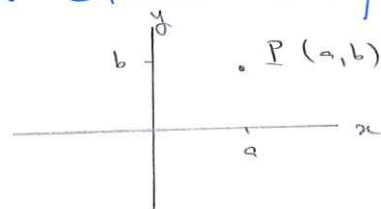


Ch 12. Vectors and The Geometry of Space:

12.1 Three-dimensional Coordinate Systems:

- The point $P(a, b) \in \mathbb{R}^2$ "Two dimensional space".

where $\mathbb{R}^2 = \{(a, b) : a, b \in \mathbb{R}\}$
(4 quadrants)



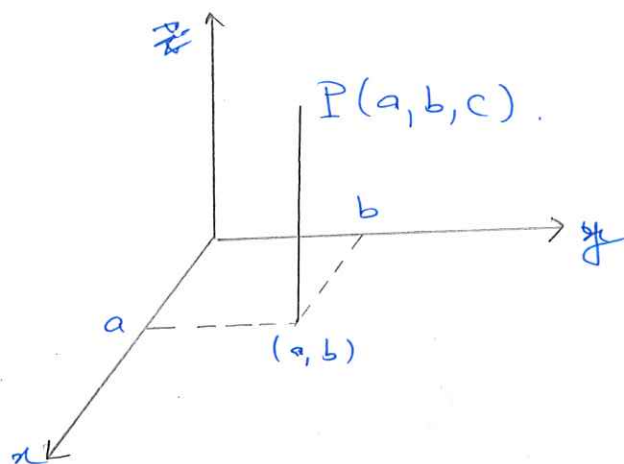
- The point $P(a, b, c) \in \mathbb{R}^3$ "Three dimensional space".

a : x-coordinate

b : y-coordinate

c : z-coordinate

(8 octant)



Geometric description in $\mathbb{R}^3$:

- $x = -3$, The plane perpendicular to x -axis at $x = -3$ and parallel to yz -plane.

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- $y = 4$, The plane $\perp$ to y -axis at $y = 4$ and $\parallel$ to xz -plane.

- $z = 5$, The plane $\perp$ to z -axis at $z = 5$ and $\parallel$ to xy -plane. (91)

4. $x = 2$ and $y = 3$:

The line in which the planes $x = 2$ & $y = 3$ intersect. This line passes through $(2, 3, 0)$ and $\parallel$ z -axis & $\perp$ to xy -plane.

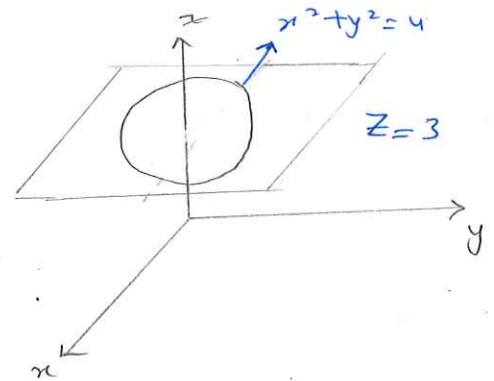
5. $z \geq 0$: The half-space consisting of the points on the above the xy -plane.

6. $x \geq 0, y \geq 0, z \geq 0$: The first octant.

7. $-1 \leq y \leq 1$: The slab ^{as a plate} between $y = -1$ & $y = 1$

8. $x^2 + y^2 = 4$ & $z = 3$:

Circle in the plane $z = 3$
center $(0, 0, 3)$



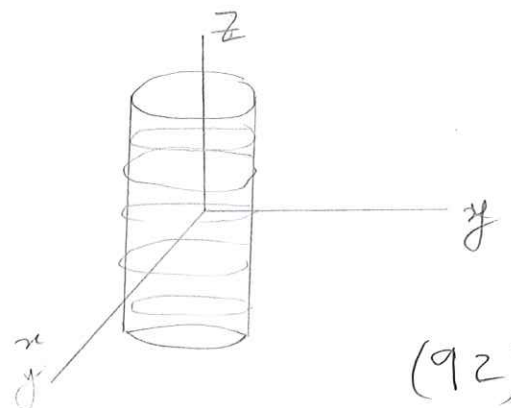
9. $x^2 + y^2 \leq 1, z = 3$:

Disk in the plane $z = 3$, with center $(0, 0, 3)$

10. $x^2 + y^2 \leq 1$

cylindrical solid with $r = 1$

and z axis $\perp$ center.



11. $z = 0, x \leq 0, y \geq 0$

second quadrant
in xy -plane

Distance and spheres in Space :

The distance between $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$|P_1 P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Example: Find the distance between $P_1(1, 1, 1)$ & $P_2(2, 3, 5)$

$$|P_1 P_2| = \sqrt{(1-2)^2 + (1-3)^2 + (1-5)^2} = \sqrt{1+4+16} = \sqrt{21}$$

The standard equation for the sphere of Radius a and Center (x_0, y_0, z_0) is

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = a^2$$

Example: Find the center and radius of

$$x^2 + y^2 + z^2 + 3x - 4z + 1 = 0$$

$$\left(x + \frac{3}{2}\right)^2 + y^2 + (z - 2)^2 = -1 + \frac{9}{4} + 4 = \frac{21}{4}$$

$\therefore$ Center $(-\frac{3}{2}, 0, 2)$ & radius $\sqrt{21}/2$.

Example: ① $x^2 + y^2 + z^2 > 4$: Exterior of the sphere
 $x^2 + y^2 + z^2 = 4$

② $x^2 + y^2 + z^2 < 4$: The Interior of the sphere $x^2 + y^2 + z^2 = 4$

③ $x^2 + y^2 + z^2 \leq 4$: The sphere with its Interior.

④ $x^2 + y^2 + z^2 = 4$, $z \leq 0$: The lower hemisphere

cut from the sphere $x^2 + y^2 + z^2 = 4$ by xy plane
(plane $z=0$) (93)

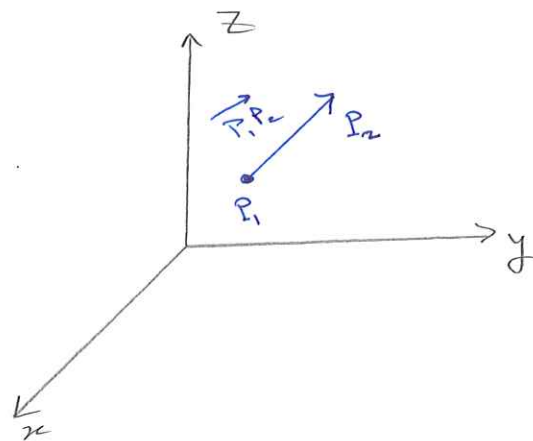
12.2 Vectors

Def: A vector is a directed line segment from Initial point $P_1(x_1, y_1, z_1)$ and terminal point $P_2(x_2, y_2, z_2)$ with length $|P_1P_2| = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$

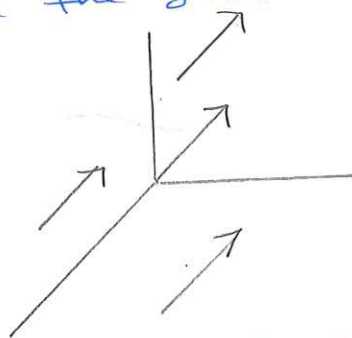
Note: we usually give names to vectors:

$\vec{u}, \vec{v}, \vec{w}, \vec{z}, \dots$

OR (Bold) u, v, w, z .



Note: Vectors are equal if they have the same length and direction.



