

1.3 linear Functions

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The linear function has the form $y = f(x) = ax + b$ where a, b are constants such that:

$m = a$ is the slope of the line $y = ax + b$

b is y -intercept obtained when $x = 0 \rightarrow (0, b)$

$-\frac{b}{a}$ is x -intercept obtained when $y = 0 \rightarrow (-\frac{b}{a}, 0)$

Exp Find intercepts and slopes and graph the following linear functions

① $12x + 3y = 24$

• To find y -intercept $\Rightarrow$ let $x = 0 \Rightarrow 3y = 24 \Rightarrow y = 8$
so y -intercept is 8

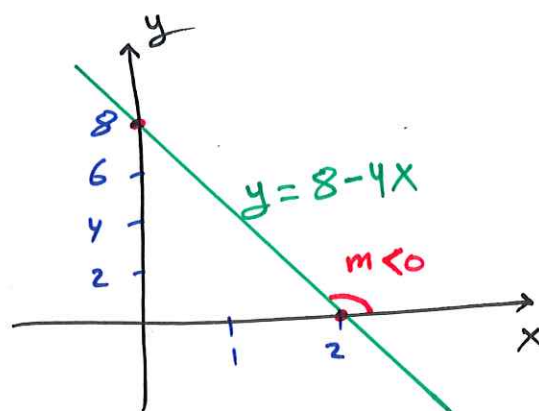
• To find x -intercept $\Rightarrow$ let $y = 0 \Rightarrow 12x = 24 \Rightarrow x = 2$
so x -intercept is 2

• To graph we use the intercepts

• To find slope $\Rightarrow$

$$3y = 24 - 12x$$

$$\boxed{y = 8 - 4x} \rightarrow \text{slope} = -4 = m$$



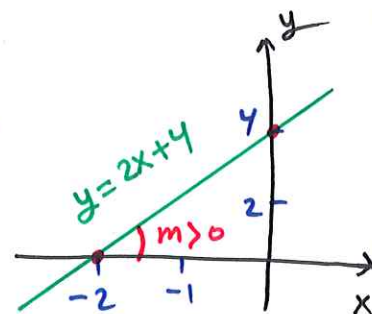
② $2x - y + 4 = 0$

• $x = 0 \Rightarrow -y + 4 = 0 \Rightarrow y = 4$ is y -intercept

• $y = 0 \Rightarrow 2x + 4 = 0 \Rightarrow 2x = -4 \Rightarrow x = -2$ is x -intercept

• To graph we use the intercepts

• To find slope $\Rightarrow \boxed{2x + 4 = y} \Rightarrow \text{slope} = 2 = m$



$$\boxed{3} \quad y - 5 = 0$$

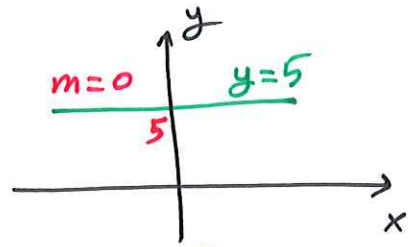
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• $x=0 \Rightarrow y-5=0 \Rightarrow y=5$ is y -intercept

• $y=0 \Rightarrow -5 \neq 0 \Rightarrow$ No x -intercept

• The graph is Horizontal line

• To find slope $\Rightarrow y = 5 + 0x \Rightarrow$ slope $= 0 = m$ for any horizontal line



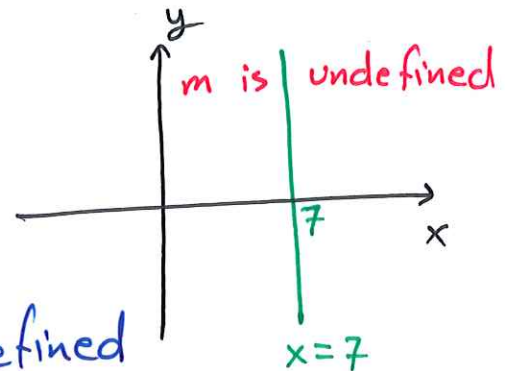
$$\boxed{4} \quad x - 4 = 3$$

• $x=0 \Rightarrow -4 \neq 3 \Rightarrow$ No y -intercept

• $y=0 \Rightarrow x=7$ is x -intercept

• The graph is Vertical line

• The slope of any vertical line is undefined



Def The slope of a non vertical line passes through the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Exp Find the slope of the line passes through the points

