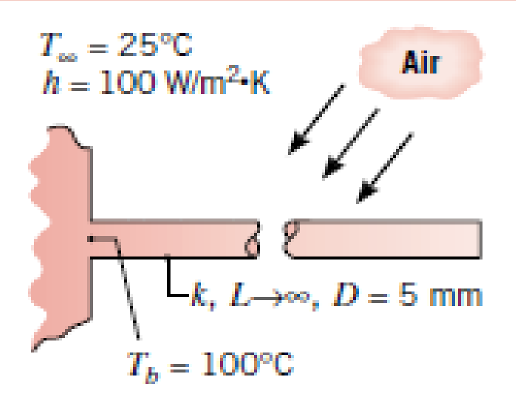
A very long rod 5 mm in diameter has one end maintained at 100°C. The cylindrical (lateral) surface of the rod is exposed to ambient air at 25°C with a convection heat transfer coefficient of 100 W/m$^2 \cdot$ K.
(a) Assuming an infinite length, determine the steady-state temperature distributions along rods constructed from pure copper, 2024 aluminum alloy, and type AISI 316 stainless steel. What are the corresponding fin heat rates from the rods?
(b) How long must the rods be for the assumption of *infinite length* to yield a reasonable estimate of the heat loss?

Solution

Known: A long, circular rod exposed to ambient air.

Find:
(a) Temperature distribution and fin heat rate when rod is fabricated from copper, an aluminum alloy, or stainless steel.
(b) How long rods must be to assume infinite length.

Schematic and Given Data:



Assumptions:
1. Steady-state conditions.
2. One-dimensional conduction along the rod.
3. Infinitely long rod.
4. Negligible radiation exchange with surroundings.
5. Uniform heat transfer coefficient.
6. Constant properties.

Figure E16.6a

Properties: Table HT-1, copper [$T = (T_b + T_\infty)/2 = 62.5°C \approx 335$ K]: $k = 398$ W/m $\cdot$ K. Table HT-1, 2024 aluminum alloy (335 K): $k = 180$ W/m $\cdot$ K. Table HT-1, stainless steel, AISI 316 (335 K): $k = 14$ W/m $\cdot$ K.

Analysis: (a) Subject to the assumption of an infinitely long fin, the temperature distributions are determined from Eq. 16.67, which may be expressed as

$$T = T_\infty + (T_b - T_\infty)e^{-mx}$$

where $m = (hP/kA_c)^{1/2} = (4h/kD)^{1/2}$. Substituting for h and D, as well as for the thermal conductivities of copper, the aluminum alloy, and the stainless steel, respectively, the values of m are 14.2, 21.2, and 75.6 m^{-1}. The temperature distributions may then be computed and plotted as shown in Fig. E16.6*b*

