

ENEE3401, COMMUNICATIONS AND DIGITAL DATA NETWORKS
Second Semester 2024-2025; Last Updated May 30, 2025

Part 1
Selected Problems on Binary and M-ary Transmission

Problem 1: Signal Space Representation

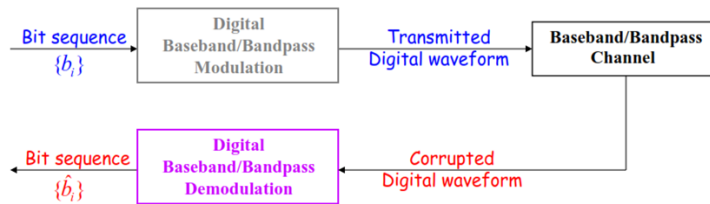
Let $\phi_1(t)$ and $\phi_2(t)$ be the basis functions of a signal space, and let the two signals $s_1(t)$ and $s_2(t)$ used to represent binary digits 0 and 1 in AWGN, with power spectral density $N_0/2$, be given by:

$$s_1(t) = \phi_1(t) - \phi_2(t) \quad s_2(t) = \phi_2(t)$$

Let $P(s_1) = P(s_2) = 0.5$. Find the correlation coefficient between $s_1(t)$ and $s_2(t)$.

Problem 2: Probability of Error in Noisy Channel

Consider the digital communication system shown in the figure where $P(b_i = 0) = 0.5$, $P(\hat{b}_i = 1|b_i = 0) = 0.05$, and $P(\hat{b}_i = 0|b_i = 1) = 0.03$. Here, b_i refers to the transmitted bit and \hat{b}_i refers to the received bit. Find the average probability of error P_b



Problem 3: B.W of an FSK Binary Signal

Data is to be transmitted over a band pass channel at a rate of 1000 bits/sec using binary frequency shift keying (BFSK). The used signals are $s_1(t) = 3\cos(2\pi(5000)t)$ and $s_2(t) = 3\cos(2\pi(4000)t)$. Find the 90% power bandwidth of the transmitted signal.

Problem 4: Data Rate in a PCM System

The signal $x(t) = 4\cos 2\pi(100)t$ is applied to a PCM system (sampler, quantizer, binary encoder) where the sampler operates at a rate of 250 samples/sec. The samples are applied to a 32-level non-uniform μ -law quantizer with $\mu=255$. Find the data rate in bits/sec at the binary encoder output.

Problem 5: Sampling Theorem

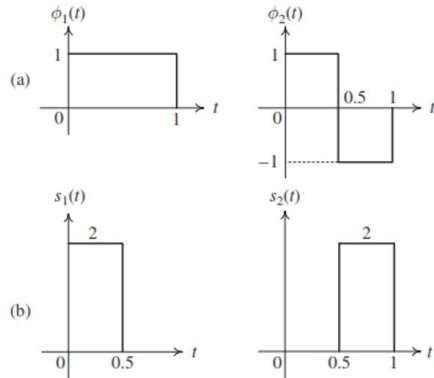
Consider the signal $x(t) = 2\cos 2\pi(25)t + 5\cos 2\pi(200)t$. Find the most appropriate sampling frequency for this signal such $x(t)$ can be reconstructed from its samples without aliasing.

Problem 6: Signal Space Representation

The coordinates of two signals $s_1(t)$ and $s_2(t)$ in the $\phi_1(t) - \phi_2(t)$ plane are respectively $(-1, 0)$ and $(2, 0)$. Find the probability of error in additive white Gaussian noise with psd $N_0/2$

Problem 7: Signal Space Representation

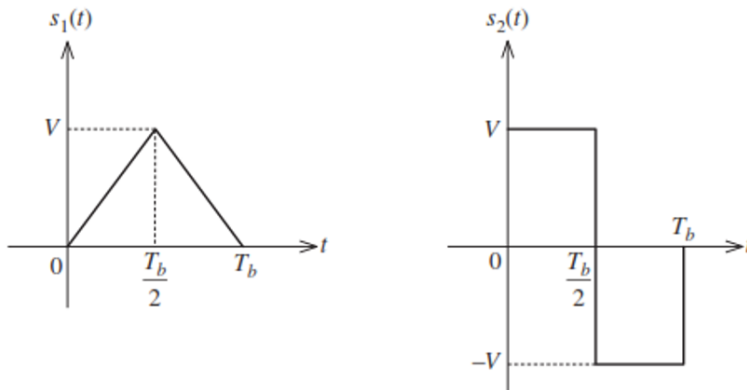
Consider the two signals $s_1(t)$ and $s_2(t)$ shown in the figure. Determine the coefficients s_{ij} of the two signals based on the two bases functions $\phi_1(t) - \phi_2(t)$



Problem 8: Signal Space Representation

Consider the two signals $s_1(t)$ and $s_2(t)$ shown in the figure.

