CHAPTER 5

Mechanical Sensors

INSTRUCTIONAL OBJECTIVES

This chapter presents various types of sensors for the measurement of mechanical phenomena such as motion, force, pressure, and position. After you have read the chapter and worked through the examples and problems you will be able to:

- Describe three types of sensors for the measurement of displacement, location, or position.
- Explain the operating principle of an LVDT for measuring displacement.
- Calculate the strain experienced by a wire under the influence of a force.
- Design the application of a strain gauge for the measurement of stress.
- Explain the operation of a strain gauge-based load cell.
- Describe the operating principles of a spring-mass accelerometer.
- Explain the operating principle of a diaphragm pressure sensor.
- Describe the operating mechanism of an orifice plate flow sensor.

5.1 INTRODUCTION

The class of sensors used for the measurement of mechanical phenomena is of special significance because of the extensive use of these devices throughout the process-control industry. In many instances, an interrelation exists by which a sensor designed to measure some mechanical variable is used to measure another variable. To learn to use mechanical sensors, it is important to understand the mechanical phenomena themselves and the operating principles and application details of the sensor.

Our purposes here are to give an overview of the essential features associated with each variable and to make the reader conversant with the principal sensors used to measure mechanical variables, the characteristics of each, and appropriate application notes. As in previous chapters, an expert understanding of a phenomenon is not required to effectively employ sensors for its measurement.

5.2 DISPLACEMENT, LOCATION, OR POSITION SENSORS

The measurement of displacement, position, or location is an important topic in the pro industries. Examples of industrial requirements to measure these variables are many varied, and the required sensors are also of greatly varied designs. To give a few exam of measurement needs: (1) location and position of objects on a conveyor system, (2) entation of steel plates in a rolling mill, (3) liquid/solid level measurements, (4) loca and position of work piece in automatic milling operations, and (5) conversion of pres to a physical displacement that is measured to indicate pressure. In the following secti the basic principles of several common types of displacement, position, and location sors are given.

5.2.1 Potentiometric Sensors

The simplest type of displacement sensor involves the action of displacement in mo the wiper of a potentiometer. This device then converts linear or angular motion in changing resistance that may be converted directly to voltage and/or current sig Such potentiometric devices often suffer from the obvious problems of mechanical w friction in the wiper action, limited resolution in wire-wound units, and high electr noise.

Figure 5.1 shows a simple mechanical picture of the potentiometric displacen sensor. You will see that there is a wire wound around a form, making a wire-wo resistor with fixed resistance, *R*, between its endpoints, 1 and 2. A wiper assembl connected in such a way that motion of an arm causes the wiper to slide across wire-wound turns of the fixed resistor. An electrical connection is made to this wi Therefore, as the arm moves back and forth, the resistance between the wiper con tion, 3, and either fixed resistor connection will change in proportion to the mot This figure is highly pictorial. In actual sensors, the coil is very tightly wound of fine wire.

In a schematic, the potentiometric sensor is simply a three-terminal variable resi The resolution of this sensor is limited to the distance, Δx, between individual turns of w with the resulting resistance change of the single turn, ΔR.

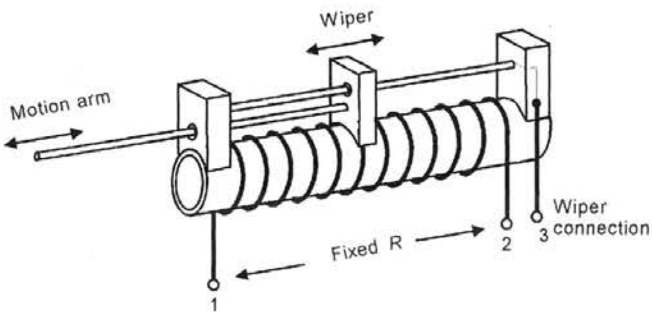


FIGURE 5.1
Potentiometric displacement sensor.

EXAMPLE 5.1

A potentiometric displacement sensor is to be used to measure work-piece motion from 0 to 10 cm. The resistance changes linearly over this range from 0 to 1 kΩ. Develop signal conditioning to provide a linear, 0- to 10-V output.

Solution

The key thing is to not lose the linearity of the resistance versus displacement. We cannot put the varying resistance in a divider to produce a varying voltage because the voltage varies nonlinearly with resistance. Remember though that the output voltage of an inverting amplifier varies linearly with the feedback resistance. Therefore, let's put the sensor in the feedback of a simple inverting amplifier. Then we would have something like

$$V_{\text{out}} = -\frac{R_2}{R_1}V_{\text{in}}$$

We can now get rid of that pesky negative by using V_{in} as a constant negative voltage, say -5.1 volts from a zener diode. Then we pick R_1 to give the desired output, 10 volts at 1 kΩ (10 cm),

$$10 = -\frac{1000}{R_1}(-5.1) \qquad \text{so} \qquad R_1 = 510\ \Omega$$

Figure 5.2 shows the circuit.

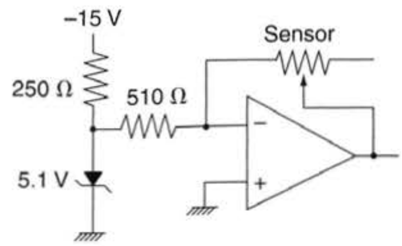


FIGURE 5.2
Circuit for Example 5.1.

5.2.2 Capacitive and Inductive Sensors

A second class of sensors for displacement measurement involves changes in capacity or inductance.

Capacitive The basic operation of a capacitive sensor can be seen from the familiar equation for a parallel-plate capacitor:

$$C = K\varepsilon_0\frac{A}{d} \tag{5.1}$$

where
K = the dielectric constant
ε_0 = permittivity = 8.85 pF/m
A = plate common area
d = plate separation

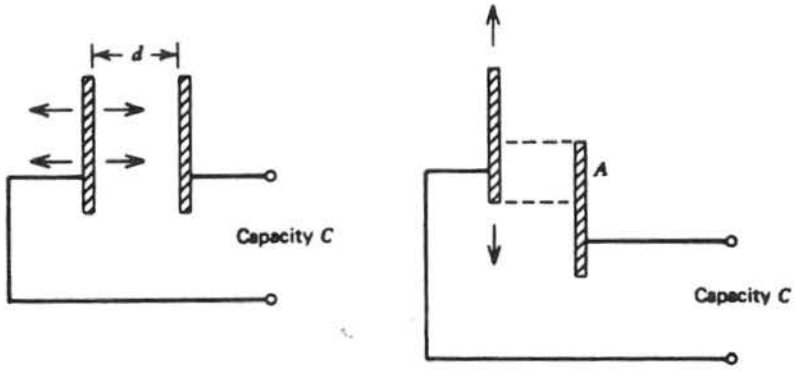


FIGURE 5.3
Capacity varies with the distance between the plates and the common area. Both effects are used in sensors.

There are three ways to change the capacity: variation of the distance between the plates (d), variation of the shared area of the plates (A), and variation of the dielectric constant (K). The former two methods are shown in Figure 5.3. The last method is illustrated in Example 5.4 later in this chapter. An ac bridge circuit or other active electronic circuit is employed to convert the capacity change to a current or voltage signal.

EXAMPLE 5.2

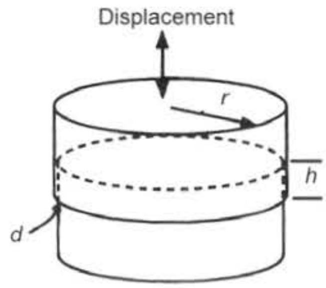
Figure 5.4 shows a capacitive-displacement sensor designed to monitor small changes in work-piece position. The two metal cylinders are separated by a plastic sheath/bearing of thickness 1 mm and dielectric constant at 1 kHz of 2.5. If the radius is 2.5 cm, find the sensitivity in pF/m as the upper cylinder slides in and out of the lower cylinder. What is the range of capacity if h varies from 1.0 to 2.0 cm?

Solution

The capacity is given by Equation (5.1). The net area is the area of the shared cylindrical area, which has a radius, r, and height, h. Thus, $A = 2\pi rh$, so the capacity can be expressed as

$$C = 2\pi K\varepsilon_0 \frac{rh}{d}$$

FIGURE 5.4
Capacitive-displacement sensor for Example 5.2.



The sensitivity with respect to the height, h, is defined by how C changes with h; that is, it is given by the derivative

$$\frac{dC}{dh} = 2\pi K\varepsilon_0\frac{r}{d}$$

Substituting for the given values, we get

$$\frac{dC}{dh} = 2\pi(2.5)(8.85\,\text{pF/m})\,\frac{2.5 \times 10^{-2}\text{m}}{10^{-3}\text{m}} = 3475\,\text{pF/m}$$

Since the function is linear with respect to h, we find the capacity range as $C_{min} = (3475\,\text{pF/m})(10^{-2}\text{m}) = 34.75$ pF to $C_{max} = (3475\,\text{pF/m})(2 \times 10^{-2}\text{m}) = 69.50$ pF.

Inductive If a permeable core is inserted into an inductor as shown in Figure 5.5, the net inductance is increased. Every new position of the core produces a different inductance. In this fashion, the inductor and movable core assembly may be used as a displacement sensor. An ac bridge or other active electronic circuit sensitive to inductance then may be employed for signal conditioning.

5.2.3 Variable-Reluctance Sensors

The class of variable-reluctance displacement sensors differs from the inductive in that a moving core is used to vary the magnetic flux coupling between two or more coils, rather than changing an individual inductance. Such devices find application in many circumstances for the measure of both translational and angular displacements. Many configurations of this device exist, but the most common and extensively used is called a *linear variable differential transformer* (LVDT).

LVDT The LVDT is an important and common sensor for displacement measurement in the industrial environment. Figure 5.6 shows that an LVDT consists of three coils of wire wound on a hollow form. A core of permeable material can slide freely through the center of the form. The inner coil is the primary, which is excited by some ac source as

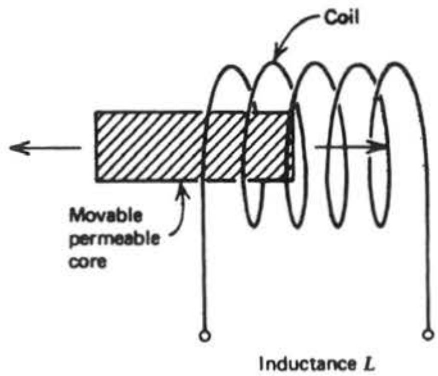


FIGURE 5.5
This variable-reluctance displacement sensor changes the inductance in a coil in response to core motion.

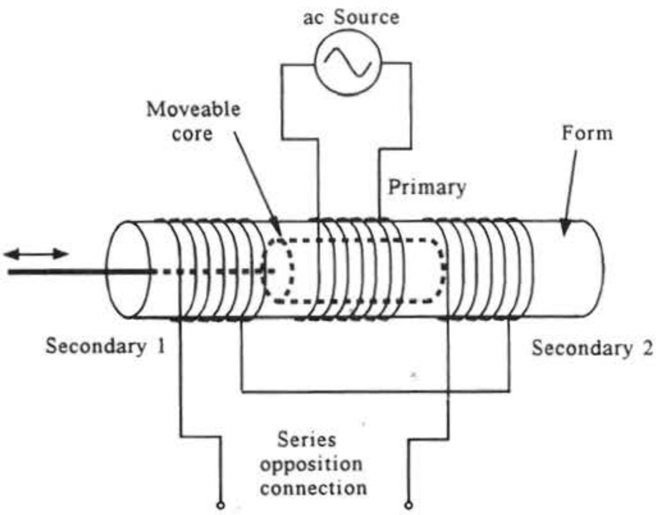
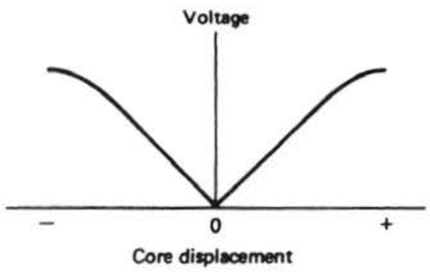


FIGURE 5.6
The LVDT has a movable core with the three coils as shown.

FIGURE 5.7
The LVDT secondary voltage amplitude for a series-opposition connection varies linearly with displacement.



shown. Flux formed by the primary is linked to the two secondary coils, inducing an ac voltage in each coil.

When the core is centrally located in the assembly, the voltage induced in each primary is equal. If the core moves to one side or the other, a larger ac voltage will be induced in one coil and a smaller ac voltage in the other because of changes in the flux linkage associated with the core.

If the two secondary coils are wired in series opposition, as shown in Figure 5.6, then the two voltages will subtract; that is, the differential voltage is formed. When the core is centrally located, the net voltage is zero. When the core is moved to one side, the net voltage amplitude will increase. In addition, there is a change in phase with respect to the source when the core is moved to one side or the other.

A remarkable result, shown in Figure 5.7, is that the differential amplitude is found to increase linearly as the core is moved to one side or the other. In addition, as noted, there is a phase change as the core moves through the central location. Thus, by measurement of the voltage amplitude and phase, one can determine the direction and extent of the core motion—that is, the displacement.

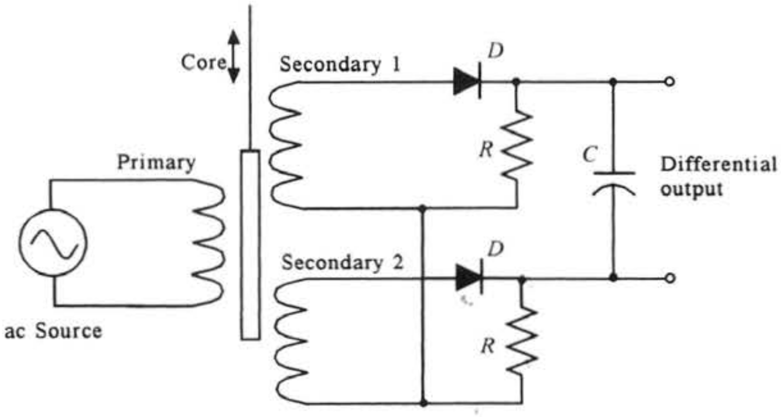


FIGURE 5.8
This simple circuit produces a bipolar dc voltage that varies with core displacement.

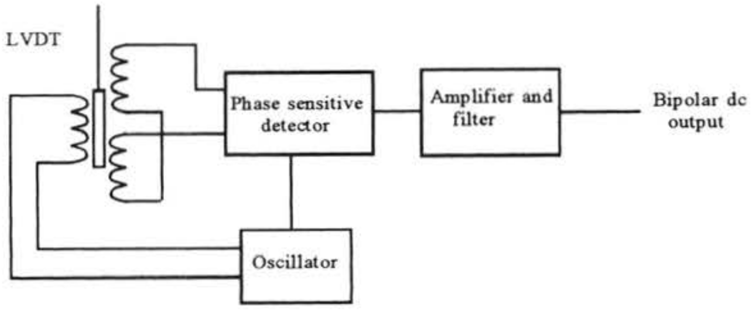


FIGURE 5.9
A more sophisticated LVDT signal-conditioning circuit uses phase-sensitive detection to produce a bipolar dc voltage output.

It turns out that a carefully manufactured LVDT can provide an output linear within $\pm 0.25\%$ over a range of core motion and with a very fine resolution, limited primarily by the ability to measure voltage changes.

The signal conditioning for LVDTs consists primarily of circuits that perform a phase-sensitive detection of the differential secondary voltage. The output is thus a dc voltage whose amplitude relates the extent of the displacement, and the polarity indicates the direction of the displacement. Figure 5.8 shows a simple circuit for providing such an output. An important limitation of this circuit is that the differential secondary voltage must be at least as large as the forward voltage drop of the diodes. The use of op amp detectors can alleviate this problem.

Figure 5.9 shows a more practical detection scheme, typically provided as a single integrated circuit (IC) manufactured specifically for LVDTs. The system contains a signal generator for the primary, a phase-sensitive detector (PSD), and amplifier/filter circuitry.

A variety of LVDTs are available with linear ranges at least from ± 25 cm down to ± 1 mm. The time response is dependent on the equipment to which the core is connected. The static transfer function is typically given in millivolts per millimeter (mV/mm) for a given primary amplitude. Also specified are the range of linearity and the extent of linearity.

EXAMPLE 5.3 An LVDT has a maximum core motion of ± 1.5 cm with a linearity of $\pm 0.3\%$ over that range. The transfer function is 23.8 mV/mm. If used to track work-piece motion from -1.2 to $+1.4$ cm, what is the expected output voltage? What is the uncertainty in position determination due to nonlinearity?

Solution

Using the known transfer function, the output voltages can easily be found,

$$V(-1.2\,\text{cm}) = (23.8\,\text{mV/mm})(-12\,\text{mm}) = -285.6\,\text{mV}$$

and

$$V(1.4\,\text{cm}) = (23.8\,\text{mV/mm})(14\,\text{mm}) = 333\,\text{mV}$$

The linearity deviation shows up in deviations of the transfer function. Thus, the transfer function has an uncertainty of

$$(\pm 0.003)(23.8\,\text{mV/mm}) = \pm 0.0714\,\text{mV/mm}$$

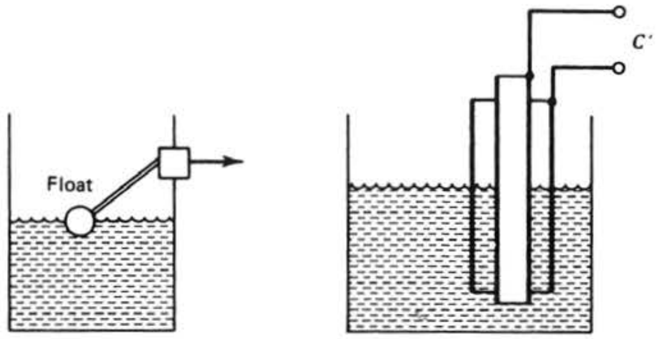
This means that a measured voltage, V_m (in mV), could be interpreted as a displacement that ranges from $V_m/23.73$ to $V_m/23.87$ mm, which is approximately $\pm 0.3\%$, as expected. Thus, if the sensor output was 333 mV, which is nominally 1.4 cm, the actual core position could range from 1.40329 to 1.39506 cm.

