PHYS I

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Chapter 3 :- "Vectors."

Physical Quantities.

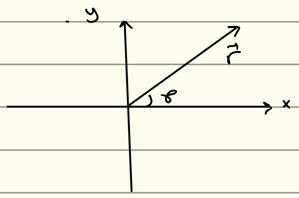
- Scalars
  - magnitude
  - unit
- Vectors.
  - magnitude
  - unit
  - direction.

ex:- Position : $(\vec{r})$.

O : refrnce Point

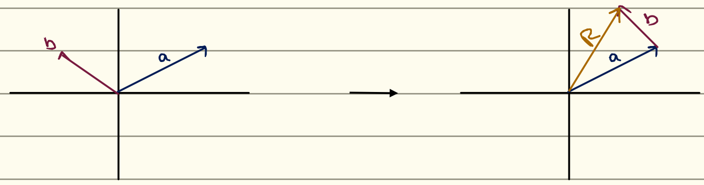
$|\vec{r}|$: magnitude

θ : direction.



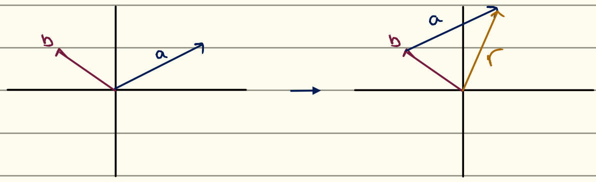
1.. Adding vectors Geometrically :

$\vec{r} = \vec{a} + \vec{b}$



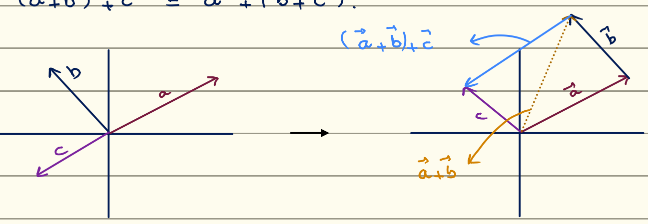
* Properties of vector addition :-

1. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$



2. $\vec{a} + \vec{b} + \vec{c}$

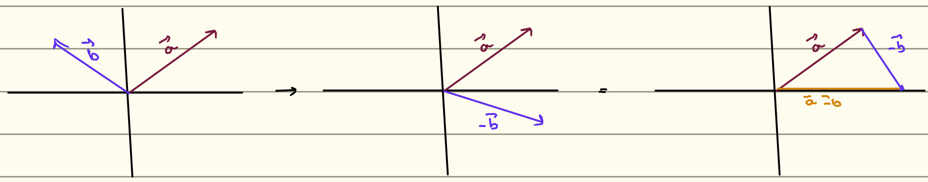
$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$.



2.. Vectors subtraction:

$\vec{r} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b})$

لـ الأول + مقلوب المتجه الثاني.



ex:- $\vec{a} = 3$ east

$\vec{b} = 4$ east

$\vec{a}+\vec{b}$? $\quad \vec{a}+\vec{b} = 3+4$

$= 7.$ east

$\vec{a} = 3$ east

$\vec{b} = 4$ west

$\vec{a}+\vec{b}$? $\quad \vec{a}+\vec{b} = 3-4$

$= -1$, or 1 west.

3 .. Component of vectors :

↳ A component of a vector is the Projection of the vector on an axis.

Note :-

CW → clock wise (مع عقارب الساعة) (-).

CCW → counter clock wise (عكس عقارب الساعة) (+).

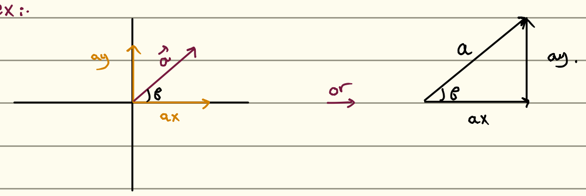
* طول المتجه يمثل مقدار الكمية

* المحور القريب ع الزاوية بجد $\underline{\underline{\cos}}$.