Note: The vector is in standard position if the Initial point is the origin. (i.e.) $P_1 = (0, 0, 0)$

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$P_2(x_2, y_2, z_2)$

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Component form:

Def: If $\vec{v}$ is a vector in 2D equal to the vector in the standard position with terminal point (v_1, v_2) , then the Component form of $\vec{v}$ is

$$\vec{v} = \langle v_1, v_2 \rangle.$$

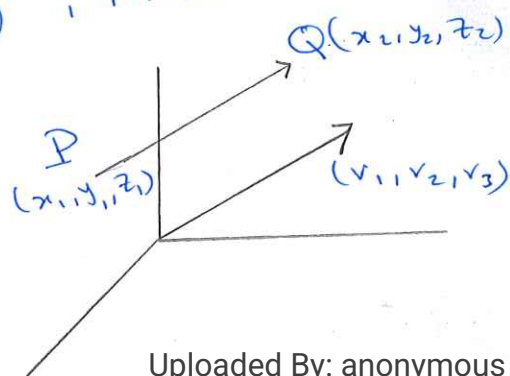
Similarly: If $\vec{v}$ is a three dimensional vector equal to a vector in the standard position with terminal point (v_1, v_2, v_3) , then the Component form of $\vec{v}$ is

$$\vec{v} = \langle v_1, v_2, v_3 \rangle.$$

Note: If $\vec{v} = \langle v_1, v_2, v_3 \rangle$ is represented by $\overrightarrow{PQ}$

where $P(x_1, y_1, z_1)$ & $Q(x_2, y_2, z_2)$, then:

$$\left. \begin{array}{l} x_1 + v_1 = x_2 \\ y_1 + v_2 = y_2 \\ z_1 + v_3 = z_2 \end{array} \right\} \Rightarrow \begin{array}{l} v_1 = x_2 - x_1 \\ v_2 = y_2 - y_1 \\ v_3 = z_2 - z_1 \end{array}$$



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(i.e) $\vec{v} = \langle v_1, v_2, v_3 \rangle = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle.$

Note: If $\vec{v} = \langle v_1, v_2, v_3 \rangle$ & $\vec{u} = \langle u_1, u_2, u_3 \rangle$

then $\vec{v} = \vec{u}$ if $v_1 = u_1, v_2 = u_2$ & $v_3 = u_3.$

Note: If $\vec{v} = \langle v_1, v_2, v_3 \rangle = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$

then: $|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Note: The only vector with length zero is $\vec{0}$ with no specific direction.

Example: Find (a) The Component form
(b) The length

of the vector with Initial point $P(-3, 4, 1)$ & $Q(-5, 2, 2)$

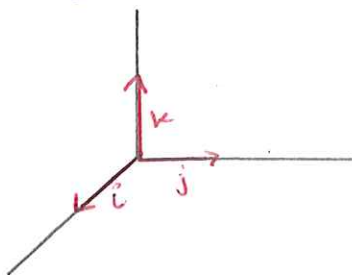
(a) $\vec{PQ} = \langle v_1, v_2, v_3 \rangle = \langle -5 - (-3), 2 - 4, 2 - 1 \rangle = \langle -2, -2, 1 \rangle$

(b) length of $\vec{PQ} = \sqrt{(-2)^2 + (-2)^2 + (1)^2} = \sqrt{9} = 3$.

Def: A vector v of length 1 is called
a Unit vector.

The standard Unit vectors are:

$$\hat{i} = \langle 1, 0, 0 \rangle, \quad \hat{j} = \langle 0, 1, 0 \rangle, \quad \hat{k} = \langle 0, 0, 1 \rangle.$$



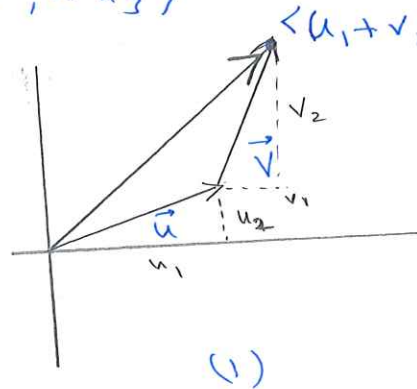
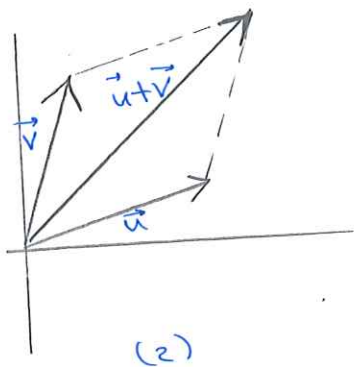
Vectors Algebra Operations:

Def: Let $\vec{u} = \langle u_1, u_2, u_3 \rangle$ & $\vec{v} = \langle v_1, v_2, v_3 \rangle$

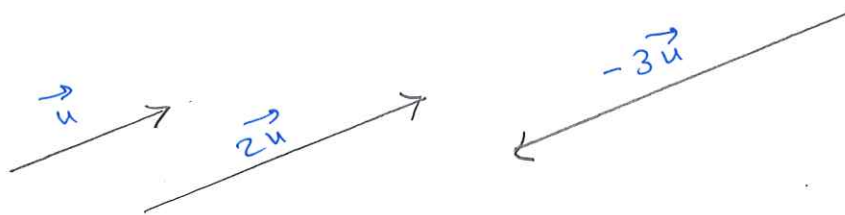
& k be a scalar, then:

$$\vec{u} + \vec{v} = \langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle$$

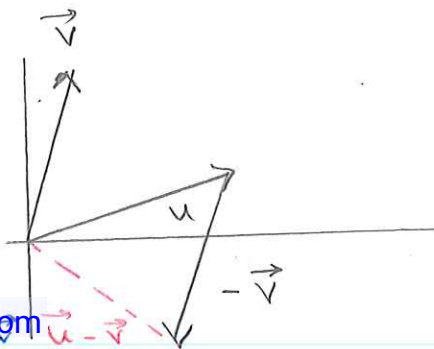
& $k \vec{u} = \langle k u_1, k u_2, k u_3 \rangle$



addition



Scalar multiplication



difference

$$\begin{aligned} (*) \cdot |\alpha \vec{u}| &= |\langle \alpha u_1, \alpha u_2, \alpha u_3 \rangle| = \sqrt{(\alpha u_1)^2 + (\alpha u_2)^2 + (\alpha u_3)^2} \\ &= \sqrt{\alpha^2 (u_1^2 + u_2^2 + u_3^2)} = |\alpha| \sqrt{u_1^2 + u_2^2 + u_3^2} \end{aligned}$$

$$\Rightarrow \underbrace{|\alpha \vec{u}|}_{\text{length}} = |\alpha| \underbrace{|\vec{u}|}_{\text{length}}$$

Example: Let $\vec{u} = \langle -1, 3, 1 \rangle$ & $\vec{v} = \langle 4, 7, 0 \rangle$.

Find the components of

(a) $2\vec{u} + 3\vec{v}$

(b) $\vec{u} - \vec{v}$

(c) $|\frac{1}{2}\vec{u}|$.

(a) $2\vec{u} + 3\vec{v} = \langle -2, 6, 2 \rangle + \langle 12, 21, 0 \rangle = \langle 10, 27, 2 \rangle$

(b) $\vec{u} - \vec{v} = \langle -1-4, 3-7, 1-0 \rangle = \langle -5, -4, 1 \rangle$.

(c) $|\frac{1}{2}\vec{u}| = |\frac{1}{2}| |\vec{u}| = \frac{1}{2} \sqrt{(1) + (9) + (1)} = \frac{1}{2}\sqrt{11}$

Properties of vector Operations:

Let $\vec{u}, \vec{v}, \vec{w}$ be vectors, a, b and c be scalars

1. $\vec{u} + \vec{v} = \vec{v} + \vec{u}$.

2. $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$.

3. $(\vec{u}) + \vec{0} = \vec{u}$

4. $0\vec{u} = \vec{0}$

5. $\vec{u} + (-\vec{u}) = \vec{0}$.

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6. $1\vec{u} = \vec{u}$.

7. $a(b\vec{u}) = a(b\vec{u})$

8. $a(\vec{u} + \vec{v}) = a\vec{u} + a\vec{v}$.

9. $(a+b)\vec{u} = a\vec{u} + b\vec{u}$.

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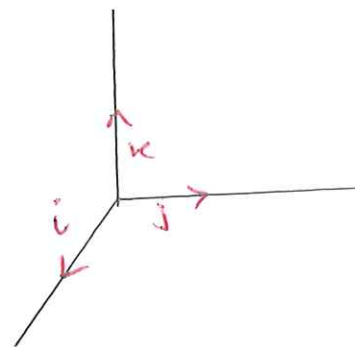
Def: A unit vector $\vec{v}$ is a vector of length 1.