5.2.4 Level Sensors

The measurement of solid or liquid level calls for a special class of displacement sensors. The level measured is most commonly associated with material in a tank or hopper. A great variety of measurement techniques exist, as the following representative examples show.

Mechanical One of the most common techniques for level measurement, particularly for liquids, is a float that is allowed to ride up and down with level changes. This float, as shown in Figure 5.10a, is connected by linkages to a secondary displacement measuring system such as a potentiometric device or an LVDT core.

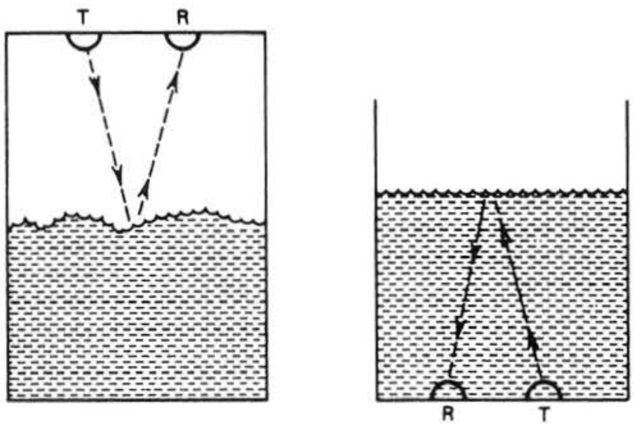
Electrical There are several purely electrical methods of measuring level. For example, one may use the inherent conductivity of a liquid or solid to vary the resistance seen by probes inserted into the material. Another common technique is illustrated in Figure 5.10b. In this case, two concentric cylinders are contained in a liquid tank. The level of the liquid partially occupies the space between the cylinders, with air in the remaining part. This device acts like two capacitors in parallel, one with the dielectric constant of air



a) Level measurement by a float and linkage

b) Level measurement by a concentric cylindrical capacitor

FIGURE 5.10
There are many level-measurement techniques.



a) Solid or liquid, above surface measurement

b) Liquid material, below surface material

FIGURE 5.11
Ultrasonic level measurement needs no physical contact with the material, just a transmitter, *T*, and receiver, *R*.

(≈ 1) and the other with that of the liquid. Thus, variation of liquid level causes variation of the electrical capacity measured between the cylinders.

Ultrasonic The use of ultrasonic reflection to measure level is favored because it is a "noninvasive" technique; that is, it does not involve placing anything in the material. Figure 5.11 shows the external and internal techniques. Obviously, the external technique is better suited to solid-material level measurement. In both cases, the measurement depends on the length of time taken for reflections of an ultrasonic pulse from the surface of the material. Ultrasonic techniques based on reflection time also have become popular for ranging measurements.

Pressure For liquid measurement, it is also possible to make a noncontact measurement of level if the density of the liquid is known. This method is based on the well-known relationship between pressure at the bottom of a tank and the height and density of the liquid. This is addressed further in Section 5.5.1.

EXAMPLE 5.4 The level of ethyl alcohol is to be measured from 0 to 5 m using a capacitive system such as that shown in Figure 5.10b. The following specifications define the system:

$$\text{for ethyl alcohol: } K = 26 \text{ (for air, } K = 1)$$
$$\text{cylinder separation: } d = 0.5\,\text{cm}$$
$$\text{plate area: } A = 2\pi RL$$

where $R = 5.75\,\text{cm}$ = average radius
L = distance along cylinder axis

Find the range of capacity variation as the alcohol level varies from 0 to 5 m.

Solution

We saw earlier that the capacity is given by $C = K\varepsilon_0(A/d)$. Therefore, all we need to do is find the capacity for the entire cylinder with *no* alcohol and then multiply that by 26.

$$A = 2\pi RL = 2\pi(0.0575\,\text{m})(5\,\text{m}) = 1.806\,\text{m}^2$$

Thus, for air,

$$C = (1)(8.85\,\text{pF/M})(1.806\,\text{m}^2/0.005\,\text{m})$$
$$C = 3196\,\text{pF} \approx 0.0032\,\mu\text{F}$$

With the ethyl alcohol, the capacity becomes

$$C = 26(0.0032\,\mu\text{F})$$
$$C = \mathbf{0.0832\,\mu F}$$

The range is 0.0032 to 0.0832 μF.

5.3 STRAIN SENSORS

Although not obvious at first, the measurement of strain in solid objects is common in process control. The reason it is not obvious is that strain sensors are used as a secondary step in sensors to measure many other process variables, including flow, pressure, weight, and acceleration. Strain measurements have been used to measure pressures from over a million pounds per square inch to those within living biological systems. We will first review the concept of strain and how it is related to the forces that produce it, and then discuss the sensors used to measure strain.

5.3.1 Strain and Stress

Strain is the result of the application of forces to solid objects. The forces are defined in a special way described by the general term *stress*. For those readers needing a review of force principles, Appendix 4 discusses elementary mechanical principles, including force. In this section, we will define stress and the resulting strain.

Definition A special case exists for the relation between force applied to a solid object and the resulting deformation of that object. Solids are assemblages of atoms in which the atomic spacing has been adjusted to render the solid in equilibrium with all external forces acting on the object. This spacing determines the physical dimensions of the solid. If the applied forces are changed, the object atoms rearrange themselves again to come into equilibrium with the new set of forces. This rearrangement results in a change in physical dimensions that is referred to as a *deformation* of the solid.

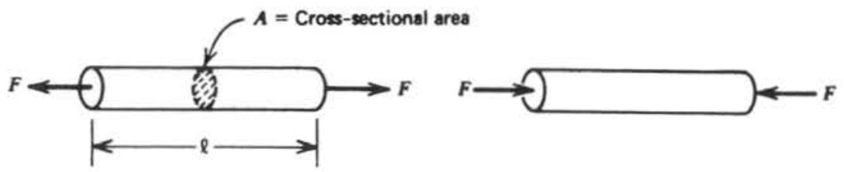
The study of this phenomenon has evolved into an exact technology. The effect of applied force is referred to as a *stress*, and the resulting deformation as a *strain*. To facilitate a proper analytical treatment of the subject, stress and strain are carefully defined to emphasize the physical properties of the material being stressed and the specific type of stress applied. We delineate here the three most common types of stress-strain relationships.

Tensile Stress-Strain In Figure 5.12a, the nature of a tensile force is shown as a force applied to a sample of material so as to elongate or pull apart the sample. In this case, the stress is defined as

$$\text{tensile stress} = \frac{F}{A} \tag{5.2}$$

where F = applied force in N
A = cross-sectional area of the sample in m^2

We see that the units of stress are N/m^2 in SI units (or $lb/in.^2$ in English units), and they are like a pressure.



a) Tensile stress applied to a rod

b) Compressional stress applied to a rod

FIGURE 5.12
Tensile and compressional stress can be defined in terms of forces applied to a uniform rod.

The strain in this case is defined as the *fractional change in length* of the sample:

$$\text{tensile strain} = \frac{\Delta l}{l} \tag{5.3}$$

where Δl = change in length in m (in.)
l = original length in m (in.)

Strain is thus a unitless quantity.

Compressional Stress-Strain The only differences between *compressional* and *tensile* stress are the direction of the applied force and the polarity of the change in length. Thus, in a compressional stress, the force presses in on the sample, as shown in Figure 5.12b. The compressional stress is defined as in Equation (5.2):

$$\text{compressional stress} = \frac{F}{A} \tag{5.4}$$

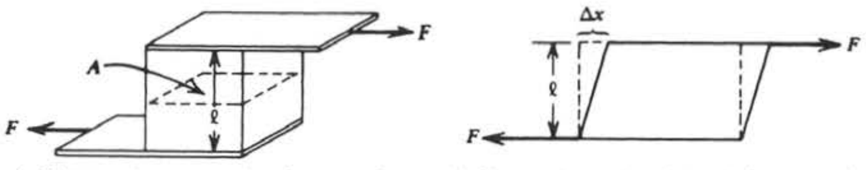
The resulting strain is also defined as the fractional change in length as in Equation (5.3), but the sample will now decrease in length:

$$\text{compressional strain} = \frac{\Delta l}{l}$$

Shear Stress-Strain Figure 5.13a shows the nature of the shear stress. In this case, the force is applied as a *couple* (that is, *not* along the same line), tending to shear off the solid object that separates the force arms. In this case, the stress is again

$$\text{shear stress} = \frac{F}{A} \tag{5.5}$$

where F = force in N
A = cross-sectional area of sheared member in m^2



a) Shear stress results from a force couple

b) Shear stress tends to deform an object as shown

FIGURE 5.13
Shear stress is defined in terms of forces not acting in a line (a couple), which deform a member linking the forces.

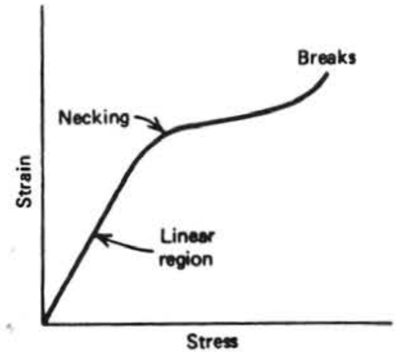


FIGURE 5.14
A typical stress-strain curve showing the linear region, necking, and eventual break.

The strain in this case is defined as the fractional change in dimension of the sheared member. This is shown in the cross-sectional view of Figure 5.13b.

$$\text{shear strain} = \frac{\Delta x}{l} \quad \textbf{(5.6)}$$

where Δx = deformation in m (as shown in Figure 5.13b)
l = width of a sample in m

Stress-Strain Curve If a specific sample is exposed to a range of applied stress and the resulting strain is measured, a graph similar to Figure 5.14 results. This graph shows that the relationship between stress and strain is linear over some range of stress. If the stress is kept within the linear region, the material is essentially *elastic* in that if the stress is removed, the deformation is also gone. But if the elastic limit is exceeded, permanent deformation results. The material may begin to "neck" at some location and finally break. Within the linear region, a specific type of material will always follow the same curves, despite different physical dimensions. Thus, we can say that the linearity and slope are a constant of the type of material only. In tensile and compressional stress, this constant is called the *modulus of elasticity,* or *Young's modulus,* as given by

$$E = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta l/l} \quad \textbf{(5.7)}$$

where stress = F/A in N/m^2 (or $lb/in.^2$)
strain = $\Delta l/l$ unitless
E = modulus of elasticity in N/m^2

The modulus of elasticity has units of stress—that is, N/m^2. Table 5.1 gives the modulus of elasticity for several materials. In an exactly similar fashion, the shear modulus is defined for shear stress-strain as

$$M = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta x/l} \quad \textbf{(5.8)}$$

where Δx is defined in Figure 5.13b and all other units have been defined in Equation (5.7).

TABLE 5.1
Modulus of elasticity

Material	Modulus (N/m^2)
Aluminum	6.89×10^{10}
Copper	11.73×10^{10}
Steel	20.70×10^{10}
Polyethylene (plastic)	3.45×10^{8}

EXAMPLE 5.5

Find the strain that results from a tensile force of 1000 N applied to a 10-m aluminum beam having a 4×10^{-4}-m^2 cross-sectional area.

Solution

The modulus of elasticity of aluminum is found from Table 5.1 to be $E = 6.89 \times 10^{10}\,N/m^2$. Now we have, from Equation (5.7),

$$E = \frac{F/A}{\Delta l/l}$$

so that

$$\begin{aligned} \text{strain} &= \frac{F}{EA} \\ &= \frac{10^3 N}{(4 \times 10^{-4} m^2)(6.89 \times 10^{10} N/m^2)} \\ &= \mathbf{3.63 \times 10^{-5}} \quad \text{or} \quad \mathbf{36.3\ \mu m/m} \qquad \text{(see next paragraph)} \end{aligned}$$

Strain Units Although strain is a unitless quantity, it is common practice to express the strain as the ratio of two length units, for example, as m/m or in./in.; also, because the strain is usually a very small number, a micro (μ) prefix is often included. In this sense, a strain of 0.001 would be expressed as $1000\,\mu$in./in., or $1000\,\mu$m/m. In the previous example, the solution is stated as $36.3\,\mu$m/m. In general, the smallest value of strain encountered in most applications is $1\,\mu$m/m. Because strain is a unitless quantity, it is not necessary to do unit conversions. A strain of $153\,\mu$m/m could also be written in the form of $153\,\mu$in./in. or even $153\,\mu$furlongs/furlong. Modern usage often just gives strain in "micros."

5.3.2 Strain Gauge Principles

In Section 4.2.1, we saw that the resistance of a metal sample is given by

$$R_0 = \rho\frac{l_0}{A_0} \qquad \textbf{(4.7)}$$

where R_0 = sample resistance Ω
ρ = sample resistivity $\Omega \cdot$ m
l_0 = length in m
A_0 = cross-sectional area in m^2

Suppose this sample is now stressed by the application of a force, F, as shown in Figure 5.12a. Then we know that the material elongates by some amount, Δl, so that the new length is $l = l + \Delta l$. It is also true that in such a stress-strain condition, although the sample lengthens, its volume will remain nearly constant. Because the volume unstressed is $V = l_0 A_0$, it follows that if the volume remains constant and the length increases, then the area must decrease by some amount, ΔA:

$$V = l_0 A_0 = (l_0 + \Delta l)(A_0 - \Delta A) \tag{5.9}$$

Because both length and area have changed, we find that the resistance of the sample will have also changed:

$$R = \rho \frac{l_0 + \Delta l}{A_0 - \Delta A} \tag{5.10}$$

Using Equations (5.9) and (5.10), the reader can verify that the new resistance is approximately given by

$$R \simeq \rho \frac{l_0}{A_0}\left(1 + 2\frac{\Delta l}{l_0}\right) \tag{5.11}$$

from which we conclude that the change in resistance is

$$\Delta R \simeq 2R_0 \frac{\Delta l}{l_0} \tag{5.12}$$

Equation (5.12) is the basic equation that underlies the use of metal strain gauges because it shows that the strain $\Delta l / l$ converts directly into a *resistance change.*

EXAMPLE 5.6 Find the approximate change in a metal wire of resistance 120 Ω that results from a strain of 1000 μm/m.

Solution

We can find the change in gauge resistance from

$$\Delta R \simeq 2R_0 \frac{\Delta l}{l_0}$$

$$\Delta R \simeq (2)(120)(10^{-3})$$

$$\Delta R = \mathbf{0.24}\ \Omega$$

Example 5.6 shows a significant factor regarding strain gauges. The change in resistance is very small for typical strain values. For this reason, resistance change measurement methods used with strain gauges must be highly sophisticated.

Measurement Principles The basic technique of strain gauge (SG) measurement involves attaching (gluing) a metal wire or foil to the element whose strain is to be measured. As stress is applied and the element deforms, the SG material experiences the same deformation, if it is securely attached. Because strain is a fractional change in length, the change in SG resistance reflects the strain of both the gauge and the element to which it is secured.

Temperature Effects If not for temperature compensation effects, the aforementioned method of SG measurement would be useless. To see this, we need only note that the metals used in SG construction have linear temperature coefficients of $\alpha \cong 0.004/°\text{C}$, typical for most metals. Temperature changes of 1°C are not uncommon in measurement conditions in the industrial environment. If the temperature change in Example 5.6 had been 1°C, substantial change in resistance would have resulted. Thus, from Chapter 4,

$$R(T) = R(T_0)[1 + \alpha_0 \Delta T]$$

or

$$\Delta R_T = R_0 \alpha \Delta T$$

where
ΔR_T = resistance change because of temperature change
$\alpha_0 \simeq 0.004/°\text{C}$ in this case
$\Delta T \simeq 1°\text{C}$ in this case
$R(T_0) = 120\,\Omega$ nominal resistance

Then, we find $\Delta R_T = 0.48\,\Omega$, which is *twice* the change because of strain! Obviously, temperature effects can mask the strain effects we are trying to measure. Fortunately, we are able to compensate for temperature and other effects, as shown in the signal-conditioning methods in the next section.

5.3.3 Metal Strain Gauges

Metal SGs are devices that operate on the principles discussed earlier. The following items are important to understanding SG applications.

Gauge Factor The relation between strain and resistance change [Equation (5.12)] is only approximately true. Impurities in the metal, the type of metal, and other factors lead to slight corrections. An SG specification always indicates the correct relation through statement of a *gauge factor* (GF), which is defined as

$$\text{GF} = \frac{\Delta R/R}{\text{strain}} \tag{5.13}$$

where
$\Delta R/R$ = fractional change in gauge resistance because of strain
strain = $\Delta l/l$ = fractional change in length

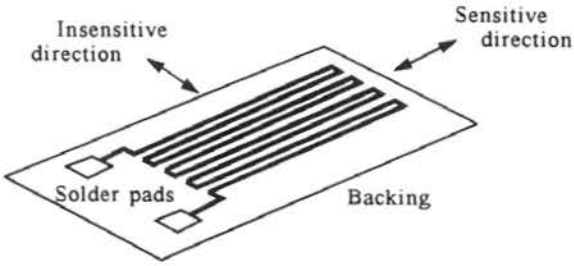


FIGURE 5.15
A metal strain gauge is composed of thin metal deposited in a pattern on a backing or carrier material.

For metal gauges, this number is always close to 2. For some special alloys and carbon gauges, the GF may be as large as 10. A high gauge factor is desirable because it indicates a larger change in resistance for a given strain and is easier to measure.

Construction Strain gauges are used in two forms, wire and foil. The basic characteristics of each type are the same in terms of resistance change for a given strain. The design of the SG itself is such as to make it very long in order to give a large enough nominal resistance (to be practical), and to make the gauge of sufficiently fine wire or foil so as not to resist strain effects. Finally, the gauge sensitivity is often made *unidirectional*; that is, it responds to strain *in only one direction*. In Figure 5.15, we see the common pattern of SGs that provides these characteristics. By folding the material back and forth as shown, we achieve a long length to provide high resistance. Further, if a strain is applied transversely to the SG length, the pattern will tend to unfold rather than stretch, with no change in resistance. These gauges are usually mounted on a paper backing that is bonded (using epoxy) to the element whose strain is to be measured. The nominal SG resistance (no strain) available are typically 60, 120, 240, 350, 500, and 1000 Ω. The most common value is 120 Ω.

