

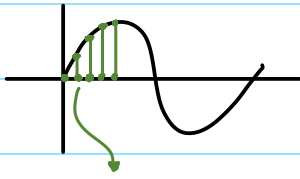
(لا تقعدوا فراغاً فان الموت يطلبكم) .

التلاميذ غير كافي

لهذا التثاقف

sampling of continuous - time signal

Periodic Sampling



- T_s :- Sampling period
- F_s :- Sampling Rate / Frequency (number of Samples / sec) Hz
- $\omega_s = \frac{2\pi}{T}$ (Sampling frequency) (radians / sec).
- $F_s = \frac{1}{T}$ (Sampling frequency) (samples/sec)

$F_s \uparrow \Rightarrow \text{quality} \uparrow \Rightarrow \text{memory} \uparrow$

$F_s \downarrow \Rightarrow \text{quality} \downarrow \Rightarrow \text{memory} \downarrow$

Higher $F_s \rightarrow$ Higher quality * 1

لما تزيد عدد العينات ، أنت تلفظ تفاصيل أدق
وبالتالي الجودة تتحسن

Higher $F_s \rightarrow$ More Memory Storage * 2

لأنك بتخزن نقاط أكثر في نفس الفترة الزمنية ،
حجم البيانات يمتد و بالتالي تحتاج مساحة أكبر
سواء للذاكرة أو القرص

مثال :-

← لو تسجل صوتك عند 8KHz (8000 عينة/ثانية)
الجودة متوسطة ، والحجم صغير

← لو تسجل صوتك عند 44.1KHz (44100 عينة/ثانية)
الجودة ممتازة لكن حجم أكبر

* To determine the best F_s , **Nyquist Rate** is presented.

Nyquist Rate $\gg 2$ (maximum frequency of the signal)

$$F_s \gg 2 BW$$

* if $F_s < \text{Nyquist Rate} \rightarrow$ **Aliasing in Sampling** $\rightarrow$ لما نأخذ عينات لا إشارة لكن بمعدل قليل

جداً ، فالإشارة به أخذ العينات بكونه

مش صحيحة وفيها تشويه

* إذا كان تردد أخذ العينات F_s أقل من ضعف

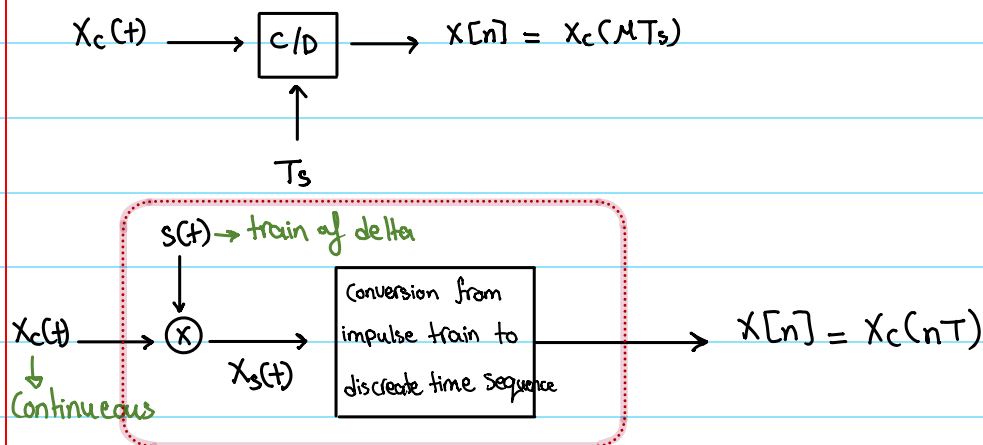
أعلى تردد موجود في الإشارة (Nyquist Rate)

بالتالي راجع ليس عنا تداخل بين الترددات (overlap)

في المجال الترددي وهذا ببولد Aliasing ..

