Birzeit University
Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Modern Communication Systems ENEE 3306
Quiz # 1

Instructors: Dr. Wael Hashlamoun, Date: March 13, 2018

Problem

The signal $5\cos 2\pi(150)t$ is sampled at the Nyquist rate. The samples are applied to an 8-level uniform quantizer with a dynamic range of (-5, 5) V. The quantized levels are then assigned binary digits following the natural binary encoding scheme.

a. Find the Nyquist rate
b. Find the signal to quantization noise ratio
c. Find the binary representation corresponding to the sample -1.14V.

3

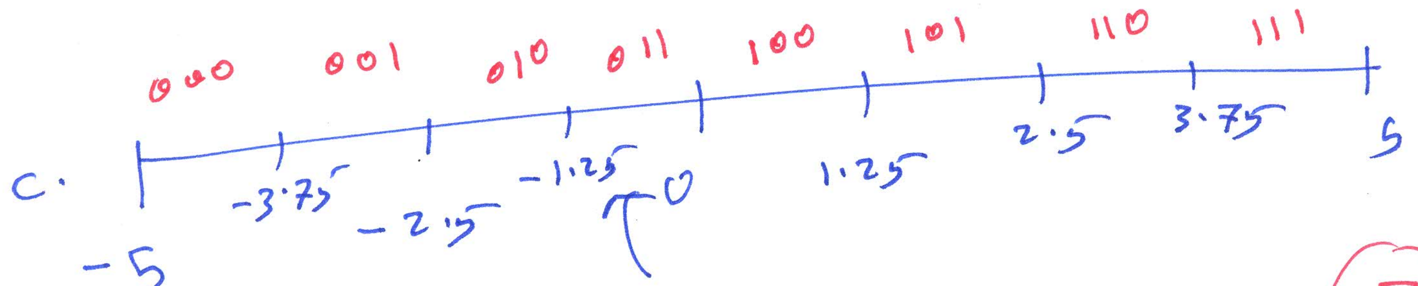
a. $f_s = 2W = 2 * 150 = 300$ sample/sec

b. $SQNR = \dfrac{A_m^2/2}{\Delta^2/12} = \dfrac{(5)^2/2}{(1.25)^2/12} = 96$

4

$\Delta = \dfrac{5-(-5)}{8} = \dfrac{10}{8} = 1.25$

c.



$-1.14 \rightarrow \dfrac{-1.25}{2} = 0.625 \Rightarrow 011$

3

Birzeit University
Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Modern Communication Systems ENEE 3306
Quiz # 1

Instructors: Dr. Wael Hashlamoun, Date: March 13, 2018

Problem

The signal $2\cos 2\pi(150)t$ is sampled at a rate of 400 samples/sec. The samples are applied to an 8-level uniform quantizer with a dynamic range of (-10, 10) V. The quantized levels are then assigned binary digits following the natural binary encoding scheme.

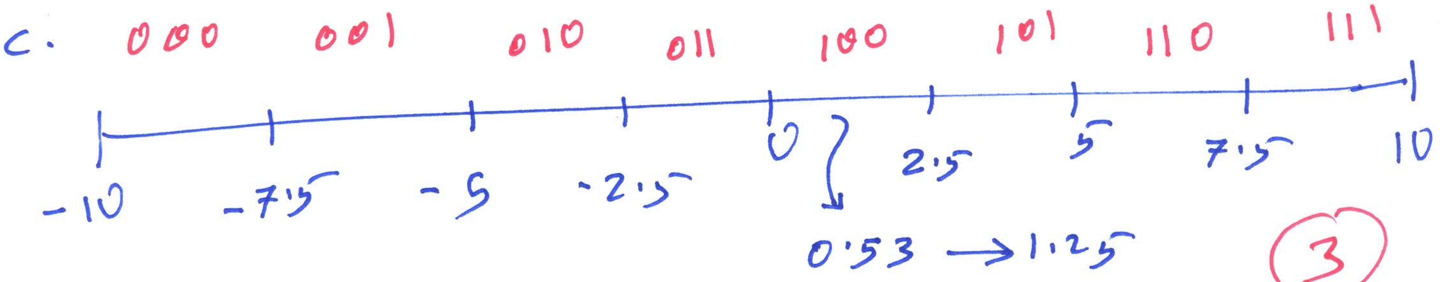
a. Find the data rate in bits/sec at the encoder output.
b. Find the quantizer step size
c. Find the binary representation corresponding to the sample 0.53V.

Levels $M = 8 = 2^3 \Rightarrow$ # of bits/sample = 3 → 3

a. data rate r_b = # of samples/sec * # bits/sample
= 400 * 3 = 1200 bits/sec → 2

b. $\Delta = \dfrac{10-(-10)}{8} = \dfrac{20}{8} = 2.5$ V 2

c.



3

0.53 → 1.25 V ⇒ 100

Birzeit University
Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Modern Communication Systems ENEE 3306
Quiz # 2

Instructors: Dr. Wael Hashlamoun, Date: April 5, 2018

Problem

Consider an FSK system that uses the signals $s_1(t) = A\cos(2\pi f_1 t)$ and $s_2(t) = B\cos(2\pi f_2 t)$, $0 \le t \le \tau$, where $s_1(t)$ and $s_2(t)$ are orthogonal.

a. Find the system probability of error.
b. Sketch the optimum receiver detailing the parameters of each unit.

a. $P_b = Q\left(\sqrt{\dfrac{\int_0^{\tau} (s_1(t) - s_2(t))^2 dt}{2N_0}}\right)$

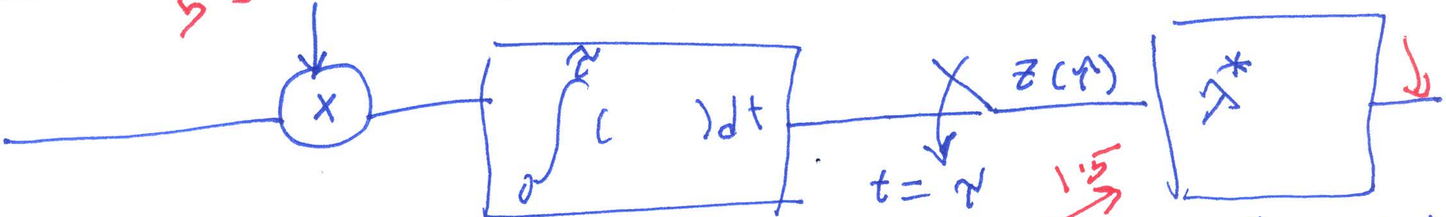
$$\int_0^{\tau} (s_1(t) - s_2(t))^2 dt = \int_0^{\tau} A^2 \cos 2\pi f_1 t \, dt + \int_0^{\tau} B^2 \cos 2\pi f_2 t \, dt - 2\underbrace{\int_0^{\tau} A \cos 2\pi f_1 t \; B \cos 2\pi f_2 t \, dt}_{=0 \text{ due to orthogonality}}$$

$$= \frac{A^2}{2}\tau + \frac{B^2 \tau}{2} = E_1 + E_2$$

$$P_b = Q\left(\sqrt{\frac{\frac{A^2\tau}{2} + \frac{B^2\tau}{2}}{2N_0}}\right)$$

5/10

b. 1.5/5



$$\lambda^* = \frac{1}{2}(E_1 - E_2)$$

1.5/5 $z(\tau) \ge \lambda^* \Rightarrow$ digit 1

$z(\tau) < \lambda^* =$ " 0

2/5 $$= \frac{1}{2}\left[\frac{A^2\tau}{2} - \frac{B^2\tau}{2}\right]$$

Birzeit University
Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Modern Communication Systems ENEE 3306
Quiz # 2

Instructors: Dr. Wael Hashlamoun, Date: April 5, 2018

Problem

Consider an FSK system that uses the signals $s_1(t) = A\cos(2\pi f_1 t)$ and $s_2(t) = A\cos(2\pi f_2 t)$, $0 \le t \le \tau$.

a. Show that $s_1(t)$ and $s_2(t)$ are orthogonal when $f_1 = nR_b$ and $f_2 = mR_b$ where n and m are integers, $n \ne m$.

b. Find the system probability of error.

a. $\int_0^{\tau} A\cos 2\pi f_1 t \cdot A\cos 2\pi f_2 t \, dt$

$= \frac{A^2}{2}\int_0^{\tau} \cos 2\pi(f_1+f_2)t \, dt + \frac{A^2}{2}\int_0^{\tau} \cos 2\pi(f_1-f_2)t \, dt$

$= \frac{A^2}{2(f_1+f_2)(2\pi)}\left(\sin 2\pi(f_1+f_2)t\Big|_0^{\tau} + \frac{A^2}{2}\frac{\sin 2\pi(f_1-f_2)t}{2\pi(f_1-f_2)}\Big|_0^{\tau}\right.$

$= 0 + 0$

$\sin 0 = 0$; $\sin k\pi = 0$; k integer

4/10

b. $P_b = Q\left(\sqrt{\frac{\int_0^{\tau}(s_1(t)-s_2(t))^2 dt}{2N_0}}\right.$ 2/10

$\int_0^{\tau}[s_1(t) \pm s_2(t)]^2 dt = \int_0^{\tau} s_1(t)^2 dt + \int_0^{\tau} s_2(t)^2 dt - 2\int_0^{\tau} s_1(t) s_2(t) dt$

$= 0$ from part a

$= \frac{A^2\tau}{2} + \frac{A^2\tau}{2} - 0 = \frac{2A^2\tau}{2} = A^2\tau$ 2/10

$\Rightarrow P_b = Q\left(\sqrt{\frac{A^2\tau}{2N_0}}\right)$ 2/10

Birzeit University
Faculty of Engineering
Department of Electrical and Computer Engineering
Modern Communication Systems ENEE 3306
Quiz # 3

Instructor: Dr. Wael Hashlamoun   Date: May 15, 2018

Consider a digital communication system that transmits one of three signals every T_s seconds over a channel corrupted by AWGN with zero mean and power spectral density $N_0/2$. The signals occur with equal probabilities. Assume $T_s = n\,T_c$; n an integer. The transmitted signals are:

$$s_1(t) = \Delta\sqrt{\frac{2}{T_s}}\cos(2\pi f_c t),\ s_2(t) = 2\Delta\sqrt{\frac{2}{T_s}}\cos(2\pi f_c t),\ s_3(t) = 3\Delta\sqrt{\frac{2}{T_s}}\cos(2\pi f_c t)$$

a. Find a set of basis functions for the signal space.
b. Find and sketch the signal space representation of the signals.
c. Find the average transmitted energy per symbol.

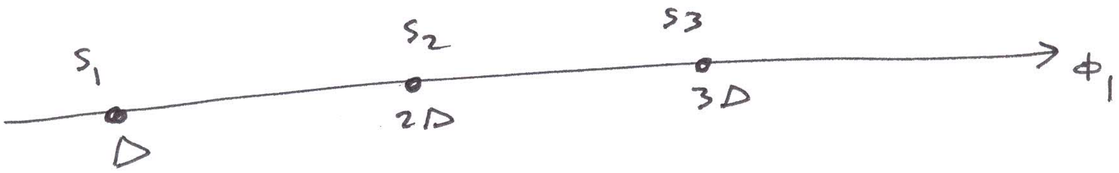
3

a. There is only one base function

$\phi_1(t) = \sqrt{\frac{2}{T_s}}\cos 2\pi f_c t$; check $\int_0^{T_s} \phi(t)^2 dt = 1$

b. $s_1(t) = \Delta\,\phi_1(t)$; $s_2(t) = 2\Delta\,\phi_1(t)$; $s_3(t) = 3\Delta\,\phi_1(t)$

4



3

c. $E_{av} = \frac{1}{3}\left[E_1 + E_2 + E_3\right]$

$= \frac{1}{3}\left[\Delta^2 + 4\Delta^2 + 9\Delta^2\right] = \frac{14\Delta^2}{3}$

STUDENTS-HUB.com

Birzeit University
Faculty of Engineering
Department of Electrical and Computer Engineering
Modern Communication Systems ENEE 3306
Quiz # 3

Instructor: Dr. Wael Hashlamoun	Date: May 15, 2018

Consider a digital communication system that transmits one of two signals every T_b seconds over a channel corrupted by AWGN with zero mean and power spectral density $N_0/2$. The signals occur with equal probabilities. Assume $T_b = n\,T_c$; n an integer. The transmitted signals are:

$$s_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t), \qquad s_2(t) = 3\sqrt{\frac{2}{T_b}}\cos(2\pi f_c t),$$

a. Verify that $\sqrt{\frac{2}{T_b}}\cos(2\pi f_c t)$ is a base function for the space.

b. Draw the block diagram of the optimum receiver.

