Mathematics Department

Math 1321 – Worksheet #2

Rasha Shadid

Name: ------

Q1 Find the Formula for the n-th term of the following Seq. and determine if the sequence Converges or Diverges

1. $1, 3, 1, 3, 1, 3, \ldots$

2. $1, \frac{-1}{4}, \frac{1}{9}, \frac{-1}{16}, \ldots$

3. $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \ldots$

Q2. Find the limit of the following sequence.

1. $a_n = \dfrac{(-1)^n}{2^n}$

2. $a_n = \sqrt[n]{3n^3}$

3. $a_n = n\left(1 - \cos\left(\frac{1}{n}\right)\right)$

4. $a_n = \dfrac{3^n\, 6^n}{2^{-n}\, n!}$

5. $a_n = \left(\dfrac{n+2}{n-1}\right)^n$

6. $a_n = \dfrac{(n+1)!}{(n+2)!}$

Q3 Determin if the sequence is monotonic and if its bounded

1. $a_n = \left(\frac{3}{4}\right)^n$

2. $a_n = \frac{\sin(n)}{n}$

3. $a_n = \frac{6^n}{n!}$

Short Answers:-

Q.1

$\boxed{1}$ $a_n = \{(-1)^n + 2\}$ Div.

$\boxed{2}$ $a_n = \left\{\dfrac{(-1)^{n+1}}{n^2}\right\}$ Conv.

$\boxed{3}$ $a_n = \left\{1 - \dfrac{1}{2^n}\right\}$ Conv.

Q2

$\boxed{1}$ $\lim\limits_{n\to\infty} \dfrac{(-1)^n}{2^n} = 0$ By Sandwich Th.

$\boxed{2}$ $\lim\limits_{n\to\infty} \sqrt[n]{3n^3} = 1$ Part 2. $\rightarrow \lim\limits_{n\to\infty} n^{1/n} = 1$

$\boxed{3}$ $\lim\limits_{n\to\infty} n\left(1 - \cos\left(\frac{1}{n}\right)\right) = 0$ By l'Hopital Rule

$\boxed{4}$ $\lim\limits_{n\to\infty} \dfrac{3^n 6^n}{2^{-n} n!} = 0$ Part 6 $\longrightarrow \lim\limits_{n\to\infty} \dfrac{x^n}{n!} = 0$

$\boxed{5}$ $\lim\limits_{n\to\infty} \left(\dfrac{n+2}{n-1}\right)^n = e^3 = \dfrac{e^2}{e^{-1}}$ Part 5 $\rightarrow \lim\limits_{n\to\infty}\left(1 + \frac{x}{n}\right)^n = e^x$

$\boxed{6}$ $\lim\limits_{n\to\infty} \dfrac{(n+1)!}{(n+2)!} = 0$

Q_3 $\boxed{1}$ $a_n = \left(\frac{3}{4}\right)^n$

- Non Increasing, Monotonic Seq. since $\frac{a_n}{a_{n+1}} \geq 1$
  $a_n \geq a_{n+1}$
- Bounded Seq.

$\therefore$ Then By monotonic seq. thm. the seq. Conv.

$\boxed{2}$ $a_n = \frac{\sin n}{n}$

- Not Monotonic seq. ↘ ↗ ↘ ↗
- Bounded Seq.
- $\lim\limits_{n\to\infty} \frac{\sin n}{n} = 0$ $\longrightarrow$ the seq. Converges

$\boxed{3}$ $a_n = \frac{6^n}{n!}$

- Not nondecreasing (We can say the seq monotonic if $n \geq 5$)
  since $\frac{a_n}{a_{n+1}} \geq 1$ only if $n \geq 5$
- Bounded Seq.
- $\lim\limits_{n\to\infty} \frac{6^n}{n!} = 0$ $\longrightarrow$ the seq conv. By part 6