* لما يعطيني الزاوية بدي أعطي الاتجاه

* كل متجه بقدر أعرف اتجاهه بالزاوية المحصورة بينه وبين محور السينات الموجب.

ex :-



* Note :-

$a_x = |a| \cos\theta$

$a_y = |a| \sin\theta$

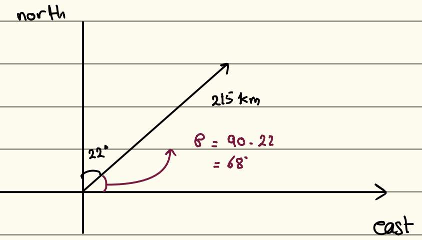
> scalar component of $\vec{a}$

$a = \sqrt{(a_x)^2 + (a_y)^2}$ → victor magnitude.

$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)$ → victor direction.

Sample Problem 3.02 **Finding components, airplane flight**

A small airplane leaves an airport on an overcast day and is later sighted 215 km away, in a direction making an angle of 22° east of due north. This means that the direction is not due north (directly toward the north) but is rotated 22° toward the east from due north. How far east and north is the airplane from the airport when sighted?



$a_x = 215 \cos 68$

$= 81$ km.

$a_y = 215 \sin 68$

$= 199$ km.

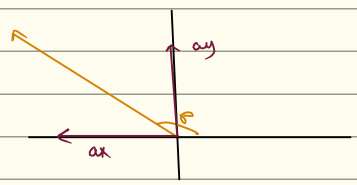
ex :- $\vec{a} = 20$, at 110° with x^+ counter clock wise

$a_x = 20 \cos 110$

$= -6.84$

$a_y = 20 \sin 110$

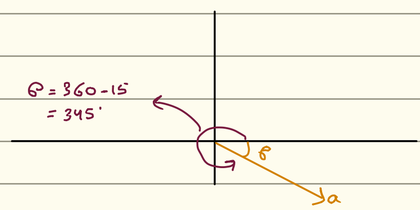
$= 18.8$.



ex :- $\vec{a} = 20$ at 15° with x^+ (clock wise).

$a_x = 20 \cos 345$

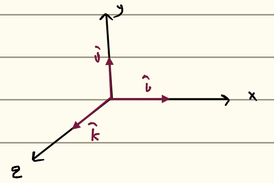
$a_y = 20 \sin 345$



4 .. Unit vectors :

is a vector that has a magnitude of exactly & Points in aparticular direction.

ex :-



$\hat{i}$: unit vector in +x-dir.

$\hat{j}$: unit vector in +y-dir.

$\hat{k}$: unit vector in +z-dir.

ex :- $\vec{a}$ = 20 at 110° with +x counter clock wise.

$a_x = 20\cos 110°$, $a_y = 20\sin 110°$

$= -6.8$. $= 18.8$.

$\therefore \vec{a} = -6.8\hat{i} + 18.8\hat{j}$

vector component.

* adding vector by components :-

$\vec{R} = \vec{a} + \vec{b}$

$= (a_x + b_x)\hat{i} + (a_y + b_y)\hat{j} + (a_z + b_z)\hat{k}$

بجمع المتشابهات مع بعض.

3 A vector has a component of 15 m in the $+x$ direction, a component of 15 m in the $+y$ direction, and a component of 10 m in the $+z$ direction. What is the magnitude of this vector?

$\vec{R} = 15\hat{i} + 15\hat{j} + 10\hat{k}$

$|\vec{R}| = \sqrt{(15)^2 + (15)^2 + (10)^2}$

$= 23.4$ m

9 Consider two vectors $\vec{a} = (5.0)\hat{i} - (4.0)\hat{j} + (2.0)\hat{k}$ and $\vec{b} = (-2.0m)\hat{i} + (2.0m)\hat{j} + (5.0m)\hat{k}$, where m is a scalar. Find (a) $\vec{a} + \vec{b}$, (b) $\vec{a} - \vec{b}$, and (c) a third vector $\vec{c}$ such that $\vec{a} - \vec{b} + \vec{c} = 0$.

