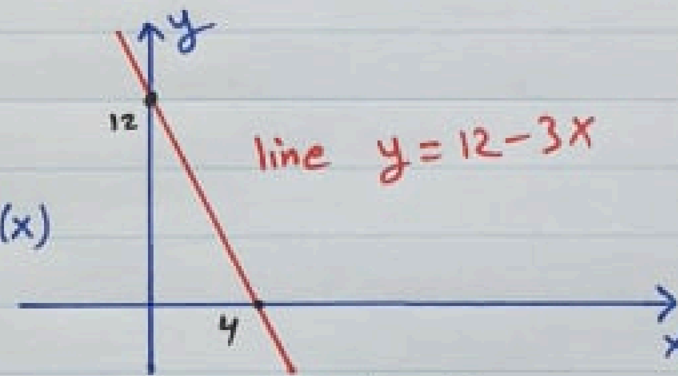
Review for Basics

- **y-intercept** is the point $(0, f(0))$ where $y = f(x)$ crosses y-axis

  Exp $y = 12 - 3x$ has y-intercept when $x = 0 \Rightarrow y = 12$
  the point of y-intercept is $(0, 12)$

- **x-intercept** is the point $(x, 0)$ where $y = f(x)$ crosses x-axis



  $y = 0 \Rightarrow 0 = 12 - 3x$
  $\Rightarrow x = 4$

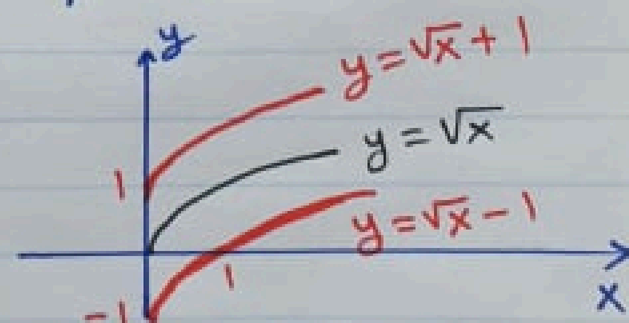
  y-intercept is $(0, 12)$
  x-intercept is $(4, 0)$

- **Shifting and Reflections**

  Given the function $y = f(x)$ and constant $c > 0$

  - $y = f(x) + c \Rightarrow$ shift the graph of $f(x)$ c units upward

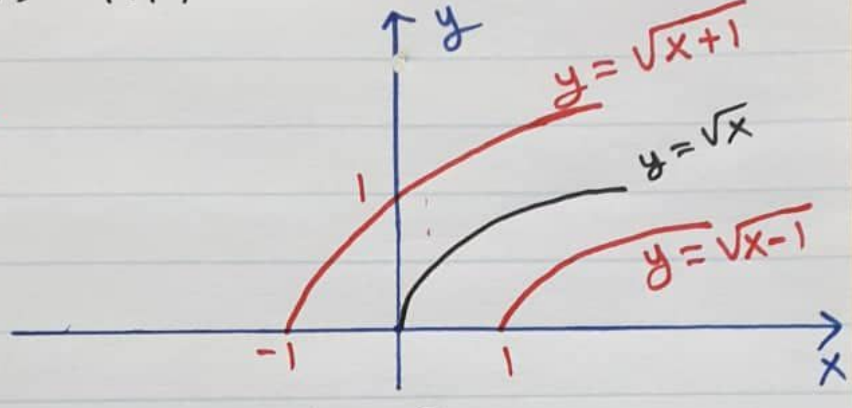
  Exp



  - $y = f(x) - c \Rightarrow$ shift the graph of $f(x)$ c units downward

- $y = f(x+c) \Rightarrow$ shift the graph of $f(x)$ c units leftward

Exp



- $y = f(x-c) \Rightarrow$ shift the graph of $f(x)$ c units rightward

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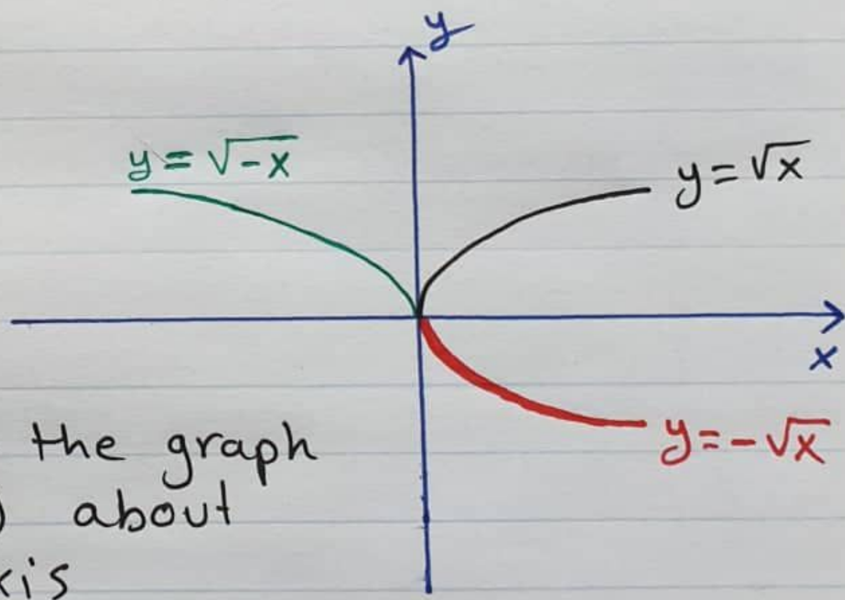
- $y = -f(x) \Rightarrow$ Reflect the graph of $f(x)$ about x-axis