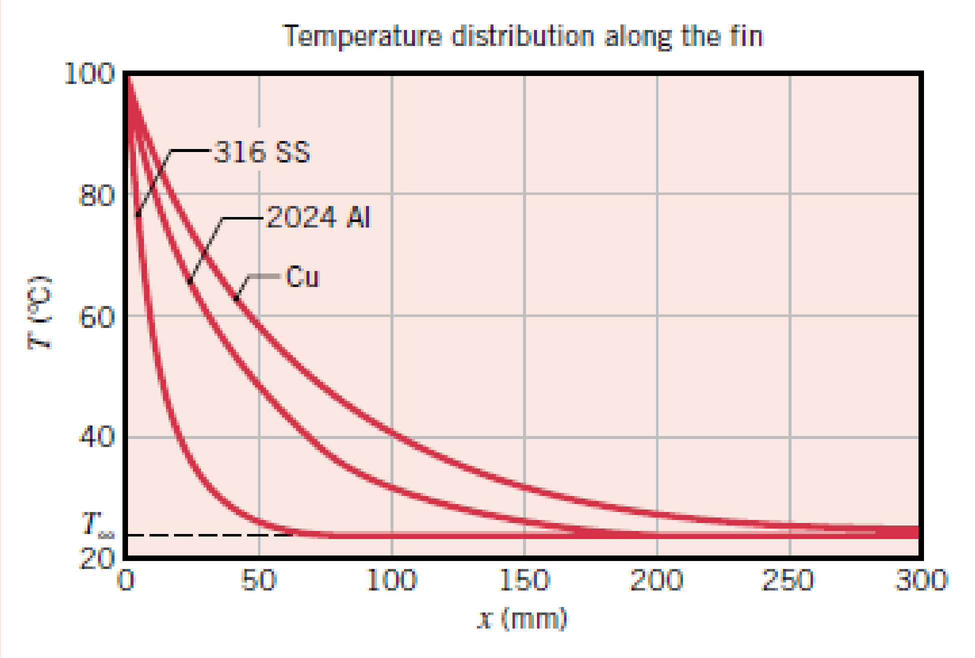


Figure E16.6b

From Eq. 16.68, the fin heat rate is

$$q_f = \sqrt{hPkA_c}\,\theta_b$$

Hence for the copper rod,

$$q_f = [100\ \mathrm{W/m^2 \cdot K}(\pi \times 0.005\ \mathrm{m})(398\ \mathrm{W/m \cdot K})(\pi/4(0.005\ \mathrm{m})^2)]^{1/2}(100 - 25)°\mathrm{C}$$

$$q_f = 8.3\ \mathrm{W} \quad \triangleleft$$

Similarly, for the aluminum alloy and stainless steel rods, respectively, the fin heat rates are q_f = 5.6 W and 1.6 W.

(b) From the temperature distributions in Fig. E16.6*b*, it is evident that there is little additional heat transfer associated with extending the length of the rod much beyond 250, 150, and 50 mm, respectively, for the copper, aluminum alloy, and stainless steel. Note also that the areas under the temperature distributions are in proportion to the fin heat rates for the three materials. (See also Fig. 16.17*d*.)

Comments: Since there is no heat loss from the tip of an infinitely long rod, an estimate of the validity of this approximation may be made by comparing Eqs. 16.70 and 16.68 (Table 16.4). To a satisfactory approximation, the expressions provide equivalent results if $\tanh mL \geq 0.99$ or $mL \geq 2.65$. Hence a rod may be assumed to be infinitely long if

$$L \geq \frac{2.65}{m} = 2.65\left(\frac{kA_c}{hP}\right)^{1/2}$$

For copper,

$$L \geq 2.65\left[\frac{398\ \text{W/m}\cdot\text{K} \times (\pi/4)(0.005\ \text{m})^2}{100\ \text{W/m}^2\cdot\text{K} \times \pi(0.005\ \text{m})}\right]^{1/2} = 187\ \text{mm}$$

Results for the aluminum alloy and stainless steel are $L \geq 126$ mm and $L \geq 35$ mm, respectively. The estimates for the infinite length, based upon inspection of the temperature distributions of Fig. E16.6*b* and summarized in part (b), are in reasonable agreement with the quantitative approach based upon the fin heat rate considered here.

Fin performance

- Fins are added to increase surface area and enhance heat transfer, however additional conductance is added by these fins, hence increase of heat transfer may not be realized.

- Fin effectiveness $\varepsilon_f = {q_{fin}}/{q_{withoutfin}}$

- Fin efficiency $\eta_f = \dfrac{q_f}{q_{\max}}$

Fin effectiveness

$$\varepsilon_f = q_{fin} / q_{withoutfin} \qquad \varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b}$$

Where cross section area at base $A_{c,b}$

- for case A: very long fin (approximate case) $$\varepsilon_f = \frac{\sqrt{hPkA_c}\,\theta_b}{hA_c\theta_b} = \sqrt{\frac{Pk}{hA_c}}$$

- Conclusion from above equation $$\varepsilon_f = \left(\frac{Pk}{hA_c}\right)^{1/2}$$
  1) Choose fin material of high conductivity e.g steel, Al
  2) Design fins to maximize $\dfrac{P}{A_c}$ for example use thin fins.
  3) Fins are used for low "h" values, for example for gases (h low) not for liquids. In case of liquid gas heat exchanger fins are place in the gas side e.g car radiator.