$\vec{a} = 5\hat{i} - 4\hat{j} + 2\hat{k}$

$\vec{b} = -2\hat{i} + 2\hat{j} + 5\hat{k}$

a) $(5\hat{i} - 4\hat{j} + 2\hat{k}) + (-2\hat{i} + 2\hat{j} + 5\hat{k})$

$= 3\hat{i} - 2\hat{j} + 7\hat{k}$

b) $(5\hat{i} - 4\hat{j} + 2\hat{k}) - (-2\hat{i} + 2\hat{j} + 5\hat{k})$

$= 7\hat{i} - 6\hat{j} - 3\hat{k}$

c) $\vec{a} - \vec{b} + c = 0$

$7\hat{i} - 6\hat{j} - 3\hat{k} + c = 0$

$c = -7\hat{i} + 6\hat{j} , 3\hat{k}$

12 A car is driven east for a distance of 40 km, then north for 30 km, and then in a direction 30° east of north for 25 km. Sketch the vector diagram and determine (a) the magnitude and (b) the angle of the car's total displacement from its starting point.

a) $D_1 = 40\hat{i}$

$D_2 = 30\hat{j}$

$D_4 = 25\sin 30\hat{i} + 25\cos 30\hat{j}$

$\therefore \vec{D} = 52.5\hat{i} + 51.6\hat{j}$.

$|\vec{D}| = \sqrt{(52.5)^2 + (51.6)^2}$

$= 73.6$ m

b) $\tan\theta = \frac{51.6}{52.5} \longrightarrow \theta = 44.4$ with x^+.

4 .. Multiplying vectors :

* vector by scalar . عدد ضرب متجه .

ex.: let $\vec{x} = 2\hat{i} + 3\hat{j}$

$2\vec{x} = 4\hat{i} + 6\hat{j}$

* vector by vector. متجه ضرب متجه .

1. Dot Product (scalar Product).

$\vec{A}.\vec{B} = AB\cos\theta$

if $\theta = 0 \longrightarrow \vec{A}.\vec{B} = AB\cos 0$

$= AB$ (Max value).

if $\theta = 90 \longrightarrow \vec{A}.\vec{B} = AB\cos 90$

$=$ zero.

In general :-

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ $\qquad$ $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$.

لـ الضرب النقطي بكون بدون اتجاهات لأنو قياسي وعشان هيك بنضرب
المتجهات اللي فيهم (متجه الوحدة) زي $\rightarrow$ $(\hat{i}, \hat{j}, \hat{k})$ ببعض
المتشابهات مع بعض لأنه الزاوية بينهم تساوي 0 و $\cos 0 = 1$
ولا نضع اتجاه $\Leftarrow$ مقدار فقط . وطبعاً صار الضرب تبادلي.

Sample Problem 3.05 **Angle between two vectors using dot products**

What is the angle ϕ between $\vec{a} = 3.0\hat{i} - 4.0\hat{j}$ and $\vec{b} = -2.0\hat{i} + 3.0\hat{k}$? (*Caution:* Although many of the following steps can be bypassed with a vector-capable calculator, you will learn more about scalar products if, at least here, you use these steps.)

$\vec{a}.\vec{b} = (3\hat{i} - 4\hat{j}) \cdot (-2\hat{i} + 3\hat{k})$

$= -6$

$|\vec{a}| = \sqrt{(3)^2 + (4)^2}$

$= 5$

$|\vec{b}| = \sqrt{(2)^2 + (3)^2}$

$= 3.61$

$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$-6 = (5)(3.61)\cos\theta$

$\cos\theta = -0.33$

$\theta = 109.4°$.