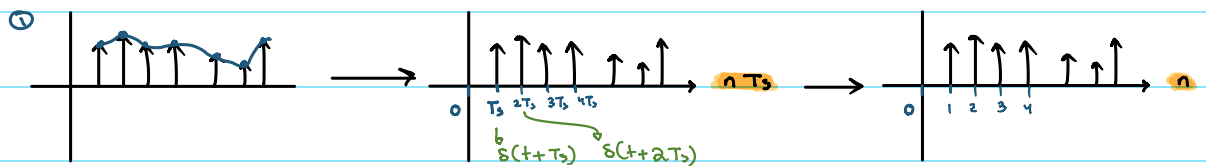
A/D $\rightarrow$ Convert from Analog to Digital

C/D $\rightarrow$ Convert from Cont. to Discrete.



sampling process $\rightarrow$ two stages

1. Impulse train Modulator. "multiplying the Cont. signal by train of impulses".
2. Normalization "convert the impulse train the Discrete sequence by divide by T_s in time domain



$$\sum_{n=-\infty}^{\infty} \delta(t+nT_s)$$

$$X_s(t) = X_c(t) \cdot s(t) \Rightarrow \text{normalization } \frac{nT_s}{T_s}$$

$$= X_c(t) \sum_{n=-\infty}^{\infty} \delta(t-nT_s)$$

$$= \sum_{n=-\infty}^{\infty} X_c(nT_s) \delta(t-nT_s) \quad \rightarrow \text{sifting property.}$$

$$X(t) \cdot \delta(t) = X(s) \delta(t)$$

$\rightarrow$ The Fourier transform of aperiodic impulse train

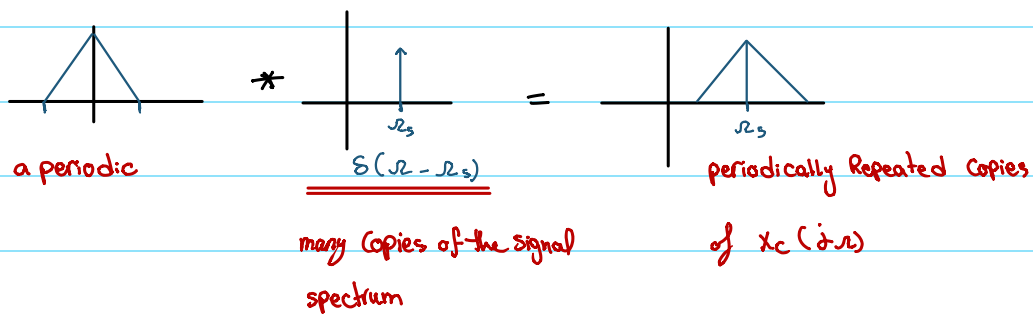
$$S(j\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_s) \rightarrow \textcircled{1}$$

$$X_s(j\omega) = \frac{1}{2\pi} X_c(j\omega) * S(j\omega)$$

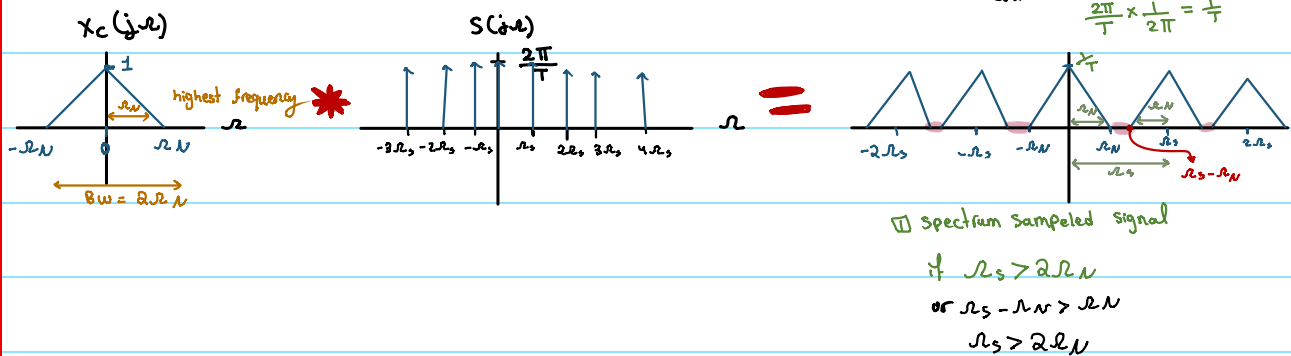
$\rightarrow$ substitute $\textcircled{1}$.