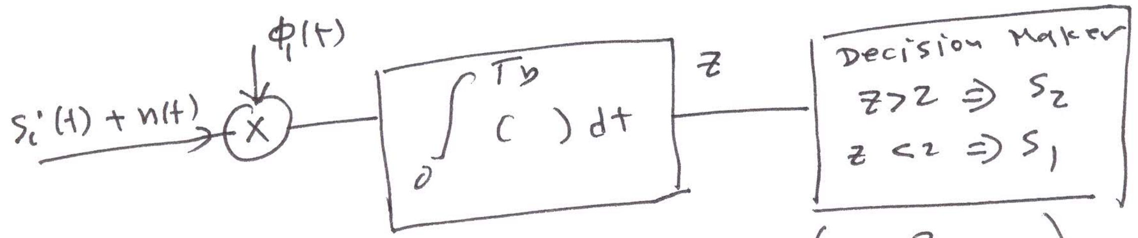
c. Find the average probability of error of the optimum receiver.

a: $\int_0^{T_b} \phi_1(t)^2 dt = \int_0^{T_b} \frac{2}{T_b}\cos^2 2\pi f_c t \, dt = 1$ 3

$s_1(t) = \phi_1(t)$; $s_2(t) = 3\phi_1(t)$

1 3 ϕ 4

b. since there is one base function, then we need only one corelator (or one matched filter)



c. $P_b = Q\left(\frac{d_{12}}{\sqrt{2N_0}}\right) = Q\left(\frac{2}{\sqrt{2N_0}}\right)$ 3

Birzeit University
Faculty of Engineering
Department of Electrical and Computer Engineering
Modern Communication Systems ENEE 3306
Quiz # 4

Instructor: Dr. Wael Hashlamoun Date: May 22, 2018

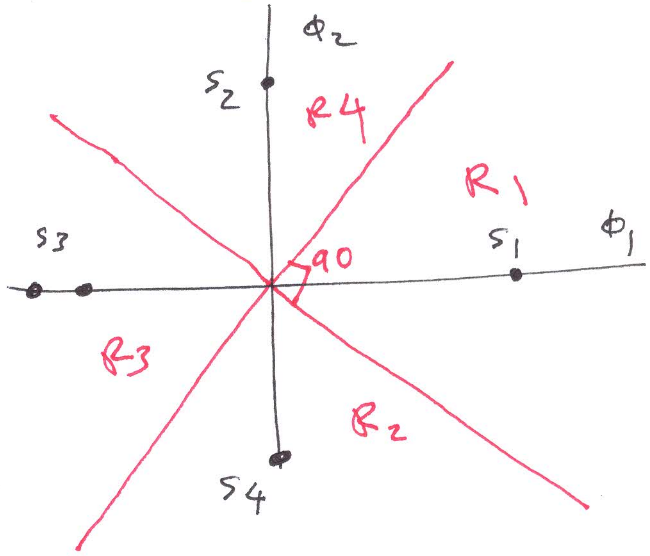
Consider a digital communication system that transmits one of four signals every T_s seconds over a channel corrupted by AWGN with zero mean and power spectral density $N_0/2$. The signals occur with equal probabilities. Assume $T_s = n\,T_c$; n an integer. The space bases functions are:

$$\phi_1(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi f_c t), \qquad \phi_2(t) = \sqrt{\frac{2}{T_s}}\sin(2\pi f_c t),$$

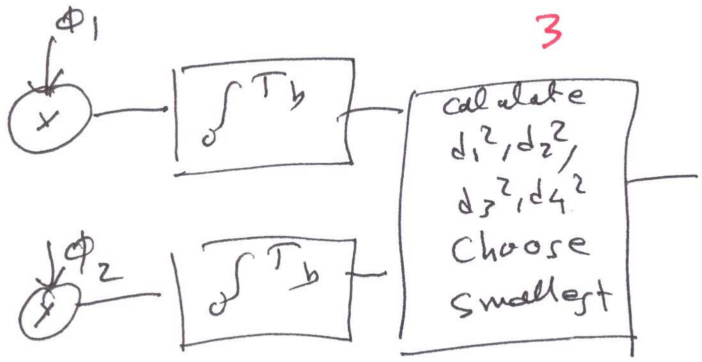
The signals have the coordinates: $s_1 = (1,0)$; $s_2 = (0,1)$; $s_3 = (-1,0)$; $s_4 = (0,-1)$;

a. Find the average energy per symbol.
b. Draw the block diagram of the optimum receiver.
c. Find the decision region corresponding to each transmitted signal.

a. $E_{av} = \frac{1+1+1+1}{4} = 1$ 3



b.



3

c. shown in Red 4

Birzeit University
Faculty of Engineering
Department of Electrical and Computer Engineering
Modern Communication Systems ENEE 3306
Quiz # 4

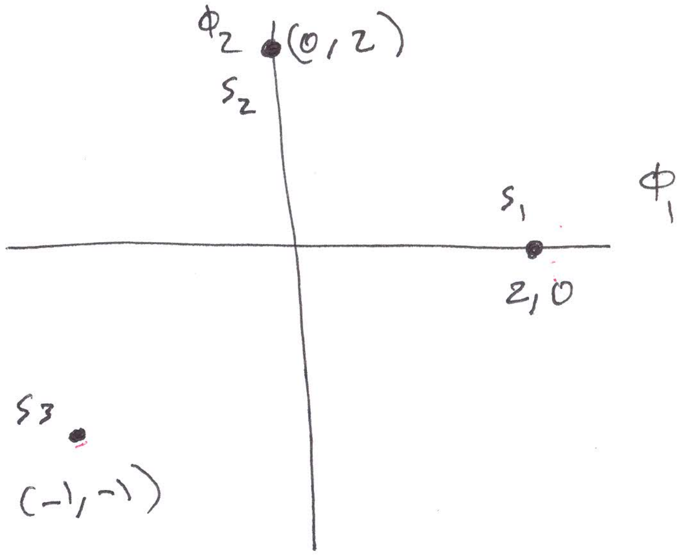
Instructor: Dr. Wael Hashlamoun Date: May 22, 2018

Consider a digital communication system that transmits one of three signals every T_s seconds over a channel corrupted by AWGN with zero mean and power spectral density $N_0/2$. The signals occur with equal probabilities. Assume $T_s = n\,T_c$; n an integer. The space bases functions are:

$$\phi_1(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi f_c t), \qquad \phi_2(t) = \sqrt{\frac{2}{T_s}}\sin(2\pi f_c t),$$

The signals have the coordinates: $s_1 = (2,0);\ s_2 = (0,2);\ s_3 = (-1,-1);$

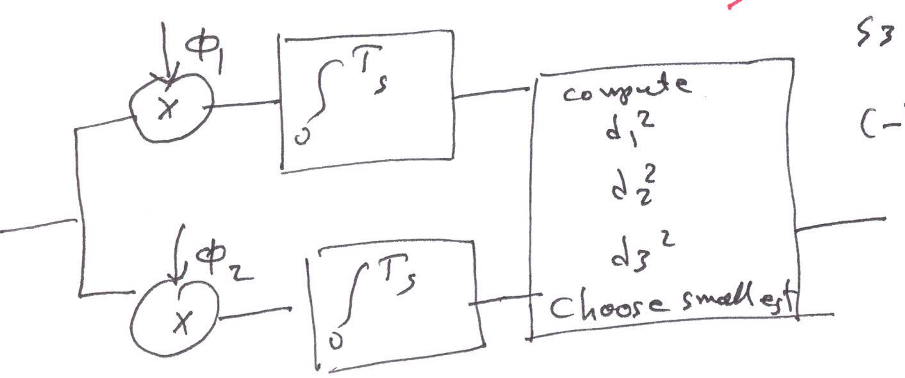
a. Find the average energy per symbol.
b. Draw the block diagram of the optimum receiver.
c. If the received correlator outputs are (0.5, -2), which signal would the demodulator decides in favor of?.



a. $E_{av} = \dfrac{(2)^2 + (2)^2 + 2}{3} = \dfrac{10}{3}$ 3

b.

3



(0.5, -2)

4

c.

(2, 0)
$d_1^2 = (2-0.5)^2 + 2^2 = 1.5^2 + 2^2 = 6.25$

(0, 2)
$d_2^2 = (0.5)^2 + (4)^2 = 16.25$

(-1, -1)
$d_3^2 = (1.5)^2 + (1)^2 = 3.25$ ✓

Select s_3

Birzeit University
Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Modern Communication Systems ENEE 3306
Quiz # 1

Instructors: Dr. Wael Hashlamoun, Date: April 14, 2022

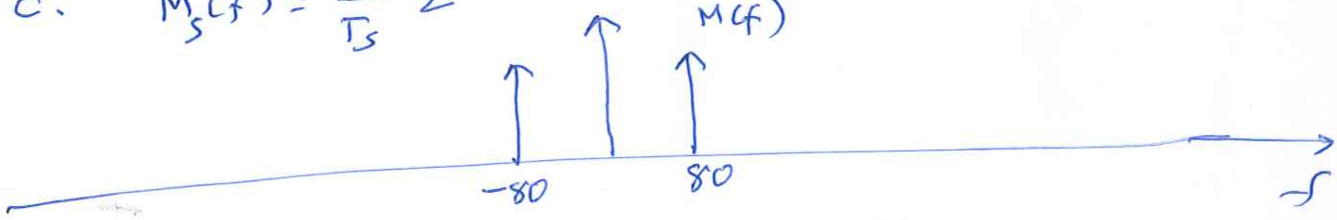
Problem
The message m(t) = 5 cos(2π80)t is ideally sampled at a rate of 140 samples/sec. The sampled signal is applied to an ideal low pass filter with bandwidth 90 Hz.

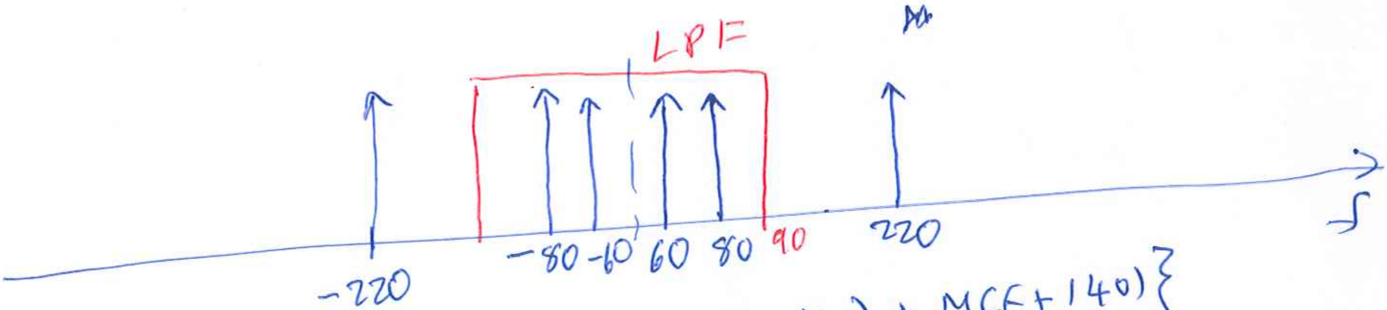
a. Find the average power in m(t)
b. Find the Nyquist rate for m(t)
c. Find the signal at the low-pass filter output.

2.5 a. $\langle m(t)^2 \rangle = \frac{5^2}{2} = 12.5$

2.5 b. $f_N = 2W = 2 * 80 = 160$ HZ

5 c. $M_s(f) = \frac{1}{T_s} \sum M(f - kf_s)$





$\frac{1}{T_s} \{ M(f) + M(f-140) + M(f+140) \}$

$y(t) = \frac{1}{T_s} \{ 5 \cos 2\pi(80)t + 5 \cos 2\pi(60)t \}$

Birzeit University
Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Modern Communication Systems ENEE 3306
Quiz # 2

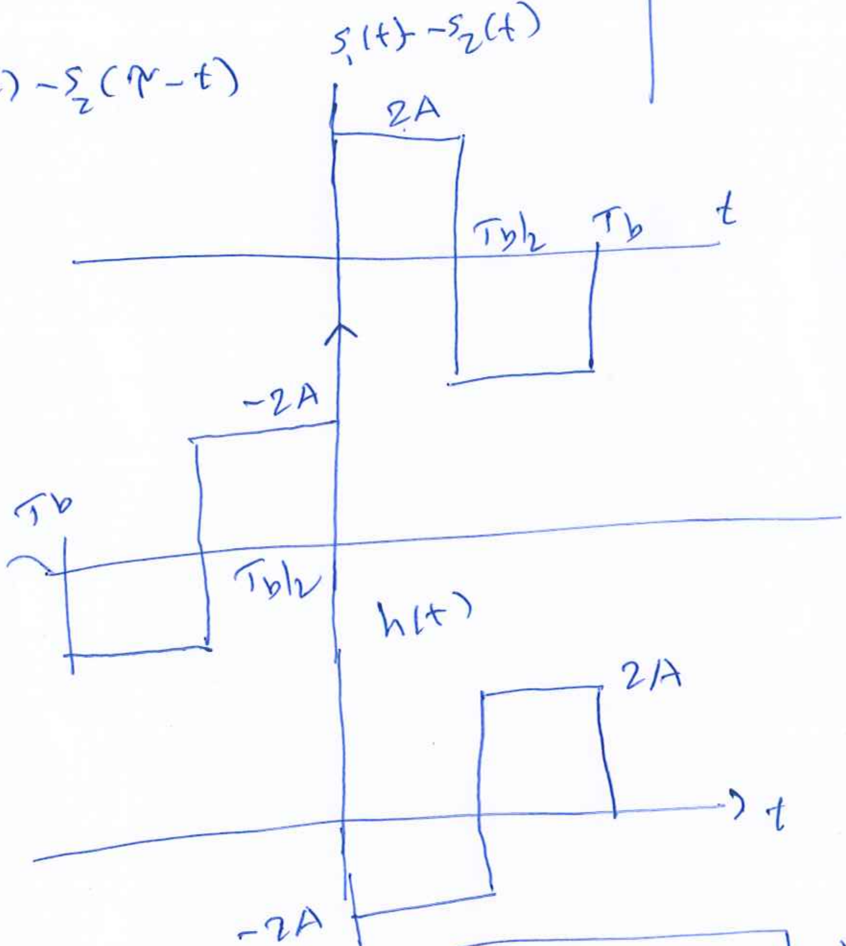
Instructors: Dr. Wael Hashlamoun, Date: May 26, 2022

Problem 1:
A digital communication signaling scheme employs the two signals $s_1(t)$ and $-s_1(t)$ to transmit binary digits 1 and 0, respectively, over a channel corrupted by AWGN with zero mean and power spectral density $N_0/2$. Let P(1) = P(0) = 1/2 and let $s_1(t)$ be defined as:

$$s_1(t) = \begin{cases} A & 0 \le t \le T_b/2 \\ -A & T_b/2 \le t \le T_b \end{cases}$$

a. Sketch h((t), the impulse response of the matched filter.
b. Find the average probability of error of the optimum receiver.

$$h(t) = s_1(\tau - t) - s_2(\tau - t)$$



$$P(E) = Q\left(\sqrt{\frac{\int_0^{T_b} (s_1 - s_2)^2 dt}{2N_0}}\right) = Q\left(\sqrt{\frac{4A^2 T_b}{2N_0}}\right)$$

$$= Q\left(\sqrt{\frac{2A^2 T_b}{N_0}}\right)$$

Birzeit University
Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Modern Communication Systems ENEE 3306
Quiz # 3

Instructors: Dr. Wael Hashlamoun, Date: June 2, 2022

Problem

Consider a binary FSK system that uses the signals $s_1(t) = Acos(2\pi f_1 t)$ and $s_2(t) = Bcos(2\pi f_2 t)$, $0 \le t \le \tau$. The data rate is 1000 bits/sec, $f_1 = 5000\ Hz$, $f_2 = 10000\ Hz$,

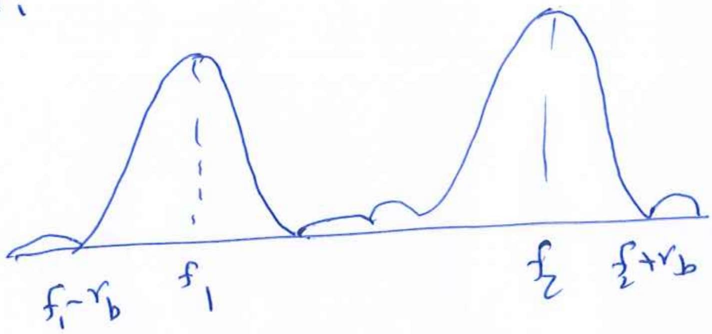
a. Are the signals $s_1(t)$ and $s_2(t)$ orthogonal?
b. Find the bandwidth of the binary FSK signal.