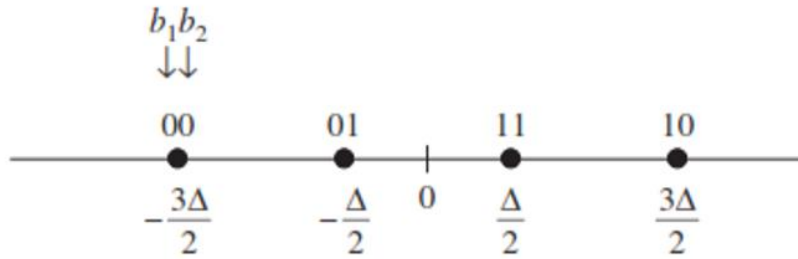
- Find the appropriate bases functions for the signal space.
- Find the probability in AWGN with power spectra density $N_0/2$



Problem 9: M-ary ASK

Consider the 4-ASK system shown in the figure. The four signals are transmitted in an AWGN channel with power spectra density $N_0/2$.

- Find the average probability of error/signal
- Find the average energy per signal.
- Find the probability that signal (11) is received if signal 00 is transmitted.
- Find the probability that signal (10) is received if signal 00 is transmitted.



Problem 10: Matched Filter in Binary Transmission

Consider a digital communication system, corrupted by AWGN with power spectral density $N_0/2$, that uses $s_1(t)$ to represent digit 1 and $s_2(t) = -s_1(t)$ to represent digit 0, where

$$s_1(t) = \begin{cases} A & 0 \leq t \leq \tau/2 \\ -A & \tau/2 \leq t \leq \tau \end{cases}$$

- Find and sketch the impulse response of the matched filter
- Find the optimum threshold used by the receiver when deciding between digits 0 and 1
- Find the system probability of error when the receiver employs the threshold of Part b.

Problem 11: Binary ASK

Consider a binary ASK modulator where the bit duration is $1 \mu s$ and the sinusoidal carrier wave used to represent symbol 1 has a frequency equal to 5 MHz.

- Draw the block diagram of the optimum coherent demodulator.
- Draw the block diagram of a noncoherent demodulator. Here, the receiver does not know the exact value of the frequency of the received signal

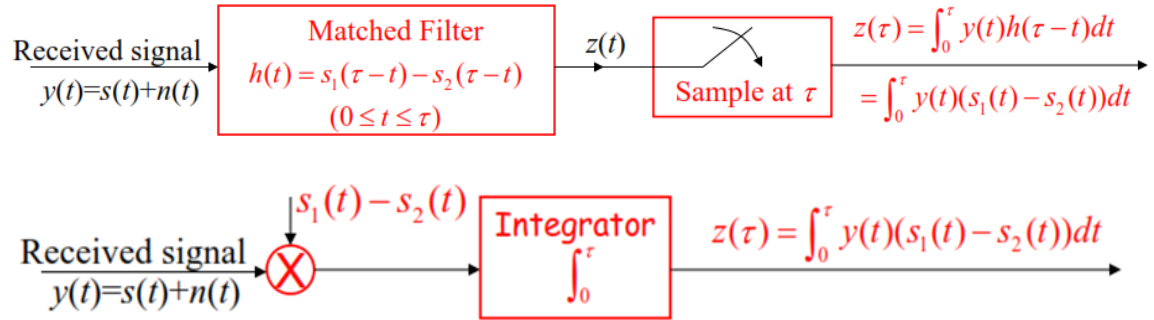
Problem 12: Matched Filter in Binary Transmission

Consider a digital communication system, corrupted by AWGN with power spectral density $N_0/2$, that uses $s_1(t)$ to represent digit 1 and $s_2(t)$ to represent digit 0, where

$$\begin{aligned} s_1(t) &= A, & 0 \leq t \leq \tau \\ s_2(t) &= 0, & 0 \leq t \leq \tau \end{aligned}$$

- Find and sketch the impulse response of the matched filter
- Find and sketch the output of the matched filter when $s_1(t)$ is applied at its input. At which time will the output be maximum?
- Find the output of the matched filter at $t = \tau$
- Find the output of the correlator at $t = \tau$

Parts c and d should have the same answer. That is, the following two receiver structures are equivalent in terms of the output at time $t = \tau$.