Exp



- $y = f(-x) \Rightarrow$ Reflect the graph of $f(x)$ about y-axis

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Lines

- Equation of line passes through the point (x_0, y_0) with slope m is

$$\boxed{y - y_0 = m(x - x_0)} \quad \Rightarrow \quad y = mx \underbrace{- mx_0 + y_0}$$

$$f(x) = mx + b$$

slope $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Exp Find the equation of line passes through the points $(\underset{x_1}{1}, \underset{y_1}{2})$ and $(\underset{x_2}{3}, \underset{y_2}{-4})$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 2}{3 - 1} = \frac{-6}{2} = -3$$

$y - y_0 = m(x - x_0)$ Take $(x_0, y_0) = (1, 2)$

$$y - 2 = -3(x - 1)$$
$$= -3x + 3 \quad \Rightarrow \quad y = -3x + 5$$

- Horizontal line $y = c$ has slope $m = 0$ since $\Delta y = y_2 - y_1 = c - c = 0$

- Vertical line $x = c$ has undefined slope since $\Delta x = x_2 - x_1 = c - c = 0$

- If L_1 is a line with slope m_1 and L_2 is a line with slope m_2 then

$L_1 \parallel L_2$ if $m_1 = m_2$ $\qquad$ $\parallel$: Parallel

and $L_1 \perp L_2$ if $m_1 m_2 = -1$ $\qquad$ $\perp$: Perpendicular (عمودي)