$$X_s(j\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X_c(j(\omega - n\omega_s)) \rightarrow \text{periodically Repeated copies of } X_c(j\omega)$$

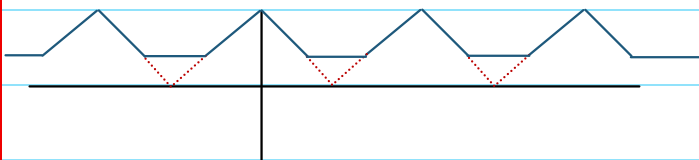
* copies of $X_c(j\omega)$ are shifted by integer multiples of sampling frequency ω_s



More Details:-



2) if $\omega_s < 2\omega_N \Rightarrow$ Aliasing

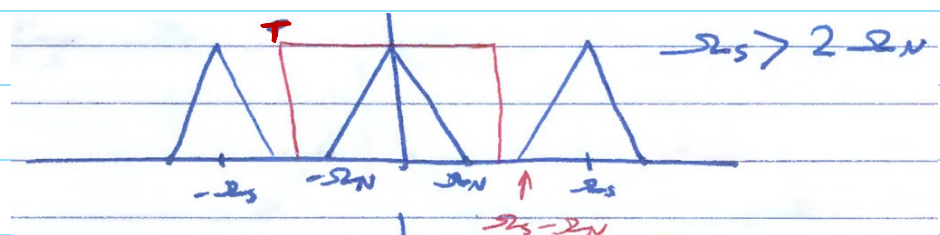


$s(t) \rightarrow \otimes \rightarrow X_s(t)$ in time domain
 $X_c(t)$
 $X_s(j\omega) = \frac{1}{2\pi} X_c(j\omega) * S(j\omega)$

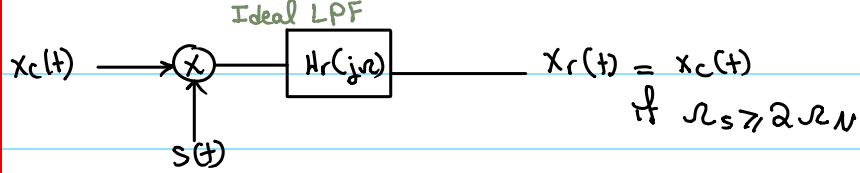
Q. How to get the Original signal?

A. By multiplying $X_s(t)$ with analog low pass filter in frequency domain, then we can get the Original signal $X_c(t)$.

* Note that, it's Analog not digital, since digital is periodic and we only need one signal no more. However, Analog low pass filter is non periodic

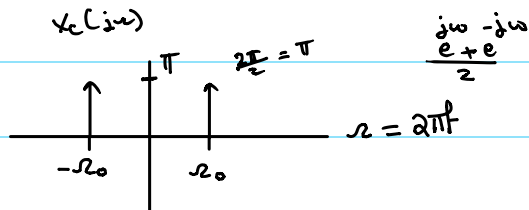


the Cut of Frequency $\omega_N < \omega_c < \omega_s - \omega_N \Rightarrow$ we often take $\omega_c = \frac{\omega_s}{2}$