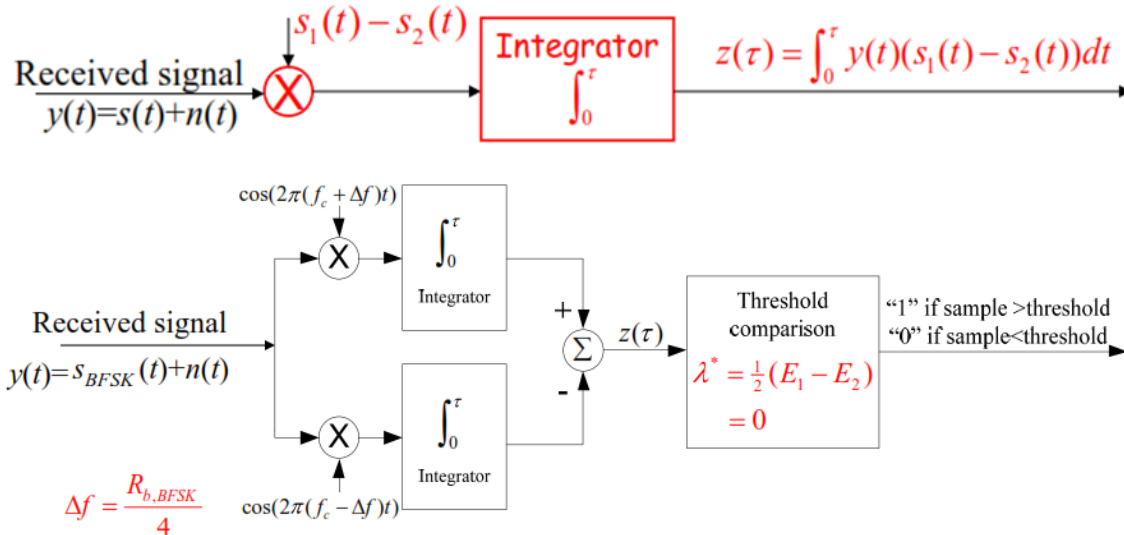
Problem 13: Binary FSK

Consider an FSK system that uses the signals $s_1(t) = A\cos(2\pi f_1 t)$ and $s_2(t) = A\cos(2\pi f_2 t)$. Show that $s_1(t)$ and $s_2(t)$ are orthogonal when $f_1 = nR_b$ and $f_2 = mR_b$ where n and m are integers, $n \neq m$, i.e., show that

$$\int_0^{1/R_b} s_1(t)s_2(t)dt = 0$$

Problem 14: Binary FSK Demodulator

Show that the following two configurations of the optimum FSK receiver are equivalent



where, $s_1(t) = A\cos(2\pi(f_c + \Delta f)t)$ and $s_2(t) = A\cos(2\pi(f_c - \Delta f)t)$.

Problem 15: PCM

The signal $m(t) = \cos(1000\pi t)$ is to be transmitted using a PCM system (a system composed of a sampler, quantizer, and binary encoder).

- If sampling is done at the Nyquist rate and a uniform quantizer with 32 levels is employed, what is the resulting data rate in bits/sec and the resulting SQNR
- Find the SQNR if the signal is sampled at 1.5 times the Nyquist rate

Problem 16: Uniform Quantization

Design a 4-bit uniform quantizer with a dynamic range $(-4, 4)$ V, i.e., find the thresholds and representation values.

Problem 17: Binary ASK

The binary amplitude shift keying (ASK) signaling scheme, discussed in class, employs the following two equally probable signals $s_1(t)$ and $s_2(t)$ to represent binary logic 1 and 0 respectively over a channel corrupted by AWGN with $N_0 = 0.001$ W/Hz:

$$s_1(t) = 4\cos(2\pi f_c t); 0 \leq t \leq T_b$$

$$s_2(t) = 0; 0 \leq t \leq T_b; T_b = n T_c; n \text{ an integer.}$$

- If the bit rate $R_b = 1000$ bps, find the bit error probability.
- If the bit error probability is not to exceed 10^{-4} , find the maximum allowable data rate R_b .

Problem 18: General Form of Binary PSK

In a binary digital communication system, the two signals $s_1(t)$ and $s_2(t)$ that are used to represent binary digits 0 and 1 over a channel corrupted by AWGN with zero mean and power spectral density $N_0/2$ are given by:

$$s_1(t) = A_c k \sin(2\pi f_c t) - A_c \sqrt{1-k^2} \cos(2\pi f_c t), 0 \leq t \leq T_b$$

$$s_2(t) = A_c k \sin(2\pi f_c t) + A_c \sqrt{1-k^2} \cos(2\pi f_c t), 0 \leq t \leq T_b$$

where $T_b = r T_c$, r is an integer, and k is a constant. This is a form of phase shift keying with transmitted carrier, in which the first term represents a carrier component included for the purpose of synchronizing the receiver to the transmitter and the second term alternates in polarity in accordance with the message signal. In the special case when $k = 0$, $s_1(t)$ and $s_2(t)$ reduce to the usual Binary PSK modulation scheme analyzed in class.

One appropriate set of basis functions is:

$$f_1(t) = \sqrt{\frac{2}{T_b}} \sin(2\pi f_c t), 0 \leq t \leq T_b$$

$$f_2(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t), 0 \leq t \leq T_b.$$

- Find and sketch the signal space representation of the signals using the given basis functions.
- Find the energy in each signal.
- Find the average probability of error assuming the signals are equally likely.

Problem 19: M-ary PSK

- An QPSK transmission is to be used on an AWGN channel with symbol energy to noise ratio $E_s/N_0 = 32$. The bit rate is 1 M bits per second. Find the symbol error probability.
- If an 8-ary PSK scheme is to be used instead, by how much should the symbol energy to noise ratio be increased (or decreased) to achieve a symbol error probability the same as that for QPSK.

Problem 20: M-ary PSK

An M-ary PSK transmission is to be used on an AWGN channel with 22.14 dB signal energy per symbol to noise ratio E/N_0 . The symbol rate is 1M symbol per second and the symbol error probability is not to exceed 0.0004

- Choose the appropriate basis functions for the signal space.
- Sketch the optimum receiver implemented in terms of correlators.
- Estimate the highest data rate in bits per second that may be achieved.

Note: $\text{erfc}(2.5) = 0.0004$, $\text{erfc}(2.0) = 0.00468$, and for $x > 3$ use the approximation

$$\text{erfc}(x) = \frac{e^{-x^2}}{x\sqrt{\pi}}$$

Problem 21: Signal Space Representation

In a binary digital communication system, two signals $s_1(t)$ and $s_2(t)$ are used to represent binary digits 0 and 1 over a channel corrupted by AWGN with zero mean and power spectral density $N_0/2$. Let $\phi_1(t)$ and $\phi_2(t)$ be the two basis functions of the signal space.