Exp ① The lines $L_1: \quad y = 2x - 1$

$L_2: \quad y = 3 + 2x$

are parallel since $m_1 = m_2 = 2$

② The lines $L_1: \quad y = 2x - 1$

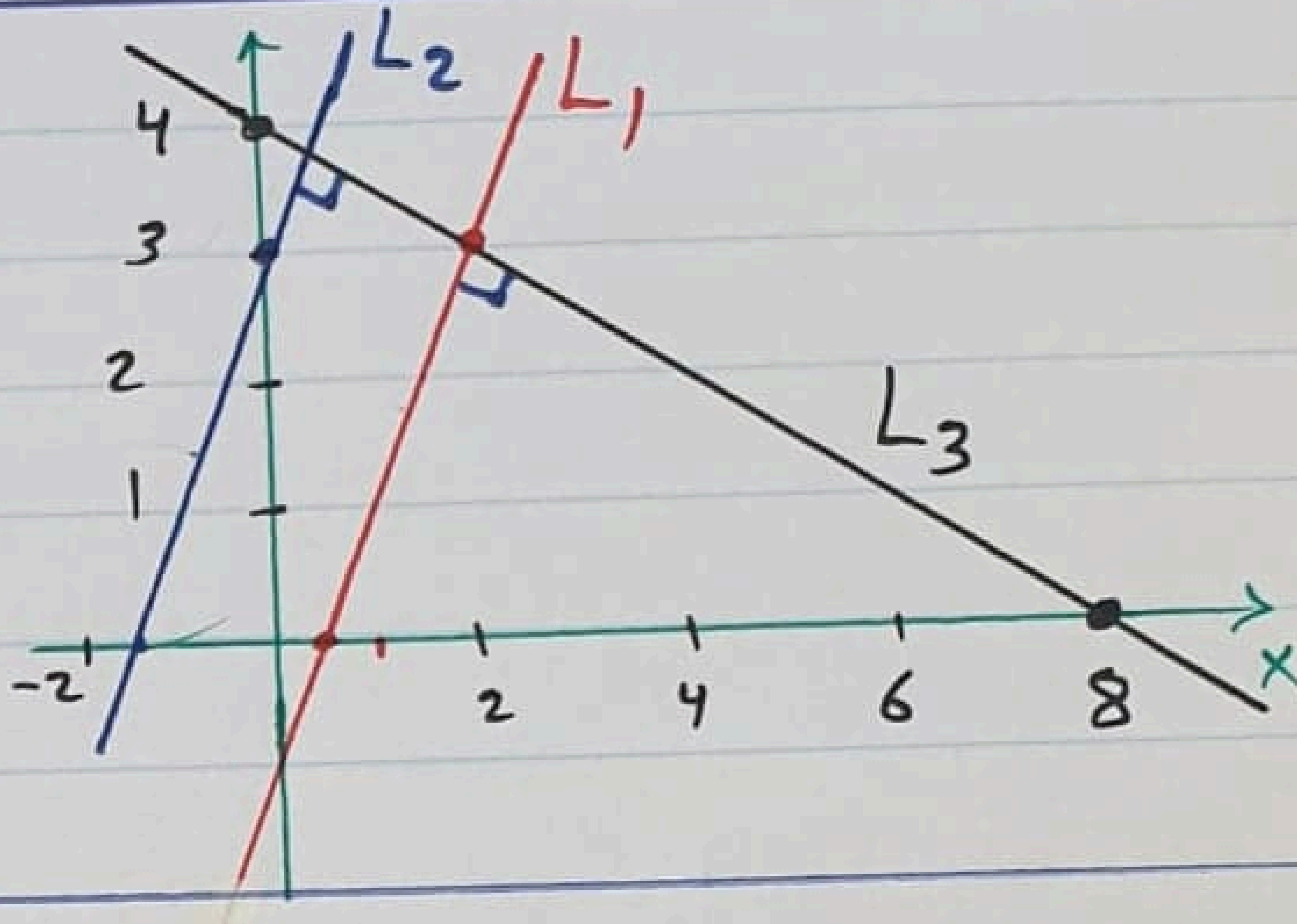
$L_3: \quad y = 4 - \frac{1}{2}x$

are perpendicular since $m_1 m_2 = (2)(-\frac{1}{2}) = -1$

$L_1 \parallel L_2$

$L_1 \perp L_3$

$L_2 \perp L_3$



Absolute Value: $\boxed{1}$ $|x| = a \Rightarrow x = a$ or $x = -a$

Exp $|x| = 7 \Rightarrow x = 7$ or $x = -7$

Exp $|x-1| = 4 \Rightarrow x-1 = 4$ or $x-1 = -4$

$\Rightarrow x = 5$ or $x = -3$

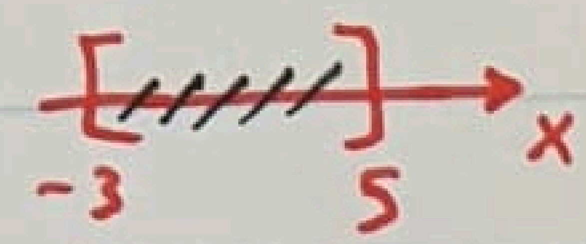
$x \in \{-3, 5\}$

$\boxed{2}$ $|x| \leq a \Rightarrow -a \leq x \leq a$

Exp $|x-1| \leq 4 \Rightarrow -4 \leq x-1 \leq 4$

$-3 \leq x \leq 5$

$\boxed{3}$ $|x| \geq a \Rightarrow x \geq a$ or $x \leq -a$



Exp $|x-1| > 4 \Rightarrow x-1 > 4$ or $x-1 < -4$

$x > 5$ or $x < -3$

$(-\infty, -3) \cup (5, \infty)$

• Factorization

- $x^2 - a^2 = (x-a)(x+a)$ — Exp $x^2 - 9 = (x-3)(x+3)$, $9 = 3^2 \Rightarrow a = 3$
- $x^3 - a^3 = (x-a)(x^2+ax+a^2)$ — Exp $x^3 - 8 = x^3 - 2^3 = (x-2)(x^2+2x+4)$
- $x^3 + a^3 = (x+a)(x^2-ax+a^2)$ — Exp $x^3 + 8 = x^3 + 2^3 = (x+2)(x^2-2x+4)$

• Quadratic Equation (parabola)

$$y = f(x) = ax^2 + bx + c = 0 \quad , \quad a \neq 0$$

- Vertex $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ where $f'(x) = 0 \Leftrightarrow 2ax + b = 0$, $x = \frac{-b}{2a}$
- Discriminant $= b^2 - 4ac$ (Discriminant = D)
  - → if $D > 0$ then f has two real different roots
  - → if $D = 0$ then f has one real root
  - → if $D < 0$ then f has no real roots
- The roots: $x = \frac{-b \pm \sqrt{D}}{2a}$
- If $a > 0$ then $f(x)$ opens upward (concave up)
- If $a < 0$ then $f(x)$ opens down (concave down)

• Square Completion $x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$

Exp.
- $x^2 - 8x + 1 = (x-4)^2 - 16 + 1 = (x-4)^2 - 15$
- $x^2 + 6x - 2 = (x+3)^2 - 9 - 2 = (x+3)^2 - 11$

• Circle of center (a, b) and radius r is $(x-a)^2 + (y-b)^2 = r^2$

Exp.
- $x^2 + y^2 = 4$ has center $(0,0)$ and radius $r = 2$
- $x^2 + 2x + y^2 = \boxed{8}$

  $(x+1)^2 + y^2 = 9$ has center $(-1,0)$ and radius $r = 3$