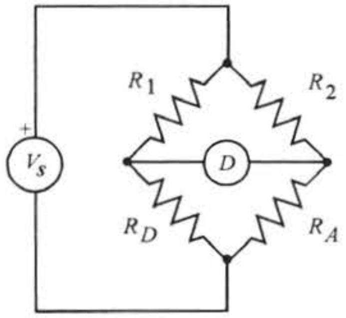
Signal Conditioning Two effects are critical in the signal-conditioning techniques used for SGs. The first is the small, fractional changes in resistance that require carefully designed resistance measurement circuits. A good SG system might require a resolution of 2 μm/m strain. From Equation (5.12), this would result in a ΔR of only $4.8 \times 10^{-4}\ \Omega$ for a nominal gauge resistance of 120 Ω.

The second effect is the need to provide some compensation for temperature effects to eliminate masking changes in strain.

The bridge circuit provides the answer to both effects. The sensitivity of the bridge circuit for detecting small changes in resistance is well known. Furthermore, by using a dummy gauge as shown in Figure 5.16a, we can provide the required temperature compensation. In particular, the dummy is mounted in an insensitive orientation (Figure 5.16b), but in the same proximity as the active SG. Then, both gauges change in resistance from temperature effects, but the bridge does not respond to a change in both strain gauges. Only the active SG responds to strain effects. This is called a *one-arm bridge*.

The sensitivity of this bridge to strain can be found by considering the equation for bridge offset voltage. Suppose $R_1 = R_2 = R_D = R$, which is the nominal (unstrained) gauge resistance. Then the active strain gauge resistance will be given by

$$R_A = R\left(1 + \frac{\Delta R}{R}\right)$$



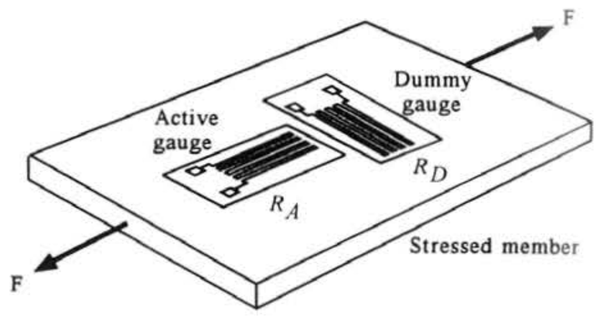


FIGURE 5.16
Strain gauges are used in pairs to provide temperature compensation. In some cases, such as this, only one gauge actually deforms during stress.

and the bridge off-null voltage will be given by

$$\Delta V = V_s\left[\frac{R_D}{R_D + R_1} - \frac{R_A}{R_A + R_2}\right]$$

If substitutions are made as defined previously, this voltage can be shown to be

$$\Delta V = -\frac{V_s}{4}\frac{\frac{\Delta R}{R}}{1 + \frac{\Delta R}{R}} \approx -\frac{V_s}{4}\frac{\Delta R}{R}$$

where the approximation is good for $(\Delta R/R) \ll 1$. Substituting from Equation (5.13) allows the expression for ΔV in terms of strain:

$$\Delta V = -\frac{V_s}{4}GF\frac{\Delta l}{l} \tag{5.14}$$

EXAMPLE 5.7 A strain gauge with GF = 2.03 and $R = 350\,\Omega$ is used in the bridge of Figure 5.16a. The bridge resistors are $R_1 = R_2 = 350\,\Omega$, and the dummy gauge has $R = 350\,\Omega$. If a tensile strain of $1450\,\mu\text{m/m}$ is applied, find the bridge offset voltage if $V_s = 10.0$ V. Find the relation between bridge off-null voltage and strain. How much voltage results from a strain of 1 micro?

Solution

With no strain, the bridge is balanced. When the strain is applied, the gauge resistance will change by a value given by

$$\text{GF} = \frac{\Delta R/R}{\text{strain}}$$

Thus,

$$\Delta R = (\text{GF})(\text{strain})(R)$$
$$\Delta R = (2.03)(1.45 \times 10^{-3})(350\,\Omega)$$
$$\Delta R = 1.03\,\Omega$$

Since it's tensile strain the resistance will increase to $R = 351\,\Omega$. The bridge offset voltage is

$$\Delta V = \frac{RV}{R_1 + R} - \frac{R_A V}{R_A + R_2}$$

Thus,

$$\Delta V = 5 - \frac{(351)(10)}{701}$$
$$\Delta V = \mathbf{-0.007\ V}$$

so that a 7-mV offset results.

The sensitivity is found from Equation (5.14):

$$\Delta V = -\frac{10}{2}(2.03)\frac{\Delta l}{l} = -10.15\frac{\Delta l}{l}$$

Thus, every micro of strain will supply only $10.15\,\mu\text{V}$.

Another configuration that is often employed uses active strain gauges in two arms of the bridge, and is thus called a *two-arm bridge*. All four arms are strain gauges, but two are for temperature compensation only. This has the added advantage of doubling the sensitivity. The bridge off-null voltage in terms of strain is given by

$$\Delta V = \frac{V_s}{2}\text{GF}\frac{\Delta l}{l} \qquad \textbf{(5.15)}$$

Obviously, the placement of the active and dummy gauges in the environment and in the bridge circuit is important. Figure 5.17 shows a common application of strain gauges to

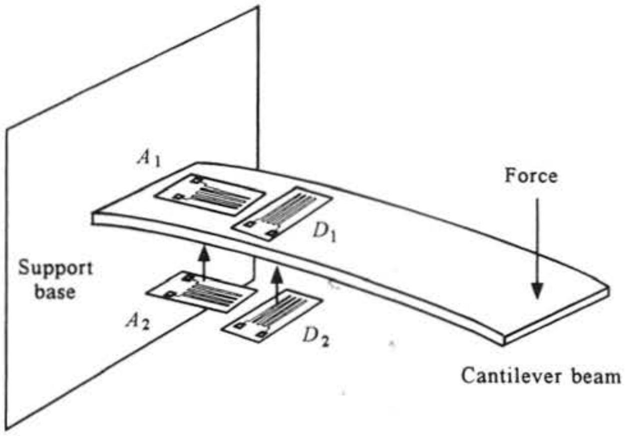


FIGURE 5.17
This structure shows how four gauges can be used to measure beam bending. Two respond to bending, and two are for temperature compensation.

measure deflections of a cantilever beam. This is a beam that is supported at only one end, and deflects as shown when a load is applied. In this application, it is common to use a two-arm bridge. One pair of active (A_1) and dummy (D_1) gauges is mounted on the top surface. The active gauge will experience tension with downward deflection of the beam, and its resistance will increase. The second pair, A_2 and D_2, are mounted on the bottom surface. The active gauge will experience compression with downward deflection, and its resistance will decrease.

EXAMPLE 5.8

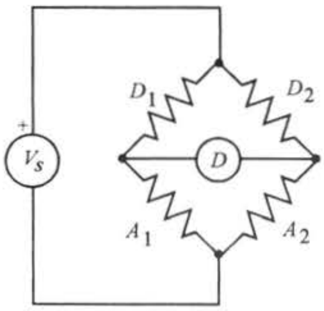
Show how the four gauges in Figure 5.17 are connected into a bridge.

Solution

The gauges must be connected so that the off-null voltage increases with strain. Thus, one divider voltage should increase and the other should decrease so that the difference grows. This can be accomplished by using the active gauges in bridge resistor positions R_3 and R_4 of the standard bridge configuration shown in Figure 5.18.

It is also possible to wire strain gauges in a four-arm bridge, where all four gauges are active and temperature compensation is still supplied. In this case, the sensitivity is increased by another factor of two, from Equation (5.15).

FIGURE 5.18
Solution to Example 5.8.



5.3.4 Semiconductor Strain Gauges (SGs)

The use of semiconductor material, notably silicon, for SG application has increased over the past few years. There are presently several disadvantages to these devices compared to the metal variety, but numerous advantages for their use.

Principles As in the case of the metal SGs, the basic effect is a change of resistance with strain. In the case of a semiconductor, the resistivity also changes with strain, along with the physical dimensions. This is due to changes in electron and hole mobility with changes in crystal structure as strain is applied. The net result is a much larger gauge factor than is possible with metal gauges.

Gauge Factor The semiconductor device gauge factor (GF) is still given by Equation (5.13):

$$GF = \frac{\Delta R/R}{\text{strain}}$$

For semiconductor strain gauges, the GF is often negative, which means the resistance decreases when a tensile (stretching) stress is applied. Furthermore, the GF can be much larger than for metal strain gauges, in some cases as large as -200 with no strain. It must also be noted, however, that these devices are highly nonlinear in resistance versus strain. In other words, the gauge factor is not a constant as the strain takes place. Thus, the gauge factor may be -150 with no strain, but drop (nonlinearly) to -50 at $5000\,\mu\text{m/m}$. The resistance change will be nonlinear with respect to strain. To use the semiconductor strain gauge to measure strain, we must have a curve or table of values of gauge factor versus resistance.

Construction The semiconductor strain gauge physically appears as a band or strip of material with electrical connection, as shown in Figure 5.19. The gauge is either bonded

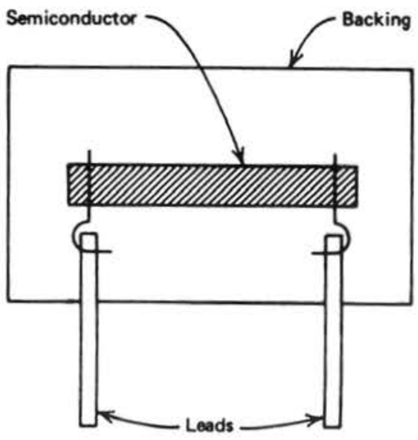


FIGURE 5.19
Typical semiconductor strain gauge structure.

directly onto the test element or, if encapsulated, is attached by the encapsulation material. These SGs also appear as IC assemblies in configurations used for other measurements.

Signal Conditioning The signal conditioning is still typically a bridge circuit with temperature compensation. An added problem is the need for linearization of the output because the basic resistance versus strain characteristic is nonlinear.

EXAMPLE 5.9

a. Contrast the resistance change produced by a 150-μm/m strain in a metal gauge with GF = 2.13 with

b. A semiconductor SG with GF = −151. Nominal resistances are both 120 Ω.

Solution
From the basic equation

$$\text{GF} = \frac{\Delta R/R}{\text{strain}}$$

a. We find for the metal gauge SG

$$\Delta R = (120\ \Omega)(2.13)(0.15 \times 10^{-3})$$
$$\Delta R = 0.038\ \Omega$$

b. For the semiconductor gauge, the change is

$$\Delta R = (120\ \Omega)(-151)(0.15 \times 10^{-3})$$
$$\Delta R = \mathbf{-2.72\ \Omega}$$

5.3.5 Load Cells

One important direct application of SGs is for the measurement of force or weight. These transducer devices, called *load cells* measure deformations produced by the force or weight. In general, a beam or yoke assembly is used that has several strain gauges mounted so that the application of a force causes a strain in the assembly that is measured by the gauges. A common application uses one of these devices in support of a hopper or feed of dry or liquid materials. A measure of the weight through a load cell yields a measure of the quantity of material in the hopper. Generally, these devices are calibrated so that the force (weight) is directly related to the resistance change. Forces as high as 5 MN (approximately 10^6 lb) can be measured with an appropriate load cell.

EXAMPLE 5.10

Figure 5.20 shows a simple load cell consisting of an aluminum post of 2.500-cm radius with a detector and compensation strain gauges. The 120 Ω strain gauges are used in the bridge of Figure 5.16, with $V = 2$ V, $R_1 = R_2 = R_D = 120.0\ \Omega$, and GF = 2.13. Find the variation of bridge offset voltage for a load of 0 to 5000 lb.

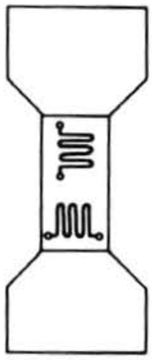


FIGURE 5.20
Load cell for Example 5.10.

Solution

We can find the strain for a 5000-lb load, then the resulting change in resistance, and from that, the bridge offset voltage. First, we change the force to newtons:

$$(5000\,\text{lb}/0.2248\,\text{lb/N}) = 22{,}240\,\text{N}$$

The cross-sectional area of the post is

$$A = \pi r^2 = \pi(0.025\,\text{m})^2 = 1.963 \times 10^{-3}\,\text{m}^2$$

From Table 5.1, the modulus of elasticity of aluminum is $E = 6.89 \times 10^{10}\,\text{N/m}^2$. From Equation (5.7), we find the strain

$$\Delta l/l = F/EA$$
$$\Delta l/l = (22{,}240\,\text{N})/[(6.89 \times 10^{10}\,\text{N/m}^2)(1.963 \times 10^{-3}\,\text{m}^2)]$$
$$\Delta l/l = 1.644 \times 10^{-4} = 164.4\,\mu\text{m/m}(or\ \mu\text{in./in.})$$

The relationship between resistance and strain is given by Equation (5.13) ($\text{GF} = (\Delta R/R)/(\Delta l/l)$, so the resistance is given by

$$\Delta R/R = 2.13(1.644 \times 10^{-4})$$
$$= 3.502 \times 10^{-4}$$

Because $R = 120.0\,\Omega$, $\Delta R = 0.04203\,\Omega$. To get the bridge offset voltage, we note that the post is under compression and, therefore, the resistance will decrease. With no strain, the bridge is nulled. Under a 5000-lb load, the active gauge has $R = 119.958\,\Omega$. Thus, the offset voltage of the bridge is

$$\Delta V = 2\frac{120}{120 + 120} - 2\frac{119.958}{120 + 119.958}$$
$$\Delta V = 1.750 \times 10^{-4}\,\text{V} = 175.0\,\mu\text{V}$$

As the force varies from 0 to 5000 lb, the offset voltage varies from 0 to 175 μV.

The form of load cell considered in Figure 5.20 is fine for illustrating principles, but real load cells cannot be made in this simple way. The problem is that forces applied to the top of the load cell may cause it to lean or bend, instead of simply compressing. In such a case, one side surface of the beam may experience compression while the other side undergoes tension. Obviously, this will alter the correct interpretation of the result.

Practical load cells are made with yoke assemblies designed so that mounted strain gauges cannot be exposed to stresses other than those caused by the compressional force applied to the cell.

5.4 MOTION SENSORS

Motion sensors are designed to measure the rate of change of position, location, or displacement of an object that is occurring. If the position of an object as a function of time is $x(t)$, then the first derivative gives the speed of the object, $v(t)$, which is called the *velocity* if a direction is also specified. If the speed of the object is also changing, then the first derivative of the speed gives the acceleration. This is also the second derivative of the position.

$$v(t) = \frac{dx(t)}{dt} \tag{5.16}$$

$$a(t) = \frac{dv(t)}{dt} = \frac{d^2x(t)}{dt^2} \tag{5.17}$$

The primary form of motion sensor is the *accelerometer.* This device measures the acceleration, $a(t)$, of an object. By integrating Equations (5.16) and (5.17), it is easy to show that the accelerometer can be used to determine both the speed and position of the object as well:

$$v(t) = v(0) + \int_0^t a(t)dt \tag{5.18}$$

$$x(t) = x(0) + \int_0^t v(t)dt \tag{5.19}$$

Thus, in the accelerometer we have a sensor that can provide acceleration, speed (or velocity), and position information.

5.4.1 Types of Motion

The design of a sensor to measure motion is often tailored to the type of motion that is to be measured. It will help you understand these sensors if you have a clear understanding of the types of motion.

The proper unit of acceleration is meters per second squared (m/s^2). Then speed will be in meters per second (m/s) and position in meters (m). Often, acceleration is expressed by comparison with the acceleration due to gravity at the Earth's surface. This amount of acceleration, which is approximately 9.8 m/s^2, is called a *gee,* which is given as a bold **g** in this text.

EXAMPLE 5.11 An automobile is accelerating away from a stop sign at 26.4 ft/s^2. What is the acceleration in m/s^2 and in **g**s?

Solution

To find the acceleration in m/s^2, we simply convert the feet to meters according to 2.54 cm/in. and 12 in./ft:

$$a = (26.4\ \text{ft/s}^2)(12\ \text{in./ft})(2.54\ \text{cm/in.})(0.01\ \text{m/cm})$$
$$a = 8.05\ \text{m/s}^2$$

In terms of **g**s, we then have

$$a_g = (8.05\ \text{m/s}^2)/(9.8\ \text{m/s}^2/\mathbf{g}) = 0.82\ \mathbf{g}\text{s}$$

where the subscript on a indicates the units are **g**s. Thus, this acceleration away from the stop sign provides an acceleration of about 80% of that caused by gravity at the Earth's surface.

Rectilinear This type of motion is characterized by velocity and acceleration which is composed of straight-line segments. Thus, objects may accelerate forward to a certain velocity, decelerate to a stop, reverse, and so on. There are many types of sensors designed to handle this type of motion. Typically, maximum accelerations are less than a few **g**s, and little angular motion (in a curved line) is allowed. If there is angular motion, then several rectilinear motion sensors must be used, each sensitive to only one line of motion. Thus, if vehicle motion is to be measured, two transducers may be used, one to measure motion in the forward direction of vehicle motion and the other perpendicular to the forward axis of the vehicle.

Angular Some sensors are designed to measure only rotations about some axis, such as the angular motion of the shaft of a motor. Such devices cannot be used to measure the physical displacement of the whole shaft, but only its rotation.