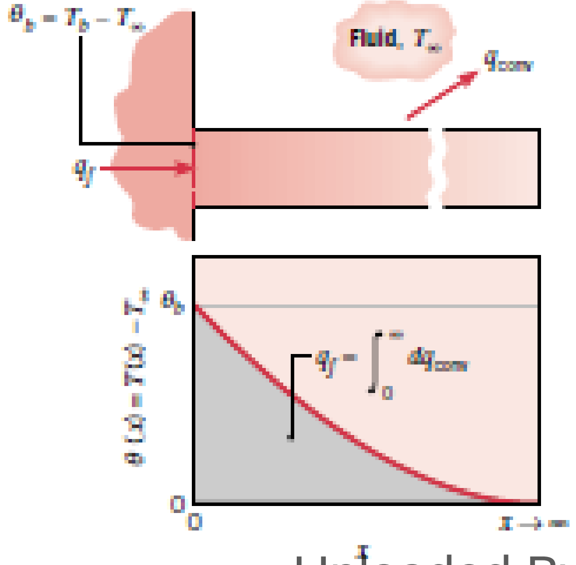
Fin efficiency η_f

- Fin efficiency = $\dfrac{q_f}{q_{\max}}$

- Maximum heat transfer from fin could occur if entire fin is at base temperature, where maximum driving for ΔT exist.
- Actually ΔT decrease along the fin and $T_{(x)} - T_\infty$ is decreasing
- Max temperature difference at the base or when entire fin remains at base temperature.