• Standard Unit vectors are:

$$\vec{i} = \langle 1, 0, 0 \rangle \Rightarrow |\vec{i}| = 1$$

$$\vec{j} = \langle 0, 1, 0 \rangle \Rightarrow |\vec{j}| = 1$$

$$\vec{k} = \langle 0, 0, 1 \rangle \Rightarrow |\vec{k}| = 1$$



Note: Any vector $\vec{v} = \langle v_1, v_2, v_3 \rangle$ can be written as a linear combination of the standard unit vectors.

$$\begin{aligned}\vec{v} = \langle v_1, v_2, v_3 \rangle &= \langle v_1, 0, 0 \rangle + \langle 0, v_2, 0 \rangle + \langle 0, 0, v_3 \rangle \\ &= v_1 \langle 1, 0, 0 \rangle + v_2 \langle 0, 1, 0 \rangle + v_3 \langle 0, 0, 1 \rangle \\ &= v_1 \vec{i} + v_2 \vec{j} + v_3 \vec{k}\end{aligned}$$

v_1 is the i -component of $\vec{v}$

v_2 is the j -component of $\vec{v}$

v_3 is the k -component of $\vec{v}$.

• If $P(x_1, y_1, z_1)$, $Q(x_2, y_2, z_2)$, then:

$$\vec{PQ} = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (z_2 - z_1)\vec{k}$$

Example: $\vec{v} = \langle 2, 5, 3 \rangle = 2\vec{i} + 5\vec{j} + 3\vec{k}$

Note: If $\vec{v}$ is any non zero vector, then

$\frac{\vec{v}}{|\vec{v}|}$ is a unit vector and is called the direction of $\vec{v}$

$$\left| \frac{\vec{v}}{|\vec{v}|} \right| = \frac{1}{|\vec{v}|} \cdot |\vec{v}| = 1$$

Note: Any non zero vector $\vec{v}$ can be written as:

$$\vec{v} = (\text{length}) (\text{Direction of } \vec{v})$$

$$\begin{array}{ccc} \downarrow & & \\ \text{velocity} & = & \underbrace{|\vec{v}| \cdot \left(\frac{\vec{v}}{|\vec{v}|} \right)}_{\text{speed} \quad \text{direction}} \end{array}$$

Example: Find a Unit vector $\vec{u}$ in the direction of the vector from $P(2, 3, 0)$ to $Q(5, 2, 4)$

$$\begin{aligned} \vec{PQ} &= (5-2)\vec{i} + (2-3)\vec{j} + (4-0)\vec{k} \\ &= 3\vec{i} - \vec{j} + 4\vec{k} \end{aligned}$$

$$|\vec{PQ}| = \sqrt{(3)^2 + (-1)^2 + (4)^2} = \sqrt{9+1+16} = \sqrt{26}$$

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$$\therefore \vec{u} = \frac{\vec{PQ}}{|\vec{PQ}|} = \frac{3}{\sqrt{26}}\vec{i} - \frac{1}{\sqrt{26}}\vec{j} + \frac{4}{\sqrt{26}}\vec{k}$$

Example: If $\vec{v} = 3\vec{i} - 4\vec{j}$ is a velocity vector.

express $\vec{v}$ as a product of its speed times a unit vector in the direction of motion.

• Speed is the length of $\vec{v}$.

$$\therefore |\vec{v}| = \sqrt{(3)^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5.$$

$$\text{Unit vector } \frac{\vec{v}}{|\vec{v}|} = \frac{3}{5}\vec{i} - \frac{4}{5}\vec{j}.$$

$$\text{So } \underline{\vec{v} = 5 \left(\frac{3}{5}\vec{i} - \frac{4}{5}\vec{j} \right)}.$$

Example: Let $\vec{u} = 2\vec{i} + \vec{j}$, $\vec{v} = \vec{i} + \vec{j}$, $\vec{w} = \vec{i} - \vec{j}$.

Find a & b (scalars) (s.t.) $\vec{u} = a\vec{v} + b\vec{w}$.

$$2\vec{i} + \vec{j} = a(\vec{i} + \vec{j}) + b(\vec{i} - \vec{j})$$

$$2\vec{i} + \vec{j} = (a+b)\vec{i} + (a-b)\vec{j}$$

$$\left. \begin{array}{l} a+b \\ 1 = a-b \end{array} \right\} \Rightarrow a = \frac{3}{2}, \quad b = \frac{1}{2}.$$

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Def: The midpoint of the line PQ , where $P(x_1, y_1, z_1)$

& $Q(x_2, y_2, z_2)$ is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2} \right)$.

$$\vec{OM} = \frac{x_1+x_2}{2}\vec{i} + \frac{y_1+y_2}{2}\vec{j} + \frac{z_1+z_2}{2}\vec{k}.$$

12.3 The dot product:

← (Number)

Def: The dot product " $\vec{u} \cdot \vec{v}$ " of the vectors

$$\vec{u} = \langle u_1, u_2, u_3 \rangle \quad \& \quad \vec{v} = \langle v_1, v_2, v_3 \rangle \text{ is}$$

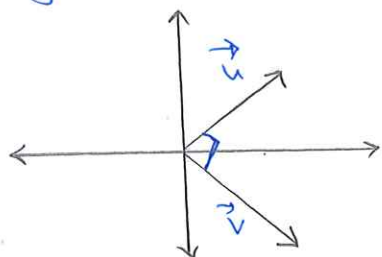
$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3.$$

Example: If $\vec{u} = \langle 2, -5, 0 \rangle$ & $\vec{v} = \frac{1}{2}\vec{i} + \frac{1}{3}\vec{j} + \vec{k}$

$$\text{then } \vec{u} \cdot \vec{v} = 2\left(\frac{1}{2}\right) + (-5)\left(\frac{1}{3}\right) + 0(1) = 1 - \frac{5}{3} = -\frac{2}{3}.$$

Example: If $\vec{u} = \vec{i} + \vec{j}$, $\vec{v} = \vec{i} - \vec{j}$, then

$$\vec{u} \cdot \vec{v} = (1)(1) - (1)(1) = 0$$



Def: vectors $\vec{u}$ & $\vec{v}$ are orthogonal (perpendicular)

$$\text{iff } \vec{u} \cdot \vec{v} = 0.$$

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Example: $\vec{0} \perp$ for any nonzero vector $\vec{v}$ since $\vec{0} \cdot \vec{v} = 0$

$$\vec{u} = \vec{i}, \quad \vec{v} = \vec{k}$$

Example:

$$\vec{u} \cdot \vec{v} = 1(0) + 0(0) + 0(1) = 0 \Rightarrow \vec{i} \perp \vec{k}.$$

Thm: The Angle between two vectors

The angle θ between two non-zero vectors $\vec{u} = \langle u_1, u_2, u_3 \rangle$ and $\vec{v} = \langle v_1, v_2, v_3 \rangle$ is given by:

$$\theta = \cos^{-1} \left(\frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|} \right).$$

Example: Find the angle between $\vec{u}$ & $\vec{v}$ in the following:

$$1) \quad \vec{u} = 2\vec{i} - 3\vec{j} + \sqrt{12}\vec{k}$$

$$\vec{v} = -2\vec{i} + 3\vec{j} - \sqrt{12}\vec{k}.$$

$$\theta = \cos^{-1} \left(\frac{2(-2) + (3)(-3) + (\sqrt{12})(-\sqrt{12})}{\sqrt{(2)^2 + (-3)^2 + (\sqrt{12})^2} \cdot \sqrt{(-2)^2 + (3)^2 + (-\sqrt{12})^2}} \right) = \cos^{-1} \left(\frac{-25}{25} \right) = \cos^{-1}(-1)$$

$$\Rightarrow \theta = \pi$$

$$2) \quad \vec{u} = \vec{i} + \vec{j}, \quad \vec{v} = \vec{i} - \vec{j}.$$

$$\theta = \cos^{-1} \left(\frac{0}{\sqrt{2} \cdot \sqrt{2}} \right) = \cos^{-1}(0) = \frac{\pi}{2}. \quad (\vec{u} \perp \vec{v})$$

Properties of the Dot Product:

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If $\vec{u}$ & $\vec{v}, \vec{w}$ are vectors & c is a scalar, then

$$1. \quad \vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$$

$$2. \quad (c\vec{u}) \cdot \vec{v} = \vec{u} \cdot (c\vec{v}) = c(\vec{u} \cdot \vec{v}).$$

$$3. \quad \vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$$

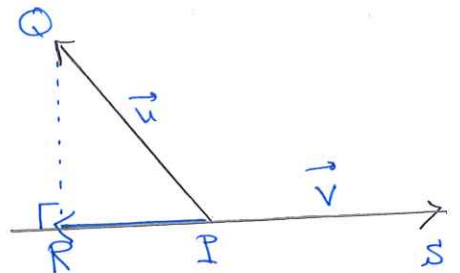
$$4. \vec{u} \cdot \vec{u} = |\vec{u}|^2 \quad (\text{length})^2.$$

$$5. \vec{0} \cdot \vec{u} = 0.$$

proof [4]: $\vec{u} \cdot \vec{u} = \langle u_1, u_2 \rangle \cdot \langle u_1, u_2 \rangle = u_1^2 + u_2^2 = |\vec{u}|^2$

Vector projection:

The vector projection of $\vec{u} = \vec{PQ}$ onto a non-zero vector $\vec{v} = \vec{PS}$ is the vector



$\vec{PR}$ determined by dropping a perpendicular from Q to $\vec{PS}$

The projection of $\vec{u}$ onto $\vec{v}$ is defined by:

$$\text{proj}_{\vec{v}} \vec{u} = \underbrace{(|\vec{u}| \cos \theta)}_{\text{Scalar Component of } \vec{u}} \underbrace{\frac{\vec{v}}{|\vec{v}|}}_{\text{Direction of } \vec{v}}, \quad \theta \text{ between } \vec{u} \text{ \& } \vec{v}$$

Note: $\text{proj}_{\vec{v}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|} \cdot \frac{\vec{v}}{|\vec{v}|} = \left(\frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \right) \vec{v}$

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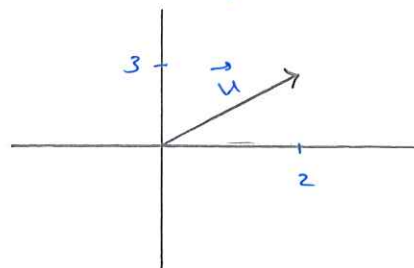
Example: Find the vector projection of $\vec{u} = 6\vec{i} + 3\vec{j} + 2\vec{k}$

onto $\vec{v} = \vec{i} - 2\vec{j} - 2\vec{k}$.

$$\begin{aligned} \text{proj}_{\vec{v}} \vec{u} &= (\text{scalar component of } \vec{u}) \cdot (\text{Direction of } \vec{v}) \\ &= \left(\frac{\vec{u} \cdot \vec{v}}{|\vec{v}|} \right) \frac{\vec{v}}{|\vec{v}|} = \frac{6(1) + 3(-2) + 2(-2)}{\sqrt{1+4+4}} \cdot \frac{\vec{i} - 2\vec{j} - 2\vec{k}}{\sqrt{1+4+4}} \\ &= \frac{-4}{3} \cdot \frac{\vec{i} - 2\vec{j} - 2\vec{k}}{3} = -\frac{4}{9}\vec{i} + \frac{8}{9}\vec{j} + \frac{8}{9}\vec{k}. \end{aligned}$$

Example: For $\vec{u} = a\vec{i} + b\vec{j}$ find the slope.

(for example: $\vec{u} = 2\vec{i} + 3\vec{j}$):



The slope of $\vec{u}$ is $\frac{3}{2}$

(i.e) The slope of $\vec{u} = a\vec{i} + b\vec{j}$ is $\frac{b}{a}$, $a \neq 0$

Example (32) show that the vector $\vec{v} = a\vec{i} + b\vec{j}$ is parallel to the line $bx - ay = c$.

sol: Slope of $\vec{v} = \frac{b}{a}$, $a \neq 0$

& slope of $ay = bx - c \Leftrightarrow$ slope of $y = \frac{b}{a}x - \frac{c}{a}$
which is $\frac{b}{a}$, $a \neq 0$

$\therefore \vec{v} \parallel bx - ay = c$.

Example: (37) Find an equation of a line that is

parallel to the vector $\vec{v} = \vec{i} - \vec{j}$ & passes through $(-2, 1)$.
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$$\text{slope} = \frac{-1}{1} = -1 = \frac{b}{a}$$

$$\therefore y - 1 = -1(x - (-2))$$

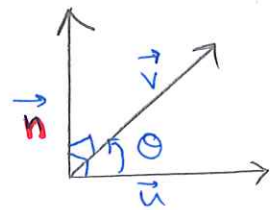
$$\therefore y = -x - 1$$

12.4 The Cross Product.

Def: If $\vec{u}$ & $\vec{v}$ are any non zero vectors, and if we select a unit vector $\vec{n}$ that is perpendicular to the plane $\vec{u}\vec{v}$ by the right hand rule, then:

The cross product denoted by $\vec{u} \times \vec{v}$ is defined by:

$$\vec{u} \times \vec{v} = \underbrace{(|\vec{u}||\vec{v}|\sin\theta)}_{\text{number}} \underbrace{\vec{n}}_{\text{vector}}$$



Note: Non Zero vectors $\vec{u}$ & $\vec{v}$ are parallel iff $\vec{u} \times \vec{v} = \vec{0}$.

Since $\vec{u} \times \vec{v} = \vec{0}$ iff $\theta = 0$ or $\theta = \pi$.

Properties of the Cross Product:

If $\vec{u}, \vec{v}$ & $\vec{w}$ are any vectors, r, s are scalars, then:

1) $(r\vec{u}) \times (s\vec{v}) = rs(\vec{u} \times \vec{v})$

2) $\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \vec{v} + \vec{u} \times \vec{w}$

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3) $\vec{u} \times \vec{v} = -(\vec{v} \times \vec{u})$

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4) $(\vec{v} + \vec{w}) \times \vec{u} = \vec{v} \times \vec{u} + \vec{w} \times \vec{u}$

5) $\vec{0} \times \vec{u} = \vec{0}$

6) $\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w}$

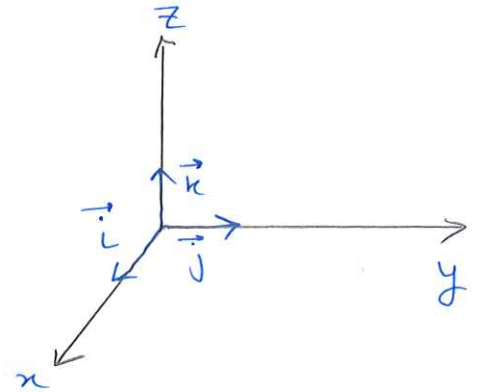
Example: $\vec{u} = 2\vec{i} + \vec{j} - \vec{k}$, $\vec{v} = \vec{i} - 2\vec{j} + 3\vec{k}$
 $\vec{w} = \vec{j} + \vec{k}$.

$$\begin{aligned}\vec{u} \times (\vec{v} \times \vec{w}) &= (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w} \\ &= ((2)(0) + (1)(1) + (-1)(1))[\vec{i} - 2\vec{j} + 3\vec{k}] - [2(-2) - 3] [\vec{j} + \vec{k}] \\ &= 0\vec{v} + 3\vec{w} = 3\vec{j} + 3\vec{k}.\end{aligned}$$

Note: $\vec{i} \times \vec{j} = (|\vec{i}||\vec{j}|\sin\theta)\vec{n}$
 $= (1 \cdot 1 \cdot \sin 90^\circ) \cdot \vec{k} = \vec{k}$

② $\vec{j} \times \vec{k} = \vec{i} = -\vec{k} \times \vec{j}$

③ $\vec{k} \times \vec{i} = \vec{j} = -\vec{i} \times \vec{k}$.