Example: Assume $x_c(t) = \cos(\Omega_0 t)$

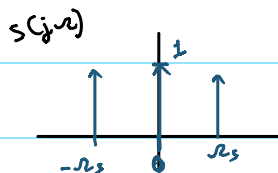
* periodic $\rightarrow$ in time domain



$$\frac{j\omega - j\omega}{e + e}$$

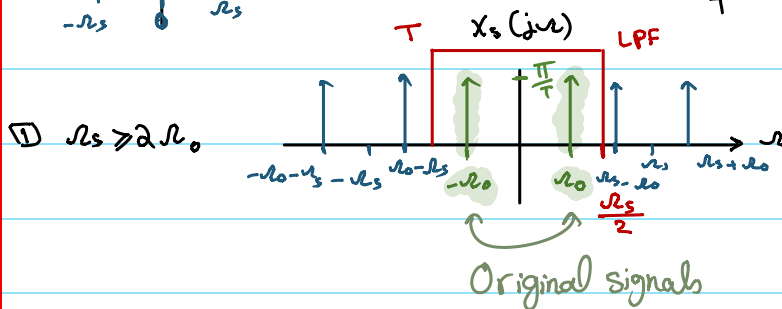


Aperiodic $\rightarrow$ in frequency domain



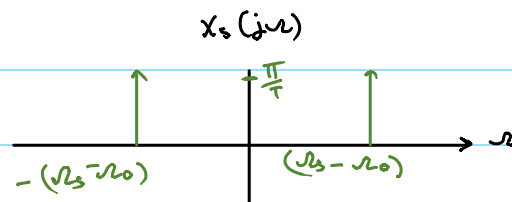
$$x_r(t) = \frac{1}{T} (x_c(j\omega) * s(j\omega))$$

$$= \frac{1}{T} \times 1 \times \pi = \pi/T \rightarrow \text{magnitude}$$



② $\Omega_s < 2\Omega_0 \rightarrow$ Aliasing.

$$\Omega_0 > \frac{\pi}{T}$$

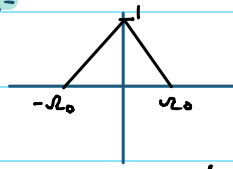


$$\Omega = \frac{\omega}{T} \rightarrow \omega = \Omega T$$

$$x[n] = x_c[nT]$$

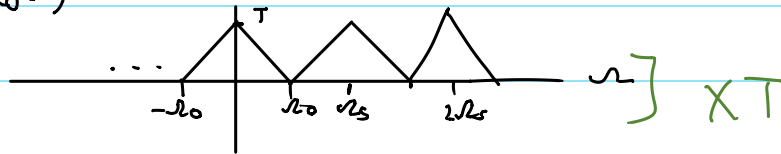
Example 8-

$X_c(j\Omega)$



Ans:- $X_s(j\Omega) = \frac{1}{T} X_c(j\Omega) * S(j\Omega)$

$X_s(j\Omega)$

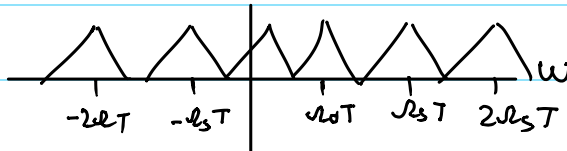


$$\Omega = \frac{\omega}{T}$$

$$\downarrow$$

$$\omega = \Omega T$$

$X_s(e^{j\omega})$



Example:- $X_c(t) = \cos(4000\pi t)$, $T = \frac{1}{6000}$ sec

Find, $X_c(j\Omega)$, $X_s(j\Omega)$, $X(e^{j\omega})$, $S(j\Omega)$

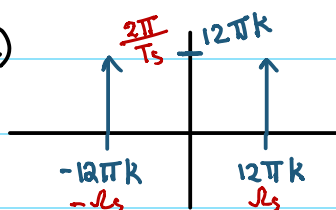
① $\Omega_0 = 4000\pi \rightarrow f_0 = \frac{\Omega_0}{2\pi} = \frac{4000\pi}{2\pi} = 2000 \text{ Hz}$

② $F_s = 6000 \text{ Hz}$, $T_s = \frac{1}{6000}$ sec.