$$\eta_f = \frac{q_f}{hA_f\theta_b}$$

A_f is the fin surface area



- Hence $q_f = \eta_f \, q_{max}$

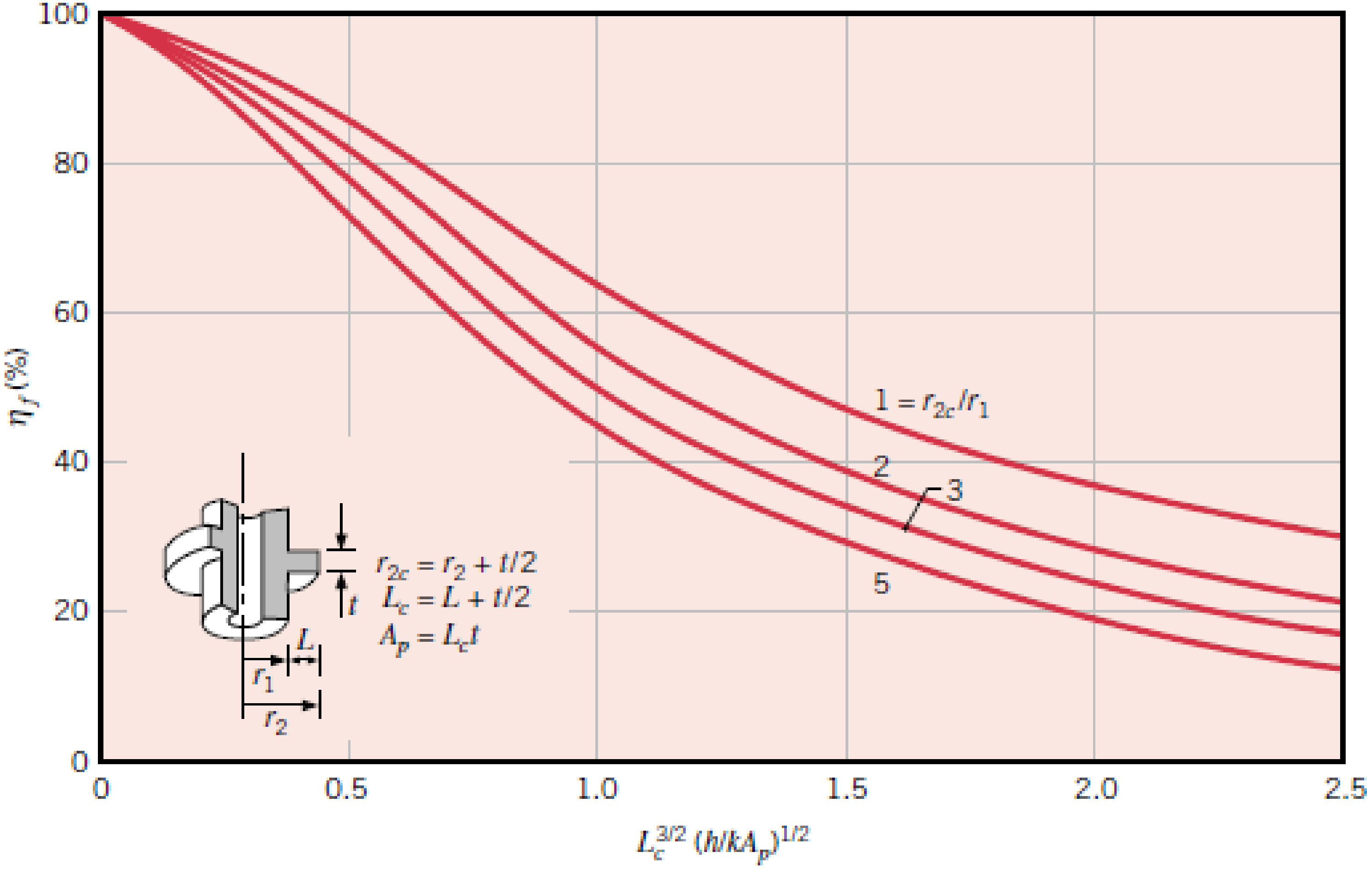
Corrected length

- Converting Active fin to adiabatic using corrected length.
- For rod Correction = D/4
- For rectangular correction thickness/2.

Table 16.4 Temperature Distribution and Heat Rate for Fins of Uniform Cross Section

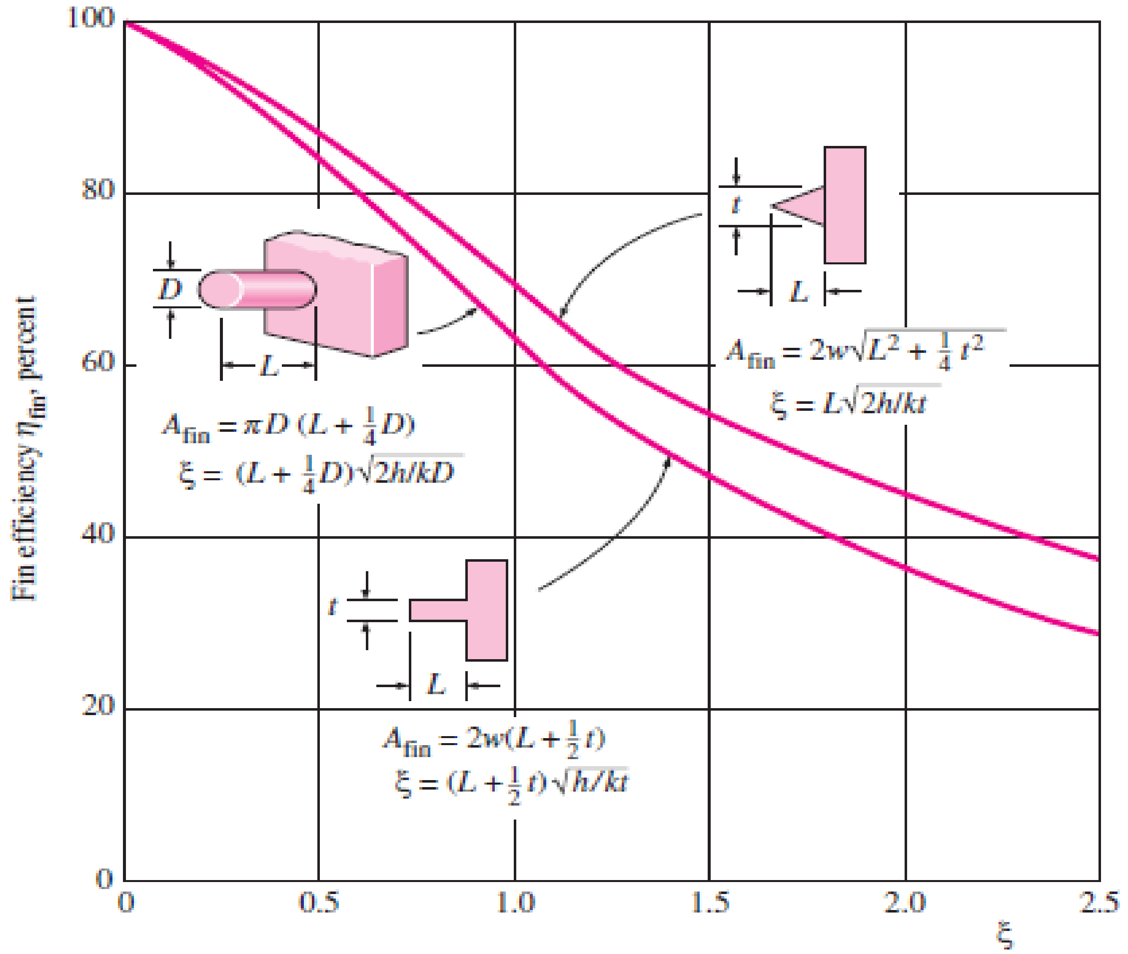
Case	Tip Condition[a] ($x = L$)	Temperature Distribution θ/θ_b[b]		Fin Heat Transfer Rate q_f[c]	
D	Active, convection heat transfer: $h\theta(L) = -kd\theta/dx\vert_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	(16.73)	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$ $M = \sqrt{hPkA_c}\,\theta_b$	(16.74)[e]
B	Adiabatic: $d\theta/dx\vert_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$	(16.69)[d]	$M \tanh mL$	(16.70)[d]