4

a. $\int_0^{T_b} A \cos 2\pi f_1 t \cdot B \cos 2\pi f_2 t \, dt = 0$

orthogonal.

b.



6

$B.W = (f_2 - f_1) + 2 r_b$

$= (10,000 - 5000) + 2 \times 1000$

$= 5000 + 2000 = 7000\ Hz$

Birzeit University
Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Modern Communication Systems ENEE 3306
Quiz # 4

Instructors: Dr. Wael Hashlamoun, Date: June 14, 2022

Problem

Consider a digital communication system that transmits one of two signals $s_1(t)$ and $s_2(t)$ every T_b seconds over a channel corrupted by AWGN with zero mean and power spectral density $N_0/2$. The signals occur with equal probabilities. Assume $T_b = nT_c$; n an integer. The space bases functions are:

$$\emptyset_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t)\,, \qquad \emptyset_2(t) = \sqrt{\frac{2}{T_b}}\sin(2\pi f_c t)$$

The signals have the coordinates: $s_1 = (2, 0)$; $s_3 = (0, 3)$;

5 a. Find the energy in $s_1(t)$ and in $s_2(t)$
5 b. Find the average probability of error.

$$E_1 = \int_0^{T_b} (2\,\phi_1(t))^2 dt$$

$$= 4\int_0^{T_b} \phi_1(t)^2 dt = 4$$

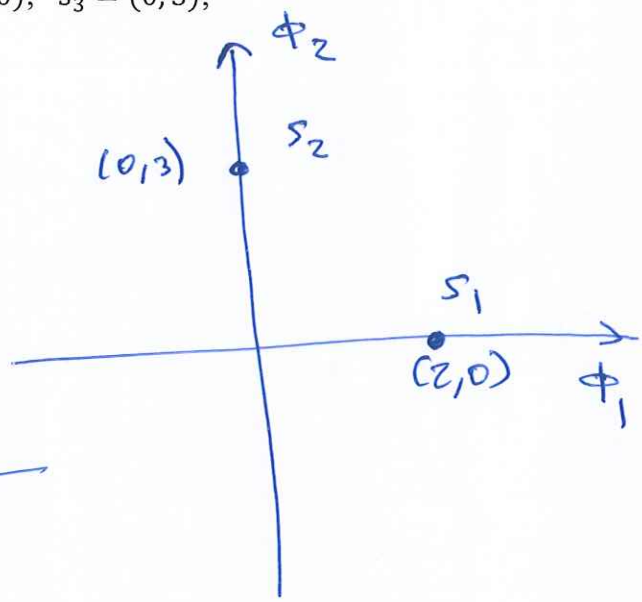
$$E_2 = \int_0^{T_b} (3\,\phi_2(t))^2 dt = 9$$

$$d_{12} = \sqrt{4+9} = \sqrt{13}$$

$$P(E) = Q\left(\frac{d_{12}}{\sqrt{2N_0}}\right)$$

$$= Q\left(\frac{\sqrt{13}}{2N_0}\right)$$

$$P(E) = Q\left(\sqrt{\frac{13}{2N_0}}\right)$$



جامعة بيرزيت
BIRZEIT UNIVERSITY

Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Modern Communication Systems ENEE 3306

Instructor: Dr. Wael Hashlamoun

Midterm Exam
Second Semester 2017-2018

Date: Sunday 15/4/2018 | Time: 75 minutes
Name: | Student #:

Opening Remarks:

- Calculators are allowed, however, mobile phones, books, notes, formula sheets, and other aids are not allowed.
- You are required to show all your work and provide the necessary explanations everywhere to get full credit.

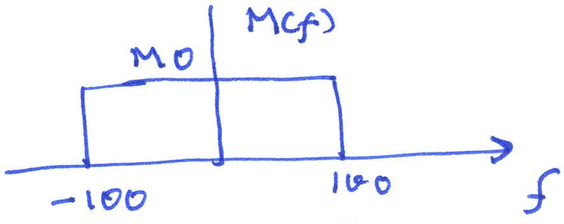
Problem 1: 25 Points
The signal $m(t)$, with spectrum $M(f)$, given below, is ideally sampled at a rate of f_s samples/sec to generate the signal $m_s(t)$

$$M(f) = \begin{cases} M_0, & -100 \le f \le 100 \\ 0, & otherwise \end{cases}$$

10 a. Sketch $M_s(f)$, the spectrum of $m_s(t)$, when $f_s = 180$ samples/sec.
b. If $m_s(t)$ in Part a is passed through an ideal low pass filter with a bandwidth of 100 Hz to produce an output $y_1(t)$.
- Sketch $Y_1(f)$, the spectrum of the filter output.
- Is $y_1(t)$ *proportional to* $m(t)$? What does that mean in terms of reconstructing $m(t)$.

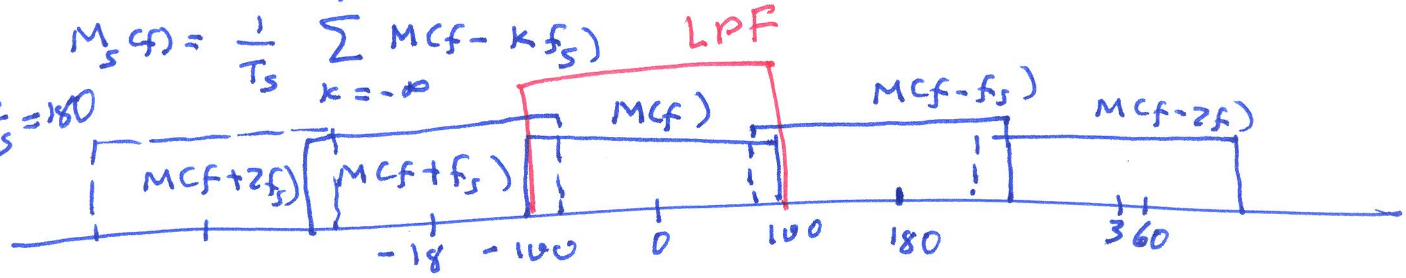
c. Sketch $M_s(f)$, the spectrum of $m_s(t)$, when $f_s = 240$ samples/sec.
10 d. If $m_s(t)$ in Part c is passed through an ideal low pass filter of bandwidth 110 Hz to produce an output $y_2(t)$.
- Sketch $Y_2(f)$, the spectrum of the filter output.
- Is $y_2(t)$ *proportional to* $m(t)$? What does that mean in terms of reconstructing $m(t)$.

5 e. Which one of the above two sampling frequencies would you recommend and why?

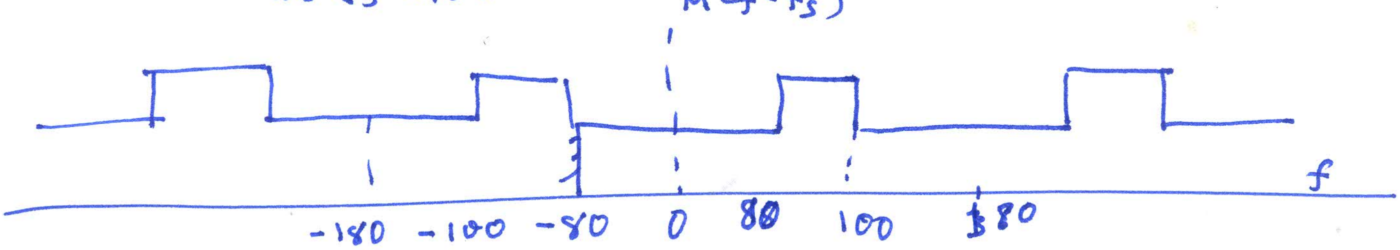


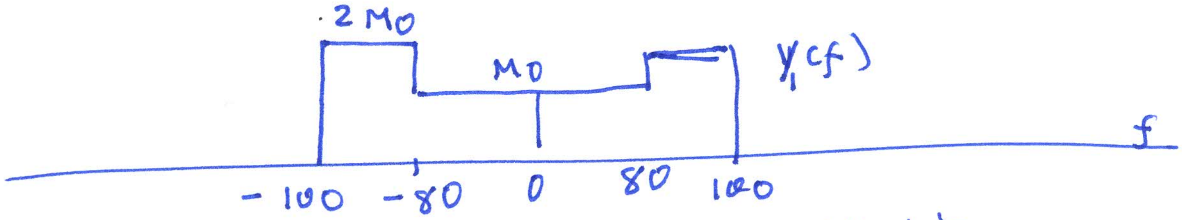
$$M_s(f) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} M(f - k f_s)$$

$f_s = 180$



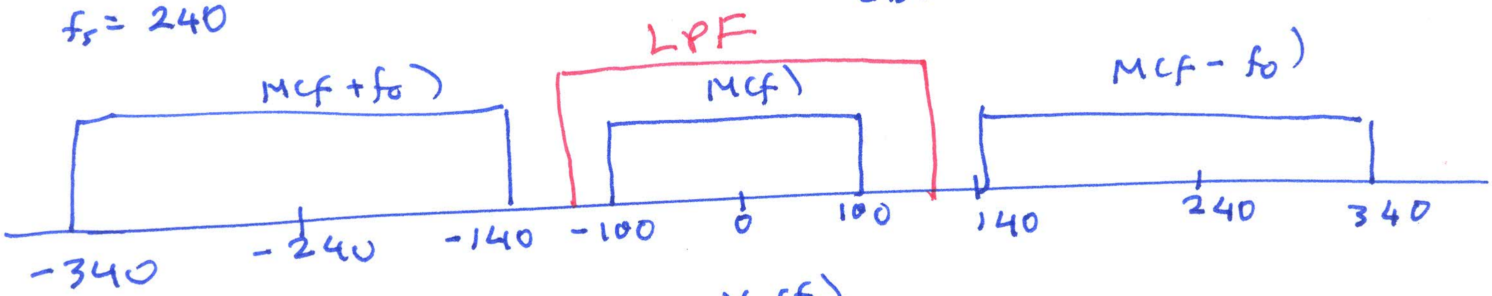
For $-100 < f < 100$

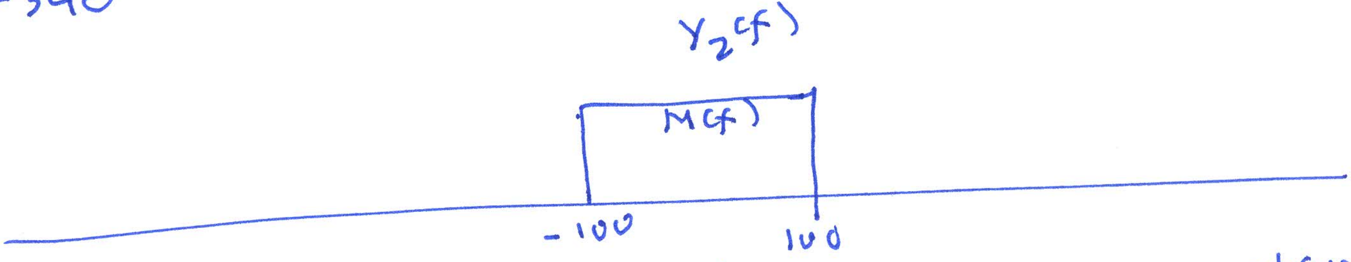




$Y_1(f) \neq K_1 M(f)$ ⇒ Aliasing ⇒ distortion
⇒ $m(t)$ cannot be reconstructed without distortion

$f_s = 240$





$Y_2(f) = K_2 M(f)$ ⇒ $m(t)$ can be reconstructed from $m_s(t)$ without distortion

⇒ second case: $f_s \geq 2W$

$240 = f_s \geq 2(100) = 200$

⇒ select second sampling rate since it is higher than the Nyquist rate
⇒ No distortion.

Problem 2: 25 Points

The signal $m(t) = \cos(2\pi(150)t)$ is to be transmitted using a PCM system (a system composed of a sampler, quantizer, and a binary encoder).

a. If sampling is done at the Nyquist rate and a uniform quantizer with 32 levels and a dynamic range between (-1, 1) is employed,

- What is the resulting data rate in bits/sec
- What is the resulting SQNR?
- If the encoder output is modulated using binary phase shift keying, find the 90% modulated signal bandwidth.
- If the encoder output is converted into polar non-return to zero format, find the 90% modulated signal bandwidth.

b. Find the SQNR if the signal is sampled at 1.2 times the Nyquist rate.

c. Find the data rate in bits per second if a nonuniform quantizer with 32 levels is used.

a. 6 $R_b = f_s \log_2 M = 2(150) \times 5 = 1500$ bits/sec

7 $SQNR = \frac{\langle m(t)^2 \rangle}{\Delta^2/12} = \frac{A_m^2/2}{[2/32]^2/12} = \frac{0.5}{3.255 \times 10^{-4}} = 1536$

$A_m = 1$

$\Delta = \frac{m_{max} - (m)_{min}}{M} = \frac{2}{32} = 0.0625$

3 BPSK $B.W = 2R_b = 3{,}000$ Hz

3 PNRZ $B.W = R_b = 1500$ Hz

3 b. $SQNR = \frac{A_m^2/2}{\Delta^2/12} = 1536$ (same as above)

3 c. $R_b = f_s \log_2 M = 1500$ bits/sec

Problem 3: 25 Points

Consider the signal $x(t) = 4\sin(2\pi t)$.

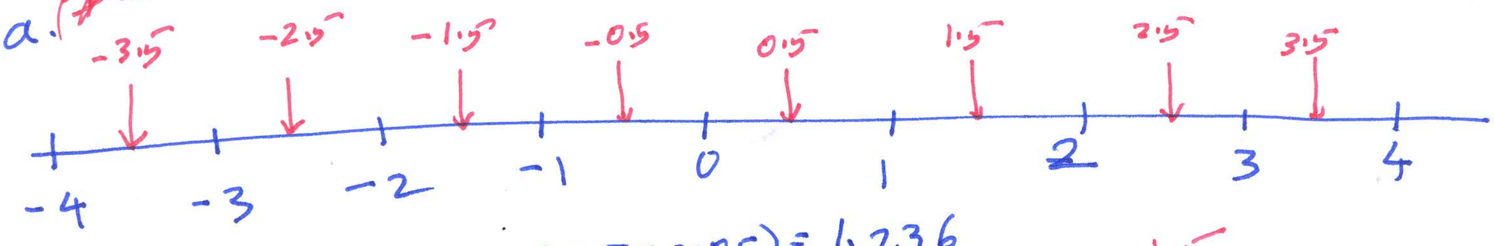
8 a. Design an 8-level uniform quantizer with a dynamic range (-4, 4) V, i.e., find the thresholds and representation values.

7.5 b. One sample is taken from the signal $x(t)$ at time t=0.05 and applied to the uniform quantizer of Part a. Find the received signal value corresponding to this sample.