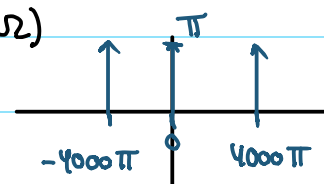
③ $\Omega_s = \frac{2\pi}{T_s} = 12000\pi$

④ $\omega = T_s \Omega = \frac{4000\pi}{6000} = \frac{2}{3}\pi$

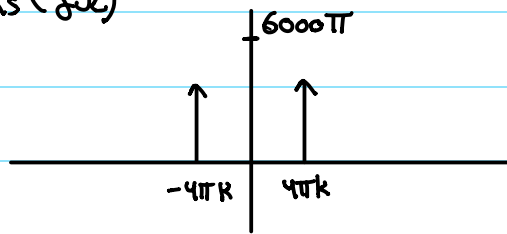
① $S(j\Omega)$



② $X_c(j\Omega)$



③ $X_s(j\omega)$



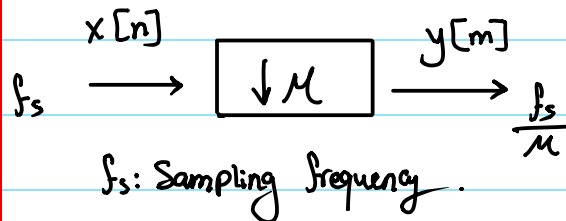
$$X_s(j\omega) = \frac{1}{2\pi} X_c(j\omega) * S(\omega)$$

$$\begin{aligned} \text{mag} &= \frac{1}{2\pi} \cdot \frac{2\pi}{T_s} \cdot \pi = \frac{\pi}{T_s} \\ &= \frac{\pi}{\frac{1}{6000}} = 6000\pi \end{aligned}$$

Decimation:

two step process → ① low pass filtering
② down sampling by M .

$$* f_{s, \text{new}} = \frac{f_{s, \text{old}}}{M}$$



* to ensure that the Nyquist Condition is valid, a low pass filter is applied directly before the down sampling, the cut off frequency of the low pass filter should be $\frac{1}{2}$ of the new Sampling Rate.

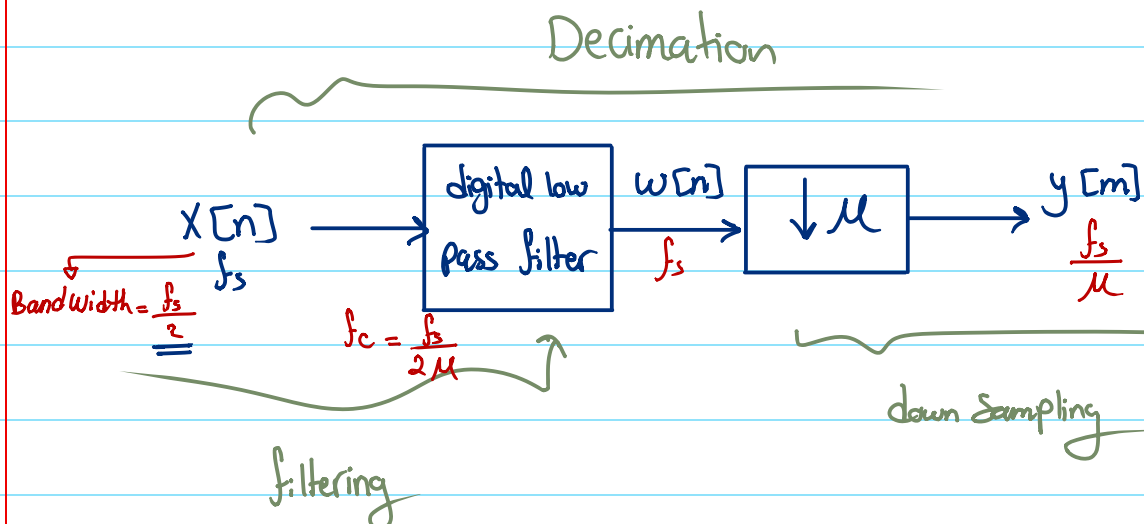
Example: $x[n] = \{1, 2, 3, 4, 5, -6, -8, 2, -3, 2\}$

1) Down Sample by 2.