- Show that the coefficients of the signals in the $\phi_1(t) - \phi_2(t)$ plane are given as:

$$s_{11} = \sqrt{E_1}, s_{12} = 0, s_{21} = \rho\sqrt{E_2}, s_{22} = \sqrt{1-\rho^2}\sqrt{E_2}$$

E_1 is the energy in $s_1(t)$, E_2 is the energy in $s_2(t)$ and ρ is the correlation coefficient

$$\text{defined as: } \rho = \frac{1}{\sqrt{E_1}\sqrt{E_2}} \int_0^{T_b} s_1(t)s_2(t)dt$$

Problem 22: Nyquist First Pulse Shaping Criterion

The bandwidth of a channel is 1800 Hz and the data rate over the channel is 2400 bits/sec. Sketch the Fourier transform of the pulse at the receiver side which removes inter-symbol interference. Mark all critical points.

Problem 23: Bandpass Signal

The energy in the signal $s(t) = A\cos(2\pi f_c t)$, $0 \leq t \leq \tau$, $T_c = n\tau$, n is an integer, is

- $A^2/2$
- A^2T_c
- $A^2\tau/2$
- $A^2\tau$

Problem 24: Binary FSK

The binary orthogonal frequency shift keying (FSK) signaling scheme, discussed in class, employs the following two equally probable signals $s_1(t)$ and $s_2(t)$ to represent binary logic 0 and 1 respectively over a channel corrupted by AWGN with $N_0 = 0.001$ W/Hz:

$$s_1(t) = 4 \cos(2\pi f_1 t), \quad 0 \leq t \leq T_b$$

$$s_2(t) = 4 \cos(2\pi f_2 t) \quad 0 \leq t \leq T_b,$$

- If the bit error probability is not to exceed 10^{-4} , find the maximum allowable data rate R_b in bits per second. (8)
- Sketch the optimum demodulator (4)

Problem 25: Nyquist Criterion

Figure 2-a shows the system model for a binary baseband antipodal signaling over a band-limited channel. Each binary digit is represented as $\pm 0.005d(t - kT_b)$. The source rate is $r_b = 1/T_b$ and the overall response $S_R(f)$ of the system is shown in Figure 2-b in which K is an arbitrary constant. The system parameters are:

Channel Transfer Function:
$$H_c(f) = \begin{cases} 1 & |f| \leq 3000 \text{ Hz} \\ 0 & |f| > 3000 \text{ Hz} \end{cases}$$

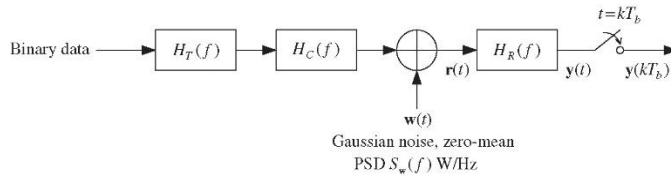


Figure 2.a

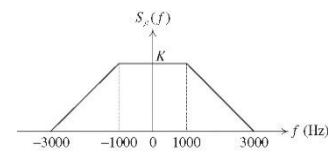


Figure 2.b

At what rate would you transmit data if you desire the ISI terms to be zero at the sampling instants? Explain.

Problem 26: Matched Filter in Binary Transmission

Consider the signal

$$s(t) = \begin{cases} A \frac{2t}{\tau} & 0 \leq t \leq \tau/2 \\ A & \tau/2 \leq t \leq \tau \end{cases}$$

- Find and sketch the impulse response, $h(t)$, of the filter matched to the signal $s(t)$ in AWGN, $n(t)$, with power spectral density $N_0/2$.
- If $g(t)$ where

$$g(t) = \begin{cases} A & 0 \leq t \leq \tau \\ 0 & \text{otherwise} \end{cases}$$

- is applied to the matched filter of Part a, find the filter output at $t = \tau$.
- If the signal $s(t)$ is passed through a correlator that correlates the input $s(t)$ with $s(t)$, find the value of the correlator output at $t = \tau$.

Problem 27: Nyquist criterion for zero ISI

A baseband digital communication system is designed to transmit binary data over a channel with a bandwidth of 3 kHz. The system employs ideal Nyquist pulse shaping (zero intersymbol interference, ISI), using a raised cosine filter with a roll-off factor $\alpha=0.25$.

- a. According to the Nyquist first criterion, what is the maximum symbol rate (baud rate) that can be supported by this channel without ISI?
- b. What is the maximum data rate (in bits per second) that can be achieved, assuming:
 - a. Binary signaling (1 bit per symbol)?
 - b. 4-level PAM signaling (2 bits per symbol)?
 - c. If the system instead uses an ideal rectangular pulse (i.e., sinc pulses with no excess bandwidth), what would be the maximum symbol rate for zero ISI in that case?
 - d. Explain how the roll-off factor influences the required bandwidth and the resulting tradeoff between bandwidth efficiency and practical pulse shaping.