Circumferential fins



- Converting Active fin to adiabatic using corrected length.
- For rod Correction = D/4
- For rectangular correction thickness/2.

Fins efficiency curves



Outline

- Heat diffusion equation
- SS one dimension no heat source
  - plane wall
  - Cylinder
  - Sphere
- SS one dimension with heat source
- Extended surfaces (fins)
- Transient conduction

Transient Conduction

- Many of heat transfer problems are time dependent and unsteady ; they are called transient .
- So far in this chapter only steady state 1- dimension problems were discussed.
- To determine transient temperature distribution within a solid, we can begin by solving the transient heat diffusion equation developed in chapter two.

$$\frac{\partial^{2T}}{\partial x^2} + \dot{q}/k = 1/\alpha \frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Simple case

- In some cases conduction thermal resistance can be very small when compared with the convection from outer surface,
- or the conductivity is very high such that uniform temperature may be assumed at any given instant.
- In such case a simple approach is used which is known as the lumped capacitance method LCM.
- Transient conduction resulting from a change in convection conditions. e.g quenching of a ball.
- Objective to develop temperature as function of time and rate of heat transfer and total heat transfer.

Lumped Capacitance Method

- Solid at some initial temperature experience a sudden change in its thermal environment. e.g. Hot ball is quenched in a liquid at $T\infty$.

- L.C.M assumes a spatially uniform temperature at any instant. This implies, or infinite conductivity or neglected conduction thermal resistance in the solid when compared with heat transfer with the surrounding.

$$\frac{dT}{dx} = 0$$

LCM

- Overall energy balance on the object:
- Initially at time=t =0 temperature is Ti =T (t=0), the rate of heat lost at surface by convection is equal to the rate of change of stored energy (internal energy) of solid.

$$\dot{E}_{st} = -\dot{E}_{out}$$

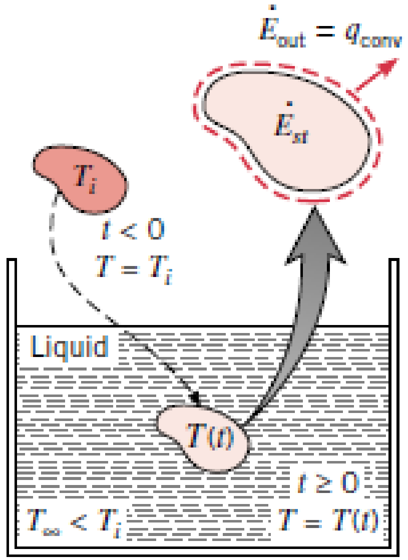
$$\dot{E}_{out} = q_{conv} = hA_s[T(t) - T_\infty]$$