Vibration In the normal experiences of daily living, a person rarely experiences accelerations that vary from 1 **g** by more than a few percent. Even the severe environments of a rocket launching involve accelerations of only 1 **g** to 10 **g**. However, if an object is placed in periodic motion about some equilibrium position, as in Figure 5.21, very large *peak* accelerations may result that reach to 100 **g** or more. This motion is called *vibration*. Clearly, the measurement of acceleration of this magnitude is very important to industrial environments, where vibrations are often encountered from machinery operations. Often, vibrations are somewhat random in both the frequency of periodic motion and the magnitude of displacements from equilibrium. For analytical treatments, vibration is defined in terms of a regular periodic motion where the position of an object in time is given by

$$x(t) = x_0 \sin \omega t \tag{5.20}$$

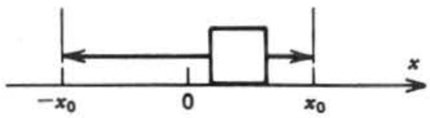


FIGURE 5.21
An object in periodic motion about an equilibrium at $x = 0$. The peak displacement is x_0.

where $x(t)$ = object position in m
x_0 = peak displacement from equilibrium in m
ω = angular frequency in rad/s

The definition of ω as angular frequency is consistent with the reference to ω as angular speed. If an object rotates, we define the time to complete one rotation as a period T, that corresponds to a frequency $f = 1/T$. The frequency represents the number of revolutions per second and is measured in hertz (Hz), where $1\,\text{Hz} = 1$ revolution per second. An angular rate of one revolution per second corresponds to an angular velocity of 2π rad/s, because one revolution sweeps out 2π radians. From this argument, we see that f and ω are related by

$$\omega = 2\pi f \tag{5.21}$$

Because f and ω are related by a constant, we refer to ω as both angular frequency and angular velocity.

Now we can find the vibration velocity as a derivative of Equation (5.20):

$$v(t) = \omega x_0 \cos \omega t \tag{5.22}$$

and we can get the vibration acceleration from a derivative of Equation (5.22):

$$a(t) = -\omega^2 x_0 \sin \omega t \tag{5.23}$$

Vibration position, velocity, and acceleration are all periodic functions having the same frequency. Of particular interest is the *peak* acceleration:

$$a_{\text{peak}} = \omega^2 x_0 \tag{5.24}$$

We see that the peak acceleration is dependent on ω^2, the angular frequency squared. This may result in very large acceleration values, even with modest peak displacements, as Example 5.12 shows.

EXAMPLE 5.12 A water pipe vibrates at a frequency of 10 Hz with a displacement of 0.5 cm. Find **(a)** the peak acceleration in m/s^2, and **(b)** **g** acceleration.

Solution

The peak acceleration will be given by

a. $a_{\text{peak}} = \omega^2 x_0$

where $\omega = 2\pi f = 20\pi$ rad/s and $x_0 = 0.5\,\text{cm} = 0.005\,\text{m}$

$$a_{\text{peak}} = (20\pi)^2(0.005)$$
$$a_{\text{peak}} = \mathbf{19.7\ m/s^2}$$

b. Noting that $1\mathbf{g} = 9.8$ m/s^2, we get

$$a_{\text{peak}} = (19.7\,\text{m/s}^2)\left(\frac{1\,\text{g}}{9.8\,\text{m/s}^2}\right)$$
$$a_{\text{peak}} = \mathbf{2.0\ g}$$

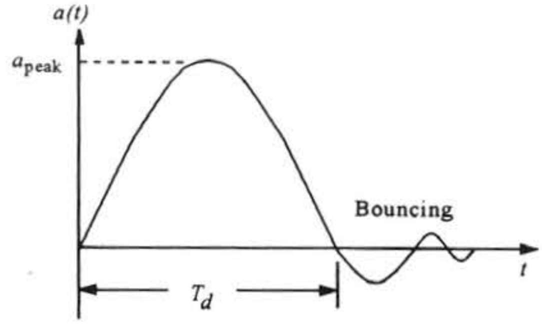


FIGURE 5.22
Typical shock acceleration profile.

A 2-**g** vibrating excitation of any mechanical element can be destructive, yet this is generated under the modest conditions of Example 5.12. A special class of sensors has been developed for measuring vibration acceleration.

Shock A special type of acceleration occurs when an object that may be in uniform motion or modestly accelerating is suddenly brought to rest, as in a collision. Such phenomena are the result of very large accelerations, or actually *decelerations,* as when an object is dropped from some height onto a hard surface. The name *shock* is given to decelerations that are characterized by very short times, typically on the order of milliseconds, with peak accelerations over 500 **g.** In Figure 5.22, we have a typical acceleration graph as a function of time for a shock experiment. This graph is characterized by a maximum or peak deceleration, a_{peak}, a shock duration, T_d, and bouncing. We can find an average shock by knowing the velocity of the object and the shock duration, as considered in Example 5.13.

EXAMPLE 5.13 A TV set is dropped from a 2-m height. If the shock duration is 5 ms, find the average shock in **g.**

Solution
The TV accelerates at 9.8 m/s^2 for 2 m. We find the velocity as

$$\begin{aligned} v^2 &= 2\,gx \\ v^2 &= (2)(9.8\,\text{m/s}^2)(2\,\text{m}) \\ v &= 6.3\,\text{m/s} \end{aligned}$$

If the duration is 5 ms, we have

$$\begin{aligned} \bar{a} &= \frac{6.3\,\text{m/s}}{5 \times 10^{-3}\text{s}} \\ \bar{a} &= 1260\,\text{m/s}^2 \end{aligned}$$

or 128 **g.** No wonder that the TV breaks apart when it hits the ground!

5.4.2 Accelerometer Principles

There are several physical processes that can be used to develop a sensor to measure acceleration. In applications that involve flight, such as aircraft and satellites, accelerometers are based on properties of rotating masses. In the industrial world, however, the most common design is based on a combination of Newton's law of mass acceleration and Hooke's law of spring action.

Spring-Mass System Newton's law simply states that if a mass, m, is undergoing an acceleration, a, then there must be a force, F, acting on the mass and given by $F = ma$. Hooke's law states that if a spring of spring constant k is stretched (extended) from its equilibrium position for a distance Δx, then there must be a force acting on the spring given by $F = k\Delta x$.

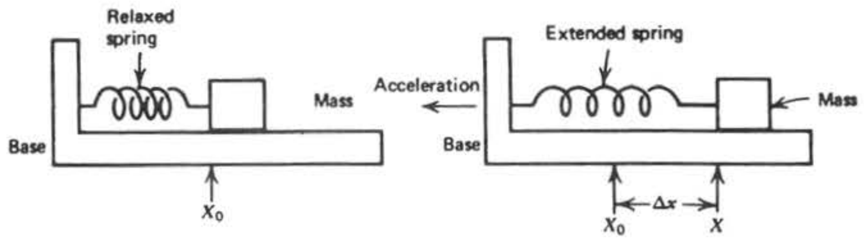
In Figure 5.23a we have a mass that is free to slide on a base. The mass is connected to the base by a spring that is in its unextended state and exerts no force on the mass. In Figure 5.23b, the whole assembly is accelerated to the left, as shown. Now the spring extends in order to provide the force necessary to accelerate the mass. This condition is described by equating Newton's and Hooke's laws:

$$ma = k\Delta x \tag{5.25}$$

where
k = spring constant in N/m
Δx = spring extension in m
m = mass in kg
a = acceleration in m/s^2

Equation (5.25) allows the measurement of acceleration to be reduced to a measurement of spring extension (linear displacement) because

$$a = \frac{k}{m}\Delta x \tag{5.26}$$



a) Spring-mass system with no acceleration

b) Spring-mass system with acceleration

FIGURE 5.23
The basic spring-mass system accelerometer.

If the acceleration is reversed, the same physical argument would apply, except that the spring is compressed instead of extended. Equation (5.26) still describes the relationship between spring displacement and acceleration.

The *spring-mass principle* applies to many common accelerometer designs. The mass that converts the acceleration to spring displacement is referred to as the *test mass*, or *seismic mass*. We see, then, that acceleration measurement reduces to linear displacement measurement; most designs differ in how this displacement measurement is made.

Natural Frequency and Damping On closer examination of the simple principle just described, we find another characteristic of spring-mass systems that complicates the analysis. In particular, a system consisting of a spring and an attached mass always exhibits oscillations at some characteristic *natural frequency*. Experience tells us that if we pull a mass back and then release it (in the absence of acceleration), it will be pulled back by the spring, overshoot the equilibrium, and oscillate back and forth. Only friction associated with the mass and base eventually brings the mass to rest. Any displacement measuring system will respond to this oscillation as if an actual acceleration occurs. This natural frequency is given by

$$f_N = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{5.27}$$

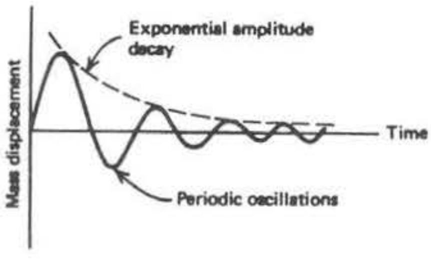
where f_N = natural frequency in Hz
k = spring constant in N/m
m = seismic mass in kg

The friction that eventually brings the mass to rest is defined by a *damping coefficient* α, which has the units of s^{-1}. In general, the effect of oscillation is called *transient response*, described by a periodic damped signal, as shown in Figure 5.24, whose equation is

$$X_T(t) = X_0 e^{-\alpha t}\sin(2\pi f_N t) \tag{5.28}$$

where $X_T(t)$ = transient mass position
X_0 = peak position, initially
α = damping coefficient
f_N = natural frequency

FIGURE 5.24
A spring-mass system exhibits a natural oscillation with damping as a response to an impulse input.



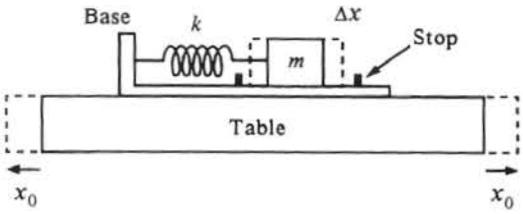


FIGURE 5.25
A spring-mass accelerometer has been attached to a table, which is vibrating. The table peak motion is x_0, and the mass motion is Δx.

The parameters, natural frequency, and damping coefficient in Equation (5.28) have a profound effect on the application of accelerometers.

Vibration Effects The effect of natural frequency and damping on the behavior of spring-mass accelerometers is best described in terms of an applied vibration. If the spring-mass system is exposed to a vibration, the resultant acceleration of the base is given by Equation (5.23):

$$a(t) = -\omega^2 x_0 \sin \omega t$$

If this is used in Equation (5.25), we can show that the mass motion is given by

$$\Delta x = -\frac{m x_0}{k} \omega^2 \sin \omega t \tag{5.29}$$

where all terms were previously defined, and $\omega = 2\pi f$, with f the applied frequency.

To make the predictions of Equation (5.29) clear, consider the situation presented in Figure 5.25. Our model spring-mass accelerometer has been fixed to a table that is vibrating. The x_0 in Equation (5.29) is the peak amplitude of the table vibration, and Δx is the vibration of the seismic mass within the accelerometer. Thus, Equation (5.29) predicts that the seismic-mass vibration peak amplitude varies as the vibration frequency squared, but linearly with the table-vibration amplitude. However, this result was obtained without considering the spring-mass system natural vibration. When this is taken into account, something quite different occurs.

Figure 5.26a shows the actual seismic-mass vibration peak amplitude versus table-vibration frequency compared with the simple frequency-squared prediction.

You can see that there is a resonance effect when the table frequency equals the natural frequency of the accelerometer—that is, the value of Δx goes through a peak. The amplitude of the resonant peak is determined by the amount of damping. The seismic-mass vibration is described by Equation (5.29) only up to about $f_N/2.5$.

Figure 5.26b shows two effects. The first is that the actual seismic-mass motion is limited by the physical size of the accelerometer. It will hit "stops" built into the assembly that limit its motion during resonance. The figure also shows that for frequencies well above the natural frequency, the motion of the mass is proportional to the table peak motion, x_0, but not to the frequency. Thus, it has become a displacement sensor. To summarize:

1. $f < f_N$—For an applied frequency less than the natural frequency, the natural frequency has little effect on the basic spring-mass response given by Equations (5.25) and (5.29). A rule of thumb states that a safe maximum applied frequency is $f < 1/2.5 f_N$.

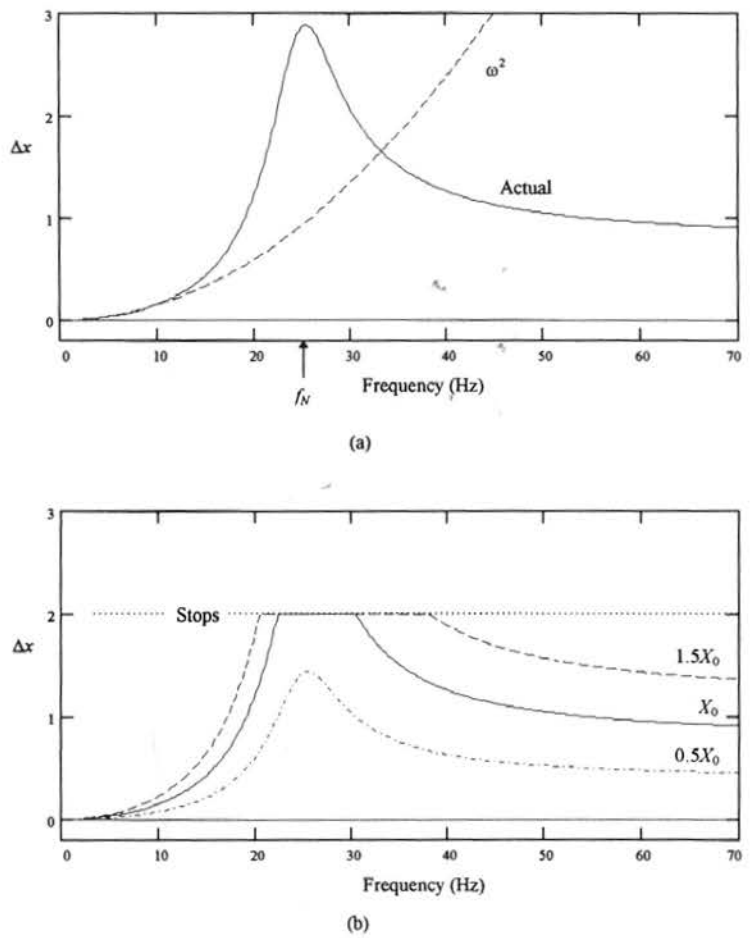


FIGURE 5.26
(a) The actual response of a spring-mass system to vibration is compared to the simple ω^2 prediction. (b) The effect of actual response with stops and various table peak motions is shown.

2. $f > f_N$—For an applied frequency much larger than the natural frequency, the accelerometer output is independent of the applied frequency. As shown in Figure 5.26b, the accelerometer becomes a measure of *vibration displacement*, x_0, of Equation (5.20) under these circumstances. It is interesting to note that the seismic mass is stationary in space in this case, and the housing, which is driven by the vibration, moves about the mass. A general rule sets $f > 2.5f_N$ for this case.

Generally, accelerometers are not used near the resonance at their natural frequency because of high nonlinearities in output.

EXAMPLE 5.14 An accelerometer has a seismic mass of 0.05 kg and a spring constant of 3.0×10^3 N/m. Maximum mass displacement is ± 0.02 m (before the mass hits the stops). Calculate **(a)** the maximum measurable acceleration in **g**, and **(b)** the natural frequency.

Solution

We find the maximum acceleration when the maximum displacement occurs, from Equation (5.26):

a.

$$a = \frac{k}{m}\Delta x$$

$$a = \left(\frac{3.0 \times 10^3\ \mathrm{N/m}}{0.05\ \mathrm{kg}}\right)(0.02\ \mathrm{m})$$

$$a = 1200\ \mathrm{m/s^2}$$

or because

$$1\ \mathbf{g} = 9.8\ \mathrm{m/s^2}$$

$$a = (1200\ \mathrm{m/s^2})\left(\frac{1\ \mathbf{g}}{9.8\ \mathrm{m/s^2}}\right)$$

$$a = 122\ \mathbf{g}$$

b. The natural frequency is given by Equation (5.27):

$$f_N = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$f_N = \frac{1}{2\pi}\sqrt{\frac{3.0 \times 10^3\ \mathrm{N/m}}{0.05\ \mathrm{kg}}}$$

$$f_N = \mathbf{39\ Hz}$$

5.4.3 Types of Accelerometers

The variety of accelerometers used results from different applications with requirements of range, natural frequency, and damping. In this section, various accelerometers with their special characteristics are reviewed. The basic difference is in the method of mass displacement measurement. In general, the specification sheets for an accelerometer will give the natural frequency, damping coefficient, and a scale factor that relates the output to an acceleration input. The values of test mass and spring constant are seldom known or required.

Potentiometric This simplest accelerometer type measures mass motion by attaching the spring mass to the wiper arm of a potentiometer. In this manner, the mass posi-

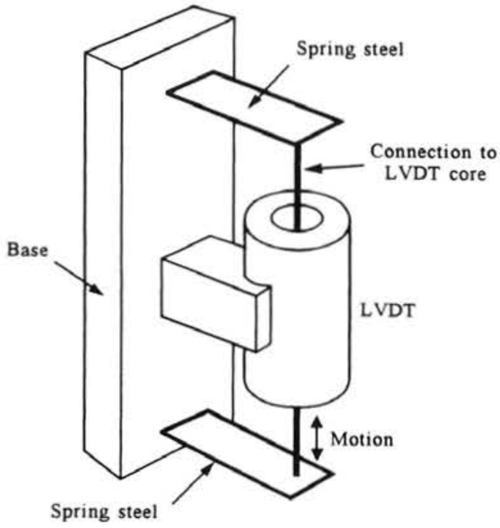


FIGURE 5.27
An LVDT is often used as an accelerometer, with the core serving as the mass.

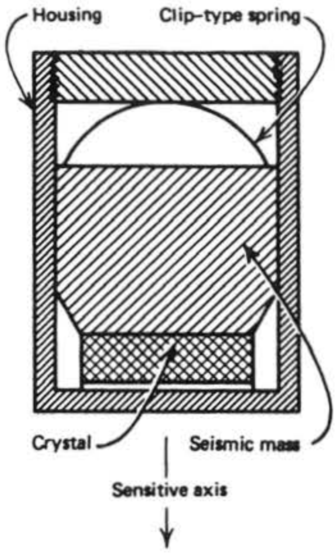
tion is conveyed as a changing resistance. The natural frequency of these devices is generally less than 30 Hz, limiting their application to *steady-state* acceleration or *low-frequency* vibration measurement. Numerous signal-conditioning schemes are employed to convert the resistance variation into a voltage or current signal.