7.5 c. One sample is taken from the signal $x(t)$ at time t=0.05 and applied to a μ-law companding system with $\mu = 255$ (a compressor followed by the uniform quantizer of Part a and an expander). Find the received signal value corresponding to this sample.

2 d. Compare the amount of distortion incurred on the sample by the two quantizers.

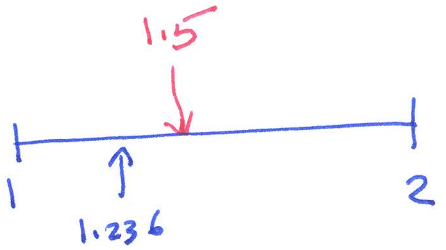
a.



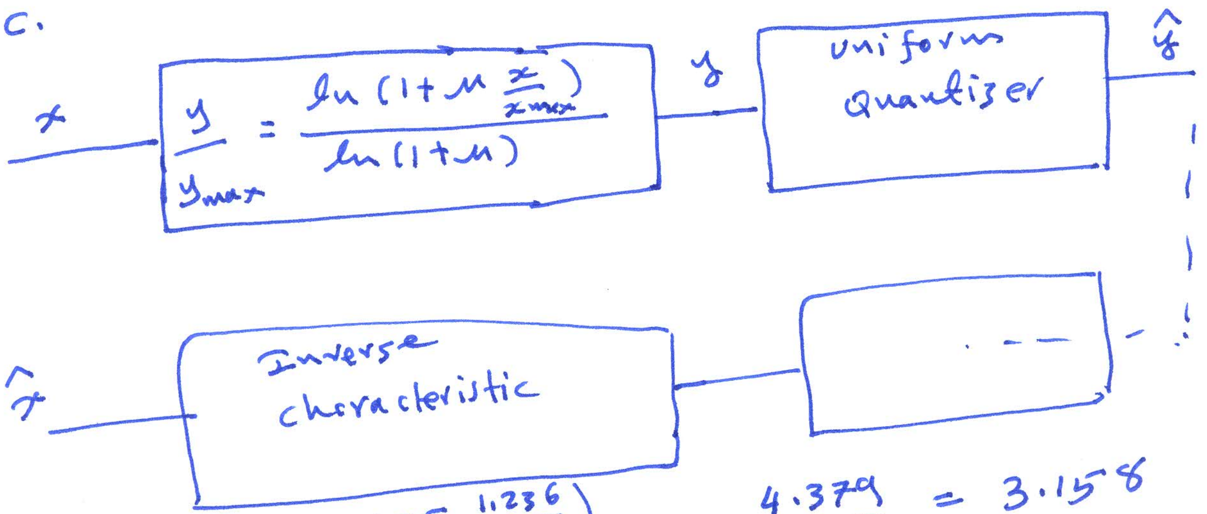
b. $x(0.05) = 4\sin(2\pi * 0.05) = 1.236$

$\boxed{\hat{x} = 1.5} \Rightarrow e = |x - \hat{x}| = |1.236 - 1.5|$

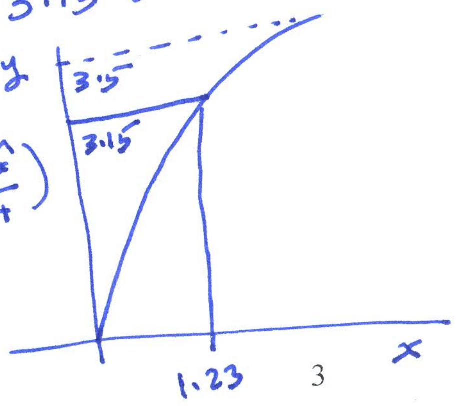
$= 0.264$



c.



$$y = 4\,\frac{\ln\left(1 + 255\,\frac{1.236}{4}\right)}{\ln(1+255)} = 4 * \frac{4.379}{5.545} = 3.158$$



$\Rightarrow \boxed{\hat{y} = 3.5}$

$$\frac{3.5}{4} = \frac{\ln\left(1 + 255\,\frac{\hat{x}}{4}\right)}{\ln(1+255)} \Rightarrow 4.85 = \ln\left(1 + 255\,\frac{\hat{x}}{4}\right)$$

$\Rightarrow \boxed{\hat{x} = 1.98}$

$|e| = |x - \hat{x}| = |1.236 - 1.98| = \boxed{0.744}$

Problem 4: 25 Points

A binary digital signaling scheme employs the following two equally probable signals $s_1(t)$ and $s_2(t)$ to represent binary logic 1 and 0, respectively, over a channel corrupted by AWGN with power spectral density $N_0/2$ W/Hz:

$$s_1(t) = A\cos\left(\frac{\pi t}{\tau}\right), \quad 0 \le t \le \tau$$

$$s_2(t) = B\cos\left(\frac{\pi t}{\tau}\right), \quad 0 \le t \le \tau$$

a. Find the energy, E_1, in $s_1(t)$ and the energy, E_2, in $s_2(t)$.
b. Find the average probability of error of the optimum receiver.
c. What is the probability of error when $A = B$?
d. Find the relationship between A and B such that the probability of error is minimized.
e. Draw the optimum receiver, implemented in terms of a correlator, for the general case when $A \neq B$, indicating the parameters of the main receiver units.

Good Luck

a. $E_1 = \int_0^\tau s_1(t)^2 dt = \int_0^\tau A^2 \cos^2\left(\frac{\pi t}{\tau}\right) dt = \frac{A^2}{2}\int_0^\tau \left[1 + \cos\left(\frac{2\pi t}{\tau}\right)\right] dt = \frac{A^2 \tau}{2}$

6

$E_2 = \int_0^\tau s_2(t)^2 dt = \frac{B^2 \tau}{2}$

b. $\int_0^\tau (s_1(t) - s_2(t))^2 dt = \int_0^\tau (A-B)^2 \cos^2 \frac{\pi t}{\tau} dt = \frac{(A-B)^2 \tau}{2}$

6

$P_b = Q\left(\sqrt{\frac{\int_0^\tau (s_1 - s_2)^2 dt}{2N_0}}\right) = Q\left(\sqrt{\frac{(A-B)^2 \tau}{4 N_0}}\right)$

4 c. $P_b = Q(0) = 0.5$

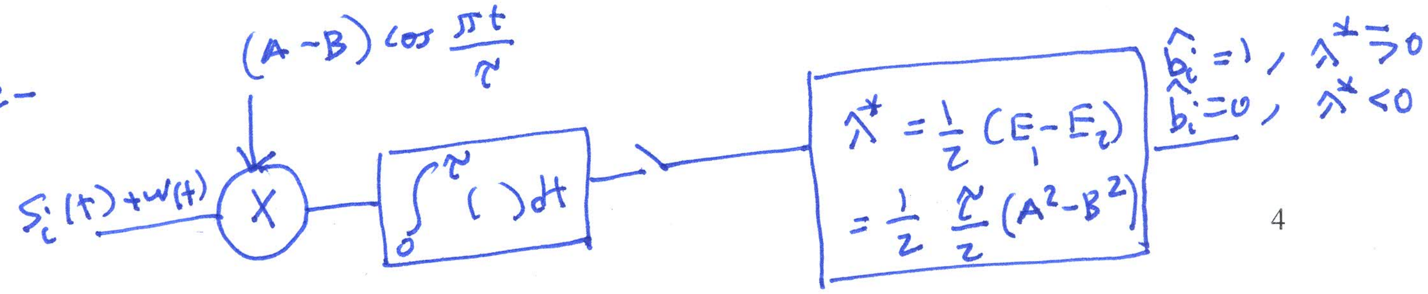
4 d. maximize: $(A-B)^2$

Let $g = (A-B)^2 \Rightarrow$

g is max when $\boxed{B = -A}$

e-

5



Problem 2: 20 Points

Consider a digital communication system that transmits one of four signals every T_s seconds over a channel corrupted by AWGN with zero mean and power spectral density $N_0/2$. The signals occur with equal probabilities. Assume $T_s = nT_c$; n an integer. The space bases functions are:

$$\emptyset_1(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi f_c t) \qquad \emptyset_2(t) = \sqrt{\frac{2}{T_s}}\sin(2\pi f_c t)$$

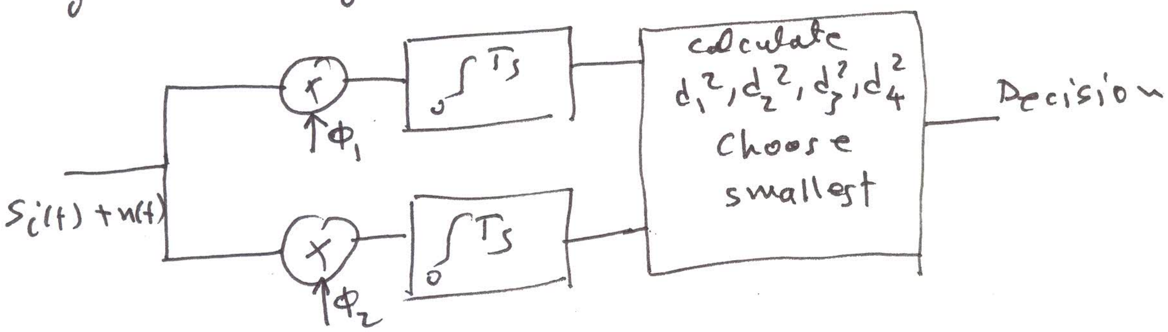
The transmitted signals are given by:

$$s_1(t) = 2\emptyset_1(t) + 2\emptyset_2(t) \qquad s_2(t) = -2\emptyset_1(t) + 2\emptyset_2(t)$$
$$s_3(t) = -2\emptyset_1(t) - 2\emptyset_2(t) \qquad s_4(t) = 2\emptyset_1(t) - 2\emptyset_2(t)$$

4 a. Find the energy in $s_4(t)$.
6 b. Draw the block diagram of the optimum receiver, showing the details of each block
4 c. Find the distance d_{13}^2 between $s_1(t)$ and $s_3(t)$
6 d. Find the average probability of error

a. $E_4 = \int_0^{T_s} s_4(t)^2 dt = \int_0^{T_s} [2\phi_1 + 2\phi_2]^2 dt = 8$

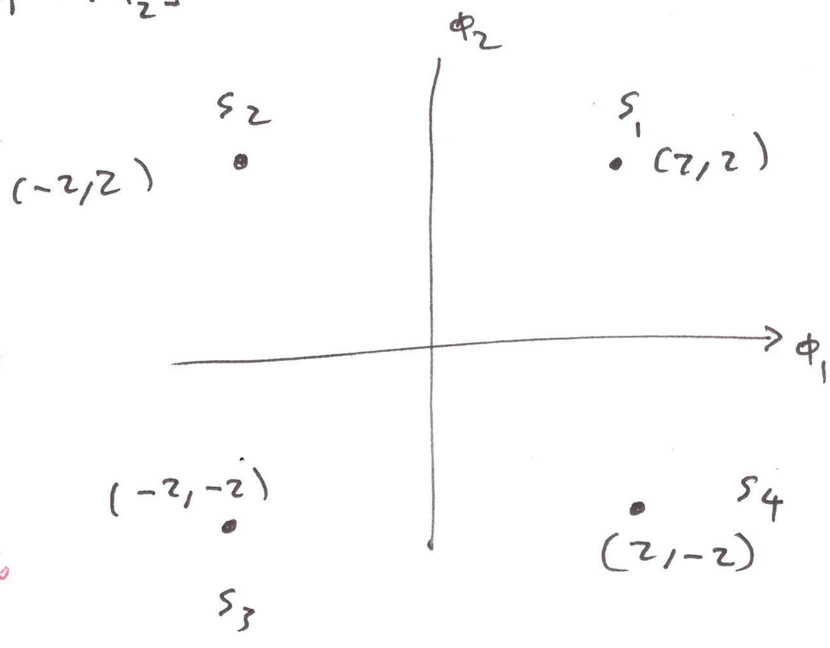
b.



c. $d_{13}^2 = \int_0^{T_s} (s_1 - s_3)^2 dt = \int_0^{T_s} [4\phi_1 + 4\phi_2]^2 dt$

$d_{13}^2 = 16 + 16 = 32$

$d_{13} = \sqrt{32} = 4\sqrt{2}$



d. $P_s = 2Q\left(\frac{d_{12}}{\sqrt{2N_0}}\right)$ 3/6

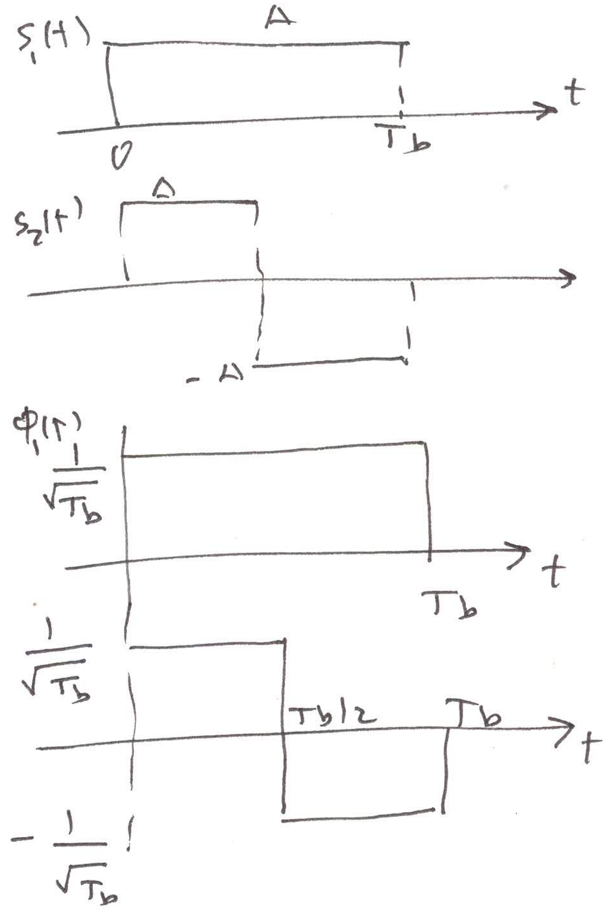
$P_s = 2Q\left(\frac{4}{\sqrt{2N_0}}\right)$ 3/6

$= 2Q\left(\sqrt{\frac{8}{N_0}}\right)$

Problem 3:
Consider a binary digital communication system that transmits one of two possible symbols every T_b seconds over a channel corrupted by AWGN with zero mean and power spectral density $N_0/2$. The signals occur with equal probabilities. The transmitted signals are:

$$s_1(t) = \begin{cases} A, & 0 \le t \le T_b \\ 0, & otherwise \end{cases} \qquad s_2(t) = \begin{cases} A, & 0 \le t \le T_b/2 \\ -A, & T_b/2 \le t \le T_b \end{cases}$$

5 a. Are $s_1(t)$ and $s_2(t)$ orthogonal? Prove your answer
5 b. Make use of the result of Part a to find the set of bases functions for the signal space.
5 c. Find and sketch the signal space representation of the signals.
5 d. Find the system average probability of error



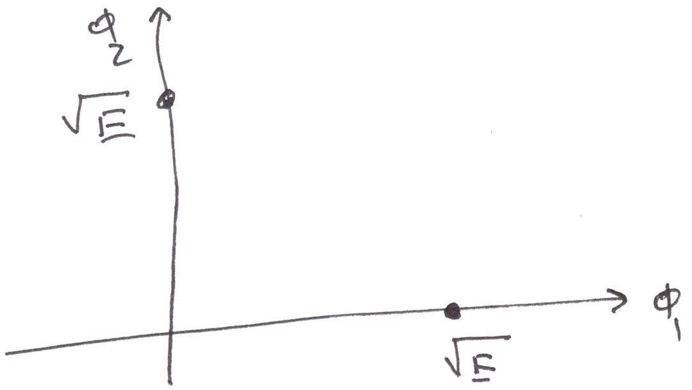
a. $\int_0^{T_b} s_1(t)\, s_2(t)\, dt = 0$

$\Rightarrow$

b. $\phi_1(t) = \dfrac{s_1(t)}{\sqrt{E_1}} = \dfrac{s_1(t)}{\sqrt{A^2 T_b}} = \dfrac{A}{\sqrt{A^2 T_b}}$

$\phi_2(t) = \dfrac{s_2(t)}{\sqrt{E_2}}$

c.