$$\dot{E}_{st} = mc\frac{dT}{dt} = \frac{dU}{dt} = \rho Vc\frac{dT}{dt}$$

Hence

$$-hA_s[T(t) - T_\infty] = \rho Vc\frac{dT}{dt}$$

$$\theta = T - T_\infty \Rightarrow \frac{d\theta}{dt} = \frac{dT}{dt}$$



$$\frac{\rho Vc}{hA_s}\frac{d\theta}{dt} = -\theta$$

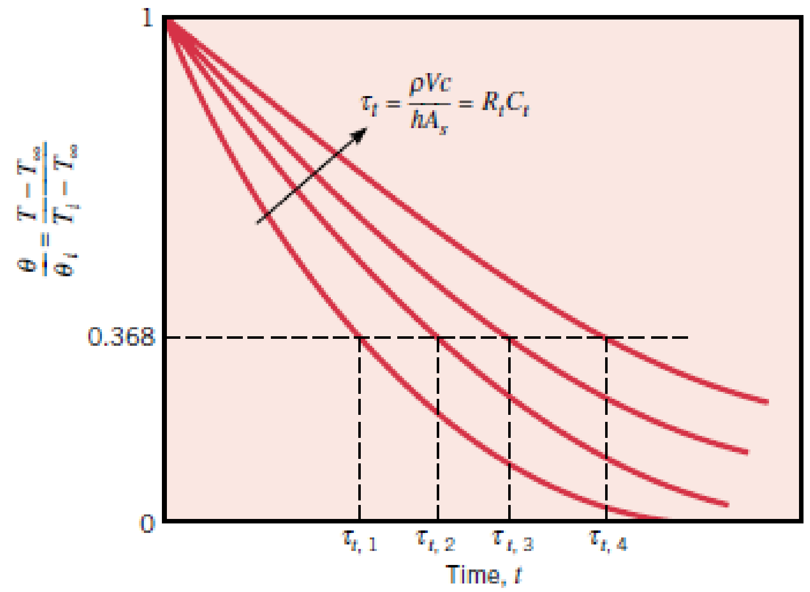
$$\frac{\rho Vc}{hA_s}\int_{\theta i}^{\theta}\frac{d\theta}{\theta} = -\int_{t=0}^{t}dt$$

$$-\frac{\rho Vc}{hA_s}\ln\frac{\theta}{\theta_i} = t \rightarrow \ln\frac{\theta}{\theta_i} = -t\left(\frac{hA_s}{\rho Vc}\right)$$

$$\frac{\theta}{\theta_i} = \frac{T(t) - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_s}{\rho Vc}\right)t\right]$$

LCM

- Thermal time constant $\dfrac{\rho Vc}{hA_s} = \tau_t =$ $\tau_t = \dfrac{1}{hA_s}\rho Vc = R_t C_t$



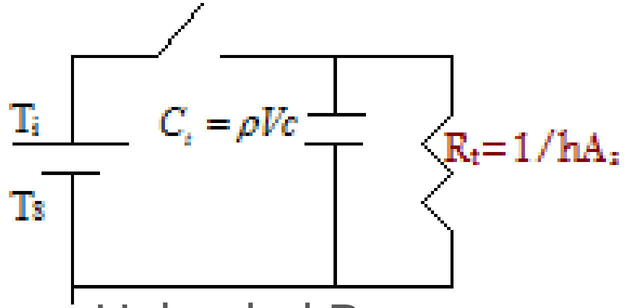
Where (Rt=1/hAs) thermal resistance of convection.

And Lumped thermal capacitance: $C_t = \rho Vc$

Larger values of $R_t C_t = \tau_t = \dfrac{1}{hA_s}\rho Vc$

Means slower response of system it takes longer to cool down (T $\rightarrow$ T∞ or Ɵ =0).

Analogous to voltage decay that occurs when a capacitor is discharged through a resistor in an electrical RC circuit.