2. Cross Product (Vector Product).

$\vec{A}.\vec{B} = \vec{A}\times\vec{B}\sin\theta$

if $\theta = 0 \longrightarrow \vec{A}.\vec{B} = AB\sin 0$

$=$ zero

if $\theta = 90 \longrightarrow \vec{A}.\vec{B} = AB\sin 90$

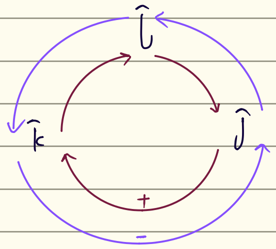
$= AB$ (Max value).

لـ الضرب التقاطعي مو ضرب متجهي يعني لازم يكون عندي مقدار واتجاه وعشان
اطلع الاتجاه بستخدم المصفوفات او باستخدام اليد اليمنى وهذا الضرب مش
تبادلي $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$ ، والمتجه الناتج من الضرب بكون عامودي على المتجهين

* Right hand tools :-

نحط أصابعي على المتجه الأول ونلف ايدي
على المتجه الثاني والإبهام بيمثل الاتجاه .

* $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$.



* $\vec{a} \times \vec{b} = \begin{array}{c} \\ \\ \\ \end{array}\begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ \end{array}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$= (a_y b_z - a_z b_y)\hat{i} - (a_x b_z - a_z b_x)\hat{j} + (a_x b_y - a_y b_x)\hat{k}$.

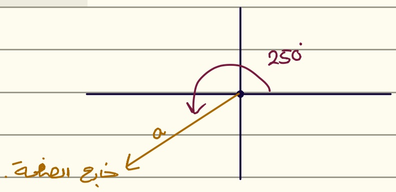
Sample Problem 3.06 **Cross product, right-hand rule**

In Fig. 3-20, vector $\vec{a}$ lies in the xy plane, has a magnitude of 18 units, and points in a direction 250° from the positive direction of the x axis. Also, vector $\vec{b}$ has a magnitude of 12 units and points in the positive direction of the z axis. What is the vector product $\vec{c} = \vec{a} \times \vec{b}$?

$\vec{c} = \vec{a} \times \vec{b} \sin\theta$

$= (18)(12) \sin(90)$

$= 216$.



$\theta = 250 - 90$

$= 160°$ with $x+$

Sample Problem 3.07 **Cross product, unit-vector notation**

If $\vec{a} = 3\hat{i} - 4\hat{j}$ and $\vec{b} = -2\hat{i} + 3\hat{k}$, what is $\vec{c} = \vec{a} \times \vec{b}$?

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 0 \\ -2 & 0 & 3 \end{vmatrix}$$

$= (-12-0)\hat{i} - (9-0)\hat{j} + (0-8)\hat{k}$

$= -12\hat{i} - 9\hat{j} - 8\hat{k}$.

Note :-

$\hat{a} = \dfrac{\vec{A}}{|A|}$ → لو طلب متجه يوازي المتجه $\vec{a}$ بستخدم هاد القانون

Lecture problems:

3 A vector has a component of 15 m in the $+x$ direction, a component of 15 m in the $+y$ direction, and a component of 10 m in the $+z$ direction. What is the magnitude of this vector?

$\vec{R} = 15\hat{i} + 15\hat{j} + 10\hat{k}$

$|\vec{R}| = \sqrt{(15)^2 + (15)^2 + (10)^2}$

$= 23.4$ m

9 Consider two vectors $\vec{a} = (5.0)\hat{i} - (4.0)\hat{j} + (2.0)\hat{k}$ and $\vec{b} = (-2.0m)\hat{i} + (2.0m)\hat{j} + (5.0m)\hat{k}$, where m is a scalar. Find (a) $\vec{a} + \vec{b}$, (b) $\vec{a} - \vec{b}$, and (c) a third vector $\vec{c}$ such that $\vec{a} - \vec{b} + \vec{c} = 0$.