Solution:

- Channel bandwidth: $B=3 \text{ kHz}$
- Roll-off factor (raised cosine): $\alpha=0.25$

a. Maximum symbol rate (baud rate) using raised cosine (Nyquist pulse shaping)

The Nyquist bandwidth requirement for **zero ISI** with a **raised cosine filter** is:

$$B = (R_s / 2) (1 + \alpha)$$

$$R_s = 2B / (1 + \alpha) = 2 \times 3000 / (1 + 0.25) = 6000 / 1.25 = 4800 \text{ baud}$$

b. Maximum data rate

$$R_b = R_s \times \log_2 M = 4800 \times \log_2 2 = 4800 \text{ bps}; M = 2$$

$$R_b = 4800 \times \log_2 M = 4800 \times 2 = 9600 \text{ bps}; M = 4$$

c. Maximum symbol rate with ideal sinc pulses (Nyquist first criterion)

For ideal sinc pulses (i.e., zero roll-off, $\alpha=0$), the minimum bandwidth required is:

$$B = R_s / 2 \Rightarrow R_s = 2B = 2 \times 3000 = 6000 \text{ baud}$$

d. Effect of roll-off factor α on bandwidth and data rate

- The roll-off factor α controls excess bandwidth beyond the Nyquist minimum.
- Higher $\alpha \rightarrow$ more bandwidth used, but easier pulse shaping and better time localization.
- **Lower α** \rightarrow more spectrally efficient (uses less bandwidth), but requires longer, more ideal pulse shapes (harder to implement).

Problem 28: Binary FSK

In a coherent binary FSK system, the two equally probable signals $s_1(t)$ and $s_2(t)$ are given by:

$$s_1(t) = V \cos 2\pi(f_c + \Delta f / 2)t, \quad 0 \leq t \leq T_b$$

$$s_2(t) = V \cos 2\pi(f_c - \Delta f / 2)t, \quad 0 \leq t \leq T_b$$

where $f_c > \Delta f$ and the correlation coefficient between the signals can be approximated by $\rho \cong \text{sinc}(2\Delta f T_b)$.

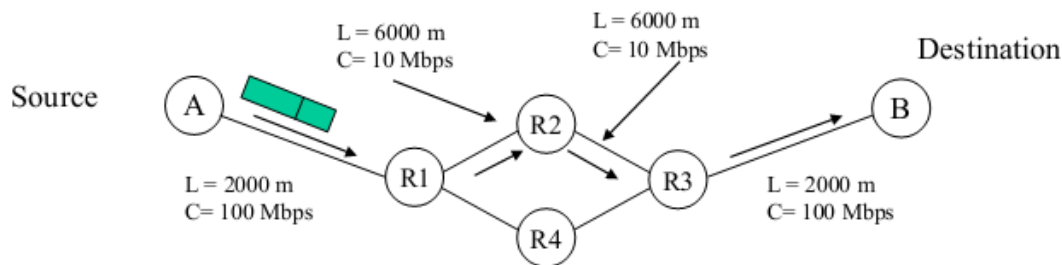
- Find the distance between the signals in terms of the correlation coefficient and the signals energy. (4)
- What is the minimum value of Δf for which the signals are orthogonal? (4)
- What is the value of Δf that minimizes the average probability of error. (6)
- For the condition in Part (c), find the minimum value of the average probability of error. (4)

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Part 2
Selected Problems on Data Communications

Problem 1: Packet Switching

You are given a communication link as shown in the figure. The objective is to transmit a file of length 2 M bytes from node A to node B, through R1, R2, and R3, as a series of packets. Node A breaks the file into frames each of size 1000 bytes. A header of size 100 bits is added to each frame before it is transmitted over the link. The lengths and capacity of the links are as indicated in the figure. The propagation speed over all links is 2.0×10^8 m/s. Find the total time needed to deliver the file to the destination neglecting the waiting times at the routers.



Problem 2: Packet switching

A message of 6,000 bytes is to be sent from Host A to Host C via an intermediate node B (two-hop path: $A \rightarrow B \rightarrow C$). The message is segmented into packets of 1,500 bytes of payload. Each packet has a header of 100 bytes. Packets are transmitted using store-and-forward pipelining.

Network characteristics:

- Link A–B:
 - Propagation delay = 5 ms
 - Data rate = 1 Mbps
- Link B–C:
 - Propagation delay = 10 ms
 - Data rate = 500 kbps

Assume:

- Node B introduces a queuing delay of 2 ms per packet before forwarding.
 - Packets are forwarded as soon as the full packet is received and queued.
- a. How many packets are generated?
- b. What is the transmission time per packet on each link?

- c. What is the total time from when the first bit of the message leaves Host A until the last bit of the message is received at Host C?

Problem 3: Packet switching

A message of 6,000 bytes is to be sent from Host A to Host C using packet switching through an intermediate node B (i.e., two hops: $A \rightarrow B \rightarrow C$). The message is divided into packets of 1,500 bytes each, with no header overhead for simplicity. Packets are forwarded as soon as they are received (i.e., store-and-forward with pipelining).

