Mathematics Department

Math 1321 – Worksheet # 3

"10.2 – 10.3"

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. Name : ______

Q. Which of the following series converge, and which diverge. If the series converges, find its sum.

1) $\sum_{n=0}^{\infty} \frac{2^{n+3}}{3^n}$

2) $\sum_{n=1}^{\infty} (-1)^{n-1}$

3) $\sum_{n=1}^{\infty} \left(\frac{n}{n-1}\right)^n$

4) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}\,(\sqrt{n}+1)}$

5) $\sum_{n=3}^{\infty} \frac{1}{n \ln n \sqrt{(\ln n)^2 - 1}}$

6) $\sum_{n=1}^{\infty} \frac{1}{2n-1}$

7) $\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$

8) $\sum_{n=1}^{\infty} \frac{n+1}{\sqrt{4n^2+3}}$

9) $\sum_{n=0}^{\infty} \left(\frac{e}{\pi}\right)^n + \frac{n}{n+3}$

*** Short Answers:

Q1 $\boxed{1}$ Converge geometric series because $|R| = \frac{2}{3} < 1$

Sum = 24

$\boxed{2}$ Diverge geometric series because $|R| = 1$

OR By using n^{th} term test $\lim_{n \to \infty} a_n = DNE$

$\longrightarrow$ Then $\sum_{n=1}^{\infty} (-1)^{n-1}$ Div.

OR By using the sequence of partial sum $\lim_{n \to \infty} S_n = DNE$

$\{S_n\} = \{1, 0, 1, 0, 1, \ldots\}$

$\longrightarrow$ Then $\sum_{n=1}^{\infty} (-1)^{n-1}$ Div.

$\boxed{3}$ By n^{th} term test $\lim_{n \to \infty} a_n = e$

$\longrightarrow$ Then $\sum_{n=1}^{\infty} \left(\frac{n}{n-1}\right)^n$ Div.

$\boxed{4}$ By using Integral Test

$f(x) = \frac{1}{\sqrt{x}(\sqrt{x}+1)}$ is positive / cont. / decreasing for $x \geq 1$

$\longrightarrow$ Then $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(\sqrt{n}+1)}$ Div.

$\boxed{5}$ By using Integral Test

$f(x) = \frac{1}{x \ln x \sqrt{(\ln x)^2 - 1}}$ is positive / cont. / Decreasing for $x \geq 3$

$\longrightarrow$ Then the series converge. لا يمكن معرفة Sum من خلال IT

6 By using Harmonic series

$$\sum_{n=1}^{\infty} \frac{1}{2n-1} = \frac{1}{2} \sum_{m=1}^{\infty} \frac{1}{m}$$

/ I.T

$$\int_{n=1}^{\infty} \frac{1}{2x-1} \, dx$$

Div

$\longrightarrow$ Then $\sum_{n=1}^{\infty} \frac{1}{2n-1}$ Div

7 By using the sequence of partial sum

$$S_n = 1 - \frac{1}{4n+1} \qquad \lim_{n\to\infty} S_n = 1$$

$\longrightarrow$ Then $\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$ Conv.

$\longrightarrow$ $\boxed{\text{Sum} = 1}$

8 By using nth term test

$$\lim_{n\to\infty} a_n = \frac{1}{2} \neq 0$$

$\longrightarrow$ The the series $\sum_{n=1}^{\infty} \frac{n+1}{\sqrt{4n^2+4}}$ Div.

9 Converge geometric Series + Diverge Series = Div.