$\vec{a} = 5\hat{i} - 4\hat{j} + 2\hat{k}$

$\vec{b} = -2\hat{i} + 2\hat{j} + 5\hat{k}$

a) $(5\hat{i} - 4\hat{j} + 2\hat{k}) + (-2\hat{i} + 2\hat{j} + 5\hat{k})$

$= 3\hat{i} - 2\hat{j} + 7\hat{k}$

b) $(5\hat{i} - 4\hat{j} + 2\hat{k}) - (-2\hat{i} + 2\hat{j} + 5\hat{k})$

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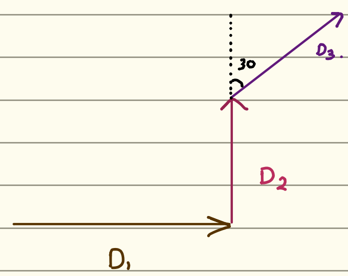
c) $\vec{a} - \vec{b} + c = 0$

$7\hat{i} - 6\hat{j} - 3\hat{k} + c = 0$

$c = -7\hat{i} + 6\hat{j} + 3\hat{k}$

12 A car is driven east for a distance of 40 km, then north for 30 km, and then in a direction 30° east of north for 25 km. Sketch the vector diagram and determine (a) the magnitude and (b) the angle of the car's total displacement from its starting point.



a) $D_1 = 40\hat{i}$

$D_2 = 30\hat{j}$

$D_4 = 25\sin 30\hat{i} + 25\cos 30\hat{j}$

$\therefore \vec{D} = 52.5\hat{i} + 51.6\hat{j}$.

$|\vec{D}| = \sqrt{(52.5)^2 + (51.6)^2}$

$= 73.6$ m

b) $\tan\theta = \frac{51.6}{52.5} \longrightarrow \theta = 44.4$ with x^+.

23 If $\vec{b} = (3.0)\hat{i} + (4.0)\hat{j}$ and $\vec{a} = \hat{i} - \hat{j}$, what is the vector having the same magnitude as that of $\vec{b}$ and parallel to $\vec{a}$?

* the vector have the same magnitude as $\vec{b}$:.

$|\vec{b}| = \sqrt{(3)^2 + (4)^2}$

$= 5.$

* the vector Parallel $\vec{a}$:.

$\hat{a} = \frac{\vec{a}}{|\vec{a}|} \longrightarrow \frac{\hat{i} - \hat{j}}{\sqrt{2}}$

$\therefore \vec{R} = 5\left(\frac{\hat{i} - \hat{j}}{\sqrt{2}}\right)$

$= \frac{5\hat{i}}{\sqrt{2}} - \frac{5\hat{j}}{\sqrt{2}}.$

36 Consider two vectors $\vec{p}_1 = 4\hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{p}_2 = -6\hat{i} + 3\hat{j} - 2\hat{k}$. What is $(\vec{p}_1 + \vec{p}_2)\cdot(\vec{p}_1 \times 5\vec{p}_2)$?

$\vec{P}_1 + \vec{P}_2 \longrightarrow (4\hat{i} - 3\hat{j} + 5\hat{k}) + (-6\hat{i} + 3\hat{i} - 2\hat{k})$

$= -2\hat{i} + 3\hat{k}$

$\vec{P}_1 \times 5\vec{P}_2 \longrightarrow (4\hat{i} - 3\hat{j} + 5\hat{k}) \times 5(-6\hat{i} + 3\hat{i} - 2\hat{k})$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -3 & 5 \\ -30 & 15 & -10 \end{vmatrix} \Rightarrow (30 - 75)\hat{i} - (-40 + 150)\hat{j} + (60 - 90)\hat{k}$

$= -45\hat{i} - 110\hat{j} - 30\hat{k}$

$\therefore (\vec{P}_1 + \vec{P}_2)\cdot(\vec{P}_1 \times 5\vec{P}_2) \longrightarrow (-2\hat{i} + 3\hat{k})\cdot(-45\hat{i} - 110\hat{j} - 30\hat{k})$

$= (90 - 90)$

$= 0.$

41 Use the definition of scalar product, $\vec{a}\cdot\vec{b} = ab\cos\theta$, and the fact that $\vec{a}\cdot\vec{b} = a_xb_x + a_yb_y + a_zb_z$ to calculate the angle between the following two vectors: $\vec{a} = 4.0\hat{i} + 4.0\hat{j} + 4.0\hat{k}$ and $\vec{b} = 3.0\hat{i} + 2.0\hat{j} + 4.0\hat{k}$.