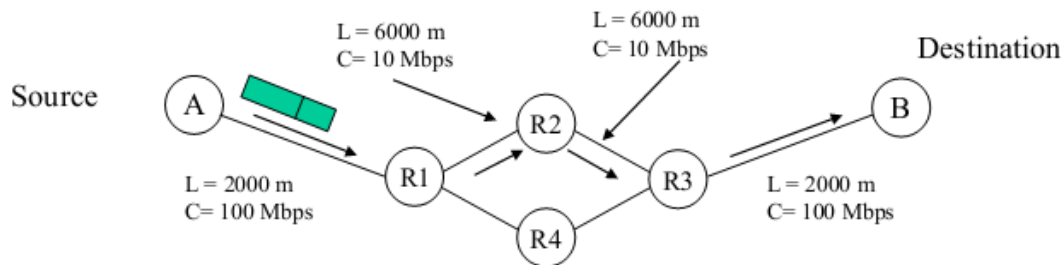
The characteristics of the links are:

- Link A–B: Propagation delay = 5 ms, Data rate = 1 Mbps
- Link B–C: Propagation delay = 10 ms, Data rate = 500 kbps

Calculate the total time from when the first bit of the message leaves Host A until the last bit of the message is received at Host C. (Answer: 123 ms)

Problem 4: Message Switching

You are given a communication link as shown in the figure. The objective is to transmit a file of length 2 M bytes from node A to node B, through R1, R2, and R3. A header of size 100 bytes bits is added to the file before it is transmitted over the link. The lengths and capacity of the links are as indicated in the figure. The propagation speed over all links is 2.0×10^8 m/s. Find the total time needed to deliver the file to the destination neglecting the waiting times at the routers. Please note that this is a message switching mechanism.



Problem 5: Message Switching

A message of 1,000 bytes is to be sent from Host A to Host E through a network using message switching. The message passes through two intermediate nodes (i.e., a total of 3 hops: $A \rightarrow B \rightarrow C \rightarrow E$). The characteristics of each link are as follows:

- Link A–B: Propagation delay = 10 ms, Data rate = 1 Mbps
- Link B–C: Propagation delay = 20 ms, Data rate = 500 kbps
- Link C–E: Propagation delay = 5 ms, Data rate = 2 Mbps

Calculate the total end-to-end delay from Host A to Host E using message switching (Answer: 63 ms)

Problem 6: CRC Code

Suppose a CRC scheme uses the generator polynomial $g(x) = x^3 + x + 1$

- Generate the CRC bits for the bit string 1101
- If $s(x)$ is the transmitted sequence, $r(x)$ the received sequence, and $e(x)$ the error sequence, then $r(x) = s(x) + e(x)$. You know that: remainder $(s(x)/g(x)) = 0$. Use this information to find out if this polynomial will be able to detect the error pattern 0000011

Problem 7: CRC Code

Suppose a cyclic redundancy check (CRC) code uses the prime generator polynomial

$$g(x) = x^3 + x + 1.$$

Find the codeword representing the message 1000

Problem 8: Stop and Wait ARQ Protocol

Consider an automatic repeat request protocol with the following parameters:

Size of the packet is 1550 bits, the data rate over the channel is 29600 bits/sec, the round trip propagation delay is 100 ms, the system timeout is 500 ms, and the probability of successfully accepting the packet is 0.9. The processing times at both the transmitter and the receiver are ignored. In addition, the size of the acknowledgement packet is ignored.

- Find the probability that the packet is successfully transmitted on the third transmission.
- Find the time, in seconds, it takes to transmit the file successfully on attempt number 6.

Problem 9: CDMA System

In a 4-station CDMA system, the binary chip sequences (signature waveforms) assigned for users A, B, C, and D are

$$A = \{1 \ 1 \ 1 \ 1\} \quad B = \{1 \ 0 \ 1 \ 0\} \quad C = \{0 \ 0 \ 1 \ 1\} \quad D = \{0 \ 1 \ 1 \ 0\}.$$

The chip duration $= T_c$ and the bit duration $= T_b = 4T_c$

- Find and sketch the transmitted signal for $0 \leq t \leq T_b$ when each one of the four stations transmits digit 1.
- If the receiver observes the following signal $[+2 \ -2 \ +2 \ +2]$ for $0 \leq t \leq T_b$, find the bit sent by station B.

Problem 10: Multiple Choice Question of CDMA

Walsh sequences

- are random signals of the form $y = A\cos(2\pi f_c t + \theta)$, θ is a random variable
- are deterministic signals of the form $y = A\cos(2\pi f_c t + \theta)$, θ is a constant
- are the components of the Fourier series expansion of a period square function.

D. are perfectly orthogonal set of square deterministic functions.

Problem 11: Stop and wait ARQ protocol

A file of size 24,000 bytes is to be sent from Host A to Host B using the Stop-and-Wait ARQ protocol. The file is divided into frames of 1,000 bytes each. Assume that each frame is successfully delivered on the first attempt (i.e., no retransmissions are needed), and the acknowledgment (ACK) is sent immediately after a frame is received.

The following parameters are given:

- a. Transmission rate: 2 Mbps (megabits per second)
 - b. One-way propagation delay between A and B: 20 ms
 - c. ACK size: negligible (i.e., assume zero transmission time)
 - d. Processing and queuing delays: negligible
-
- a. How many frames are required to transmit the entire file? (24 frames)
 - b. What is the transmission time for each frame? (4 ms); (44 ms)
 - c. What is the total time required to send the entire file using Stop-and-Wait protocol? (24*44 ms)
 - d. What is the link utilization (efficiency) of the protocol in this scenario? (4/44)

Problem 12: Stop and wait protocol

A file of 18,000 bytes is to be sent from Host A to Host B using the Stop-and-Wait ARQ protocol over a point-to-point link. The file is divided into frames of 1,500 bytes each. Each frame includes a header of 100 bytes, and the rest is payload.