LVDT A second type of accelerometer takes advantage of the natural linear displacement measurement of the LVDT (see Section 5.2.3) to measure mass displacement. In these instruments, the LVDT core itself is the seismic mass. Displacements of the core are converted directly into a linearly proportional ac voltage. These accelerometers generally have a natural frequency of less than 80 Hz and are commonly used for steady-state and low-frequency vibration. Figure 5.27 shows the basic structure of such an accelerometer.

Variable Reluctance This accelerometer type falls in the same general category as the LVDT in that an inductive principle is employed. Here, the test mass is usually a permanent magnet. The measurement is made from the voltage induced in a surrounding coil as the magnetic mass moves under the influence of an acceleration. This accelerometer is used in vibration and shock studies only, because it has an output *only* when the mass is in motion. Its natural frequency is typically less than 100 Hz. This type of accelerometer often is used in oil exploration to pick up vibrations reflected from underground rock strata. In this form, it is commonly referred to as a *geophone*.

Piezoelectric The piezoelectric accelerometer is based on a property exhibited by certain crystals where a voltage is generated across the crystal when stressed. This property is also the basis for such familiar sensors as crystal phonograph cartridges and crystal microphones. For accelerometers, the principle is shown in Figure 5.28. Here, a piezoelectric

FIGURE 5.28
A piezoelectric accelerometer has a very high natural frequency.



crystal is spring-loaded with a test mass in contact with the crystal. When exposed to an acceleration, the test mass stresses the crystal by a force ($F = ma$), resulting in a voltage generated across the crystal. A measure of this voltage is then a measure of the acceleration. The crystal per se is a very high-impedance source, and thus requires a high-input-impedance, low-noise detector. Output levels are typically in the millivolt range. The natural frequency of these devices may exceed 5 kHz, so that they can be used for vibration and shock measurements.

5.4.4 Applications

A few notes about the application of accelerometers will help in understanding how the selection of a sensor is made in a particular case.

Steady-State Acceleration In steady-state accelerations, we are interested in a measure of acceleration that may vary in time but that is nonperiodic. Thus, the stop-go motion of an automobile is an example of a steady-state acceleration. For these steady-state accelerations, we select a sensor having (1) adequate range to cover expected acceleration magnitudes and (2) a natural frequency sufficiently high that its period is shorter than the characteristic time span over which the measured acceleration changes. By using electronic integrators, the basic accelerometer can provide both velocity (first integration) and position (second integration) information.

EXAMPLE 5.15 An accelerometer outputs 14 mV per **g**. Design a signal-conditioning system that provides a velocity signal scaled at 0.25 V for every m/s, and determine the gain of the system and the feedback resistance ratio.

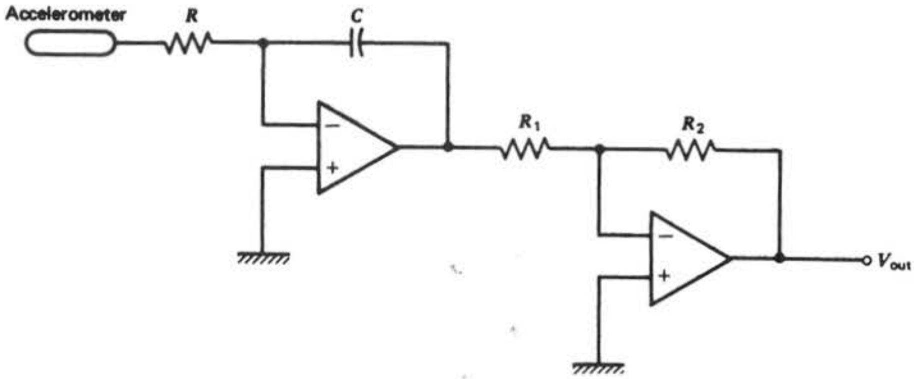


FIGURE 5.29
An integrator can be used to obtain velocity information from an accelerometer.

Solution
First we note that 14 mV/**g** becomes

$$\left(14\,\frac{\text{mV}}{\mathbf{g}}\right)\left(\frac{1\,\mathbf{g}}{9.8\,\text{m/s}^2}\right) = 1.43\,\frac{\text{mV}}{\text{m/s}^2}$$

Thus we can write the output voltage of the sensor as $V_a = Ka$, where K is the 1.43 mV(m/s^2) and a is the acceleration in m/s^2. If this is used as input to an op amp integrator, the result is:

$$V_v = -\frac{1}{RC}\int V_a dt = -\frac{K}{RC}\int a dt = -\frac{K}{RC}v$$

where V_v is the integrator output voltage and v is the velocity in m/s. We must get rid of the negative sign and provide the correct scale factor of 0.25 V/(m/s). Therefore, we can use an inverting amplifier on the output of the integrator. The circuit is shown in Figure 5.29. The output is $V_{out} = \left(\frac{R_2}{R_1}\right)\frac{K}{RC}v$. Notice the output is now positive as required. Any combination of quantities can be used that give the desired result. For example, we can take $R = 1\ \text{M}\Omega$ and $C = 1\ \mu\text{F}$, which makes $RC = 1$. Then we use a gain (R_2/R_1) to provide the correct scale factor,

$$0.25 = \left(\frac{R_2}{R_1}\right)\frac{1.43x10^{-3}}{1}\ \text{so,}\ \frac{R_2}{R_1} = 175$$

Therefore, so we could use $R_1 = 1\ \text{k}\Omega$ and then $R_2 = 175\ \text{k}\Omega$.

Vibration The application of accelerometers for vibration first requires that the applied frequency is less than the natural frequency of the accelerometer. Second, one must be sure the stated range of acceleration measured will never exceed that of the specification for the device. This assurance must come from a consideration of Equation (5.29) under circumstances of maximum frequency and vibration displacement.

Shock The primary elements of importance in shock measurements are that the device has a natural frequency that is greater than 1 kHz and a range typically greater than 500 **g**. The primary accelerometer that can satisfy these requirements is the piezoelectric type (see Section 5.4.3).

5.5 PRESSURE SENSORS

The measurement and control of fluid (liquid and gas) pressure has to be one of the most common in all the process industries. Because of the great variety of conditions, ranges, and materials for which pressure must be measured, there are many different types of pressure sensor designs. In the following paragraphs, the basic concepts of pressure are presented, and a brief description is given of the most common types of pressure sensors. You will see that pressure measurement is often accomplished by conversion of the pressure information to some intermediate form, such as displacement, which is then measured by a sensor to determine the pressure.

5.5.1 Pressure Principles

Pressure is simply the force per unit area that a fluid exerts on its surroundings. If it is a gas, then the pressure of the gas is the force per unit area that the gas exerts on the walls of the container that holds it. If the fluid is a liquid, then the pressure is the force per unit area that the liquid exerts on the container in which it is contained. Obviously, the pressure of a gas will be uniform on all the walls that must enclose the gas completely. In a liquid, the pressure will vary, being greatest on the bottom of the vessel and zero on the top surface, which need not be enclosed.

Static Pressure The statements made in the previous paragraph are explicitly true for a fluid that is not moving in space, that is not being pumped through pipes or flowing through a channel. The pressure in cases where no motion is occurring is referred to as *static* pressure.

Dynamic Pressure If a fluid is in motion, the pressure that it exerts on its surroundings *depends* on the motion. Thus, if we measure the pressure of water in a hose with the nozzle closed, we may find a pressure of, say, 40 lb per square inch (*Note:* force per unit area). If the nozzle is opened, the pressure in the hose will drop to a different value, say, 30 lb per square inch. For this reason, a thorough description of pressure must note the circumstances under which it is measured. Pressure can depend on flow, compressibility of the fluid, external forces, and numerous other factors.

Units Since pressure is force per unit area, we describe it in the SI system of units by newtons per square meter. This unit has been named the *pascal* (Pa), so that $1\ \text{Pa} = 1\ \text{N/m}^2$. As will be seen later, this is not a very convenient unit, and it is often used in conjunction with the SI standard prefixes as kPa or MPa. You will see the combination N/cm^2 used, but use of this combination should be avoided in favor of Pa with the appropriate prefix. In the English system of units, the most common designation is the pound per square inch, lb/in^2, usually written *psi*. The conversion is that 1 psi is approximately 6.895 kPa. For very low pressures, such as may be found in vacuum systems, the unit *torr* is often used. One torr is approximately 133.3 Pa. Again, use of the pascal with an appropriate prefix is preferred. Other units that you may encounter in the pressure description are the *atmosphere* (atm), which is 101.325 kPa or $\simeq$ 14.7 psi, and the *bar,* which is 100 kPa. The use of inches or feet of water and millimeters of Mercury will be discussed later.

Gauge Pressure In many cases, the absolute pressure is not the quantity of major interest in describing the pressure. The atmosphere of gas that surrounds the earth exerts a pressure, because of its weight, at the surface of the earth of approximately 14.7 psi, which defines the "atmosphere" unit. If a closed vessel at the earth's surface contained a gas at an absolute pressure of 14.7 psi, then there would be no *net* pressure on the walls of the container because the atmospheric gas exerts the same pressure from the outside. In cases like this, it is more appropriate to describe pressure in a relative sense—that is, compared to atmospheric pressure. This is called *gauge pressure* and is given by

$$p_g = p_{abs} - p_{at} \tag{5.30}$$

where p_g = gauge pressure
p_{abs} = absolute pressure
p_{at} = atmospheric pressure

In the English system of units, the abbreviation *psig* is used to represent the gauge pressure.

Head Pressure For liquids, the expression *head pressure,* or *pressure head,* is often used to describe the pressure of the liquid in a tank or pipe. This refers to the static pressure produced by the weight of the liquid above the point at which the pressure is being described. This pressure depends *only* on the height of the liquid above that point and the liquid density (mass per unit volume). In terms of an equation, if a liquid is contained in a tank, the pressure at the bottom of the tank is given by

$$p = \rho g h \tag{5.31}$$

where p = pressure in Pa
ρ = density in kg/m^3
g = acceleration due to gravity ($9.8\,m/s^2$)
h = depth in liquid in m

This same equation could be used to find the pressure in the English system, but it is common to express the density in this system as the weight density, ρ_w, in lb/ft^3, which includes the gravity term of Equation (5.31). In this case, the relationship between pressure and depth becomes

$$p = \rho_w h \tag{5.32}$$

where p = pressure in lb/ft^2
ρ_w = weight density in lb/ft^3
h = depth in ft

If the pressure is desired in psi, then the ft^2 would be expressed as $144\,in^2$. Because of the common occurrence of liquid tanks and the necessity to express the pressure of such systems, it has become common practice to describe the pressure directly in terms of the *equivalent* depth of a particular liquid. Thus, the term *mm of Mercury* means that the pressure is equivalent to that produced by so many millimeters of Mercury depth, which could be calculated from Equation (5.31) using the density of Mercury. In the same sense, the expression "inches of water" or "feet of water" means the pressure that is equivalent to some particular depth of water using its weight density.

Now you can see the basis for level measurement on pressure mentioned in Section 5.2.4. Equation (5.31) shows that the level of liquid of density p is directly related to the pressure. From level measurement we pass to pressure measurement, which is usually done by some type of displacement measurement.

EXAMPLE 5.16 A tank holds water with a depth of 7.0 ft. What is the pressure at the tank bottom in psi and Pa (density $= 10^3\ \mathrm{kg/m^3}$)?

Solution

We can find the pressure in Pa directly by converting the 7.0 ft into meters; thus, (7.0 ft) (0.3048 m/ft) = 2.1 m. From Equation (5.31),

$$p = (10^3\ \mathrm{kg/m^3})(9.8\ \mathrm{m/s^2})(2.1\ \mathrm{m})$$
$$p = 21\ \mathrm{kpa} \qquad \text{(note significant figures)}$$

To find the pressure in psi, we can convert the pressure in Pa to psi or use Equation (5.32). Let's use the latter. The weight density is found from

$$\rho_w = (10^3\ \mathrm{kg/m^3})(9.8\ \mathrm{m/s^2}) = 9.8 \times 10^3\ \mathrm{N/m^3}$$

or

$$\rho_w = (9.8 \times 10^3\ \mathrm{N/m^3})(0.3048\ \mathrm{m/ft^3})(0.2248\ \mathrm{lb/N})$$
$$\rho_w = 62.4\ \mathrm{lb/ft^3}$$

The pressure is

$$p = (62.4\ \mathrm{lb/ft^3})(7.0\ \mathrm{ft}) = 440\ \mathrm{lb/ft^2}$$
$$p = \mathbf{3\ psi}$$

5.5.2 Pressure Sensors (p > 1 atmosphere)

In general, the design of pressure sensors employed for measurement of pressure higher than one atmosphere differs from those employed for pressure less than 1 atmosphere (atm). In this section the basic operating principles of many types of pressure sensors used for the higher pressures are considered. You should be aware that this is not a rigid separation, because you will find many of these same principles employed in the lower (vacuum) pressure measurements.

Most pressure sensors used in process control result in the transduction of pressure information into a physical displacement. Measurement of pressure requires techniques for producing the displacement and means for converting such displacement into a proportional electrical signal. This is *not* true, however, in the very low pressure region ($p < 10^{-3}$ atm), where many purely electronic means of pressure measurement may be used.

Diaphragm One common element used to convert pressure information into a physical displacement is the diaphragm (thin, flexible piece of metal) shown in Figure 5.30.

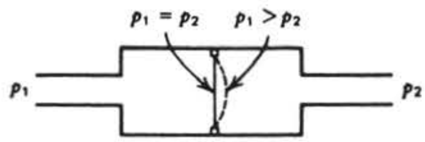


FIGURE 5.30
A diaphragm is used in many pressure sensors. Displacement varies with pressure difference.

If a pressure p_1 exists on one side of the diaphragm and p_2 on the other, then a net force is exerted given by

$$F \doteq (p_2 - p_1)A \tag{5.33}$$

where A = diaphragm area in m^2
p_1, p_2 = pressure in N/m^2

A diaphragm is like a spring and therefore extends or contracts until a Hooke's law force is developed that balances the pressure difference force. This is shown in Figure 5.30 for p_1 greater than p_2. Notice that since the force is greater on the p_1 side of the diaphragm, it has deflected toward the p_2 side. The extent of this deflection (i.e., the diaphragm displacement) is a measure of the pressure difference.

A *bellows*, shown in Figure 5.31, is another device much like the diaphragm that converts a pressure differential into a physical displacement, except that here the displacement is much more a straight-line expansion. The accordion-shaped sides of the bellows are made from thin metal. When there is a pressure difference, a net force will exist on the flat, front surface of the bellows. The bellows assembly will then collapse like an accordion if p_2 is greater than p_1 or expand if p_2 is less than p_1. Again, we have a displacement which is proportional to pressure difference. This conversion of pressure to displacement is very nearly linear. Therefore, we have suggested the use of an LVDT to measure the displacement. This sensor will output an LVDT voltage amplitude that is linearly related to pressure.

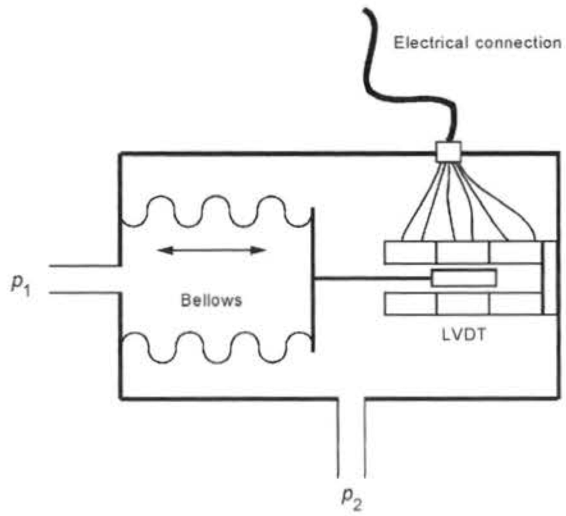


FIGURE 5.31
A bellows is another common method of converting pressure to displacement. Here an LVDT is used to convert the displacement to voltage amplitude.

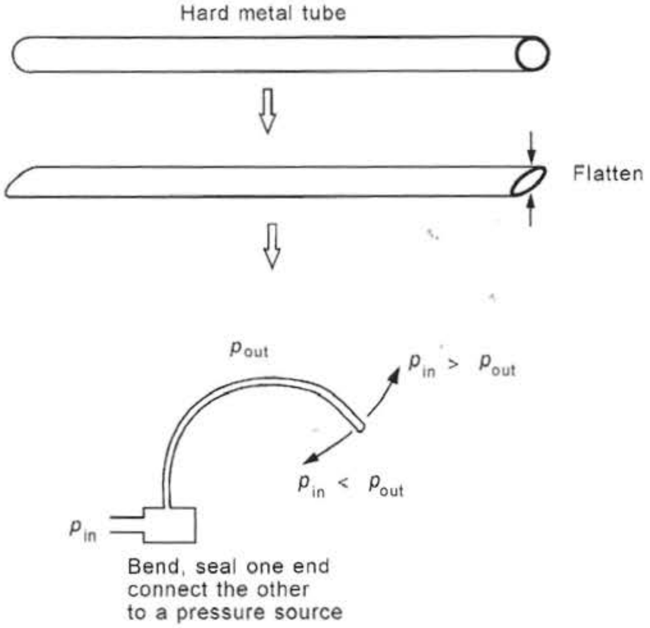


FIGURE 5.32
The Bourdon tube is probably the most common pressure-to-displacement element.

Figure 5.31 also shows how an LVDT can be connected to the bellows so that pressure measurement is converted directly from displacement to a voltage. In addition, the displacement and pressure are nearly linearly related, and because the LVDT voltage is linear with displacement, the voltage and pressure are also linearly related.