$E_1 = A^2 T_b, \quad E_2 = A^2 T_b$

d. $P_b = Q\left(\dfrac{d_{12}}{\sqrt{2N_0}}\right) = Q\left(\dfrac{\sqrt{2E}}{\sqrt{2N_0}}\right) = Q\left(\sqrt{\dfrac{E}{N_0}}\right)$

Problem 4: 20 Points

In a 4-station CDMA system, the binary chip sequences (signature waveforms) assigned for users A, B, C, and D are

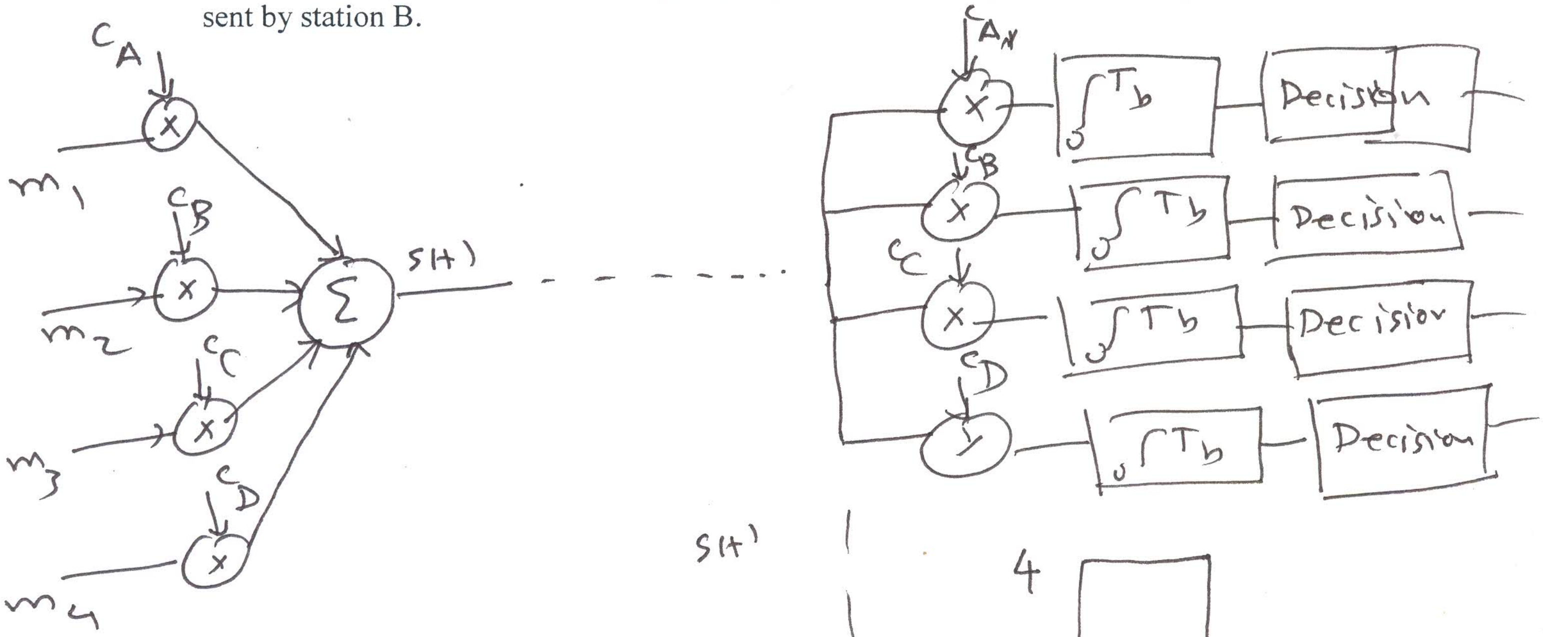
A = {+1 +1 +1 +1} B = {+1 -1 +1 -1} C = {-1 -1 +1 +1} D = {-1 +1 +1 -1}.

The chip duration = T_c and the bit duration = T_b

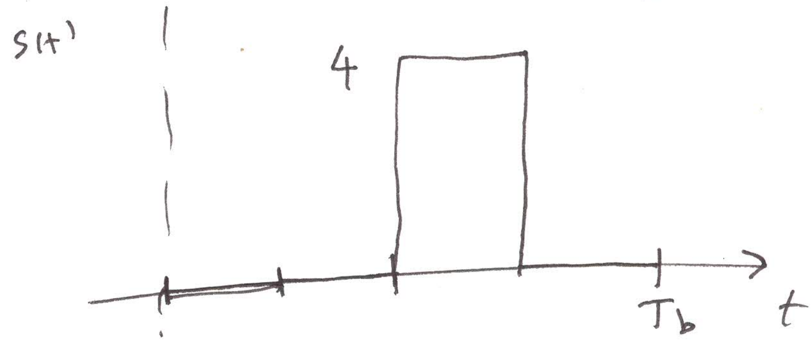
8 a. Draw the block diagram of the transmitter and the receiver, showing the details of each block.

6 b. Find and sketch the transmitted signal for $0 \le t \le T_b$ when each one of the four stations transmits digit 1.

6 c. If the receiver observes the following chip signal [+2 -2 +2 +2] for $0 \le t \le T_b$, find the bit sent by station B.



b.



c.

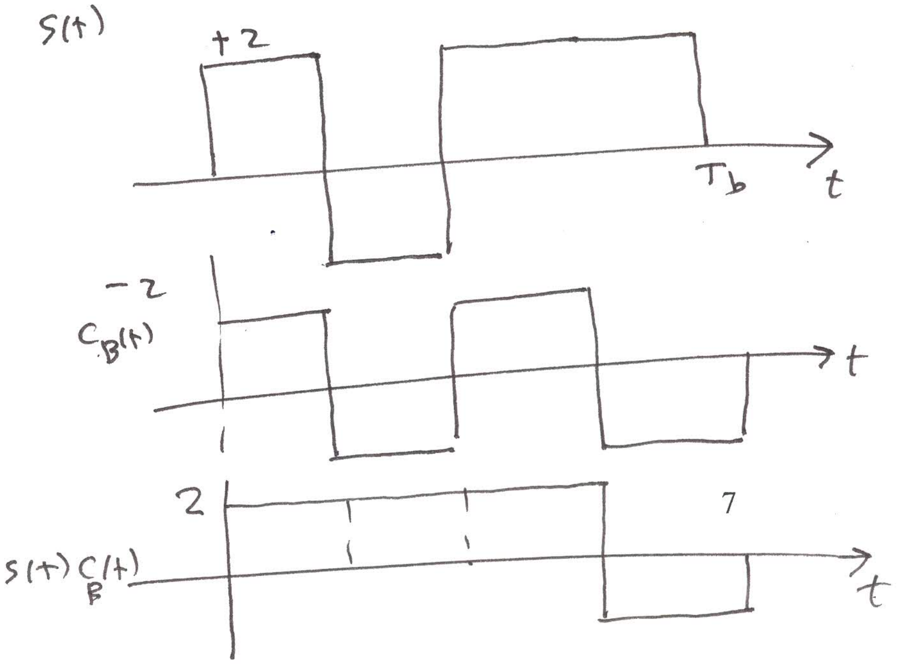
$$\int_0^{T_b} s(t)\, C_B(t)\, dt \Rightarrow$$

$$= 2\left(\frac{3T_b}{4}\right) - 2\left(\frac{T_b}{4}\right)$$

$$= \frac{6T_b - 2T_b}{4} = \frac{4T_b}{4}$$

$$= T_b > 0$$

$\Rightarrow$ digit 1



BIRZEIT UNIVERSITY
Faculty of Engineering and Technology
Department of Electrical and Computer Engineering

Modern Communication Systems ENEE 3306

Instructor: Dr. Wael Hashlamoun

Midterm Exam

Second Semester 2018-2019

Date: Sunday April 7, 2019 Time: 75 minutes

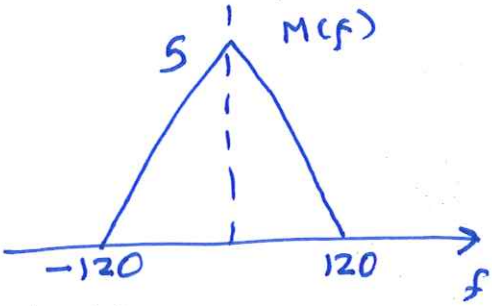
Name: Student #:

Opening Remarks:

- Calculators are allowed, however, mobile phones, books, notes, formula sheets, and other aids are not allowed.
- You are required to show all your work and provide the necessary explanations everywhere to get full credit.

Problem 1: 25 Points

Consider the signal $m(t)$, with spectrum $M(f)$, given by,



$$M(f) = \begin{cases} 5 - f/120 \ , & 0 < f \leq 120 \\ 5 + f/120, & -120 \leq f \leq 0 \\ 0 & , \ otherwise \end{cases}$$

This signal is multiplied by $c(t)$ to get signal $m_s(t)$, where

$$c(t) = A_0 + A_1 cos(2\pi(240)t) + A_2 cos(2\pi(480)t)$$

a. What is the absolute bandwidth of $m(t)$?

b. Sketch $M_s(f)$, the spectrum of $m_s(t)$.

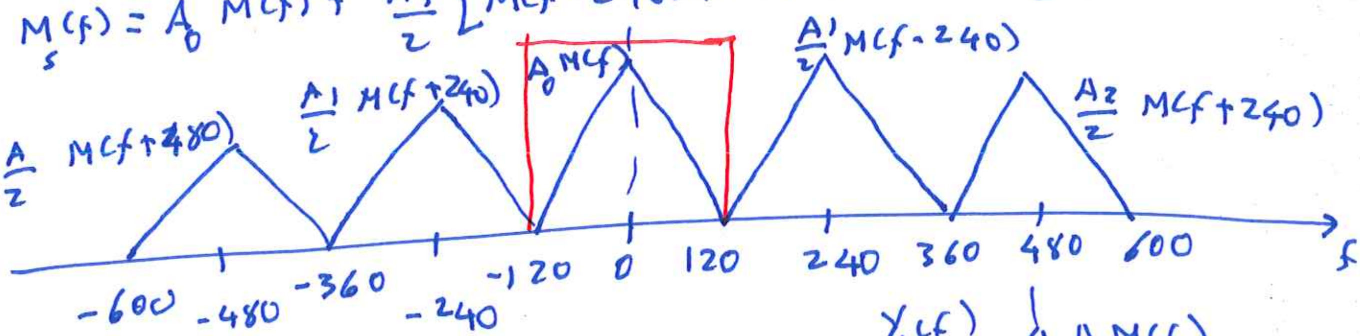
c. If $m_s(t)$ is passed through an ideal low pass filter with a bandwidth of 120 Hz to produce an output $y_1(t)$. Sketch $Y_1(f)$.

d. Is $y_1(t)$ *proportional to* $m(t)$? Explain why.

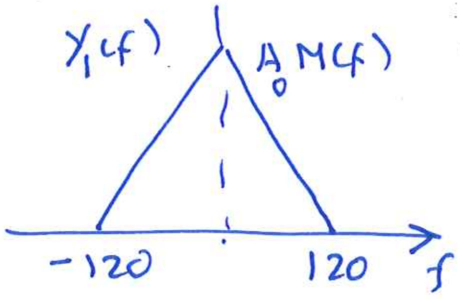
a. $B.W = (120 - 0) = 120$ Hz

b. $m_s(t) = m(t)\, c(t) = m(t)[A_0 + A_1 \cos 2\pi(240)t + A_2 \cos 2\pi(480)$

$M_s(f) = A_0 M(f) + \frac{A_1}{2}[M(f-240) + M(f+240)] + \frac{A_2}{2}[M(f-480) + M(f+480)]$



$Y_1(f) = M_s(f) H(f) = A_0 M(f)$



$Y_1(f) \propto M(f)$

$\Rightarrow y_1(t) \propto m(t)$

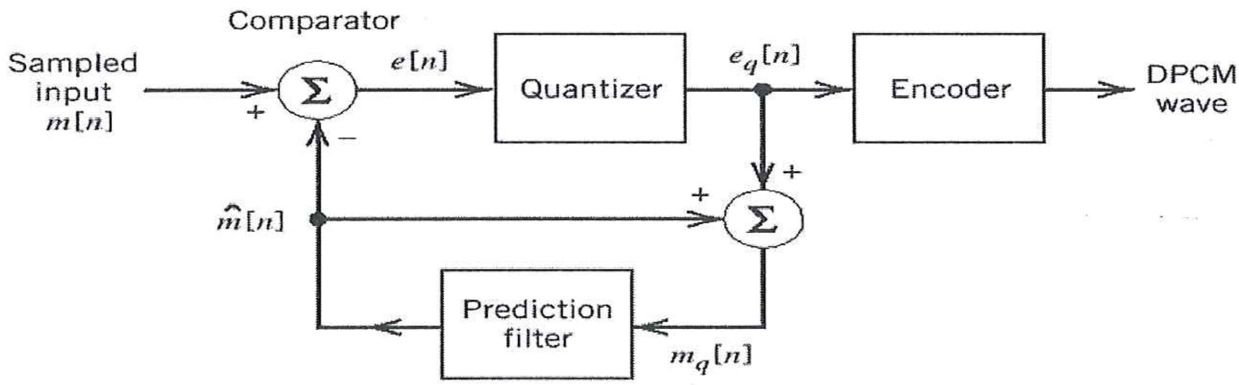
$\Rightarrow$ No distortion

Problem 2: 25 Points

Consider a differential pulse code modulator similar to the one shown in the figure below. A signal $m(t)$ with bandwidth 400 Hz is sampled at its Nyquist rate. The error $e(t)$ is applied to a 16-level uniform quantizer. The prediction is made based only on the previous sample, i.e., $\hat{m}(nT_s) = w_1 m((n-1)T_s)$. The autocorrelation function of $m(t)$ is given by

$$R_m(\tau) = \begin{cases} 3 + \left(\dfrac{\tau}{3T_s}\right), -3T_s \le \tau \le 0 \\ 3 - \left(\dfrac{\tau}{3T_s}\right), 0 \le \tau \le 3T_s \\ 0, \quad otherwise \end{cases}$$

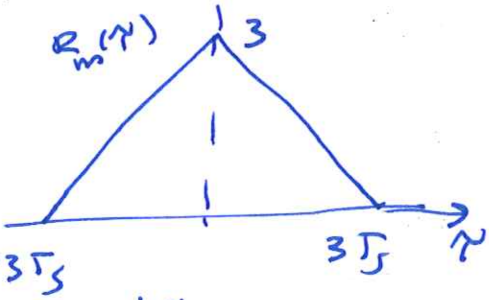
a. Find w_1 that minimizes the mean square error between the sample and its predicted value.
b. Find the sampling frequency.
c. Find the data rate in bits/sec.



