3.6 The weighted Mean and working with Grouped Data.

Weighted Mean : (sample)

$$\bar{X}_w = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$ where

x_i = value of observation i

w_i = weight for observation i

weighted population Mean

$$\mu_w = \frac{\sum_{i=1}^{N} w_i x_i}{\sum_{i=1}^{N} w_i}$$

Computing a grade point average is a good example of the use of a weighted mean

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Example (Q52 page 122) Consider the following data and crossponding weights:

x_i	Weight (w_i)	$w_i x_i$
3.2	6	19.2
2.0	3	6.0
2.5	2	5.0
5.0	8	40.0
12.7	19	70.2

[a] Compute the weighted mean

$$X_w = \frac{\sum w_i x_i}{\sum w_i} = \frac{70.2}{19} = 3.69$$

[b] Compute the sample mean without weighting:

$$\bar{X} = \frac{\sum x_i}{n} = \frac{12.7}{4} = 3.175$$

Grouped Data

Data available in class intervals that are summarized by a frequency distribution.

Example: Consider the following frequency distribution of the audit time for 20 clients.

Audit time (days)	(fi) Frequency	Class Midpoint (Mi)	$f_i M_i$	$M_i - \bar{X}_g$	$(M_i - \bar{X}_g)^2$	$f_i (M_i - \bar{X}_g)^2$
10–14	4	$(10+14)/2 = 12$	$4 \times 12 = 48$	$12-19=-7$	49	196
15–19	8	$(15+19)/2 = 17$	$8 \times 17 = 136$	$17-19=-2$	4	32
20–24	5	22	$5 \times 22 = 110$	$22-19=3$	9	45
25–29	2	27	$2 \times 27 = 54$	$27-19=8$	64	128
30–34	1	32	$1 \times 32 = 32$	$32-19=13$	169	169
	20		380			570

(a) Find the sample mean for the grouped data $\bar{X}_g = \frac{\sum f_i M_i}{n} = \frac{380}{20} = 19$

(b) Find the sample variance for the grouped data $s^2 = \frac{\sum f_i (M_i - \bar{X}_g)^2}{n-1} = \frac{570}{19} = 30$

(c) Find the sample standard deviation $s = \sqrt{30} \approx 5.48$

- Sample Mean for grouped data $\bar{X}_g = \frac{\sum_{i=1}^{n} f_i M_i}{n}$ where

M_i = the middle point of class i

f_i = the frequency of class i

n = the sample size

Population Mean for grouped data $\mu_g = \frac{\sum_{i=1}^{N} f_i M_i}{N}$

- Sample Variance for grouped data $s_g^2 = \frac{\sum f_i (M_i - \bar{X}_g)^2}{n-1}$

Population Variance for grouped data $\sigma_g^2 = \frac{\sum_{i=1}^{N} f_i (M_i - \mu_g)^2}{N}$

$\bar{X}_g$ is an estimator for μ_g

s_g^2 is an estimator for σ_g^2