The following parameters are known:

- a. Transmission rate: 1 Mbps
 - b. One-way propagation delay: 15 ms
 - c. Acknowledgment (ACK) size: negligible (no transmission time)
 - d. Processing and queuing delays: negligible
 - e. Probability of frame acceptance (i.e., no error): $P_{\text{success}} = 0.9$
 - f. Stop-and-Wait ARQ is used with retransmission of lost or corrupted frames.
-
- a. How many frames are required to send the entire file (12)
 - b. What is the transmission time for each frame? (12 ms)
 - c. What is the expected number of transmissions per frame? $(1/0.9) = 1.1111$
 - d. What is the expected total time to send one frame? $(1/0.9) * (42 \text{ ms})$
 - e. What is the expected total time to send the file? (12 frames) * $(1/0.9) * (42 \text{ ms})$
 - f. What is the expected link utilization? $(12 \text{ ms} / 42 * 1.1111 \text{ ms}) = 25.7\%$

Problem 13: Stop and wait ARQ protocol

A file of 30,000 bytes is to be sent from Sender A to Receiver B using the Stop-and-Wait ARQ protocol over a full-duplex link. The file is divided into data frames, each consisting of:

- 1,000 bytes of payload
- 100 bytes of header
- ACKs are 100 bits long and require transmission just like regular frames.

Channel and protocol characteristics:

- a. Transmission rate (both directions): 2 Mbps
- b. One-way propagation delay: 25 ms
- c. Bit error rate (BER): 10^{-5}
- d. A frame is considered lost if any bit is corrupted
- e. Stop-and-Wait ARQ is used with retransmission of erroneous or lost frames
- f. Assume errors are independent for each bit, and ACKs are never lost or corrupted
- g. No processing or queuing delays

- a. How many frames are needed to transmit the file? (30)
- b. Calculate the probability that a data frame is successfully received. ($P_{\text{success}} = (1 - 10^{-5})^{8800} \approx 0.9157$)
- c. Compute the expected number of transmissions per frame. ($1/0.9157$)
- d. Determine the expected transmission time per frame. (59.5 ms)
- e. Determine the expected total transmission time to send the entire file. (1785 ms)
- f. Compute the expected link utilization. (0.0739)

Problem 14: CDMA

In a synchronous CDMA system, four users A, B, C, and D are assigned the following orthogonal signature codes of length 4:

- A = [1, 1, 1, 1]
- B = [1, -1, 1, -1]
- C = [1, 1, -1, -1]
- D = [1, -1, -1, 1]

Assume:

- All codes are normalized (chip duration T_c , bit duration $T_b = 4T_c$)
- During a bit interval, the transmitted bits are:
 - A: +1
 - B: +1
 - C: -1
 - D: -1

The received signal vector due to noise is: $\mathbf{r} = [0, 2, 2, 0]$

- a. Verify that the codes are pairwise orthogonal.
- b. Compute the combined transmitted signal vector (before noise).
- c. Determine the noise vector added to the channel; $\mathbf{n} = \mathbf{r} - \mathbf{s}$ (received vector – transmitted vector)
- d. Use correlation detection to recover the bit sent by each user.
- e. Discuss how orthogonality ensures minimal multiple access interference (MAI) in this scenario.

Problem 15: FDM

A communication system uses Frequency Division Multiplexing (FDM) to transmit 4 analog voice signals simultaneously over a single channel. Each voice signal has a bandwidth of 3 kHz and requires a guard band of 1 kHz between adjacent channels to avoid interference.

Given:

- Total available channel bandwidth: (20 kHz)
 - Each voice channel: (3 kHz)
 - Guard band between adjacent channels: (1 kHz)
 - Carrier frequencies used for modulation are chosen to be non-overlapping.
- a. Determine whether it is possible to transmit all 4 signals within the available 20 kHz bandwidth using FDM.
 - b. If it is possible, sketch the frequency allocation diagram showing the positions of the four channels and the guard bands.
 - c. Determine the minimum required bandwidth if the number of channels were increased to 6, keeping the same voice and guard band requirements.
 - d. Explain how the receiver can recover a specific user's signal in an FDM system.

Problem 16: FDMA

A satellite communication system uses FDMA to allow 5 users to simultaneously transmit data over a shared satellite uplink. Each user is allocated a separate frequency band. Each user transmits a signal that occupies 2 kHz of bandwidth. To avoid inter-user interference, a guard band of 500 Hz is required between adjacent channels.

- a. Determine the minimum total bandwidth required to support all 5 users.
- b. Sketch the frequency allocation diagram showing the position of each user's band and the guard bands.
- c. If the available bandwidth is limited to 15 kHz, what is the maximum number of users the system can support while maintaining the same guard band and user signal bandwidth?

Problem 17: TDM

A communication system employs Time Division Multiplexing (TDM) to transmit data from 4 sensors (A, B, C, and D) to a central processor over a single shared link. Each sensor samples its analog signal at a rate of 1,000 samples per second, and each sample is quantized into 8 bits. The system uses synchronous TDM, where each sensor is assigned a dedicated time slot in a repeating frame.