$\hat{x}(n) = w_1 \; x(n-1) \quad \Rightarrow \epsilon = E\{[x(n) - \hat{x}(n)]^2\}$

11 a. $R\,W = R_1$ or $R_x(0)\, w_1 = R_x(T_s)$

$\Rightarrow w_1 = \dfrac{R_x(T_s)}{R_x(0)}$



$w_1 = \dfrac{3 - \left(\frac{T_s}{3T_s}\right)}{3}$

$= \dfrac{3 - 1/3}{3} = \dfrac{8/3}{3} = \dfrac{8}{9} = 8/9$

7 b. $f_s = 2W = 2(400) = 800$ samples/sec

7 c. $L = 16 \Rightarrow L = 16 = 2^r = 4$ digits $= r$

$R_b = (2W)$ samples/sec $\times$ 4 digits/sample

$= 8W$ bits/sec

$= 3200$ bits/sec

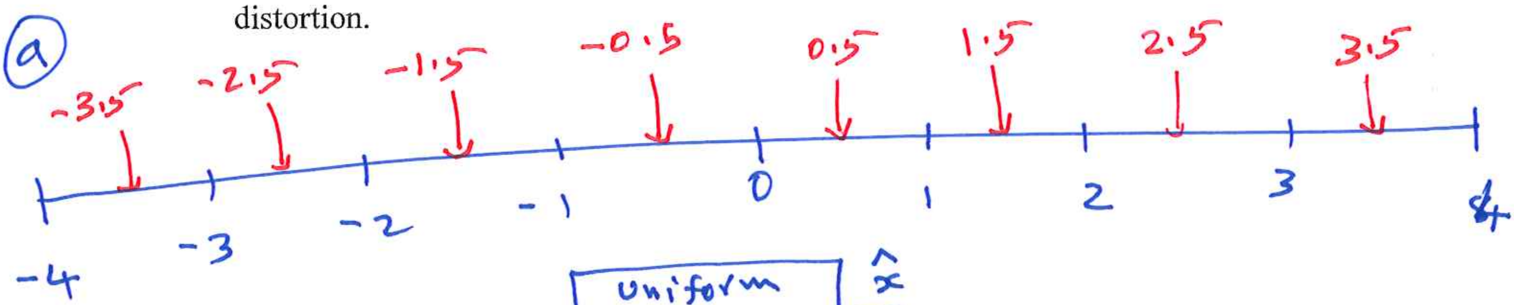
Problem 3: 25 Points

8 a. Design an 8-level uniform quantizer with a dynamic range (-4, 4) V, i.e., find the thresholds and representation values.

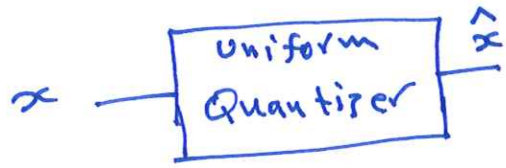
6 b. If a sample with a 0.15 V value is applied to the uniform quantizer of Part a, find the received signal value corresponding to this sample, and the amount of distortion affecting this sample.

11 c. If a sample with a 0.15 V value is applied to a μ-law companding system with μ = 255 (a compressor followed by the uniform quantizer of Part a and then an expander), find the received signal value corresponding to this sample, and the amount of distortion.

(a)



b.

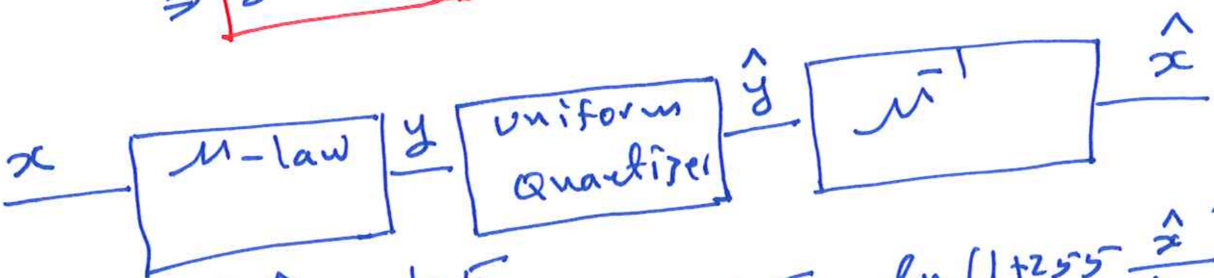


$x = 0.15 \Rightarrow \hat{x} = 0.5 \Rightarrow \epsilon = |x - \hat{x}| = 0.35$

c. $\dfrac{y}{y_{max}} = \dfrac{\ln\left(1+255\dfrac{x}{x_{max}}\right)}{\ln(1+255)}$; Let $x = 0.15$

$\dfrac{y}{4} = \dfrac{\ln\left(1+255\dfrac{0.15}{4}\right)}{\ln(256)} = \dfrac{2.357}{5.545} = 0.425$

$\Rightarrow \boxed{y = 1.7}$



$\Rightarrow \hat{y} = 1.5$

Need to solve for $\hat{x}$ $\quad \dfrac{1.5}{4} = \dfrac{\ln\left(1+255\dfrac{\hat{x}}{4}\right)}{\ln(256)}$

$0.375 * 5.545 = \ln\left(1+255\dfrac{\hat{x}}{4}\right) \Rightarrow 1 + 63.75\hat{x} = e^{2.079}$

$\hat{x} = \dfrac{e^{2.079} - 1}{63.75} = \dfrac{7.999 - 1}{63.75} = 0.1097$

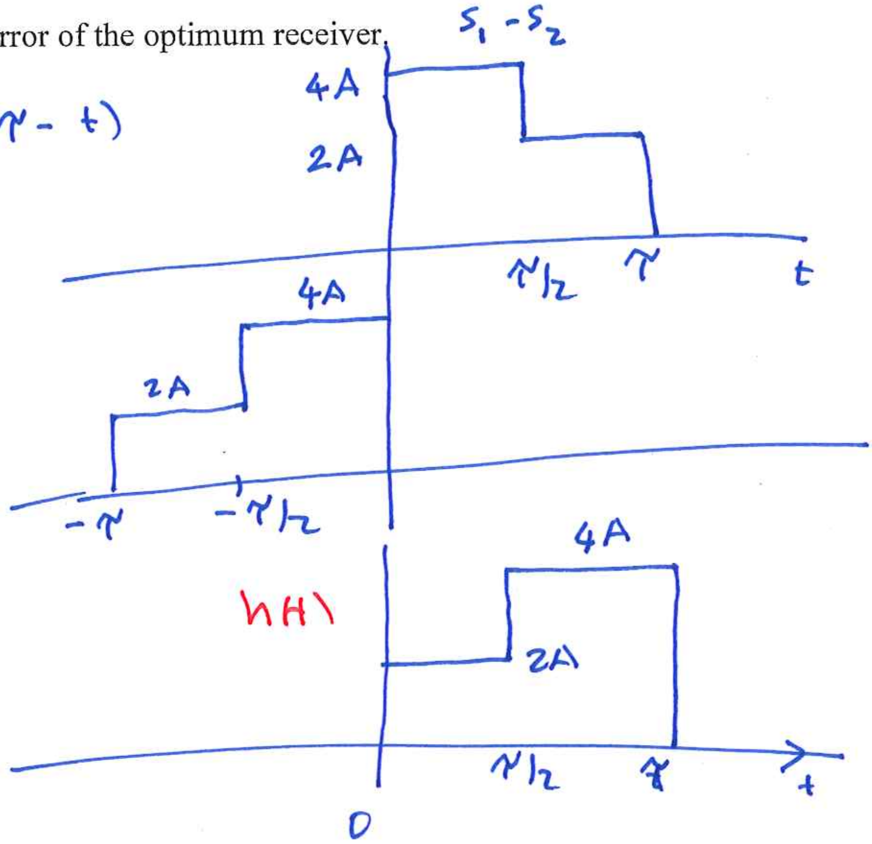
$\Rightarrow \boxed{\text{error} = |0.15 - 0.1097| = 0.04}$

Problem 4: 25 Points

A binary digital signaling scheme employs the signal $s_1(t)$ to represent digit 1 and $s_2(t) = -s_1(t)$ to represent binary digit 0, over a channel corrupted by AWGN with power spectral density $N_0/2$ W/Hz, where

$$s_1(t) = \begin{cases} 2A, & 0 \le t \le \tau/2 \\ A, & \tau/2 \le t \le \tau \\ 0, & otherwise \end{cases}$$

a. Find and sketch the optimum filter
b. Find E_1, the energy in $s_1(t)$.
c. Find the average probability of error of the optimum receiver.



a. $h(t) = s_1(\tau - t) - s_2(\tau - t)$

9

b. $E_1 = \int_0^{\tau} |s_1(t)|^2 dt$

7

$= \int_0^{\tau/2} 4A^2 dt + \int_{\tau/2}^{\tau} A^2 dt$

$= 4A^2 \cdot \frac{\tau}{2} + A^2 \frac{\tau}{2}$

$E_1 = \frac{5A^2\tau}{2}$

c. $P_b = Q\left(\sqrt{\frac{\int_0^{\tau}(s_1 - s_2)^2 dt}{2N_0}}\right)$

9

$\int_0^{\tau}(s_1 - s_2)^2 dt = \int_0^{\tau/2} 16A^2 dt + \int_{\tau/2}^{\tau} 4A^2 dt = 16A^2 \frac{\tau}{2} + 4A^2 \frac{\tau}{2}$

$= 10A^2\tau$

$P_b = Q\left(\sqrt{\frac{10A^2\tau}{2N_0}}\right) = Q\left(\sqrt{\frac{5A^2\tau}{N_0}}\right)$

Problem 2: 20 Points

The Fourier transform, $G(f)$, of a signal $g(t)$ is given as:

$$G(f) = \begin{cases} A, & -200 \le f \le 200 \\ 0, & |f| > 200 \end{cases}$$

The signal $g(t)$ is ideally sampled at a rate of 350 samples/sec to produce the samples signal $g_s(t)$.

a. Find and sketch $G_s(f)$, the Fourier transform of $g_s(t)$ for $-400 \le f \le 400$

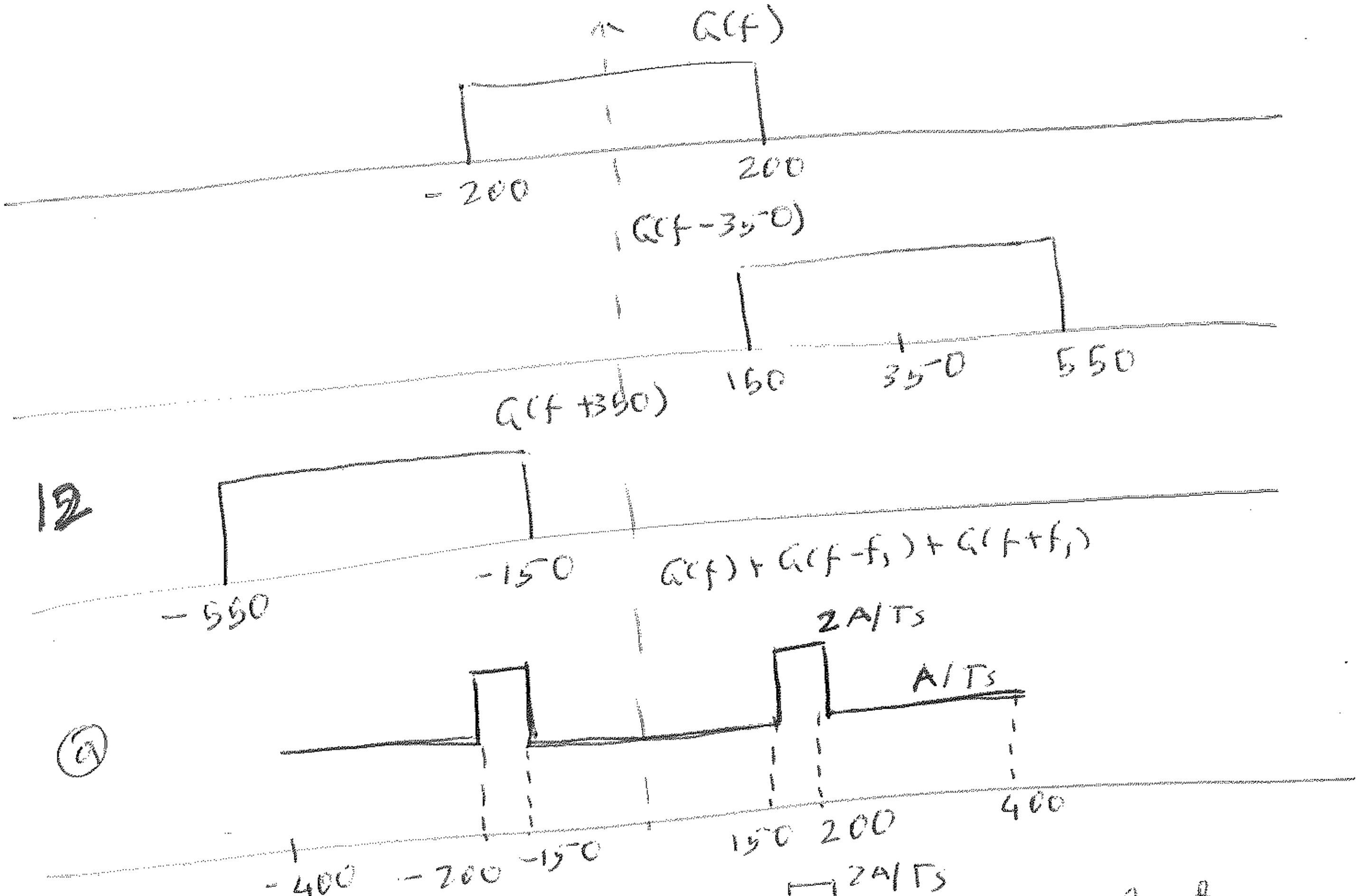
b. If $g_s(t)$ is applied to an ideal low pass filter with a bandwidth of 200 Hz, sketch the Fourier transform of the signal appearing at the output of the filter.

c. Based on the results of Part b, do you think that $g(t)$ can be recovered from $g_s(t)$ without distortion? Explain why.