Calculating the Cross product as a Determinant:

If $\vec{u} = u_1\vec{i} + u_2\vec{j} + u_3\vec{k}$
 $\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$, then:

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$$= (u_2v_3 - v_2u_3)\vec{i} - (u_1v_3 - v_1u_3)\vec{j} + (u_1v_2 - v_1u_2)\vec{k}$$

proof: $\vec{u} \times \vec{v} = (u_1\vec{i} + u_2\vec{j} + u_3\vec{k}) \times (v_1\vec{i} + v_2\vec{j} + v_3\vec{k})$

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Example: Find $\vec{u} \times \vec{v}$ if $\vec{u} = 2\vec{i} - 3\vec{j} + \vec{k}$, $\vec{v} = \vec{i} - 4\vec{k}$

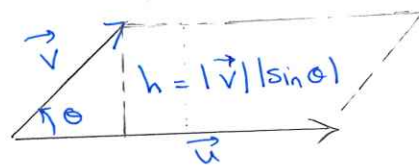
$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -3 & 1 \\ 1 & 0 & -4 \end{vmatrix} = 12\vec{i} + 9\vec{j} + 3\vec{k}.$$

Example: Find $|\vec{u} \times \vec{v}| = \sqrt{(12)^2 + (9)^2 + (3)^2} = \sqrt{234}.$

Example: $\vec{v} \times \vec{u} = -12\vec{i} - 9\vec{j} - 3\vec{k}.$

Area of Parallelogram:

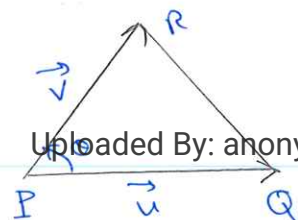
$$\begin{aligned} \text{Area} &= (\text{base}) \times (\text{height}) \\ &= |\vec{u}| \cdot |\vec{v}| \sin \theta \\ &= |\vec{u} \times \vec{v}| \end{aligned}$$



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Area of Triangle PQR = $\frac{1}{2}(\text{base})(\text{height})$

$$\begin{aligned} &= \frac{1}{2} |\vec{u}| |\vec{v}| \sin \theta \\ &= \frac{1}{2} |\vec{u} \times \vec{v}| \end{aligned}$$



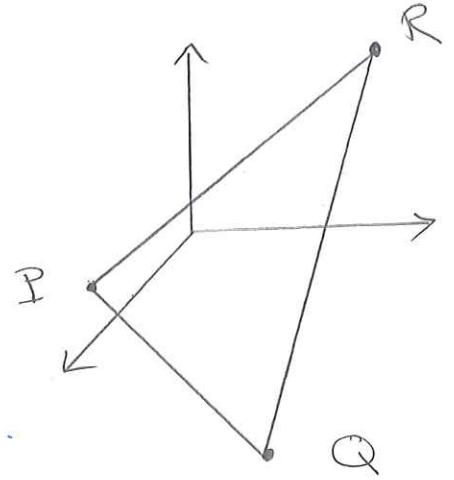
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Example: Find the area of the triangle with vertices:

$$P(1, -1, 0), Q(2, 1, -1) \text{ and } R(-1, 1, 2)$$

$$\text{Let } \vec{u} = \vec{PQ} = \vec{i} + 2\vec{j} - \vec{k}$$

$$\vec{v} = \vec{PR} = -2\vec{i} + 2\vec{j} + 2\vec{k}$$



$$\text{Then } \vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & -1 \\ -2 & 2 & 2 \end{vmatrix} = 6\vec{i} + 6\vec{k}$$

$$\Rightarrow \text{Area of triangle} = \frac{|\vec{u} \times \vec{v}|}{2} = \frac{\sqrt{(6)^2 + (6)^2}}{2} = \frac{\sqrt{72}}{2} = 3\sqrt{2}$$

Example: Find a Unit vector perpendicular to the plane of.

$$P(1, -1, 0), Q(2, 1, -1), R(-1, 1, 2)$$

The vector $\vec{PQ} \times \vec{PR} \perp$ Triangle (to the plane)

Then $\vec{PQ} \times \vec{PR}$ is the vector perpendicular.

$$\Rightarrow \text{Unit vector} = \frac{\vec{PQ} \times \vec{PR}}{|\vec{PQ} \times \vec{PR}|} = \frac{6\vec{i} + 6\vec{k}}{6\sqrt{2}}$$

$$= \frac{\vec{i}}{\sqrt{2}} + \frac{\vec{k}}{\sqrt{2}}$$

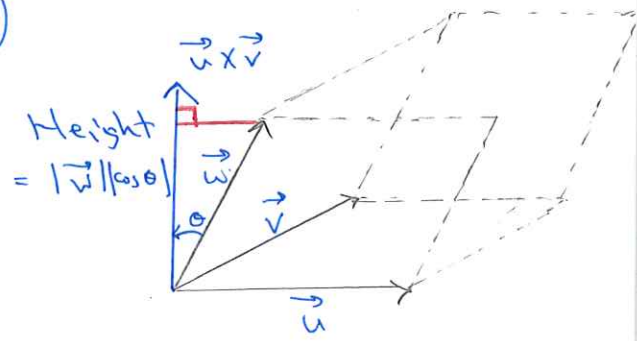
Note: Also $-\frac{\vec{PQ} \times \vec{PR}}{|\vec{PQ} \times \vec{PR}|} \perp$ plane.

Triple Scalar or Box Product:

(parallelogram-sided)
Box
(parallelepiped)

$$\text{Volume of the Box} = (\text{Area}_{\text{base}})(\text{height})$$

$$= |\vec{u} \times \vec{v}| |\vec{w}| |\cos \theta|$$



$$\Leftrightarrow |(\vec{u} \times \vec{v}) \cdot \vec{w}| = |\vec{u} \times \vec{v}| |\vec{w}| |\cos \theta|$$

Remark: $(\vec{u} \times \vec{v}) \cdot \vec{w} = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$

Example: Find the volume of the box determined by

$$\vec{u} = \vec{i} + 2\vec{j} - \vec{k}, \quad \vec{v} = -2\vec{i} + 3\vec{k}, \quad \vec{w} = 7\vec{j} - 4\vec{k}.$$

$$(\vec{u} \times \vec{v}) \cdot \vec{w} = \begin{vmatrix} 1 & 2 & -1 \\ -2 & 0 & 3 \\ 0 & 7 & -4 \end{vmatrix} = -23.$$

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The volume is $|(\vec{u} \times \vec{v}) \cdot \vec{w}| = 23$ units cubed.

12.5 Lines and planes in Space:

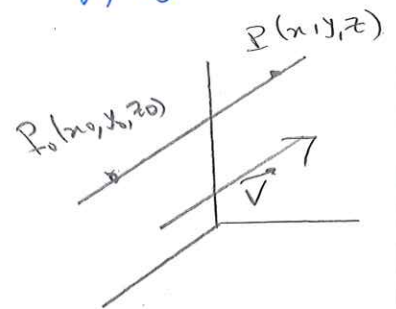
Parametrization of a Line in space.

The standard parametrization of the line through

$P_0(x_0, y_0, z_0)$ parallel to $\vec{v} = v_1 \vec{i} + v_2 \vec{j} + v_3 \vec{k}$ is

$$\left. \begin{aligned} x &= x_0 + t v_1 \\ y &= y_0 + t v_2 \\ z &= z_0 + t v_3 \end{aligned} \right\} \quad -\infty < t < \infty$$

proof: L is the set of all points $P(x, y, z)$ for which $\overrightarrow{P_0 P}$ is parallel to $\vec{v}$.



Thus: $\overrightarrow{P_0 P} = t \vec{v}$

$$\Rightarrow (x - x_0) \vec{i} + (y - y_0) \vec{j} + (z - z_0) \vec{k} = t (v_1 \vec{i} + v_2 \vec{j} + v_3 \vec{k})$$

$$\Rightarrow x - x_0 = t v_1 \quad x = x_0 + t v_1$$

$$y - y_0 = t v_2 \quad y = y_0 + t v_2$$

$$z - z_0 = t v_3 \quad z = z_0 + t v_3$$

Example: Find the ^{parametric} equation of the following:

- (1) A Line through $(-2, 4, 0)$ parallel to $\vec{v} = \underset{v_1}{2}\vec{i} + \underset{v_2}{4}\vec{j} - \underset{v_3}{2}\vec{k}$.