① $(2, 1)$ and $(-1, 1)$ $\Rightarrow m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 1}{-1 - 2} = \frac{0}{-3} = 0$

$\Rightarrow$ this line is horizontal

② $(-2, 1)$ and $(4, 3)$ $\Rightarrow m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{4 - (-2)} = \frac{2}{6} = \frac{1}{3}$

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③ $(3, 0)$ and $(4, -3)$ $\Rightarrow m_3 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 0}{4 - 3} = \frac{-3}{1} = -3$

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④ $(-1, 2)$ and $(5, 4)$ $\Rightarrow m_4 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{5 - (-1)} = \frac{2}{6} = \frac{1}{3}$

Note that $\rightarrow$ line ② and line ④ are parallel since have same slopes

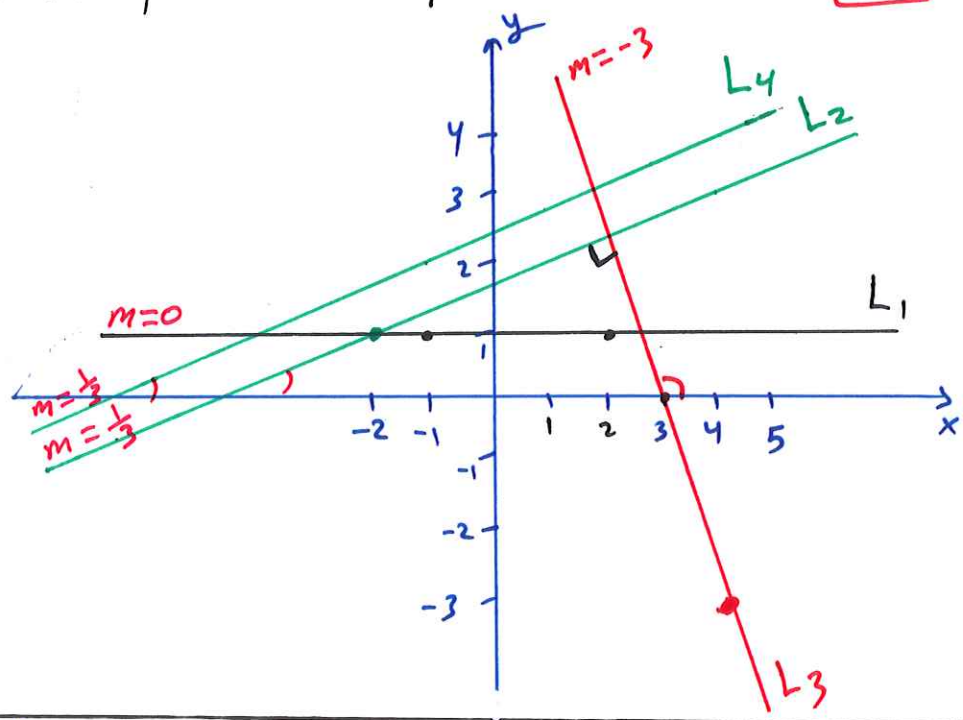
$\rightarrow$ line ② and line ③ are perpendicular since $m_2 \cdot m_3 = -1$

$\rightarrow$ line ① is horizontal since $m_1 = 0$

Exp Draw the lines in the previous example

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- ① $(2, 1)$ and $(-1, 1)$
- ② $(-2, 1)$ and $(4, 3)$
- ③ $(3, 0)$ and $(4, -3)$
- ④ $(-1, 2)$ and $(5, 4)$



Remark • Parallel line have same slopes ($L_1 \parallel L_2$ iff $m_1 = m_2$)
 • Two lines L_1 and L_2 are perpendicular (or normal or orthogonal) iff $m_1 \cdot m_2 = -1$ ($L_1 \perp L_2$ iff $m_1 \cdot m_2 = -1$)