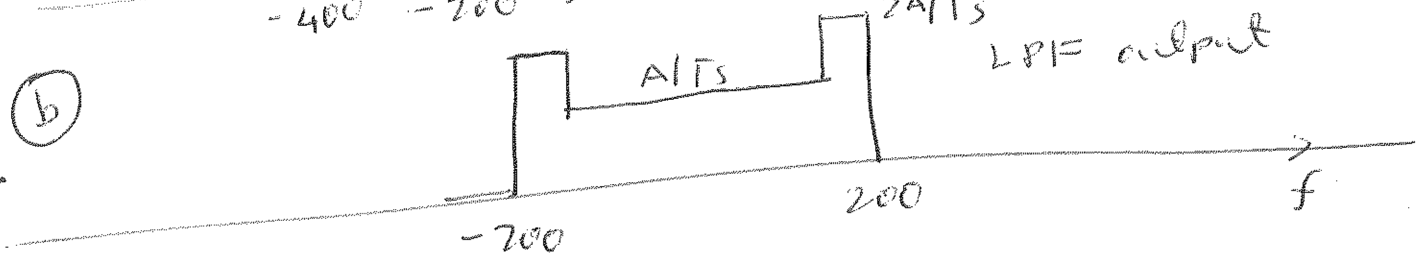
$$G_s(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} G(f - nf_s)$$

12

(a)



4 (b)



4 (c) since $f_s < 2(200) = 400 \Rightarrow$ Distortion

$\Rightarrow g(t)$ cannot be recovered.

Problem 3: 18 Points

The signal $x(t) = 4cos(2\pi f_0 t)$ is applied to a uniform quantizer with L quantization levels and a dynamic range $(-4, 4)$ V. Find the minimum value of L that will achieve a signal to quantization noise ratio $SQNR \geq 1000$.

4 $\Delta = \frac{4-(-4)}{L}$; $= \frac{8}{L}$

4 $\langle x(t)^2 \rangle = \frac{A_m^2}{2} = \frac{(4)^2}{2} = 8$; average signal power

4 quantization noise $= \frac{\Delta^2}{12}$

$SQNR = \frac{\langle x(t)^2 \rangle}{\Delta^2/12} = \frac{8}{(8/L)^2/12} = \frac{8 \times 12 \times L^2}{64}$

$SQNR = \frac{3}{2} L^2 \geq 1000$

6

$L^2 \geq \frac{2000}{3}$

$L \geq \sqrt{\frac{2000}{3}}$

$L \geq 26$

Problem 4: 22 Points

The binary digital communication signaling scheme, discussed in class, employs the following two equally probable signals $s_1(t)$ and $s_2(t) = -s_1(t)$ to represent binary logic 1 and 0 respectively over a channel corrupted by AWGN with power spectral density $N_0/2$ W/Hz. Here,

$$s_1(t) = \begin{cases} A\dfrac{t}{\tau}, & 0 \le t \le \tau \end{cases}$$

where τ is the binary symbol duration.

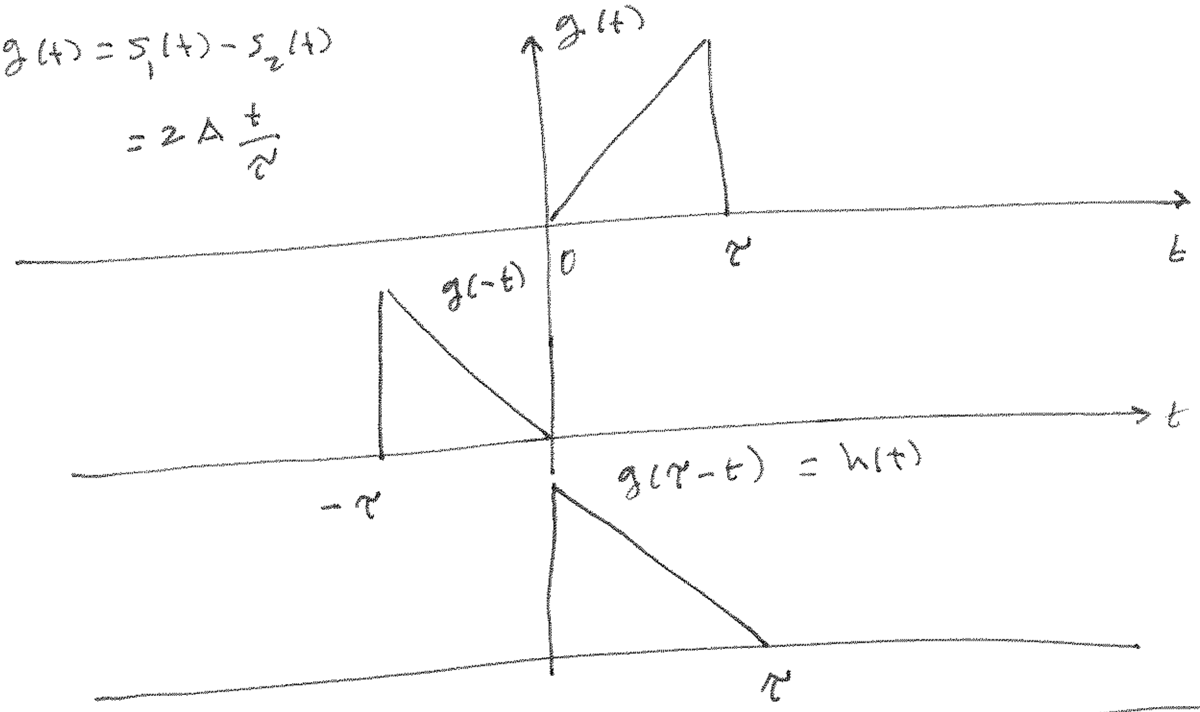
8 a. Find and sketch the impulse response, $h(t)$, of the matched filter, designed to minimize the probability of error.

6 b. Find the optimum threshold used by the threshold detector at the receiver.

8 c. Find the system average probability of error. Leave your answer in terms of the Q function.

Good Luck

$$g(t) = s_1(t) - s_2(t) = 2A\frac{t}{\tau}$$



$$\lambda^* = \frac{1}{2}(E_1 - E_2) = 0 \;;\; P_b = Q\left(\sqrt{\frac{\int_0^\tau (s_1-s_2)^2\,dt}{2N_0}}\right)$$

$$\int_0^\tau (s_1-s_2)^2\,dt = \int_0^\tau \left(\frac{2At}{\tau}\right)^2 dt = \frac{4A^2}{\tau^2}\int_0^\tau t^2\,dt$$

$$= \frac{4A^2}{\tau^2}\cdot\frac{\tau^3}{3} = \frac{4}{3}A^2\tau$$

$$P_b = Q\left(\sqrt{\frac{4A^2\tau}{6N_0}}\right) = Q\left(\sqrt{\frac{2A^2\tau}{3N_0}}\right)$$ 7

Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Communications and Digital Data Networks ENEE3401
Midterm Exam
Second Semester 2022-2023

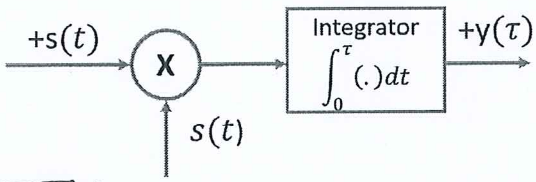
Date: Saturday June 15 2023 Time: 90 minutes
Name: Student #:

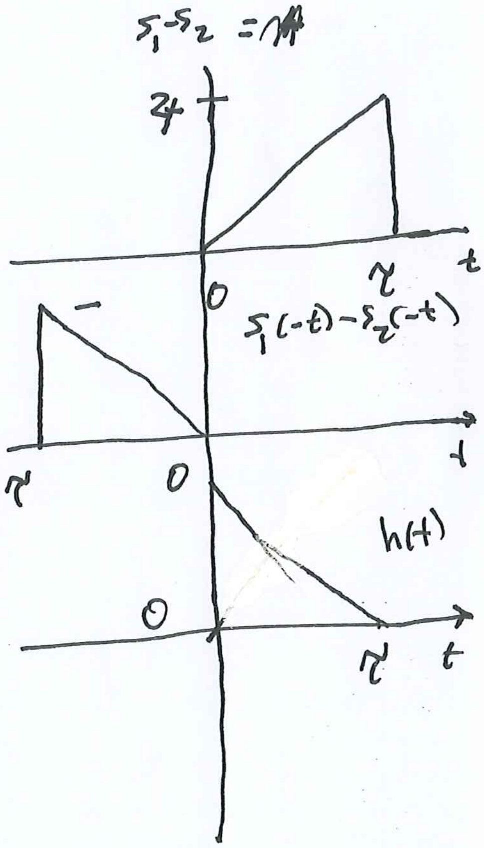
Problem 1: 25 Points

A baseband digital communication system uses the signals +s(t) and –s(t) to represent the equally probable binary digits 1 and 0, respectively, where s(t) is given as:

$$s(t) = \begin{cases} \frac{2t}{\tau} & 0 \le t \le \tau \\ 0, & otherwise \end{cases}$$

9 a. Find and sketch the impulse h(t) of the matched filter.
9 b. Find the probability of error in AWGN with psd = $N_0/2$.
7 c. If the signal s(t) is passed through a correlator that correlates the input s(t) with s(t), find the value of the correlator output y(τ)





b. $P_b = Q\left(\sqrt{\frac{\int (s_1-s_2)^2 dt}{2N_0}}\right)$

$\int_0^\tau \left(\frac{4t}{\tau}\right)^2 dt = \frac{16}{\tau^2}\int_0^\tau t^2 dt = \frac{16}{\tau^2}\frac{\tau^3}{3}$

$= \frac{16\tau}{3}$

$P_b = Q\left(\sqrt{\frac{16\tau}{6N_0}}\right)$

$y(\tau) = \int_0^\tau s(t)^2 dt = \int_0^\tau \frac{4t^2}{\tau^2} dt = \frac{4}{\tau^2}\frac{\tau^3}{3}$

$y(\tau) = \frac{4}{3}\tau$

Problem 2: 25 Points

Consider the two bases functions $\varphi_1(t)$ and $\varphi_2(t)$ defined as:

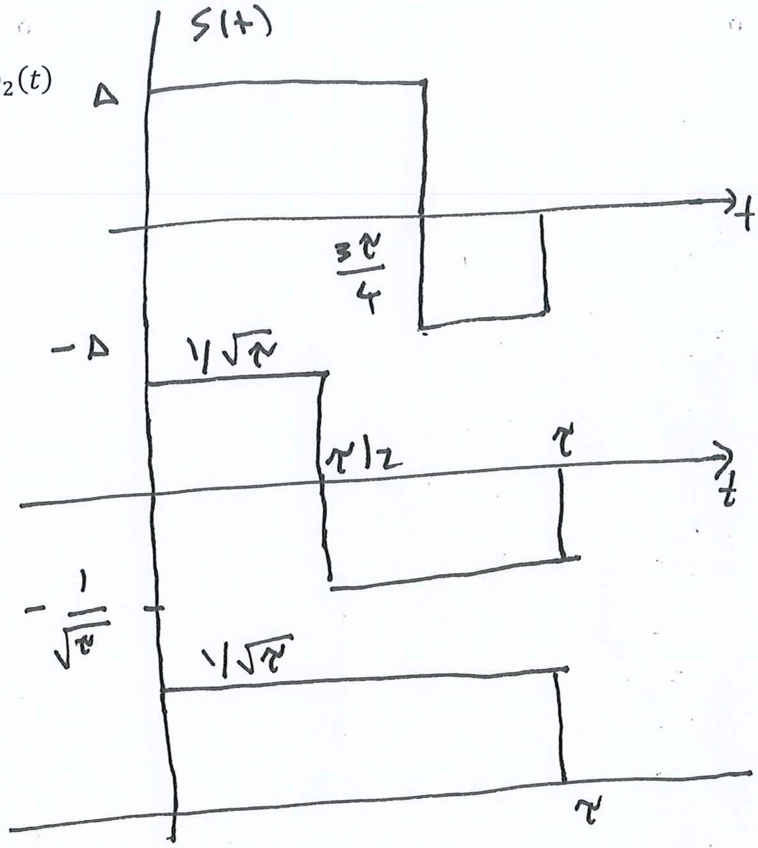
$$\varphi_1(t) = \begin{cases} \frac{1}{\sqrt{\tau}}, & 0 \le t \le \tau/2 \\ -\frac{1}{\sqrt{\tau}}, & \tau/2 \le t \le \tau \end{cases} \qquad \varphi_2(t) = \begin{cases} \frac{1}{\sqrt{\tau}}, & 0 \le t \le \tau \\ 0, & otherwise \end{cases}$$

a. Find the coordinates of the signal $s(t)$ in the $\varphi_1(t) - \varphi_2(t)$ plane, where $s(t)$ is given by

$$s(t) = \begin{cases} A, & 0 \le t \le 3\tau/4 \\ -A, & 3\tau/4 \le t \le \tau \end{cases}$$

b. Find the probability of error in additive white Gaussian noise with psd $N_0/2$ for the two signals

$s_1(t) = \emptyset_1(t) - \emptyset_2(t)$ $\qquad$ $s_2(t) = \emptyset_2(t)$



a. $s(t) = s_{11}\,\phi_1 + s_{12}\,\phi_2$

3 $\quad s_{11} = \int_0^{\tau} s(t)\,\phi_1(t)\,dt =$

$= \left(\frac{A}{\sqrt{\tau}}\right)\left(\frac{\tau}{2} + \frac{\tau}{4}\right) - \frac{A}{\sqrt{\tau}}\left(\frac{3\tau}{4} - \frac{\tau}{2}\right)$