Assume $P_0(-2, 4, 0)$, then:

$$x = -2 + 2t$$

$$y = 4 + 4t \quad -\infty < t < \infty$$

$$z = 0 - 2t$$

-
- (2) A Line passes through $P(-3, 2, -3)$ and $Q(1, -1, 4)$

The vector $\vec{PQ}$ // to the line.

$$\begin{aligned}\vec{PQ} &= (1 - (-3))\vec{i} + (-1 - 2)\vec{j} + (4 - (-3))\vec{k} \\ &= 4\vec{i} - 3\vec{j} + 7\vec{k}\end{aligned}$$

Then Assuming $P_0(x_0, y_0, z_0) = (-3, 2, -3)$, then the parametric equations are:

$$x = -3 + 4t$$

$$y = 2 - 3t$$

$$z = -3 + 7t$$

Or: If we assume $P_0(x_0, y_0, z_0) = (1, -1, 4)$, then:

$$x = 1 + 4t$$

$$y = -1 - 3t$$

$$z = 4 + 7t$$

Example: Parametrize the Line segment joining the points $P(-3, 2, -3)$ and $Q(1, -1, 4)$

1) We first parametrize a Line through the points, which ^{are} ~~are~~

$$x = -3 + 4t$$

$$y = 2 - 3t$$

$$z = -3 + 7t$$

$$\vec{PQ} = 4\vec{i} - 3\vec{j} + 7\vec{k}$$

2) we find the t -values of the endpoints and restrict t .

• when $t=0$, the parametric equation give the point

$$P(-3, 2, -3)$$

• when $t=1 \Rightarrow Q(1, -1, 4)$, so we add a restriction:

$$x = -3 + 4t, y = 2 - 3t, z = -3 + 7t, \quad 0 \leq t \leq 1$$

Distance from a point to a Line in Space

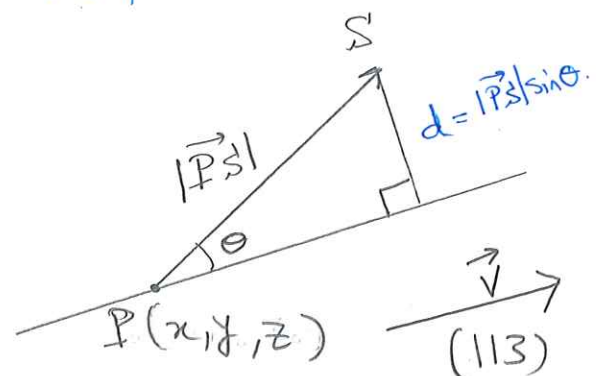
Def: The distance from a Point S to a Line through

P parallel to $\vec{v}$

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$$d = \frac{|\vec{PS} \times \vec{v}|}{|\vec{v}|} = \frac{|\vec{PS}| |\vec{v}| \sin \theta}{|\vec{v}|} |\vec{n}|$$

$$= |\vec{PS}| \sin \theta$$



Example: Find the distance from the point $S(1,1,5)$

to the line $L: \begin{aligned} x &= 1+t \leftarrow v_1 = 1 \\ y &= 3-t \leftarrow v_2 = -1 \\ z &= 2t \leftarrow v_3 = 2 \end{aligned}$

Note that when $t=0$, L passes through $P(1,3,0)$.

parallel to $\vec{v} = \vec{i} - \vec{j} + 2\vec{k}$, with:

$$\vec{PS} = (1-1)\vec{i} + (1-3)\vec{j} + (5-0)\vec{k} = -2\vec{j} + 5\vec{k}.$$

$$\therefore \vec{PS} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -2 & 5 \\ 1 & -1 & 2 \end{vmatrix} = \vec{i} + 5\vec{j} + 2\vec{k}.$$

$$\therefore \text{then } d = \frac{|\vec{PS} \times \vec{v}|}{|\vec{v}|} = \frac{\sqrt{(1)^2 + (5)^2 + (2)^2}}{\sqrt{(1)^2 + (-1)^2 + (2)^2}} = \frac{\sqrt{30}}{\sqrt{6}} = \sqrt{5}$$

An Equation for a plane in Space:

The equation of plane in space can be determined by

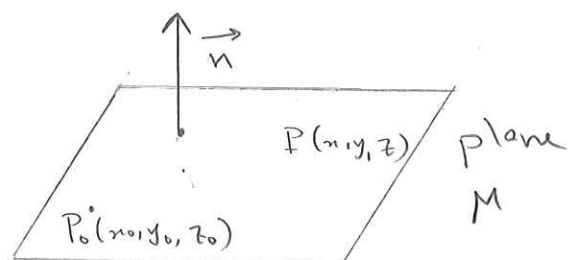
1) Point $P_0(x_0, y_0, z_0)$ on the plane

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2) vector $\vec{n} = A\vec{i} + B\vec{j} + C\vec{k}$ (s.t) orthogonal

(normal) to the plane.



The plane M is the set of all points $P(x, y, z)$

for which $\overrightarrow{P_0 P}$ is orthogonal to $\vec{n}$, i.e.

$$(x - x_0)\vec{i} + (y - y_0)\vec{j} + (z - z_0)\vec{k} \perp A\vec{i} + B\vec{j} + C\vec{k}$$

Hence $\overrightarrow{P_0 P} \cdot \vec{n} = 0$

$$\Rightarrow (x - x_0) \cdot A + (y - y_0) \cdot B + (z - z_0) \cdot C = 0$$

$$\Rightarrow \underline{Ax + By + Cz = Ax_0 + By_0 + Cz_0}$$

Equation of plane

$\vec{n} = A\vec{i} + B\vec{j} + C\vec{k}$

Example: Find Equation of plane that

(1) passes through $P_0(x_0, y_0, z_0) = (0, 2, -1)$ and normal to $\vec{n} = \overset{A}{3}\vec{i} - \overset{B}{2}\vec{j} - \overset{C}{1}\vec{k}$

$$\underline{3x - 2y - z = 3(0) + (-2)(2) + (-1)(-1) = -3}$$

(2) passes through $P_0(1, -1, 3)$ and // to the plane $3x + y + z = 7$

we find $\vec{n} \perp 3x + y + z = 7$

$$\Rightarrow \vec{n} = 3\vec{i} + \vec{j} + \vec{k}$$

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$$\Rightarrow 3x + y + z = 3(1) + (1)(-1) + (3)(3)$$

$$= 5$$

Check !!

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Example: Find an equation for the plane through:

$$A(0,0,1), B(2,0,0), C(0,3,0)$$

1. we find $\vec{n}$ (s.t.) $\vec{n} \perp \text{plane}$.

$$\vec{n} = \vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & -1 \\ 0 & 3 & -1 \end{vmatrix} = \underset{A}{3\vec{i}} + \underset{B}{2\vec{j}} + \underset{C}{6\vec{k}}$$

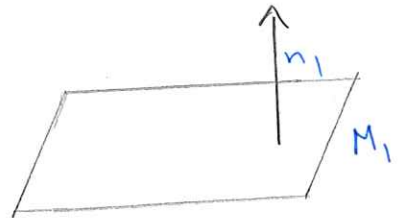
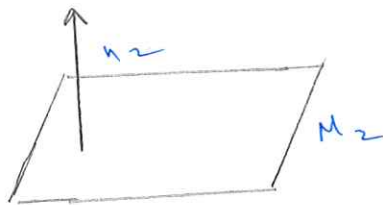
$$\vec{AB} \times \vec{AC} \perp \text{plane}$$

no y, z .

2. Use $\vec{n}$ with one of the points! , say, $A(0,0,1)$

$$\Rightarrow 3x + 2y + 6z = (0)(3) + (0)(2) + (1)(6) = 6$$

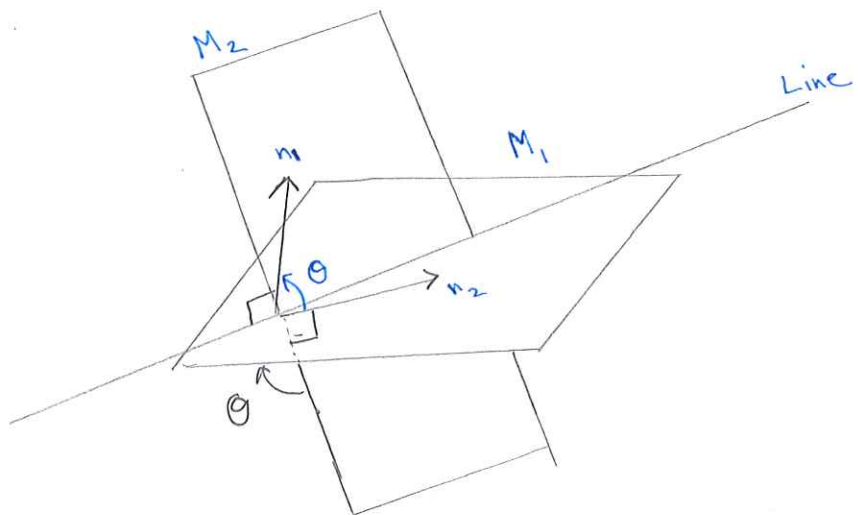
Note: Two planes in space are parallel iff their normals are parallel.