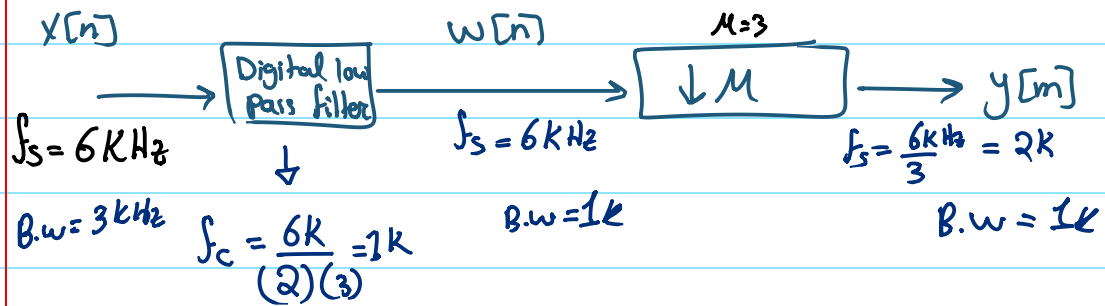
$$\text{new bandwidth} = \text{"cut off frequency"} = \frac{f_s}{2(2)} = \frac{f_s}{4}$$
$$y[m] = \{1, 3, 5, -8, -3\}$$

2) Down Sample by 3. → Band width = $\frac{f_s}{6}$

$$y[m] = \{1, 4, -8, 2\}$$



Example 8:



Example 8- Decimation of $x[n] = \{2, 6, 4, 2, 6, 8, 4, 2, 4, 4\}$

$M=2$, find $w[n]$ and $y[n]$?

$$h[n] = \{1/2, 1/2\}$$

gain = 1, $\omega_c = \frac{\pi}{2}$, for the LPF.

$$w[n] = x[n] * h[n]$$

	2	6	4	2	6	8	4	2	4	4
x	1	3	2	1	3	4	2	1	2	2
$x/2$	1	3	2	1	3	4	2	1	2	2

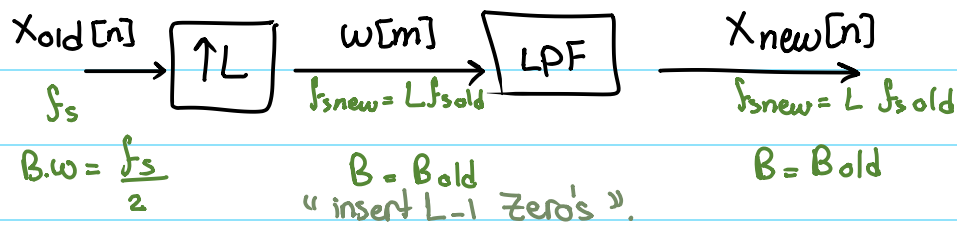
$$w[n] = \{1, 4, 5, 3, 4, 7, 6, 3, 3, 4, 2\}$$

$$y[m] = \{1, 5, 4, 6, 3, 2\}$$

$$* \quad \omega_c = \frac{2\pi f_{s,old}}{f_{s,new}} = \frac{2\pi \frac{f_s}{2M}}{f_s} = \frac{\pi}{M}$$

interpolation

Sample Rate increase by interpolation. New Sample values need to be calculated



Example:- $x[n] = \{1, 0.9, -0.5\}$, $L=3$, find $w[n]$:-

$$w[n] = \{1, 0, 0, 0.9, 0, 0, -0.5, 0, 0\}$$

Example:- $x[n] = \{1, 2, 4, 3, -5, 6, -7, 2, 4, 3\}$ find Interpolation for :-

① $L=2$.