Bourdon Tube Probably the most common pressure sensor in universal use is based upon the Bourdon tube concept. Figure 5.32 shows the process for making a Bourdon tube and how it measures pressure. A hard metal tube, usually a type of bronze or brass, is flattened, and one end is closed off. The tube is then bent into a curve or arc, sometimes even a spiral. The open end is attached to a header by which a pressure can be introduced to the inside of the tube. When this is done, the tube will deflect when the inside applied pressure is different from the outside pressure. The tube will tend to straighten out if the inside pressure is higher than the outside pressure and to curve more if the pressure inside is less than that outside.

Most of the common, round pressure gauges with a meter pointer that rotates in proportion to pressure are based on this sensor. In this case, the deflection is transformed into a pointer rotation by a system of gears. Of course, for control applications, we are interested in converting the deflection into an electrical signal. This is accomplished by various types of displacement sensors to measure the deflection of the Bourdon tube.

Electronic Conversions Many techniques are used to convert the displacements generated in the previous examples into electronic signals. The simplest technique is to use a mechanical linkage connected to a potentiometer. In this fashion, pressure is related to a resistance change. Other methods of conversion employ strain gauges directly on a di-

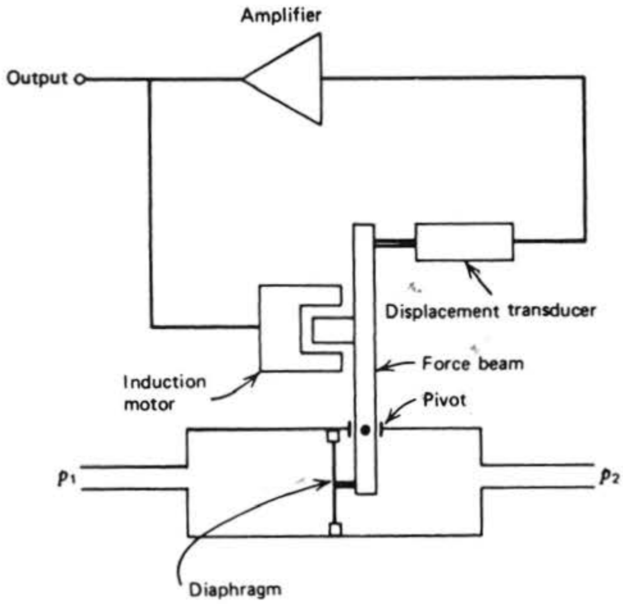


FIGURE 5.33
A differential pressure (DP) cell measures pressure difference with a diaphragm. A feedback system minimizes actual diaphragm deflection.

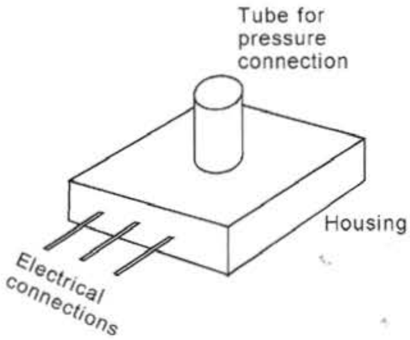
aphragm. LVDTs and other inductive devices are used to convert bellows or Bourdon tube motions into proportional electrical signals.

Often, pressure measurement is accomplished using a diaphragm in a special feedback configuration, shown in Figure 5.33. The feedback system keeps the diaphragm from moving, using an induction motor. The error signal in the feedback system provides an electrical measurement of the pressure.

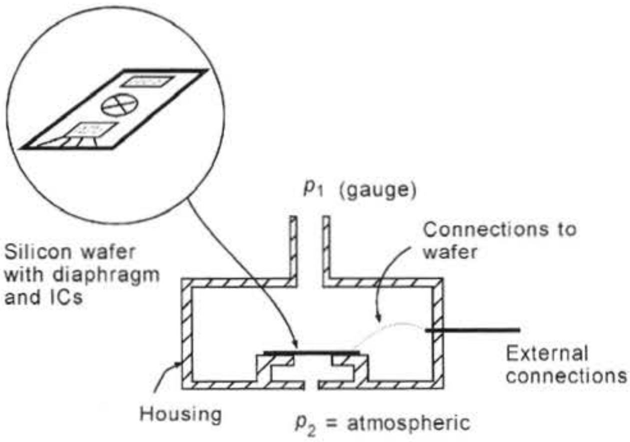
Solid-State Pressure Sensors Integrated circuit technology has led to the development of solid-state (SS) pressure sensors that find extensive application in the pressure ranges of 0 to 100 kPa (0 to 14.7 psi). These small units often require no more than three connections—dc power, ground, and the sensor output. Pressure connection is via a metal tube, as shown in Figure 5.34a. Generally, manufacturers provide a line of such sensors with various ranges of pressure and configurations.

The basic sensing element is a small wafer of silicon acting as a diaphragm that, as usual, deflects in response to a pressure difference. However, as suggested in Figure 5.34b, in this case the deflection is sensed by semiconductor strain gauges grown directly on the silicon wafer; furthermore, signal-conditioning circuitry is grown directly on the wafer as well. This signal conditioning includes temperature compensation and circuitry that provides an output voltage that varies linearly with pressure over the specified operating range.

The configuration shown in Figure 5.34b is for measuring *gauge* pressure, since one side of the diaphragm is open to the atmosphere. Figure 5.35 shows that simple modifications are used to convert the basic sensor to an absolute or differential-type gauge. For



a) Solid state pressure sensor



b) Internal structure of the pressure cell

FIGURE 5.34
Solid-state pressure sensors employ integrated circuit technology and silicon diaphragms. This example measures gauge pressure.

absolute pressure measurement, one side of the wafer is sealed off and evacuated. For differential measurement, facilities are provided to allow application of independent pressures p_1 and p_2 to the two sides of the diaphragm.

SS pressure sensors are characterized by

1. Sensitivities in the range of 10 to 100 mV/kPa.
2. Response times on the order of 10 ms. These are not first-order time-response devices, so response time is defined as the time for a change from 10% to 90% of the final value following a step change in input pressure.

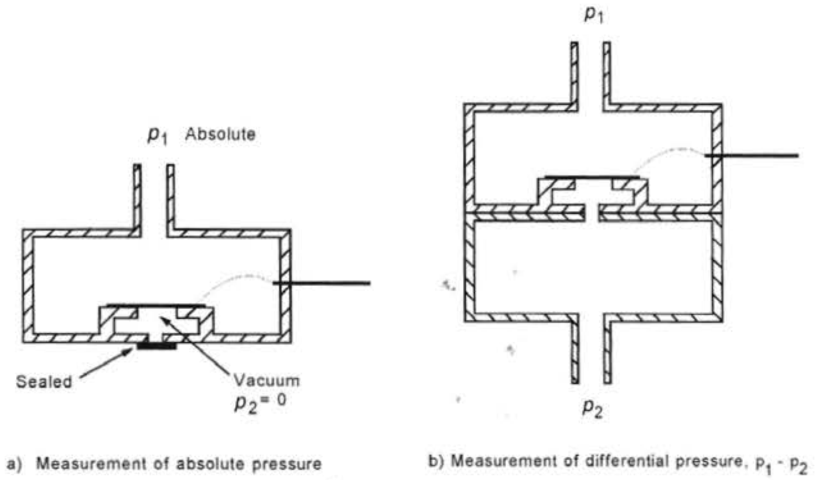


FIGURE 5.35
Simple modifications allow SS pressure sensors to measure absolute or differential pressure.

3. Linear voltage versus pressure within the specified operating range.
4. Ease of use, with often only three connections: dc power (typically 5 V), ground, and the sensor output voltage.

SS pressure sensors find application in a broad sector of industry and control, wherever low pressures are to be measured. Another important application is in the commercial field, where such sensors are employed, for example, in home appliances such as dishwashers and washing machines.

EXAMPLE 5.17

A SS pressure sensor that outputs 25 mV/kPa for a pressure variation of 0.0 to 25 kPa will be used to measure the level of a liquid with a density of $1.3 \times 10^3\,\text{kg/m}^3$. What voltage output will be expected for level variations from 0 to 2.0 m? What is the sensitivity for level measurement expressed in mV/cm?

Solution

The pressure sensor will be attached to the bottom of the tank holding the liquid. Therefore, the pressure measured will be given by Equation (5.31). Clearly, when empty the pressure will be zero and output voltage will be zero as well. At 2.0 m, the pressure will be

$$p = \rho g h = (1.3 \times 10^3\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(2.0\,\text{m}) = 25.48\,\text{kPa}.$$

Therefore, the sensor output will be

$$V = (25\,\text{mV/kPa})(25.48\,\text{kPa}) = 0.637\,\text{V}$$

So the sensitivity will be

$$S = 637\,\text{mV}/200\,\text{cm} = 3.185\,\text{mV/cm}$$

5.5.3 Pressure Sensors ($p < 1$ atmosphere)

Measurements of pressure less than 1 atm are most conveniently made using purely electronic methods. There are three common methods of electronic pressure measurements.

The first two devices are useful for pressure less than 1 atm, down to about 10^{-3} atm. They are both based on the rate at which heat is conducted and radiated away from a heated filament placed in the low-pressure environment. The heat loss is proportional to the number of gas molecules per unit volume, and thus, under constant filament current, the filament temperature is proportional to gas pressure. We have thus transduced a pressure measurement to a temperature measurement.

Pirani Gauge This gauge determines the filament temperature through a measure of filament *resistance* in accordance with the principles established in Section 4.3. Filament excitation and resistance measurement are both performed with a bridge circuit. The response of resistance versus pressure is highly nonlinear.

Thermocouple A second pressure transducer or gauge measures filament temperature using a thermocouple directly attached to the heated filament. In this case, ambient room temperature serves as a reference for the thermocouple, and the voltage output, which is proportional to pressure, is highly nonlinear. Calibration of both Pirani and thermocouple gauges also depends on the type of gas for which the pressure is being measured.

Ionization Gauge This device is useful for the measurement of very low pressures from about 10^{-3} atm to 10^{-13} atm. This gauge employs electrons, usually from a heated filament, to ionize the gas whose pressure is to be measured, and then measures the current flowing between two electrodes in the ionized environment, as shown in Figure 5.36. The number of ions per unit volume depends on the gas pressure, and hence the current also depends on gas pressure. This current is then monitored as an approximately linear indication of pressure.

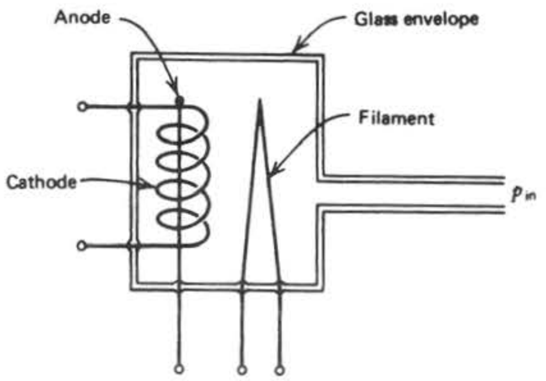


FIGURE 5.36
The ionization gauge is used to measure very low pressures, down to about 10^{-13} atm.

5.6 FLOW SENSORS

The measurement and control of flow can be said to be the very heart of process industries. Continuously operating manufacturing processes involve the movement of raw materials, products, and waste throughout the process. All such functions can be considered flow, whether automobiles through an assembly line or methyl chloride through a pipe. The methods of measurement of flow are at least as varied as the industry. It would be unreasonable to try to present every type of flow sensor, and in this section we will consider flow on three broad fronts—solid, liquid, and gas. As with pressure, we will find that flow information is often translated into an intermediate form, that is then measured using techniques developed for that form.

5.6.1 Solid-Flow Measurement

A common solid-flow measurement occurs when material in the form of small particles, such as crushed material or powder, is carried by a conveyor belt system or by some other host material. For example, if solid material is suspended in a liquid host, the combination is called a *slurry*, which is then pumped through pipes like a liquid. We will consider the conveyor system and leave slurry to be treated as liquid flow.

Conveyor Flow Concepts For solid objects, the flow usually is described by a specification of the mass or weight per unit time that is being transported by the conveyor system. The units will be in many forms—for example, kg/min or lb/min. To make a measurement of flow, it is only necessary to weigh the quantity of material on some fixed length of the conveyor system. Knowing the speed of the conveyor allows calculation of the material flow rate.

Figure 5.37 shows a typical conveyor system where material is drawn from a hopper and transported by the conveyor system. A mechanical valve controls the rate at which material can flow from the hopper onto the conveyor belt. The belt is driven by a motor system. Flow rate is measured by weighing the amount of material on a platform of length L at any instant. The conveyor belt slides over the platform, which deflects slightly due to the weight of material. A load cell measures this deflection as an indication of weight. In this case, flow rate can be calculated from

$$Q = \frac{WR}{L} \tag{5.34}$$

where Q = flow (kg/min or lb/min)
W = weight of material on section of length L (kg or lb)
R = conveyor speed (m/min or ft/min)
L = length of weighing platform (m or ft)

Flow Sensor In the example with which we are working in Figure 5.37, it is evident that the flow sensor is actually the assembly of the conveyor, hopper opening, and weighing platform. It is the actual weighing platform that performs the measurement from

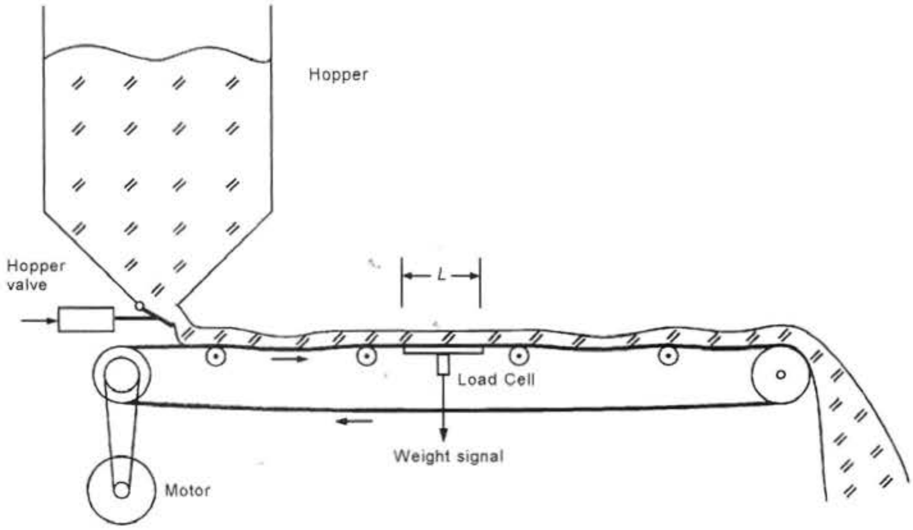


FIGURE 5.37
Conveyor system for illustrating solid-flow measurement.

which flow rate is determined, however. We see that flow measurement becomes weight measurement. In this case, we have suggested that this weight is measured by means of a load cell, which is then a strain gauge measurement. Another popular device for weight measurement of moving systems like this is an LVDT that measures the droop of the conveyor at the point of measurement because of the material that it carries.

EXAMPLE 5.18 A coal conveyor system moves at 100 ft/min. A weighing platform is 5.0 ft in length, and a particular weighing shows that 75 lb of coal are on the platform. Find the coal delivery in lb/h.

Solution

We can use Equation (5.34) directly to find the flow:

$$Q = \frac{(75\,\text{lb})(100\,\text{ft/min})}{5\,\text{ft}}$$

$$Q = 1500\,\text{lb/min}$$

Then, converting to lb/h by multiplying by 60 min/h,

$$Q = \mathbf{90{,}000\ lb/h}$$

5.6.2 Liquid Flow

The measurement of liquid flow is involved in nearly every facet of the process industry. The conditions under which the flow occurs and the vastly different types of material that flow result in a great many types of flow measurement methods. Indeed, entire books are written devoted to the problems of measuring liquid flow and how to interpret the results of flow measurements. It is impractical and not within the scope of this book to present a comprehensive study of liquid flow; only the basic ideas of liquid flow measurement will be presented.

Flow Units The units used to describe the flow measured can be of several types, depending on how the specific process needs the information. The most common descriptions are the following:

1. *Volume flow rate* Expressed as a volume delivered per unit time. Typical units are gals/min, m^3/h, or ft^3/h ($1\ \text{gal} = 231\ \text{in.}^3$).
2. *Flow velocity* Expressed as the distance the liquid travels in the carrier per unit time. Typical units are m/min or ft/min. This is related to the volume flow rate by

$$V = \frac{Q}{A} \tag{5.35}$$

where V = flow velocity
Q = volume flow rate
A = cross-sectional area of flow carrier (pipe, and so on)

3. *Mass or weight flow rate* Expressed as mass or weight flowing per unit time. Typical units are kg/h or lb/h. This is related to the volume flow rate by

$$F = \rho Q \tag{5.36}$$

where F = mass or weight flow rate
ρ = mass density or weight density
Q = volume flow rate

EXAMPLE 5.19 Water is pumped through a 1.5-in. diameter pipe with a flow velocity of 2.5 ft/s. Find the volume flow rate (ft^3/min) and weight flow rate (lb/min). The weight density is 62.4 lb/ft^3.

Solution

The flow velocity is given as 2.5 ft/s, so the volume flow rate can be found from Equation (5.35), $Q = VA$. The area is given by

$$A = \pi d^2/4$$

where the diameter $d = (1.5\ \text{in.})(1/12\ \text{ft/in.}) = 0.125\ \text{ft}$
so that

$$A = (3.14)(0.125)^2/4 = 0.0122\ \text{ft}^2$$

Then, the volume flow rate is

$$Q = (2.5\ \text{ft/s})(0.0122\ \text{ft}^2)(60\ \text{s/min})$$
$$Q = 1.8\ \text{ft}^3/\text{min} \quad \text{or} \quad 13.5\ \text{gal/min}$$

The weight flow rate is found from Equation (5.36):

$$F = (62.4\ \text{lb/ft}^3)(1.8\ \text{ft}^3/\text{min})$$
$$F = \mathbf{112\ lb/min}$$

Pipe Flow Principles The flow rate of liquids in pipes is determined primarily by the pressure that is forcing the liquid through the pipe. The concept of pressure head, or simply *head,* introduced in the previous sections is often used to describe this pressure, because it is easy to relate the forcing pressure to that produced by a depth of liquid in a tank from which the pipe exits. In Figure 5.38, flow through pipe P is driven by the pressure in the pipe, but this pressure is caused by the weight of liquid in the tank of height h (head). The pressure is found by Equation (5.31) or (5.32). Many other factors affect the actual flow rate produced by this pressure, including liquid viscosity, pipe size, pipe roughness (friction), turbulence of flowing liquid, and others. It is beyond the scope of this book to detail exactly how these factors determine the flow. Instead, it is our objective to discuss how such flow is measured, regardless of those features that may determine exactly what the flow is relative to the conditions.