What is the bit rate of the channel required to carry the combined TDM stream?

- a. Determine the duration of each time slot and the frame duration.
- b. If each frame starts with a synchronization bit pattern of 8 bits, what is the total overhead percentage introduced by synchronization?
- c. Explain how the central processor separates the individual sensor data streams.

Problem 18: Time Division Multiplexing of Voice Signals

A digital telecommunication system uses Time Division Multiplexing (TDM) to transmit multiple voice signals over a common communication channel. Each voice signal has the following characteristics:

- The bandwidth of each analog voice signal is 4 kHz.
- Each signal is sampled at the Nyquist rate.
- Each sample is quantized uniformly into 256 levels (i.e., 8 bits per sample).
- There are 60 simultaneous users.

- The system uses TDM to multiplex the encoded samples from all users into a single stream.
 - The resulting binary stream is transmitted using a baseband transmission scheme with Polar Non-Return-to-Zero (NRZ) line coding.
- a. Calculate the sampling rate per user based on the Nyquist criterion.
 - b. Determine the bit rate per user.
 - c. Compute the total TDM data rate required to transmit all 60 users over the channel.
 - d. Determine the 90% power bandwidth of the polar NRZ waveform
 - e. Assuming ideal Polar NRZ signaling, calculate the minimum channel bandwidth required to transmit the TDM data stream without intersymbol interference (Use the Nyquist criterion for zero ISI $B = R_b/2$)

Problem 19: Wireless Time Division Multiplexing (TDM)

A wireless communication system uses Time Division Multiplexing (TDM) to combine the voice signals of 60 users, each occupying a 4 kHz bandwidth. Voice signals are sampled at the Nyquist rate, quantized into 256 levels, and transmitted over a shared wireless channel.

- a. Compute the total TDM data rate (before modulation).
- b. Determine the minimum channel bandwidth if binary PSK is used
- c. Determine the minimum channel bandwidth if binary FSK is used
- d. Determine the minimum channel bandwidth if QPSK modulation is used

Problem 20: TDMA

A mobile system uses TDMA (Time Division Multiple Access) to serve users over a single 200 kHz frequency channel. The system operates as follows:

- Each TDMA frame is divided into 8 equal time slots.
 - Each active user is assigned one time slot per frame.
 - During a time slot, a user transmits at 270.8 kbps.
 - Each time slot carries 156.25 bits.
- a. What is the average data rate per user? (**33.85 kbps**)
 - b. What is the duration of one frame, based on the bits per slot and transmission rate? (**4.616 ms**)
 - c. What is the maximum number of users that can be supported simultaneously on this frequency channel? (8)

Problem 21: MAI in CDMA system with Walsh sequences

Consider a CDMA system that uses the spreading sequences:

- $A = [+1, -1, +1, -1]$
- $B = [-1, +1, -1, +1]$

Each user sends 1 bit of data per symbol interval, spread using their PN sequence.

- a. Find the received signal (sum of both).
- b. Using the correlator receiver, recover the bits sent by User A when B is received without any delay

- c. Repeat Part b when B is delayed by two chip bits.

Problem 22: MAI in CDMA system which uses PN sequences

Consider a CDMA system that uses the spreading sequences:

- $CA = [+1, +1, -1, +1, -1, -1, -1]$
- $CB = [+1, -1, -1, +1, +1, -1, +1]$

Each user sends digit 1 per symbol interval, spread using their PN sequence.

- a. Are CA and CB orthogonal?
- b. Find total transmitted signal. **$[+2, 0, -2, +2, 0, -2, 0]$**
- c. Using the correlator receiver, find the output of the A receiver (8)
- d. How do you assess the MAI component in the A receiver? (**$MAI = Output\ A - Ideal = 8 - 7 = +1$**)
- e. If signal B received delayed by one chip bit, find the received signal, new output and new MAI component. **$R = [1, 2, -2, 0, 0, 0, -2]$; $VA = 7$, $MAI = 0$**
- f. If signal B received delayed by two chip bits, find the received signal, new output and new MAI component. **$R = [1, 1, 0, 0, -2, 0, 0]$; $VA = 4$, $MAI = -3$.**

Problem 23: MAI using Gold sequences

Consider the two Gold sequences:

$GA = [+1, +1, +1, +1, +1, +1, +1, -1, +1, +1, +1, -1, +1, -1, -1, +1, +1, +1, +1, +1, +1, -1, +1, +1, -1, +1, -1, -1, +1]$

$GB = [-1, +1, +1, +1, +1, -1, -1, -1, +1, +1, -1, +1, +1, -1, -1, -1, -1, +1, +1, +1, -1, -1, -1, +1, +1, -1, -1, -1]$

- a. Find the cross correlation function $R_{AB}(\tau = 0)$; (7)
- b. Find the two autocorrelation function $R_{AA}(\tau = 0)$ and $R_{BB}(\tau = 0)$; (31, 31)
- c. If both users transmit digits 1, find the output of the A receiver (38)
- d. Find the MAI component in the A receiver, if any. (7)
- e. Find the output of A and the MAI for a delay in B by 1, 2, and 3 chip bits. (39, +8); (24, -7); (25, -6).