Note: If two planes are not parallel, then they intersect

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in a line.



Note: The angle θ between the two planes in space is the same as the angle between the corresponding Normals.

Lines of Intersection

Example: Find a vector parallel to the Line of Intersection of the planes $3x - 6y - 2z = 15$ & $2x + y - 2z = 5$

The Line of Intersection of two planes $\perp \vec{n}_1$ & $\vec{n}_2$
for plane 1 for plane 2
& $\vec{n}_1 \times \vec{n}_2 \perp \vec{n}_1$ & $\vec{n}_2$

$\Rightarrow \vec{n}_1 \times \vec{n}_2 \parallel$ Line of Intersection of two planes

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -6 & -2 \\ 2 & 1 & -2 \end{vmatrix} = 14\vec{i} + 2\vec{j} + 15\vec{k}$$

Example: Find parametric equation for the Line in which

the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$

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Intersect,

1) $\vec{v} = \vec{n}_1 \times \vec{n}_2 \parallel$ Line of Intersection

2) Choose any point $P_0(x_0, y_0, z_0)$ (by letting $z=0$,
 $\Rightarrow \begin{cases} x=3, y=-1 \end{cases} \Rightarrow P_0(x_0, y_0, z_0) = (3, -1, 0)$

$\Rightarrow x = 3 + 14t, y = -1 + 2t, z = 15t$ (117)

Example: Find the point where the Line:

$$x = \frac{8}{3} + 2t, \quad y = -2t, \quad z = 1+t$$

intersects the plane $3x + 2y + 6z = 6$.

The point $\left(\frac{8}{3} + 2t, -2t, 1+t\right)$ lies ⁱⁿ the plane
if its coordinates satisfy the equation of the plane, i.e.,

$$3\left(\frac{8}{3} + 2t\right) + 2(-2t) + 6(1+t) = 6$$

$$\text{Solve for } t \Rightarrow \boxed{t = -1}$$

$$\therefore (x, y, z) \Big|_{t=-1} = \left(\frac{8}{3} - 2, 2, 1-1\right) = \left(\frac{2}{3}, 2, 0\right)$$

The Distance from a point to a plane:

If P is a point on a plane with normal $\vec{n}$, then
the distance from any point S to the plane is
the length of the vector projection of $\vec{PS}$ onto $\vec{n}$
absolute value.

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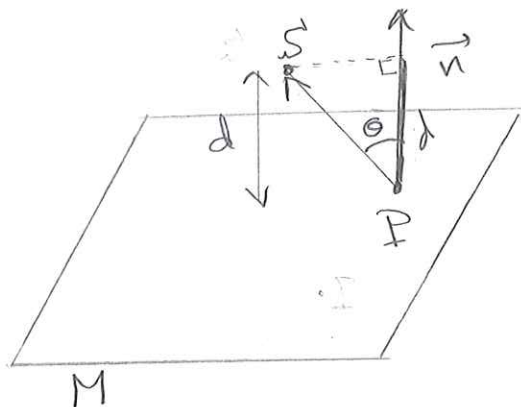
$$d = \left| \frac{\vec{PS} \cdot \vec{n}}{|\vec{n}|} \right|, \quad \vec{n} = A\vec{i} + B\vec{j} + C\vec{k}$$

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Proof:

$$\cos \theta = \frac{d}{|\vec{PS}|}$$

$$\begin{aligned} \Rightarrow d &= |\vec{PS}| \cos \theta \\ &= \frac{|\vec{n}| |\vec{PS}| \cos \theta}{|\vec{n}|} \\ &= \left| \frac{\vec{PS} \cdot \vec{n}}{|\vec{n}|} \right| \end{aligned}$$



Example: Find the distance from the point $S(0,0,0)$ to the plane $3x + 2y + 6z = 6$.

$$\vec{n} = 3\vec{i} + 2\vec{j} + 6\vec{k}$$

$$|\vec{n}| = \sqrt{9 + 4 + 36} = 7$$

we need P known, the easiest to find from the plane's equation are the intercepts

Assume P the x Intercept. (i.e.)

$$3x + 0 + 0 = 6 \Rightarrow x = 2 \Rightarrow \boxed{P(2,0,0)}$$

$$\begin{aligned}\text{Then } \vec{PS} &= (0-2)\vec{i} + (0-0)\vec{j} + (0-0)\vec{k} \\ &= -2\vec{i}\end{aligned}$$

$$\begin{aligned}\text{Hence, } \vec{PS} \cdot \vec{n} &= (-2)(3) + 0(2) + 0(6) \\ &= -6\end{aligned}$$

$$\therefore d = \left| \frac{-6}{7} \right| = \frac{6}{7}$$

Angle Between planes:

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The angle between two intersecting planes is defined to be the acute angle between their normal vectors

Example: Find the angle between the planes:

$$5x + y - z = 10 \quad \& \quad x - 2y + 3z = -1$$

$$\vec{n}_1 = 5\vec{i} + \vec{j} - \vec{k} \quad \Rightarrow \quad |\vec{n}_1| = \sqrt{25+1+1} = \sqrt{27}$$

$$\vec{n}_2 = \vec{i} - 2\vec{j} + 3\vec{k} \quad \Rightarrow \quad |\vec{n}_2| = \sqrt{1+4+9} = \sqrt{14}$$

$$\therefore \theta = \cos^{-1} \left(\frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| |\vec{n}_2|} \right) = \cos^{-1} \left(\frac{5 - 2 - 3}{\sqrt{27} \sqrt{14}} \right)$$

$$= \cos^{-1}(0) = \frac{\pi}{2}$$

Example: Discussion 10

Find parametric equation for the line passes through

$P(2, 3, 0)$ and $\perp$ to the vectors:

$$\vec{u} = \vec{i} + 2\vec{j} + 3\vec{k} \quad \& \quad \vec{v} = 3\vec{i} + 4\vec{j} + 5\vec{k}$$

$$\vec{w} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{vmatrix} = -2\vec{i} + \vec{j} - 2\vec{k}$$

$\vec{w} \parallel$ line.

$$\therefore x = 2 - 2t, \quad y = 3 + t, \quad z = -2t$$

$$t \in \mathbb{R}.$$

12.6 Cylinders and Quadric Surfaces:

Until now we studied two types of surfaces:

spheres and planes. We will extend our study & study Cylinders and Quadric Surfaces.

Cylinders:

A cylinder is a surface that is generated by moving a straight line along a given planar curve while holding the line parallel to a given fixed line.

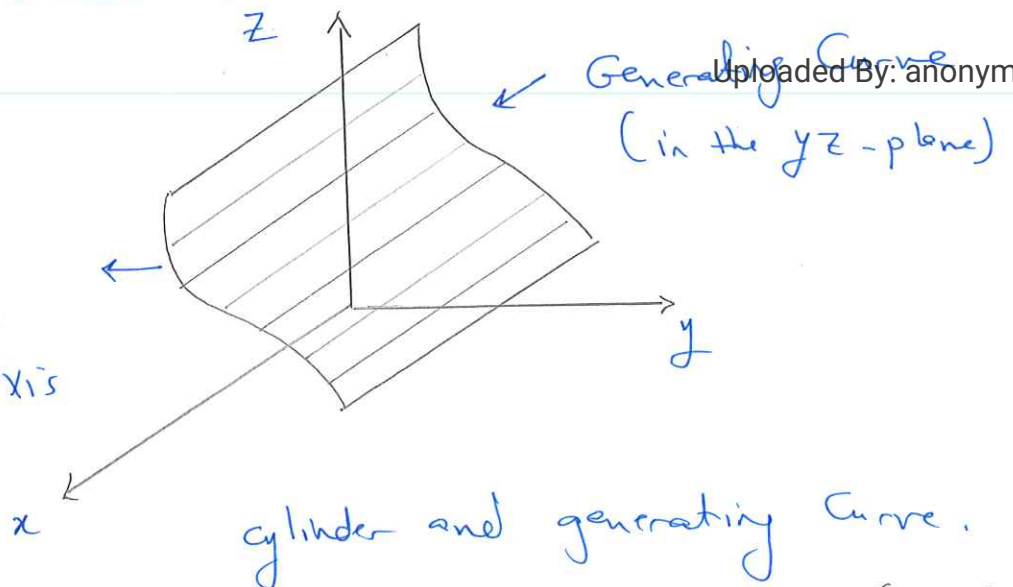
The curve is called a generating curve for the cylinder.