Restriction Flow Sensors One of the most common methods of measuring the flow of liquids in pipes is by introducing a restriction in the pipe and measuring the pressure drop that results across the restriction. When such a restriction is placed in the pipe, the velocity of the fluid through the restriction *increases*, and the pressure in the restriction *decreases*. We find that there is a relationship between the pressure drop and the rate of flow such that, as the flow increases, the pressure drops. In particular, one can find an equation of the form

$$Q = K\sqrt{\Delta p} \tag{5.37}$$

where Q = volume flow rate
K = a constant for the pipe and liquid type
Δp = drop in pressure across the restriction

The constant, K, depends on many factors, including the type of liquid, size of pipe, velocity of flow, temperature, and so on. The type of restriction employed also will change the value of the constant used in this equation. The flow rate is linearly dependent not on the

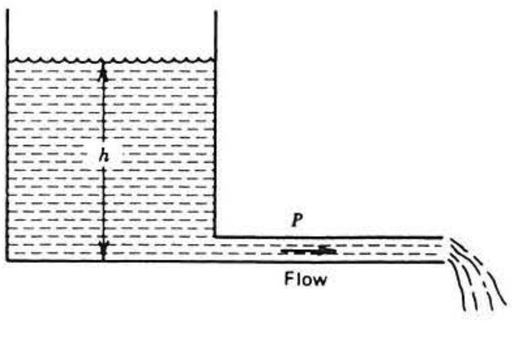


FIGURE 5.38
Flow through the pipe, P, is determined in part by the pressure due to the head, h.

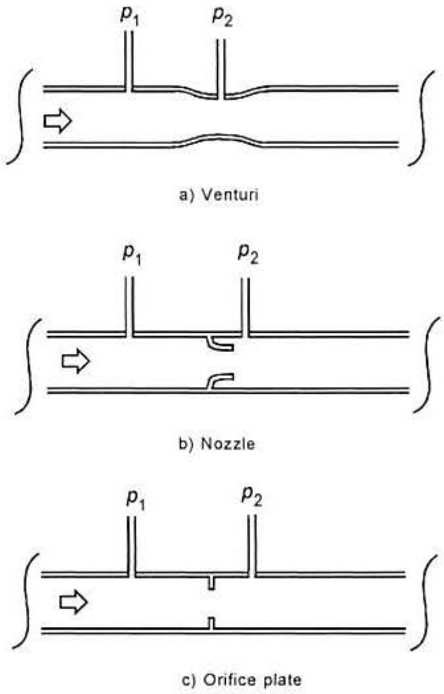


FIGURE 5.39
Three different types of restrictions are commonly used to convert pipe flow to a pressure difference, $p_1 - p_2$.

pressure drop, but on the square root. Thus, if the pressure drop in a pipe increased by a factor of 2 when the flow rate was increased, the flow rate will have increased only by a factor of 1.4 (the square root of 2). Certain standard types of restrictions are employed in exploiting the pressure-drop method of measuring flow.

Figure 5.39 shows the three most common methods. It is interesting to note that having converted flow information to pressure, we now employ one of the methods of measuring pressure, often by conversion to displacement, which is measured by a displacement sensor before finally getting a signal that will be used in the process-control loop. The most common method of measuring the pressure drop is to use a differential pressure sensor similar to that shown in Figure 5.35b. These are often described by the name *DP cell.*

EXAMPLE 5.20

Flow is to be controlled from 20 to 150 gal/min. The flow is measured using an orifice plate system such as that shown in Figure 5.39c. The orifice plate is described by Equation (5.37), with $K = 119.5(\text{gal/min})/\text{psi}^{1/2}$. A bellows measures the pressure with an LVDT so that the output is 1.8 V/psi. Find the range of voltages that result from the given flow range.

Solution

From Equation (5.37), we find the pressures that result from the given flow:

$$\Delta p = (Q/K)^2$$

For 20 gal/min,

$$\Delta p = (20/119.5)^2 = 0.0280\,\text{psi}$$

and for 150 gal/min,

$$\Delta p = (150/119.5)^2 = 1.5756 \text{ psi}$$

Because there are 1.8 V/psi, the voltage range is easily found.

$$\text{For 20 gal/min, } V = 0.0280(1.8) = 0.0504 \text{ V}$$
$$\text{For 150 gal/min, } V = 1.5756(1.8) = 2.836 \text{ V}$$

Pitot Tube The pitot tube is a common way to measure flow rate at a particular point in a flowing fluid (liquid or gas). Figure 5.40 shows a tube placed in the flowing fluid with its opening directed into the direction of flow. The principle is that the fluid will be brought to rest in the tube, and therefore its pressure will be the sum of the static fluid pressure plus the effective pressure of the flow. The pressure in the pitot tube is measured in differential to the static pressure of the flowing fluid in the same vicinity as the tube. This differential pressure will be proportional to the square root of the flow rate. The flow rate in a pipe varies across the pipe, so the pitot tube determines the flow rate only at the point of insertion.

Obstruction Flow Sensor Another type of flow sensor operates by the effect of flow on an obstruction placed in the flow stream. In a *rotameter,* the obstruction is a float that rises in a vertical tapered column. The lifting force, and thus the distance to which the float rises in the column, is proportional to the flow rate. The lifting force is produced by the differential pressure that exists across the float, because it is a restriction in the flow. This type of sensor is used for both liquids and gases. A *moving vane* flow meter has a vane target immersed in the flow region, which is rotated out of the flow as the flow velocity increases. The angle of the vane is a measure of the flow rate. If the rotating vane shaft is attached to an angle-measuring sensor, the flow rate can be measured for use in a process-control application. A *turbine* type of flow meter is composed of a freely spinning turbine blade assembly in the flow path. The rate of rotation of the turbine is proportional to the flow rate. If the turbine is attached to a tachometer, a convenient electrical signal can

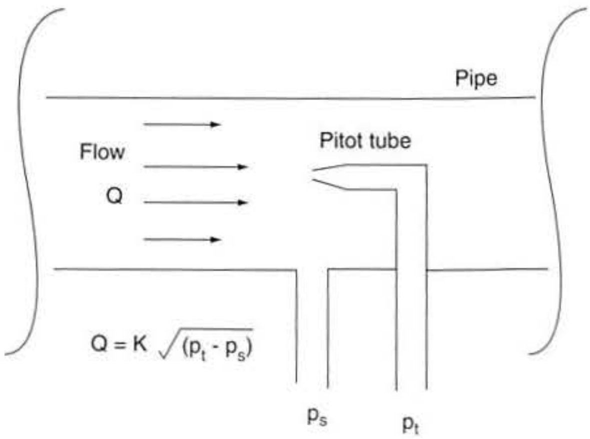


FIGURE 5.40
The pitot tube measures flow at a point in the gas or liquid.

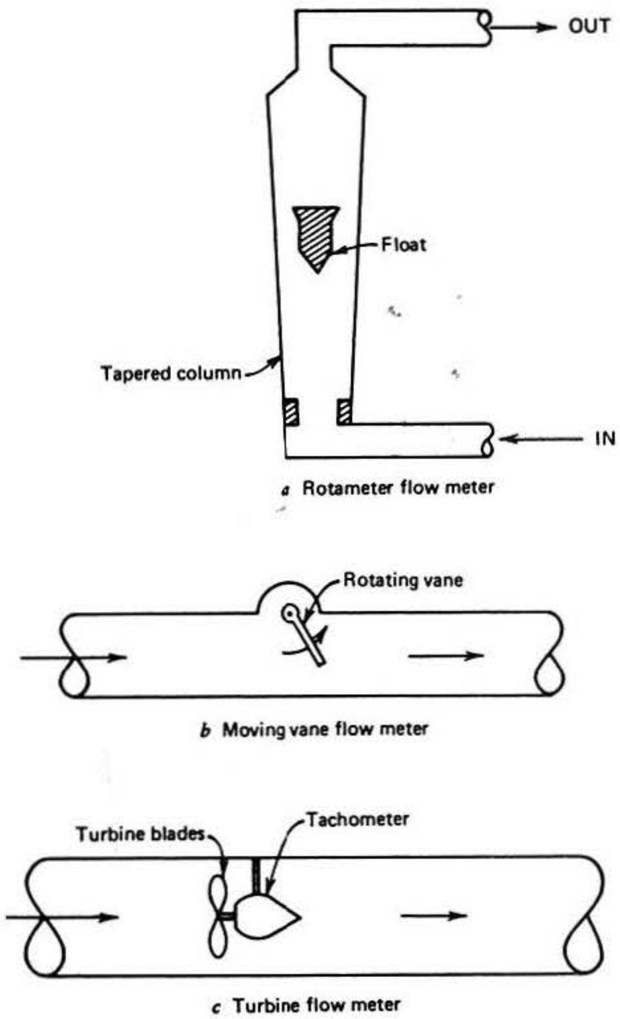


FIGURE 5.41
Three different types of obstruction flow meters.

be produced. In all of these methods of flow measurement, it is necessary to present a substantial obstruction into the flow path to measure the flow. For this reason, these devices are used only when an obstruction does not cause any unwanted reaction on the flow system. These devices are illustrated in Figure 5.41.

Magnetic Flow Meter It can be shown that if charged particles move across a magnetic field, a potential is established across the flow, perpendicular to the magnetic field. Thus, if the flowing liquid is also a conductor (even if not necessarily a good conductor) of electricity, the flow can be measured by allowing the liquid to flow through a magnetic field and measuring the transverse potential produced. The pipe section in which this measurement is made must be insulated, and a nonconductor itself, or the potential produced, will be

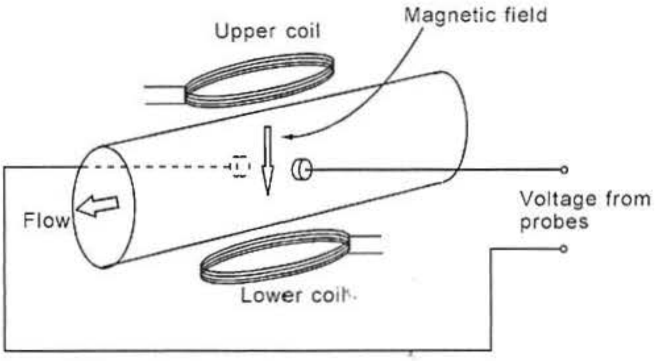


FIGURE 5.42
A magnetic flow meter will work only with conducting fluids such as blood.

cancelled by currents in the pipe. A diagram of this type of flow meter is presented in Figure 5.42. This type of sensor produces an electrical signal directly and is convenient for process-control applications involving conducting fluid flow.

SUMMARY

In this chapter, an assortment of measurement systems that fall under the general description of mechanical sensors has been studied. The objective was to gain familiarity with the essential features of the variables themselves and the typical measurement methods.

Topics covered were the following:

1. Position, location, and displacement sensors, including the potentiometric, capacitive, and LVDT. The LVDT converts displacement linearly into a voltage.
2. The strain gauge measures deformation of solid objects resulting from applied forces called stress. The strain gauge converts strain into a change of resistance.
3. Accelerometers are used to measure the acceleration of objects because of rectilinear motion, vibration, and shock. Most of them operate by the spring-mass principle, which converts acceleration information into a displacement.
4. Pressure is the force per unit area that a fluid exerts on the walls of a container. Pressure sensors often convert pressure information into a displacement. Examples include diaphragms, bellows, and the Bourdon tube. Electronic measures are often used for low pressures.
5. For gas pressures less than 1 atm, purely electrical techniques are used. In some cases, the temperature of a heated wire is used to indicate pressure.
6. Flow sensors are very important in the manufacturing world. Typically, solid flow is mass or weight per unit time.
7. Fluid flow through pipes or channels typically is measured by converting the flow information into pressure by a restriction in the flow system.

PROBLEMS

Section 5.2

5.1 A 50-kΩ, wire-wound pot is used to measure the displacement of a work piece. A linkage is employed so that as the work piece moves over a distance of 12 cm, the pot varies by the full 50-kΩ. The pot is wound on a 3-cm-diameter form, 5 cm long, and the distance between wires in the pot is 0.25 mm. What is the resolution in work-piece motion? What resistance change corresponds to this resolution?

5.2 Develop signal conditioning for Problem 5.1 so the output is −6 to +6 V as the work piece moves over the 0- to 12-cm motion limit.

5.3 A capacitive displacement sensor is used to measure rotating shaft wobble, as shown in Figure 5.43. The capacity is 880 pF with no wobble. Find the change in capacity for a +0.02- to −0.02-mm shaft wobble.

5.4 Develop an ac bridge for Problem 5.3. Use 880 pF for all the bridge capacitors, and assume a 10-V rms, 10-kHz excitation. What is the maximum bridge offset voltage amplitude?

FIGURE 5.43
Figure for Problem 5.3.

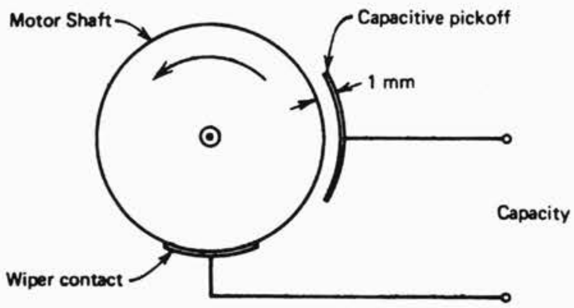
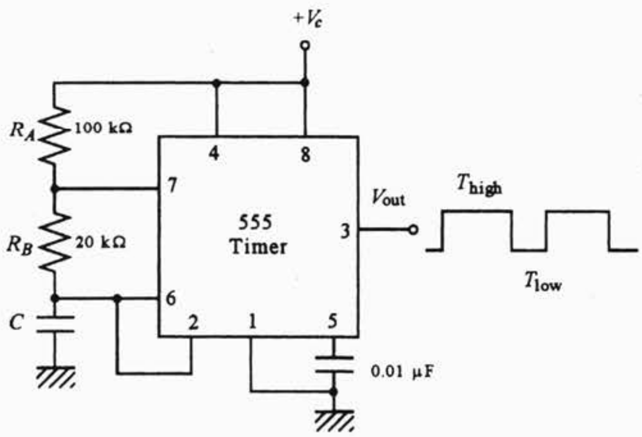


FIGURE 5.44
Figure for Problem 5.5.



5.5 Figure 5.44 shows how a 555 IC timer can be connected to make a frequency generator. The output low time is given by $T_{low} = 0.693 R_B C$, whereas the high time is given by $T_{high} = 0.693(R_A + R_B)C$. Suppose the capacitor in Example 5.2 is used in parallel with a 1000-pF fixed capacitor for C in this circuit. Then displacement will be converted to a variation of output frequency.

a. What range of frequency corresponds to the 1- to 2-cm displacement of the sensor?

b. Plot the frequency change versus displacement. Is it linear?

5.6 An LVDT with associated signal conditioning will be used to measure work-piece motion from −20 to +20 cm. The static transfer function is 2.5 mV/mm. The output will be interfaced to a computer via an ADC.

a. What is the range of output voltage?

b. If the desired resolution is 0.5 mm, how many bits must the ADC have?

c. Design analog signal conditioning to provide interface to a bipolar ADC with a 5-V reference.

5.7 Design a linkage system such that as a float for liquid level measurement moves from 0 to 1 m, an LVDT core moves over its linear range of 3 cm. Suppose the LVDT output is interfaced to a 10-bit ADC. What is the resolution in level measurement?

5.8 For Example 5.4, what capacity change would need to be measured to have a resolution of 2 cm? If the measurement will ultimately go to a computer, how many bits must the ADC have to support this resolution?

5.9 Design an ac bridge like that in Figure 2.11 to convert the capacity change of Example 5.4 into an ac offset voltage. The bridge should null at 0 m of level. Use $R_1 = R_3 = 1\ \mathrm{k\Omega}$, $C_3 = 0.02\ \mu\mathrm{F}$, and an excitation of 5 V rms at 1 kHz. Plot the voltage versus level.

Section 5.3

5.10 An aluminum beam supports a 550-kg mass. If the beam diameter is 6.2 cm, calculate **(a)** the stress and **(b)** the strain of the beam.

5.11 Using Equations (5.9) and (5.10), prove that Equation (5.11) is valid to first order in $\Delta l/l$. Note that $1/(1-x) \sim 1 + x$ for $x \ll 1$.

5.12 A strain gauge has GF = 2.06 and $R = 120\ \Omega$, and is made from wire with $\alpha_0 = 0.0034/^\circ\mathrm{C}$ at 25°C. The dissipation factor is given as $P_D = 25\ \mathrm{mW}/^\circ\mathrm{C}$. What is the maximum current that can be placed through the SG to keep self-heating errors below 1 μ of strain?

5.13 A strain gauge has GF = 2.14 and a nominal resistance of 120 Ω. Calculate the resistance change resulting from a strain of 144 μin./in.

5.14 A strain gauge with GF = 2.03 and 120 Ω nominal resistance is to be used to measure strain with a resolution of 5 μs. Design a bridge and detector that provides this over five switched ranges of 1000-μ spans (i.e., 0 to 1000 μ, 1000 to 2000 μ, etc.). The idea is that a bridge null is found by a combination of switching to the appropriate range and then making a smooth null adjustment within that range. If the strain were 3390 μ, it would be necessary to switch to the 3000- to 4000-μ range and then adjust the smooth pot until a null occurred, which would be at 390 μ.