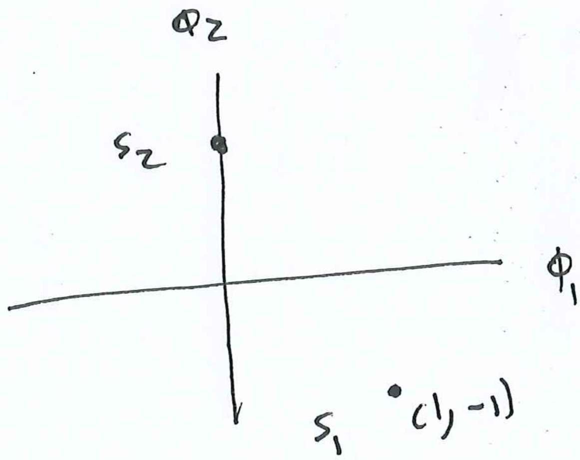
$= \frac{A}{\sqrt{\tau}} \cdot \frac{\tau}{2} = \boxed{\frac{A}{2}\sqrt{\tau}}$ 3

3 $\quad s_{12} = \int_0^{\tau} s(t)\,\phi_2(t)\,dt$

$= \frac{A}{\sqrt{\tau}}\left(\frac{3\tau}{4} - \frac{\tau}{4}\right) = \frac{A}{2\sqrt{\tau}} \cdot \tau$

3 $\quad \boxed{s_{112} = \frac{A}{2}\sqrt{\tau}}$

$d_{12} = \sqrt{5}$ 8



b. 13

$P_b = Q\left(\frac{d_{12}}{\sqrt{2N_0}}\right)$ 5

$P_b = Q\left(\frac{5}{\sqrt{2N_0}}\right)$

Problem 3: 25 Points

Consider the two bases functions $\varphi_1(t)$ and $\varphi_2(t)$ defined as:

$$\varphi_1(t) = \begin{cases} \frac{1}{\sqrt{\tau}}, & 0 \le t \le \tau/2 \\ -\frac{1}{\sqrt{\tau}}, & \tau/2 \le t \le \tau \end{cases} \qquad \varphi_2(t) = \begin{cases} \frac{1}{\sqrt{\tau}}, & 0 \le t \le \tau \\ 0, & otherwise \end{cases}$$

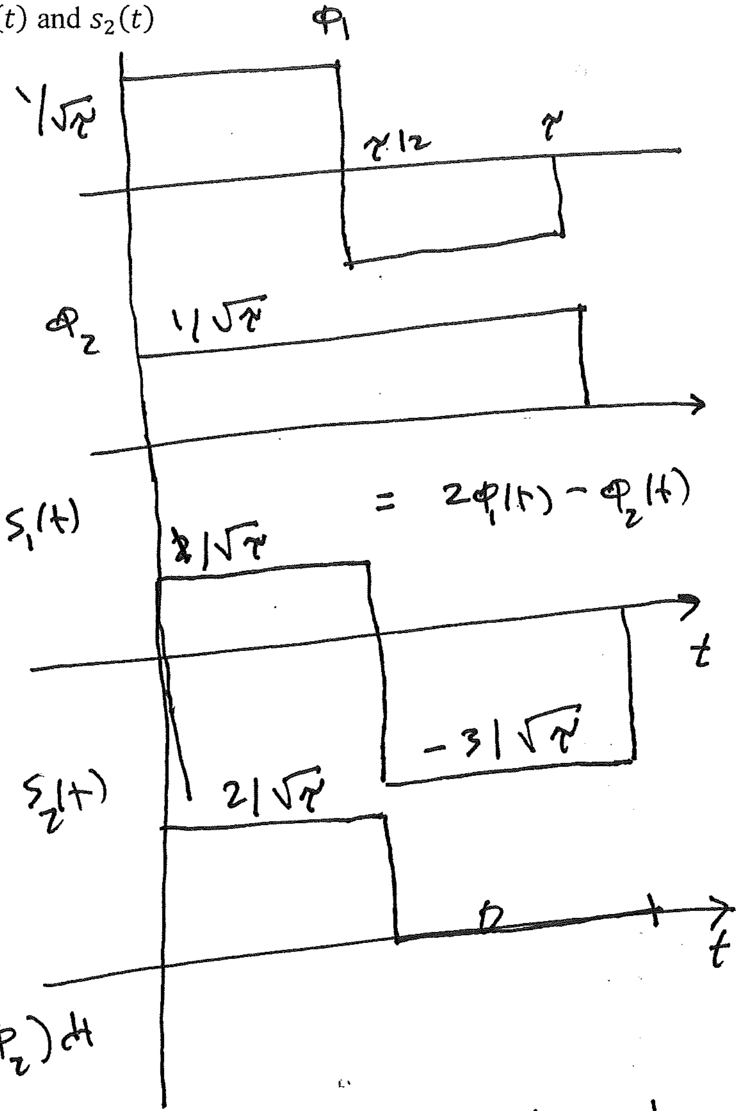
Let $s_1(t) = 2\emptyset_1(t) - \emptyset_2(t)$; $s_2(t) = \emptyset_1(t) + \emptyset_2(t)$

7 a. Sketch $s_1(t)$ and $s_2(t)$

6 b. Find E_1 and E_2

6 c. Find the correlation coefficient between $\emptyset_1(t)$ and $\emptyset_2(t)$

6 d. Find the correlation coefficient between $s_1(t)$ and $s_2(t)$



$s_1(t) = 2\phi_1(t) - \phi_2(t)$

$s_2(t) = \phi_1 + \phi_2$

$E_1 = (4 + 1) = 5$

b. $E_2 = (1) + (1) = 2$

c. $\rho = \int_0^\tau \phi_1(t)\,\phi_2(t)\,dt = 0$

d. $\rho = \dfrac{1}{\sqrt{E_1}\sqrt{E_2}} \int_0^\tau s_1 s_2\,dt$

$\int_0^\tau s_1 s_2\,dt = \int_0^\tau (2\phi_1 - \phi_2)(\phi_1 + \phi_2)\,dt$

$= \int_0^\tau (2\phi_1^2 + 2\phi_1\phi_2 - \phi_1\phi_2 - \phi_2^2)\,dt = 2 - 1 = 1$

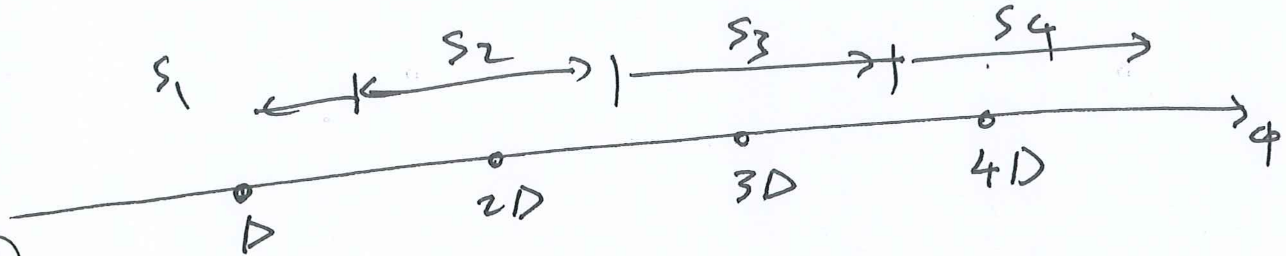
$\rho = \dfrac{1}{\sqrt{5}\,\sqrt{2}} = \dfrac{1}{\sqrt{10}}$

Problem 4: 25 Points

Consider a digital communication system that transmits one of four possible symbols every T_s seconds over a channel corrupted by AWGN with zero mean and power spectral density $N_0/2$. The transmitted signals are

$$s_i(t) = i\Delta\sqrt{\frac{2}{T_s}}\cos(2\pi f_c t)\ ;\ 0 \le t \le T_s;\ i = 1, 2, 3, 4;\ T_s = kT_c$$

a. Find the set of bases functions for this signal space.
b. Sketch the signal space representation of the signals.
c. Find the average symbol probability of error assuming signals are equally probable.
d. Sketch the optimum receiver, describing the function of each unit.



b

6

a. $\phi = \sqrt{\frac{2}{T_s}} \cos 2\pi f_c t$

b. 6

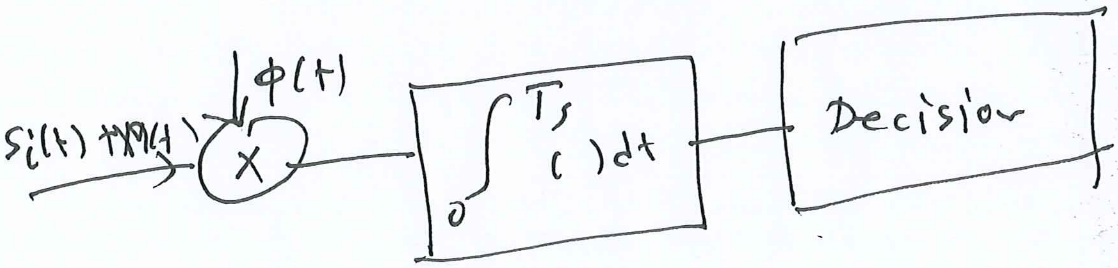
c

$$P_b = \frac{2+1+1+2}{4} Q\left(\frac{\Delta}{\sqrt{2N_0}}\right)$$

7

$$P_b = \frac{6}{4} Q\left(\frac{\Delta}{\sqrt{2N_0}}\right) = \frac{3}{2} Q\left(\frac{\Delta}{\sqrt{2N_0}}\right)$$

d.



6

Birzeit University
Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Information and Coding Theory ENEE 5304
Quiz # 4

Instructors: Dr. Wael Hashlamoun   Date: May 21, 2018

Problem

Suppose a cyclic redundancy check (CRC) code uses the prime generator polynomial

$$g(x) = x^3 + x + 1.$$

a. Generate the CRC bits for the message 1101
b. Can this code detect the error pattern 0000011? Explain

$\frac{6}{10}$

- $r=3$ in the polynomial
- Multiply $m(x)$ by x^r $\Rightarrow (x^3+x^2+1)x^3$
- Divide $m(x)x^r$ by $g(x)$ and find the remainder

$$
\begin{array}{r|l}
 & x^3 + x^2 + x + 1 \\
\hline
x^3+x+1 & x^6 + x^5 + x^3 \\
 & x^6 + x^4 + x^3 \\
\hline
 & x^5 + x^4 \\
 & x^5 + x^3 + x^2 \\
\hline
 & x^4 + x^3 + x^2 \\
 & x^4 + x^2 + x \\
\hline
 & x^3 + \cancel{x^2} + x \\
 & x^3 + x + 1 \\
\hline
 & 1
\end{array}
$$

remainder

$\Rightarrow$ CRC bits $\boxed{001}$ so that transmitted sequence is

1101 001

b. Error polynomial $x+1$ $\Rightarrow$ Is there a remainder

remainder $\left(\frac{x+1}{x^3+x^2+1}\right)$ $\Rightarrow$ 011 $\neq 0$

$\Rightarrow$ this error pattern can be detected since remainder $\neq 0$

Key

Birzeit University
Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
ENEE3401, COMMUNICATIONS AND DIGITAL DATA NETWORKS
First Quiz

Instructors: Dr. Wael Hashlamoun   Date: May 2, 2023

Problem

A digital communication signaling scheme employs the two signals $s_1(t)$ and $-s_1(t)$ to transmit binary digits 1 and 0, respectively, over a channel corrupted by AWGN with zero mean and power spectral density $N_0/2$. Let P(1) = P(0) = 1/2 and let $s_1(t)$ be defined as:

$$s_1(t) = A\cos\left(\frac{\pi t}{T_b}\right), \qquad 0 \le t \le T_b$$

a. Find the energy in $s_1(t)$
b. Find the average probability of error of the optimum receiver.
c. Find the optimum threshold of the receiver, which minimizes the probability of error.

$$E_1 = \int_0^{T_b} |s_1(t)|^2 dt = \int_0^{T_b} A^2 \cos^2\frac{\pi t}{T_b}\, dt$$

3

$$= \frac{A^2}{2}\int_0^{T_b}\left(1 + \cos\frac{2\pi t}{T_b}\right)dt = \frac{A^2}{2} T_b + \frac{A^2}{2}\,\frac{\sin(2\pi t/T_b)}{2\pi/T_b}\Bigg|_0^{T_b}$$

$$E_1 = \frac{A^2 T_b}{2}$$

4

b. $$P_b = Q\left(\sqrt{\frac{\int_0^{T_b}(s_1-s_2)^2\,dt}{2N_0}}\right) = Q\left(\sqrt{\frac{4\int_0^{T_b}|s_1(t)|^2\,dt}{2N_0}}\right)$$

$$= Q\left(\sqrt{\frac{4 * A^2 T_b/2}{2N_0}}\right) = Q\left(\sqrt{\frac{A^2 T_b}{N_0}}\right)$$

3

c. $$T.H = \frac{1}{2}(E_1 - E_2)$$

$$= 0$$

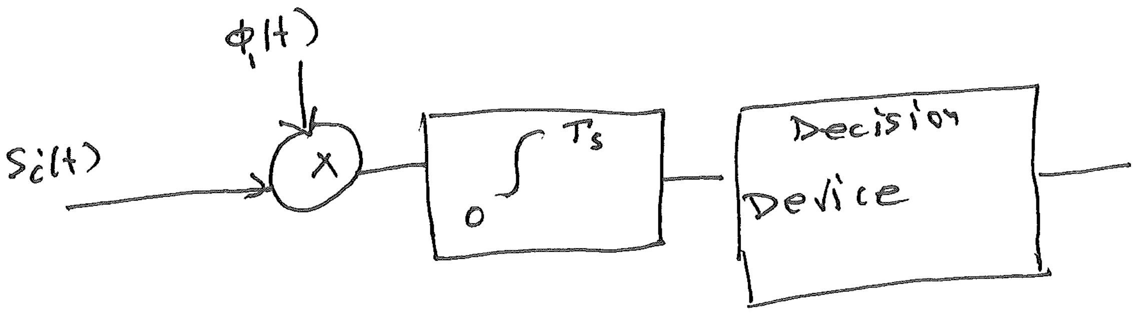
Birzeit University
Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
ENEE3401, COMMUNICATIONS AND DIGITAL DATA NETWORKS
Second Quiz

Instructors: Dr. Wael Hashlamoun   Date: June8 2, 2023

Problem
An M-ary ASK system consists of four signals with coordinates (-3Δ, -Δ, Δ, 3Δ) relative to the base function $\emptyset_1(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi f_c t)$.

a. Sketch the optimum receiver
b. Find the average energy per symbol

5 (a)



5 (b)

$$E_{av} = \frac{\Delta^2 + \Delta^2 + (3\Delta)^2 + (3\Delta)^2}{4}$$

$$= \frac{20\Delta^2}{4} = 5\Delta^2$$