$|\vec{a}| = \sqrt{(4)^2 + (4)^2 + (4)^2}$

$= 6.92.$

$|\vec{b}| = \sqrt{(3)^2 + (2)^2 + (4)^2}$

$= 5.38.$

$\vec{a}\cdot\vec{b} = (4\hat{i} + 4\hat{j} + 4\hat{k}) + (3\hat{i} + 2\hat{j} + 4\hat{k})$

$= (12\hat{i} + 8\hat{j} + 16\hat{k})$

$= 36.$

$\therefore \vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$36 = (6.92)(5.38)\cos\theta$

$\cos\theta = 0.966$

$\theta = 15^\circ$

Discussion problems:

1 If the x component of a vector $\vec{a}$, in the xy plane, is half as large as the magnitude of the vector, find the tangent of the angle between the vector and the x axis.

$$a_x = \frac{1}{2}a$$

$$a\cos\theta = \frac{1}{2}a$$

$$\cos\theta = \frac{1}{2} \longrightarrow \theta = 60^\circ$$

7 Consider two displacements, one of magnitude 3 m and another of magnitude 4 m. Show how the displacement vectors may be combined to get a resultant displacement of magnitude (a) 7 m, (b) 1 m, and (c) 5 m.

$D_1 = 3m$

$D_2 = 4m.$

a) $R = D_1 + D_2 \quad , \theta = 0$

$= 3 + 4$

$= 7m.$

b) $R = D_2 - D_1 \quad , \theta = 180$

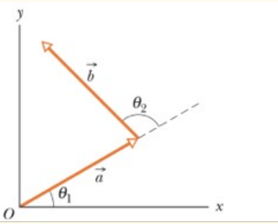
$= 4 - 3$

$= 1\ m$

c) $R = \sqrt{(D_1)^2 + (D_2)^2} \quad , \theta = 90$

$= \sqrt{(4)^2 + (3)^2} \longrightarrow 5m.$

15 The two vectors $\vec{a}$ and $\vec{b}$ in Fig. 3-23 have equal magnitudes of 10.0 m and the angles are $\theta_1 = 30^\circ$ and $\theta_2 = 105^\circ$. Find the (a) x and (b) y components of their vector sum $\vec{r}$, (c) the magnitude of $\vec{r}$, and (d) the angle $\vec{r}$ makes with the positive direction of the x axis.



$\vec{a} = 10\cos 30\,\hat{i} + 10\sin 30\,\hat{j}$

$= 8.66\,\hat{i} + 5\,\hat{j}$

$\vec{b} = 10\cos 135\,\hat{i} + 10\sin 135\,\hat{j} \longrightarrow \theta = 30 + 105.$

$= -7.07\,\hat{i} + 7.07\,\hat{j}$

$\vec{r} = \vec{a} + \vec{b}$

$= (8.66\,\hat{i} + 5\,\hat{j}) + (-7.07\,\hat{i} + 7.07\,\hat{j})$

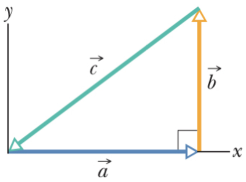
$= 1.59\,\hat{i} + 12.07\,\hat{j}$

$|\vec{r}| = \sqrt{(1.59)^2 + (12.07)^2}$

$= 12.17\ m.$

$\theta = \tan^{-1}\left(\frac{12.07}{1.59}\right) \longrightarrow \theta = 82.5^\circ$

32 For the vectors in Fig. 3-26, with $a = 4$, $b = 3$, and $c = 5$, what are (a) the magnitude and (b) the direction of $\vec{a} \times \vec{b}$, (c) the magnitude and (d) the direction of $\vec{a} \times \vec{c}$, and (e) the magnitude and (f) the direction of $\vec{b} \times \vec{c}$? (The z axis is not shown.)