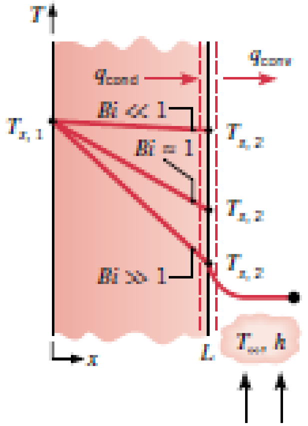
Validity of LCM

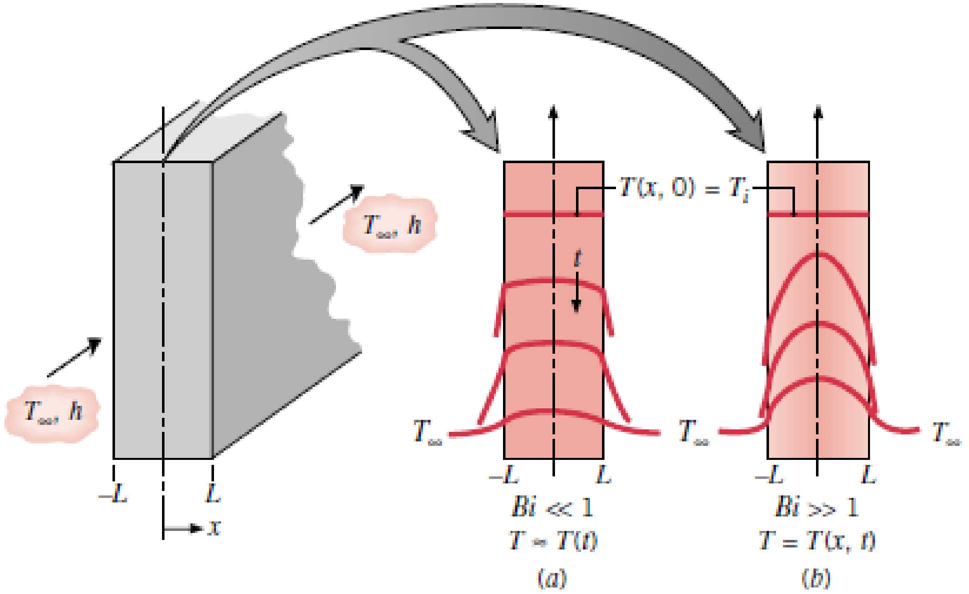
- At steady state $\qquad q_{cond} = q_{conv}.$

$$\frac{kA}{L}(T_{s1} - T_{s2}) = hA_s(T_{s2} - T\infty)$$

$$\frac{T_{s1} - T_{s2}}{T_{s2} - T_{\infty}} = \frac{L/kA}{1/hA_s} = \frac{R_{cond}}{R_{conv}} = \frac{hL}{k}$$

$$\text{Biot number} = \frac{R_{cond}}{R_{conv}} = \frac{hL}{k}$$





Biot number is the ratio of conduction resistance to the convection resistance. Bi: dimensionless group.

Bi<<1, R_{cond} << R_{conv} uniform temperature may be assumed in solid. Bi<0.1

In general

$$Bi = \frac{hl_c}{k} = \frac{hV/A_s}{k}$$

L_c: characteristics length. $L_c = V/A_s$ = Volume/Surface Area. For :

a) 2L thick wall $L_c = L$
b) Long cylinder $L_c = r_0/2$.
c) Sphere $L_c = r_0/3$.

Biot & Fourier numbers

$$\theta = \theta_i e^{\frac{-hA_s}{\rho Vc}t}$$

$$\frac{t}{\tau} = \frac{h(A_s)}{\rho Vc}t = \frac{h}{\rho c L_c}t \times \frac{L_c k}{L_c k} \Rightarrow \frac{hL_c}{k}\left(\frac{k}{\rho c}\right)\frac{t}{L_c{}^2}$$

$$\left(\frac{hL_c}{k}\right)\left(\frac{\alpha t}{L_c{}^2}\right) = \text{Bi.Fo}$$

where :

$$Bi = \frac{hL_c}{k} = \frac{hV}{kA_s}$$

$$Fo = \frac{\alpha t}{L_c{}^2}$$

Fo: Fourier number.

$$\frac{\theta}{\theta_i} = e^{\frac{-(hA_s)}{(\rho VC)}t} = e^{-Bi.Fo}$$

Heat transfer

- Energy transfer Q occurring from t=o up to time‘t’ is given as

$$Q = \int_0^t q dt = \int hA_s (T - T_\infty) dt = \int hA_s \theta dt =$$

$$Q = hA_s \int_0^t \theta_i e^{\frac{-t}{\tau}} dt = hAs\theta_i \int_0^t e^{\frac{-t}{\tau}} dt$$ = Q (t) = $\rho Vc\, \Theta_i\, [1- e^{-t/\tau}]$

Example 16.8 p. 390 Moran 2003.

