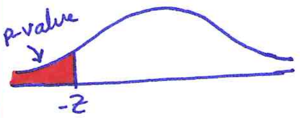
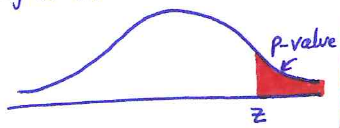
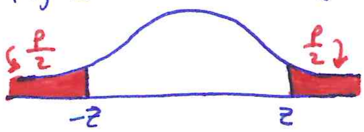
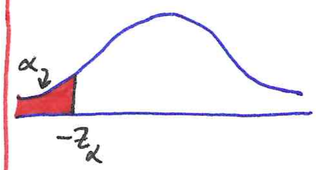
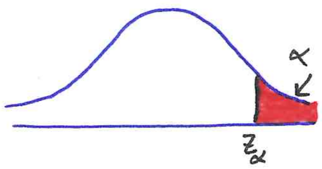
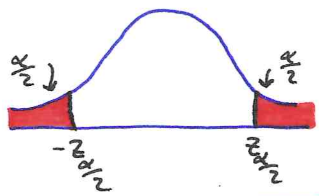
9.5 Hypothesis Testing about Proportion (p)

	Lower Tail Test	Upper Tail Test	Two Tailed Test
Hypothesis	$H_0: p \geq p_0$ $H_a: p < p_0$	$H_0: p \leq p_0$ $H_a: p > p_0$	$H_0: p = p_0$ $H_a: p \neq p_0$
Test statistic	$z = \dfrac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$	$z = \dfrac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$	$z = \dfrac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$
• Rejection Rule using p-value approach	Reject H_0 if p-value $\leq \alpha$ 	Reject H_0 if p-value $\leq \alpha$ 	Reject H_0 if p-value $\leq \alpha$ 
• Rejection Rule using critical value approach	Reject H_0 if $z \leq -z_\alpha$ 	Reject H_0 if $z \geq z_\alpha$ 	Reject H_0 if $z \leq -z_{\alpha/2}$ or $z \geq z_{\alpha/2}$ 

* The procedure used to construct hypothesis test about population proportion p is similar to the procedure used to construct hypothesis test about the population mean

* We assume $np \geq 5$ and $n(1-p) \geq 5$ so that the normal prob. dist. can be used to approximate the sampling distribution of $\bar{p}$ "which is a discreat binomial dist."

* The standard error of $\bar{p}$ is $\sigma_{\bar{p}} = \sqrt{\dfrac{p_0(1-p_0)}{n}}$

Example (Q 35 page 362) Consider the hypothesis test $H_0: p = 0.20$
$H_a: p \neq 0.20$

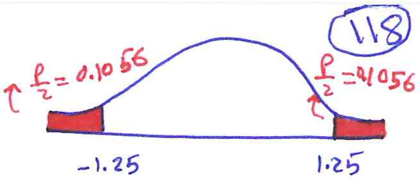
A sample of 400 provided a sample proportion $\bar{p} = 0.175$ Two Tailed Test

[a] Compute the value of the test statistic ? $p_0 = 0.2$, $\bar{p} = 0.175$, $n = 400$

$$z = \frac{\bar{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.175 - 0.20}{\sqrt{\frac{0.2(0.8)}{400}}} = \frac{-0.025}{0.02} = -1.25$$

[b] What is the p-value?

From the standard normal table, we have



$p\text{-value} = 0.1056 + 0.1056 = 0.2112$

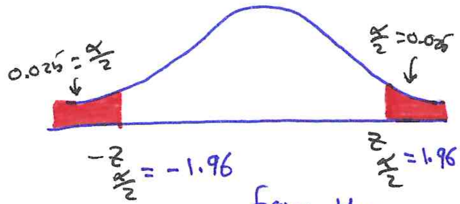
c) At $\alpha = 0.05$, what is your conclusion?

Do not reject H_0 since $p\text{-value} = 0.2112 > 0.05 = \alpha$

d) what is the rejection rule using the critical value? what is your conclusion?

Reject H_0 if $z \le -z_{\frac{\alpha}{2}} = -z_{0.025} = -1.96$ or

if $z \ge z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$



from the standard normal table.

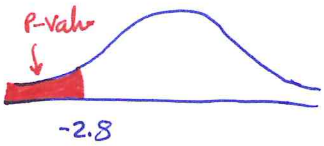
Since $z = -1.25 > -1.96$, we do not reject H_0.

Example

Q 36 page 362 Consider the hypothesis test $H_0: p \ge 0.75$
$H_a: p < 0.75$

A sample of 300 items was selected. Compute p-value and state your conclusion for each of the following results (use $\alpha = 0.05$). lower tail test

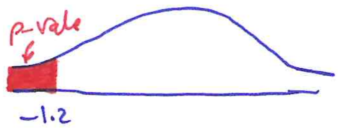
a) $\bar{p} = 0.68$ $z = \dfrac{\bar{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}} = \dfrac{0.68 - 0.75}{\sqrt{\dfrac{0.75(0.25)}{300}}} = -2.80$



From the standard normal table, we have $p\text{-value} = 0.0026$

Reject H_0 since $p\text{-value} = 0.0026 \le \alpha = 0.05$.

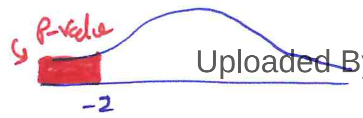
b) $\bar{p} = 0.72$ $z = \dfrac{\bar{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}} = \dfrac{0.72 - 0.75}{\sqrt{\dfrac{0.75(0.25)}{300}}} = -1.2$



From the standard normal table, we have $p\text{-value} = 0.1151$

Do not reject H_0 since $p\text{-value} = 0.1151 > 0.05$

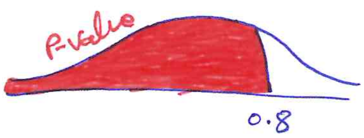
c) $\bar{p} = 0.70$ $z = \dfrac{0.70 - 0.75}{\sqrt{\dfrac{0.75(0.25)}{300}}} = -2$



From the standard normal table, we have $p\text{-value} = 0.0228$

Reject H_0 since $p\text{-value} \le 0.05$

d) $\bar{p} = 0.77$ $z = \dfrac{0.77 - 0.75}{\sqrt{\dfrac{0.75(0.25)}{300}}} = 0.8$



From the standard normal table, we have $p\text{-value} = 0.7881$

Do not reject H_0 since $p\text{-value} \ge 0.05$.