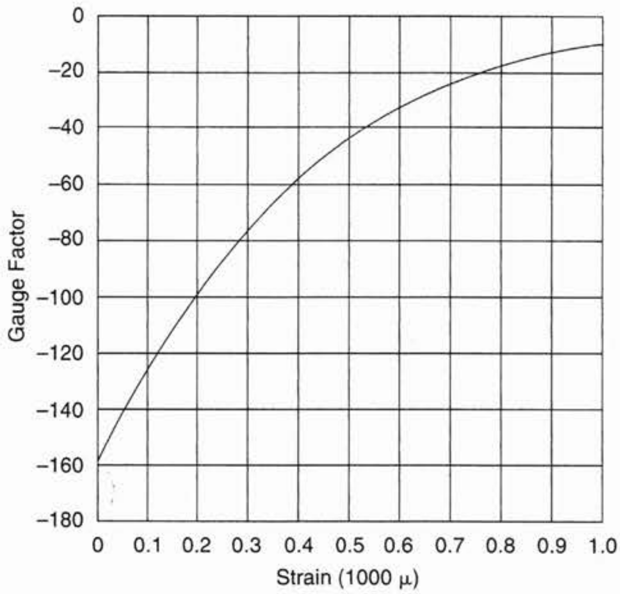


FIGURE 5.45
Semiconductor GF versus strain for Problem 5.16.

5.15 Derive the Equation (5.14), giving the approximate bridge off-null voltage versus strain. How much error does this equation have from the exact off-null voltage if the GF = 2.05, $R_A = 120\ \Omega$, $V_s = 10$ V, and the strain is 500 μ? Assume all other bridge resistors are 120 Ω also.

5.16 A semiconductor strain gauge has $R = 300\ \Omega$ and a GF versus strain given in Figure 5.45. This gauge is used in a bathroom scale for which the strain varies from 0 to 1000 μ as weight varies from 0 to 300 lb. Plot the gauge resistance change versus weight.

5.17 For Example 5.10, develop signal conditioning to provide input to a 10-bit unipolar ADC with a 5.000-V reference. How many pounds does each LSB represent? Plot the ADC output in hex versus force. Evaluate the linearity. Do not use a gain of more than 500 for any single op amp circuit stage in the design.

5.18 We will weigh objects by a strain gauge of $R = 120\ \Omega$, GF = 2.02 mounted on a copper column of 6-in. diameter. Find the change in resistance per pound placed on the column. Is this change an increase or decrease in resistance? Draw a diagram of the system showing how the active and dummy gauges should be mounted.

5.19 Figure 5.46 shows a micro-miniature cantilever beam with four strain gauges mounted. All are active gauges. Show how the gauges are wired into a bridge circuit to provide temperature compensation. If each gauge has the same gauge factor, GF, derive an equation for the bridge off-null voltage as a function of strain.

5.20 Show how to add another active gauge and dummy gauge to the system of Problem 5.18 to provide increased sensitivity. Assuming a bridge excitation of 5.0 volts, determine the bridge off-null voltage per 1000 pounds placed on the beam.

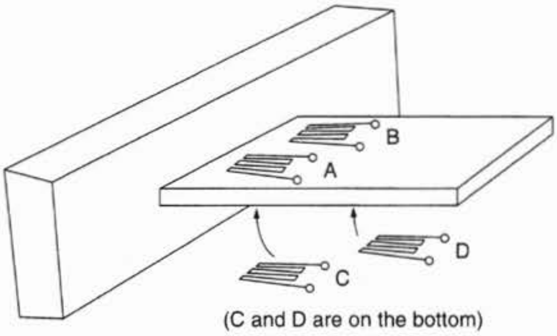


FIGURE 5.46
Four-gauge system for Problem 5.19.

Section 5.4

5.21 Calculate the rotation rate of a 10,000-rpm motor in rad/s.

5.22 An object falls from rest near the Earth's surface, accelerating downward at 1 **g**. After 5 s, what is the speed and the distance moved?

5.23 A force of 2.7 lb is applied to a 5.5-kg mass. Find the resulting acceleration in m/s^2.

5.24 Calculate the average shock in **g**s experienced by a transistor that falls 1.5 m from a tabletop, if it takes 2.7 ms to decelerate to zero when reaching the floor.

5.25 An automobile fender vibrates at 16 Hz with a peak-to-peak amplitude of 5 mm. Calculate the peak acceleration in **g**s.

5.26 A spring-mass system has a mass of 0.02 kg and a spring constant of 140 N/m. Calculate the natural frequency.

5.27 An LVDT is used in an accelerometer to measure seismic-mass displacement. The LVDT and signal-conditioning output is 0.31 mV/mm with a ± 2 cm maximum core displacement. The spring constant is 240 N/m, and the core mass is 0.05 kg. Find **(a)** the relation between acceleration in m/s^2 and output voltage, **(b)** the maximum acceleration that can be measured, and **(c)** the natural frequency.

5.28 For the accelerometer in Problem 5.27, design a signal-conditioning system that provides velocity information at 2 mV/(m/s) and position information at 0.5 V/m.

5.29 A piezoelectric accelerometer has a transfer function of 61 mV/g and a natural frequency of 4.5 kHz. In a vibration test at 110 Hz, a reading of 3.6 V-peak results. Find the vibration peak displacement.

5.30 An accelerometer for shock is designed as shown in Figure 5.47. Find a relation between strain gauge resistance change and shock in **g** (i.e., the resistance change per **g**). The force rod cross-sectional area is 2.0×10^{-4} m^2.

5.31 Design a signal-conditioning scheme for the accelerometer in Problem 5.30 using a bridge circuit. Plot the bridge offset voltage versus shock in **g** from 0 to 5000 **g**s.

Section 5.5

5.32 Calculate **(a)** the pressure in atmospheres that a water column 3.3 m high exerts on its base and **(b)** the pressure if the liquid is Mercury. Convert these results to pascals. Mercury density is 13.546 g/cm^3.

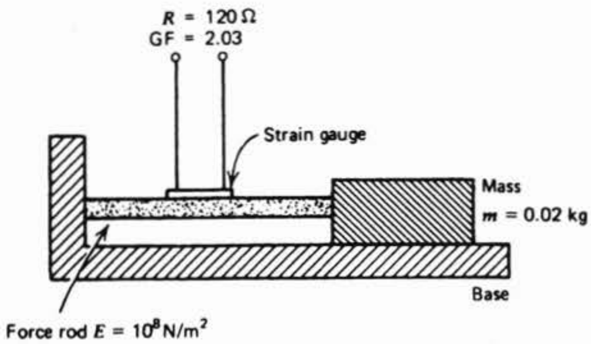


FIGURE 5.47
Figure for Problem 5.28.

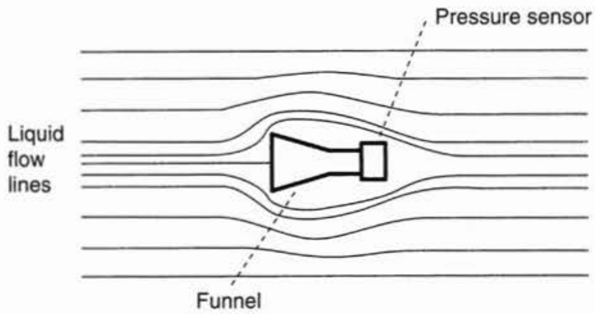


FIGURE 5.48
Figure for Problem 5.35.

5.33 A welding tank holds oxygen at 1500 psi. What is the tank pressure expressed in Pa? What is the pressure in atmospheres?

5.34 A diaphragm has an effective area of 25 cm^2. If the pressure difference across the diaphragm is 5 psi, what force is exerted on the diaphragm?

5.35 Figure 5.48 shows a proposed sensor for measuring the speed of liquid flowing in an open channel. An SS pressure sensor is connected to a funnel as shown. A pressure is formed when the funnel has its open end pointed upstream so that the liquid is brought to rest against the funnel opening. The pressure is given by $p = \frac{1}{2}\rho v^2$, where ρ is the liquid density in kg/m^3 and v is the liquid speed in m/s. The SS pressure sensor has a range of 0 to 5 kPa with a transfer function of 40 mV/kPa. Suppose the liquid is water with a density of $1 g/cm^3$. What is the maximum speed which can be measured? Plot a graph of sensor output voltage versus liquid speed. Comment on linearity.

5.36 The bellows, diaphragm, and Bourdon tube pressure sensors all exhibit second-order time response. This means that a sudden change in pressure will cause an oscillation in the displacement and, therefore, in sensor output. Because they are like springs, they have an effective spring constant and mass, so the frequency can be estimated by Equation (5.27). Consider a bellows with an effective spring constant of 3500 N/m and mass of 50 g. The effective area against which the pressure acts is 0.5 in^2. Calculate **(a)** the bellows deflection for a pressure of 20 psi and **(b)** the natural frequency of oscillation.

Section 5.6

5.37 A grain conveyor system finds the weight on a 1.0-m platform to be 258 N. What conveyor speed is needed to get a flow of 5200 kg/h?

5.38 Convert water flow of 52.2 gal/h into kg/h and velocity in m/s through a 2-in.-diameter pipe.

5.39 For an orifice plate in a system pumping alcohol, we find $K = 0.4\ m^3/min\,(kPa)^{1/2}$. Plot the pressure versus flow rate from 0 to 100 m^3/h.

SUPPLEMENTARY PROBLEMS

S5.1 An ultrasonic system will be used to measure the level of grain from 1 to 9 m in a bin, as shown in Figure 5.49. This will be done by measuring the time delay between transmitting a short ultrasonic pulse and receiving the pulse echo from the grain surface. A 50-kHz transmitter can be triggered on and off by a logic high/low input, as shown. The transmitted pulse is to have a duration of 6 ms, and the propagation speed is 300 m/s. The receiver is connected to a comparator that goes high when a signal is received. The receiver must be disabled while the transmitter is sending a pulse using a logic high input, as shown. An 8-bit counter will start counting when the pulse

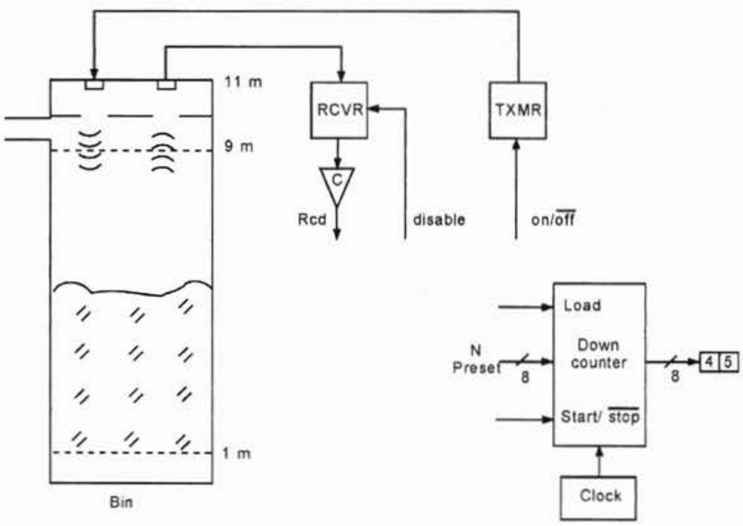


FIGURE 5.49
Ultrasonic system of level measurement for Problem S5.1.

is sent and stop when the echo is received. Its count is thus a measure of the time delay and the grain level. A reading is to be taken every second.

a. Calculate the expected time delay for grain levels between 1 m and 9 m.

b. Since the ultrasonic system measures from the bin top, the time delay will be reversed (i.e., long time is low level and short time is high level). To account for this, the counter will count down from some preset value. Determine the preset value and the correct counter clock speed so that the digital display is equal to the grain level (i.e., a level of 4.5 m would produce a binary count of $\mathbf{00101101}_2$, so the display would show 45).

c. Use one-shot pulse generators, flip/flops, and any other digital logic devices to complete the design shown in Figure 5.49.

S5.2 Show how the gauge mounting in Figure 5.17 can be changed so that all four gauges are active but temperature compensation is still provided. Show how the gauges are connected in a bridge.

S5.3 Figure 5.45 shows the variation of GF with strain for a semiconductor strain gauge with $R = 300\ \Omega$ under no strain. This gauge is used to measure solid material flow on a conveyor using a system like that in Figure 5.37. The load cell structure is shown in Figure 5.50. The conveyor speed is $R = 0.3$ m/s, and the platform length is $L = 1.5$ m.

a. Design a bridge and signal conditioning for the strain gauges that is temperature compensated and provides an output of 0.0 to 5.0 V for a strain of 0 to 500 μ.

b. Prepare a plot of output voltage versus flow rate from 0.0 to 200 kg/s.

S5.4 An SS pressure sensor will be used to measure the specific gravity of a flowing production liquid. Specific gravity is simply the ratio of liquid density to water density. Figure 5.51 shows how this can be measured by the difference in head pressure for equal heights of the liquid and water. A level control system maintains the flowing production liquid at the same level as the water. The differential SS sensor has a sensitivity of 45 mV/kPa. The specific gravity will be measured in the range of 1.0 to 2.0. The liquid/water level is 1.0 m, and the density of water is 1 g/cm^3.

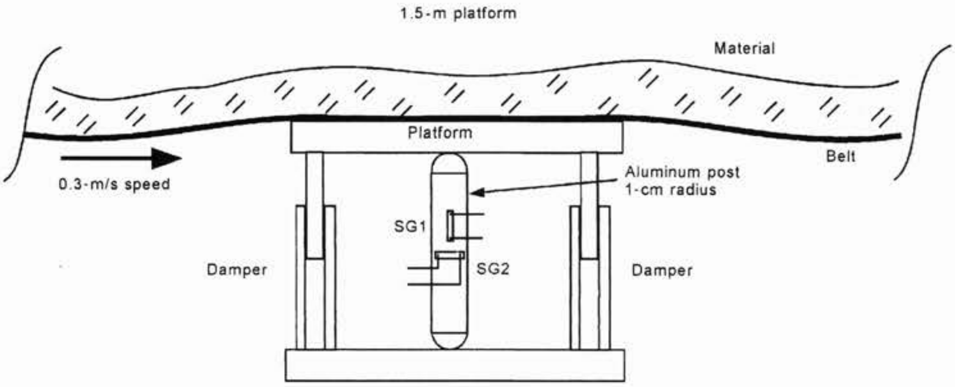


FIGURE 5.50
Load cell for Problem S5.3.

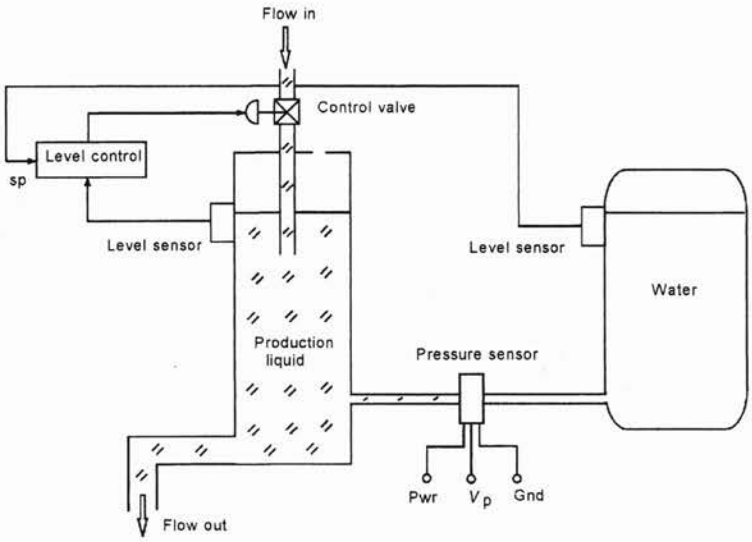
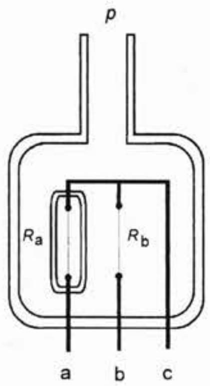


FIGURE 5.51
System for Problem S5.4.

a. What is the pressure of water at the sensor? What is the range of pressure of the liquid at the sensor?
b. What sensor voltage results for the range of 1.0 to 2.0 specific gravity?
c. Develop signal conditioning to interface the sensor output to an 8-bit ADC with a 5.00-V reference.
d. What is the digital resolution in specific gravity measurement?
e. If the level control system has a ±2-cm error about the setpoint, how much error will there be in specific gravity measurement?

FIGURE 5.52
Pirani gauge for Problem S5.5.



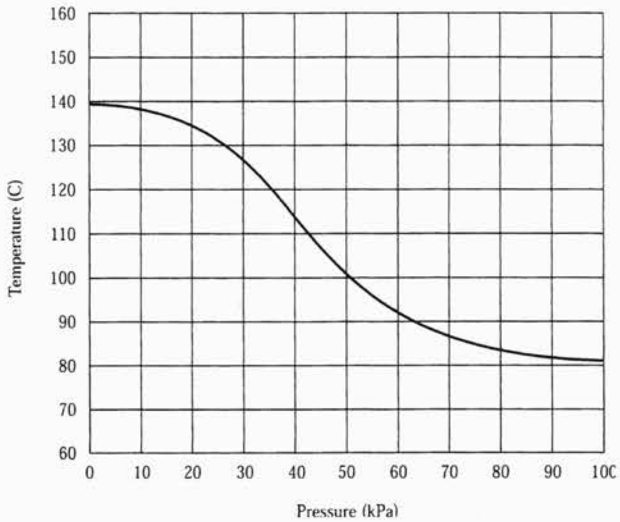


FIGURE 5.53
Resistance versus pressure for Problem S5.5.

S5.5 Figure 5.52 shows a Pirani gauge for measuring vacuum pressures from about 0 to 100 kPa (atmospheric pressure). The two resistive filaments are operated at an elevated temperature of 80°C at 1 atm pressure. The exposed filament temperature is a function of pressure, but the encapsulated filament temperature is not. Figure 5.53 shows how the exposed filament temperature varies with pressure. Variation of resistance with temperature is given by the RTD relation with $\alpha = 0.035/^\circ\mathrm{C}$ and $R = 20\ \Omega$ at 20°C and a self-heating dissipation constant of 30 mW/°C. Design a bridge circuit that provides the necessary self-heating current and an off-null voltage that depends on pressure. Plot the off-null voltage versus pressure from 0.0 to 100 kPa.