Point with slope: The equation of the line passing through the point (x_1, y_1) with slope m is

$$y - y_1 = m(x - x_1)$$

Exp Find equation for the line passes through $(1, -2)$ and

① slope is 2

$y - y_1 = m(x - x_1) \Rightarrow y - -2 = 2(x - 1) \Rightarrow y + 2 = 2x - 2 \Rightarrow \boxed{y = 2x - 4}$

② slope is 0

$y - y_1 = m(x - x_1) \Rightarrow y - -2 = 0(x - 1) \Rightarrow y + 2 = 0 \Rightarrow \boxed{y = -2}$ H.L

③ slope is undefined

This is the vertical line $\boxed{x = 1}$

④ y-intercept -3

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$$(1, -2) \text{ and } (0, -3) \Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - (-2)}{0 - 1} = \frac{-1}{-1} = 1$$

$$y - y_1 = m(x - x_1) \Rightarrow y - (-2) = 1(x - 1) \Rightarrow y + 2 = x - 1 \Rightarrow \boxed{y = x - 3}$$

⑤ x-intercept -3

$$(1, -2) \text{ and } (-3, 0) \Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (-2)}{-3 - 1} = \frac{2}{-4} = -\frac{1}{2}$$

$$y - y_1 = m(x - x_1) \Rightarrow y - (-2) = -\frac{1}{2}(x - 1) \Rightarrow y + 2 = -\frac{x}{2} + \frac{1}{2} \Rightarrow \boxed{y = -\frac{x}{2} - \frac{3}{2}}$$

Exp which of the following lines are perpendicular or parallel or neither of these:

① $5x - 2y = 8$ and $10x - 4y = 8$

$$-2y = 8 - 5x$$

$$y = -4 + \frac{5}{2}x$$

$$m_1 = \frac{5}{2}$$

$$-4y = 8 - 10x$$

$$y = -2 + \frac{10}{4}x$$

$$m_2 = \frac{10}{4} = \frac{5}{2}$$

$m_1 = m_2$ so
these lines are
parallel

② $2 - 3y = 5 - 6x$ and $4y + 2x = 3$

$$-3y = 3 - 6x$$

$$\boxed{y = -1 + 2x}$$

$$m_1 = 2$$

$$4y = 3 - 2x$$

$$y = \frac{3}{4} - \frac{1}{2}x$$

$$m_2 = -\frac{1}{2}$$

$m_1 \cdot m_2 = (2)(-\frac{1}{2}) = -1$
so these lines are
perpendicular

③ $x + y = 2$ and $y = 3 - 11x$

$$y = 2 - x$$

$$m_1 = -1$$

$$m_2 = -11$$

$m_1 \neq m_2$ not parallel
 $m_1 \cdot m_2 = 11 \neq -1$
not perpendicular
so neither of these

Exp Write the equation line passing through $(-1, 3)$ and parallel to the line $12x + 4y = 1$ 54

$$y - y_1 = m(x - x_1)$$

$$4y = 1 - 12x$$

$$y = \frac{1}{4} - 3x$$

$$y - 3 = -3(x - -1)$$

$$m = -3$$

$$y - 3 = -3x - 3$$

Both lines have

$$y = -3x$$

Same slopes since they are parallel

Exp Find the equation for the line passes through $(-2, -8)$ that is perpendicular to the line $x = 4y + 3$

$$y - y_1 = m_1(x - x_1)$$

$$y - -8 = -4(x - -2)$$

$$y + 8 = -4x - 8$$

$$m_1 \cdot m_2 = -1$$

$$m_1 \cdot \frac{1}{4} = -1$$

$$m_1 = -4$$

$$-4y + x = 3$$

$$-4y = 3 - x$$

$$y = -\frac{3}{4} + \frac{1}{4}x$$

$$m_2 = \frac{1}{4}$$

$$y = -4x - 16$$

Exp Find the slope and the intercepts of the line $x + 2y = 8$

• $x = 0 \Rightarrow 2y = 8 \Rightarrow y = 4$ is y-intercept

• $y = 0 \Rightarrow x = 8$ is x-intercept

• $x + 2y = 8$

$$2y = 8 - x$$

$$y = 4 - \frac{1}{2}x$$

$m = -\frac{1}{2}$ is the slope