$\vec{a} = 4\hat{i}$

$\vec{b} = 3\hat{j}$

$-\vec{c} = 4\hat{i} + 3\hat{j}$

a) $\vec{a} \times \vec{b}$

$= 4\hat{i} \times 3\hat{j}$

$= 12\hat{k}$

d) $\vec{a} \times \vec{c}$

$(4\hat{i}) \times (-4\hat{i} + -3\hat{j})$

$= -12\hat{k}$

f) $\vec{b} \times \vec{c}$

$= (3\hat{i}) \times (-4\hat{i} - 3\hat{i})$

$= 12\hat{k}$

35 Two vectors $\vec{p}$ and $\vec{q}$ lie in the xy plane. Their magnitudes are 3.50 and 6.30 units, respectively, and their directions are 220° and 75.0°, respectively, as measured counterclockwise from the positive x axis. What are the values of (a) $\vec{p} \times \vec{q}$ and (b) $\vec{p} \cdot \vec{q}$?

$\vec{p} = 3.5 \cos 220\,\hat{i} + 3.5 \sin 220\,\hat{j}$

$= -2.68\hat{i} - 2.24\hat{j}$

$\vec{q} = 6.3 \cos 75\,\hat{i} + 6.3 \sin 75\,\hat{j}$

$= 1.63\hat{i} + 6.08\hat{j}.$

a) $\vec{p} \times \vec{q} \longrightarrow (-2.68\hat{i} - 2.24\hat{j}) \times (1.63\hat{i} + 6.08\hat{j}).$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2.68 & -2.24 & 0 \\ 1.63 & 6.08 & 0 \end{vmatrix} \longrightarrow (0-0)\hat{i} - (0-0)\hat{j} + (-3.65 + 16.29)\hat{k}$$

$= 12.64\hat{k}$

b) $\vec{p} \cdot \vec{q} \longrightarrow (-2.68\hat{i} - 2.24) \cdot (1.63\hat{i} + 6.08\hat{j})$

$= -4.36\hat{i} - 13.61\hat{j}$

$= -17.97.$

44 In the product $\vec{F} = q\vec{v} \times \vec{B}$, take $q = 3$,

$$\vec{v} = 2.0\hat{i} + 4.0\hat{j} + 6.0\hat{k} \quad \text{and} \quad \vec{F} = 4.0\hat{i} - 20\hat{j} + 12\hat{k}.$$

What then is $\vec{B}$ in unit-vector notation if $B_x = B_y$?

$\vec{F} = q\vec{v} \times \vec{B} \longrightarrow 3\vec{v} \times \vec{B}$

$\vec{F} = 4\hat{i} - 20\hat{j} + 12\hat{k}$

$\vec{v} = 2\hat{i} + 4\hat{j} + 6\hat{k} \longrightarrow \times(3) \rightarrow 6\hat{i} + 12\hat{j} + 18\hat{k}$

$$\vec{v} \times \vec{B} \quad \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 12 & 18 \\ B & B & B_3 \end{vmatrix} = 4\hat{i} - 20\hat{j} + 12\hat{k}$$

$(12B_3 - 18B)\,\hat{i} = 4\hat{i}$

$(6B_3 - 18B)\,\hat{j} = -20\hat{j}$

$(6B - 12B)\,\hat{k} = 12\hat{k}$

$-6B = 12$

$B = -2$

$\therefore 12B_3 - 18(-2) = 4$

$12B_3 = -32 \longrightarrow B_3 = -2.6$

$\therefore \vec{B} = -2\hat{i} - 2\hat{j} - 2.6\hat{k}$