Note: In solid geometry, where cylinder means circular

cylinder, the generating curves are circles.

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Lines through
generating curve
parallel to x -axis

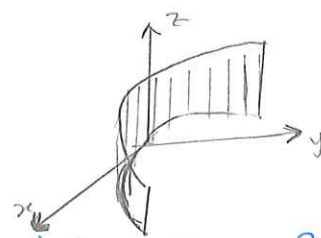


cylinder and generating curve.

Example: Find an equation for the cylinder made by the

lines parallel to the z -axis that pass through the

parabola $y = x^2$, $z = 0$.



The point $P_0(x_0, x_0^2, 0)$ lies on the parabola $y = x^2$.

in the xy -plane.

Then for any value of z , the point $Q(x_0, x_0^2, z)$

lies on the cylinder because it lies on the line

$x = x_0$, $y = x_0^2$ through P_0 parallel to z -axis.

In this case, we call the cylinder "The cylinder $y = x^2$ ".

Note: The points on the surface are the points whose coordinates satisfy " $y = x^2$ ". This makes $y = x^2$ an equation for the cylinder.

Note: Any curve $f(x, y) = c$ in xy -plane defines

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a cylinder parallel to z -axis.

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Any curve $f(x, z) = c$ in xz -plane defines

a cylinder parallel to y -axis

Note: The equation $x^2 + y^2 = 1$ defines the circular cylinder made by the lines parallel to the z axis that pass through the circle $x^2 + y^2 = 1$ in xy -plane



Quadric Surfaces:

A quadric surface is the graph in space of a second-degree equation in x, y and z .

We will focus on the special equation:

$$Ax^2 + By^2 + Cz^2 + Dx + Ey + Fz = G$$

where A, B, C, D, E, F, G are constants.

The Basic Quadric Surfaces are:

1) ellipsoids, 2) paraboloids, 3) elliptical cones, 4) hyperboloids

Note: Sphere is a special case of ellipsoid.
when $a = b = c$

Example: The ellipsoid:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

The ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Cuts the coordinate axes at $(\pm a, 0, 0)$, $(0, \pm b, 0)$

& $(0, 0, \pm c)$, then it lies within the rectangular

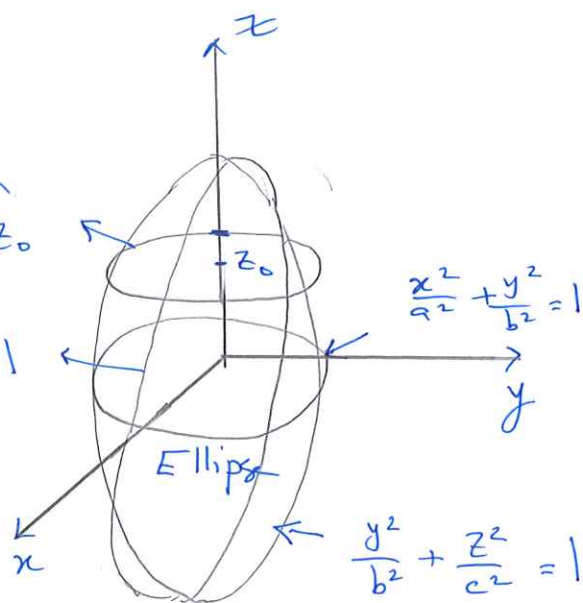
box defined by the inequalities:

$$|x| \leq a, \quad |y| \leq b \quad \text{and} \quad |z| \leq c.$$

Moreover, The surface is symmetric with respect to each of the coordinate planes because each variable in the defining equation is squared.

Elliptical Cross-section
in the plane $z = z_0$

$$\frac{x^2}{a^2} + \frac{z^2}{c^2} = 1$$



in which the three

coordinate planes cut the surface are ellipses.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{when } z = 0 \quad (\text{in } xy \text{ plane})$$

$$\frac{x^2}{a^2} + \frac{z^2}{c^2} = 1, \quad \text{when } y = 0 \quad (\text{in } xz \text{ plane})$$

$$\frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \quad \text{when } x = 0 \quad (\text{in } yz \text{ plane})$$

The Curve cut from the Surface by the plane $Z = Z_0$
with $|Z_0| < c$ is the ellipse:

$$\frac{x^2}{a^2(1 - (Z_0/c)^2)} + \frac{y^2}{b^2(1 - (Z_0/c)^2)} = 1$$

Note: If any two of the semi-axes a, b, c are equal
the surface is an ellipsoid of revolution

Note: If all of the semi-axes a, b, c are equal
the surface is sphere.

Example: The hyperbolic paraboloid:

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = \frac{z}{c}, \quad c > 0.$$

has symmetry with respect to the planes $x=0$ and $y=0$

The Cross sections are:

$x=0 \Rightarrow$ The parabola $Z = \frac{c}{b^2} y^2$. (opens upward)

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$y=0 \Rightarrow$ The parabola $Z = -\frac{c}{a^2} x^2$. (opens downward)

If we cut the surface by a plane $z = \underline{z_0} > 0$.

The cross-section is a hyperbola

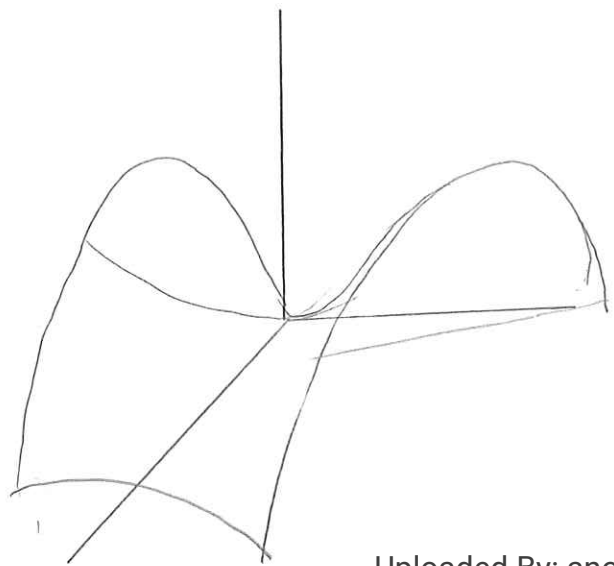
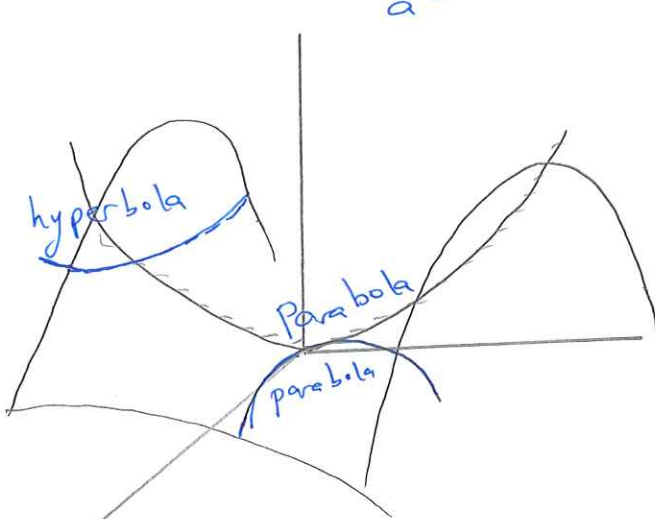
$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = \frac{z_0}{c}$$

with its focal axis parallel to the y -axis

& its vertices on the parabola $z = \frac{c}{b^2} y^2$.

• If $z_0 < 0$, the focal axis is parallel to the x -axis and the vertices lie on the parabola

$$z = -\frac{c}{a^2} x^2$$



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Note: Near the origin, the surface is shaped like a

saddle point since:

(Traveling along the surface in yz -plane, the origin is Min)
(Traveling along the surface in xz -plane, the origin is Max).

TABLE 12.1 Graphs of Quadric Surfaces