Example 16.8 Lumped Capacitance Method: Cooling Process

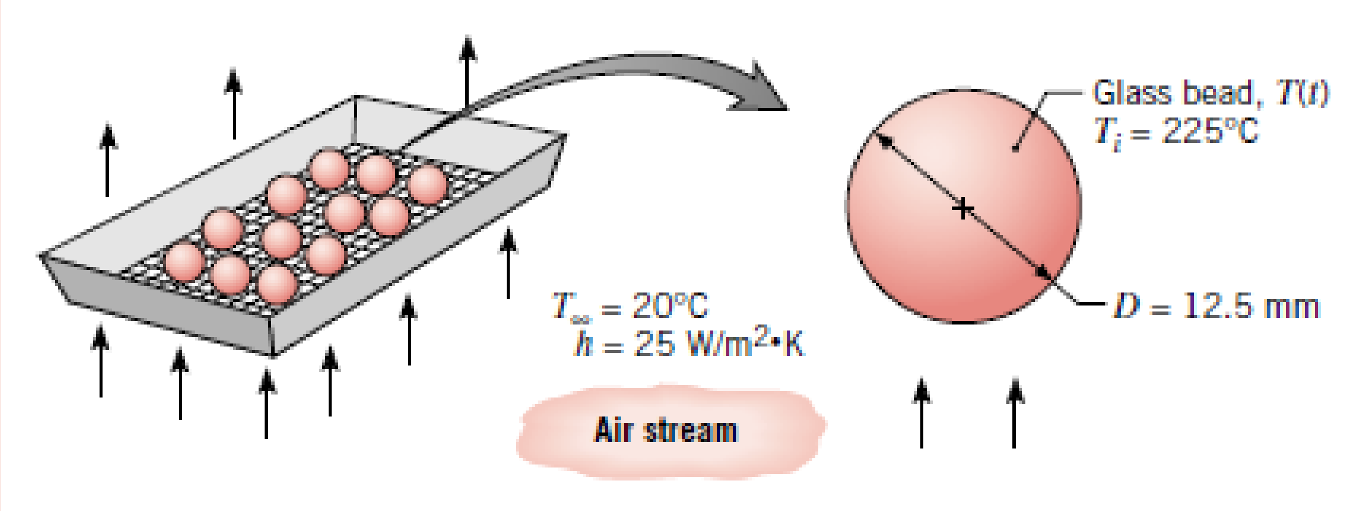
In a materials evaluation program, dielectric-coated glass beads of 12.5 mm diameter are removed from a process oven with a uniform temperature of 225°C. The beads are cooled in an air stream for which $T_\infty = 20°C$ and the convection coefficient is 25 $W/m^2 \cdot K$. What is the temperature of a bead after 6 min?

Solution

Known: A glass bead, initially at a uniform temperature, is suddenly subjected to a convection cooling process.

Find: Temperature of the glass bead after 6 min.

Schematic and Given Data:



Assumptions:

1. Temperature of the bead is uniform at any instant.
2. The coating has negligible thermal resistance and capacitance.
3. Radiation exchange with the surroundings is negligible.
4. Constant properties.

Figure E16.8

Properties: Table HT-2, glass, Pyrex (300 K): $\rho = 2225\ kg/m^3$, $c = 835\ J/kg \cdot K$, $k = 1.4\ W/m \cdot K$.

Analysis: To establish the validity of the lumped capacitance method, calculate the Biot number. From Eq. 16.90, the characteristic length of the spherical bead is

$$L_c = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6}$$

and using Eq. 16.89, determine the Biot number,

$$Bi = \frac{hL_c}{k} = \frac{h(D/6)}{1.4} = \frac{25\ \text{W/m}^2\cdot\text{K}\ (.0125\ \text{mm}/6)}{1.4\ \text{W/m}\cdot\text{K}} = 0.037$$

Accordingly, $Bi < 0.1$ so that the bead has a nearly uniform temperature during the cooling process. Using Eq. 16.85, with $L_c = D/6$ the temperature $T(t)$ after 6 min is

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{h}{\rho L_c c}\right)t\right]$$

$$\frac{T(t) - 20°\text{C}}{(225 - 20)°\text{C}} = \exp\left[-\left(\frac{25\ \text{W/m}^2\cdot\text{K}}{2225\ \text{kg/m}^3\ (0.0125\ \text{m}/6)\ 835\ \text{J/kg}}\right)360\ \text{s}\right]$$

$$T(t) = 20°\text{C} + (225 - 20)°\text{C} \times 0.0978 = 40.0°\text{C} \quad \triangleleft$$

End of conduction