$$w[n] = \{1, 0, 2, 0, 4, 0, 3, 0, -5, 0, 6, 0, -7, 0, 2, 0, 4, 0, 3, 0\}$$

② $L=3$

$$w[n] = \{1, 0, 0, 2, 0, 0, 4, 0, 0, 3, 0, 0, -5, 0, 0, 6, 0, 0, -7, 0, 0, 2, 0, 0, 4, 0, 0, 3, 0, 0\}$$

Note:-

the quality of the signal will not improved after the interpolation even the Sampling.

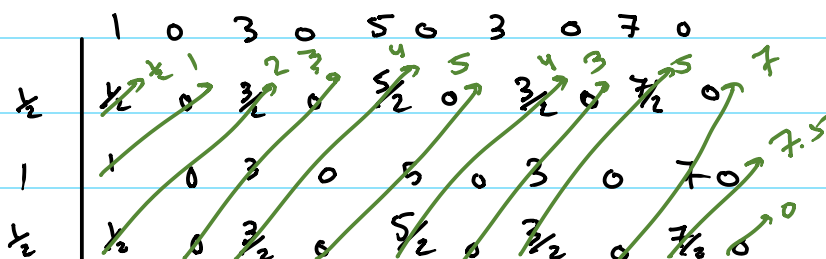
Example:- interpolation of $x[n] = \{1, 3, 5, 3, 7\}$, $h[n] = \{\frac{1}{2}, \frac{1}{2}\}$

find $w[m]$, $y[m]$?

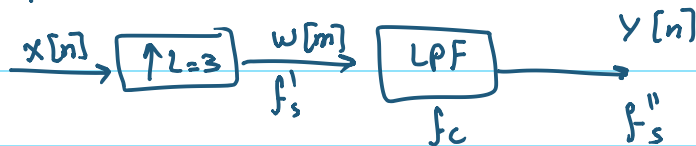
$$\text{gain} = 2, \quad \omega_c = \pi/2, \quad L = 2$$

$$w[m] = \{1, 0, 3, 0, 5, 0, 3, 0, 7, 0\}$$

$$y[m] = w[m] * h[n] = \{0.5, 1, 2, 3, 4, 5, 4, 3, 5, 7, 7.5, 0\}$$



Example 8



$$f_s = 8 \text{ KHz}$$

$$BW = 2 \text{ KHz}$$

What are the values of f_c and f_s'' ?

$$f_s'' = f_s' = L f_s = (3)(8 \text{ KHz}) = 24 \text{ KHz}$$

$$f_c = BW = 2 \text{ KHz} \quad \text{or} \quad \frac{f_s}{2} \quad \text{if the BW is not given.}$$

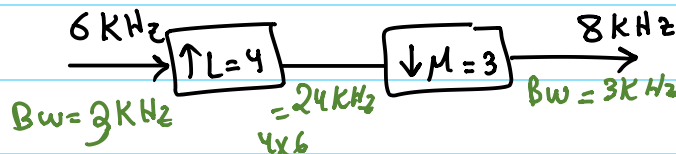
Sampling Rate conversion by non integer factor:

$$6 \text{ KHz}$$



$$\frac{8}{6} = \frac{4}{3}$$

In this case L is non integer so, we need to do the following steps.



The overall process is interpolation since the ratio is $\frac{4}{3}$ more than 1. For best quality, do up sampling then down sampling.

Example 8-



$$\frac{48 \text{ K}}{44.1 \text{ K}} = \frac{48000}{44100} = \frac{160}{147} = \frac{L}{M}$$

The overall process

