–Chapter 5
Syntax Analysis(Parser)

A Syntax analyzer is formally defined as :

An Algorithm that Groups the Set of Tokens Sent by the Scanner to Form **Syntax Structures** Such As Expressions, Statements, Blocks, etc.

Simply, the parser examines if the source code written follows the grammar(production rules) of the language.

The Syntax structure of programming languages and even spoken languages can be expressed in what is called **BNF**
notation, which stands for **B**akus **N**aur **F**orm.

For example, in spoken English, we can say the following:

sentence --> noun-phrase verb-phrase
noun-phrase --> article noun
article --> THE | A | AN ...
noun --> STUDENT | BOOK | ...
verb-phrase --> verb noun-phrase
verb --> READS | BUYS | ....

Note : The BNF Notation uses different symbols, for example, a sentence is defined as :

< sentence > ::= < noun-phrase > < verb-phrase >

But this is very cumbersome, so we use the first

notation, since it is easier to use. Now, let us derive

a sentence :

sentence --> **noun-phrase** verb-phrase
--> **article** noun verb-phrase
--> THE **noun** verb-phrase
--> THE STUDENT **verb-phrase**

--> THE STUDENT **verb** noun-phrase

--> THE STUDENT READS **noun-phrase**

--> THE STUDENT READS **article** noun

--> THE STUDENT READS A **noun**

--> THE STUDENT READS A BOOK

In the same way, the parser tries to **derive** your source program from the starting symbol of the grammar. Let us say we have these sentences :

THE BOOK BUYS A STUDENT
THE BOOK WRITES A DISH

Syntax-wise, all of these sentences are correct. However, their meaning is not correct, and they are not useful. What differentiates 2 sentences that are grammatically correct is their meaning or their **semantics**. You and I can agree that the meaning of a grammatically correct sentence is not correct, but how does the computer do it?

Grammar

A grammar G=(V_N, V_T, S, P) where:

1. V_N : A finite set of nonterminals (nonterminals set).
2. V_T : A finite set of terminals(terminals set).
3. $S \in V_N$: The Starting symbol of the grammar.
4. P = A set of **production rules**(productions).<-- Pending <==> Basically the whole grammar.

Note :

1. $V_N \cap V_T = \varnothing$.
2. $V_N \cup V_T = V$ (the vocabulary of the grammar).

Note : We will use:

1. Uppercase Letters A,B,...,Z for non-terminals.
2. Lowercase Letters a,b,...,z for terminals.
3. Greek letters α,β,γ,... for strings formed

from V_N OR V_T = V.

eg, if V_N = {S,A,B} , V_T = {0,1}

then

α = A11B
β = S110B
γ = 0010

Productions

1. A Production **α --> β** (alpha derives beta) is a rewriting rule such that the occurrence of **α** can be substituted by **β** in any string.

   Note: α must contain at least one nonterminal from V_N, i.e. $\in V_N$.

   For example, Assume we have the string

   γασ,

   > γασ --> γβσ

2. A Derivation is a sequence of strings $\alpha_0, \alpha_1, \alpha_2, ...,\alpha_n$, then :

$\alpha_0 \dashrightarrow \alpha_1 \dashrightarrow \alpha_2 \dashrightarrow .... \dashrightarrow \alpha_n$

Given a grammar G, then :

L(G) = Language Generated By the Grammar.

example, Given the Grammar, G = ({S,B,C},{a,b,c},S,P),

P :
S --> aSBC
S --> abC
CB --> BC
bB --> bb
bC --> bc
cC --> cc

What is L(G)=?

Let us do some derivations:

S --> a**bC** --> abc(all terminals) $\in$ L(G) <--- A sentence

S --> a**S**BC --> aa**bC**BC --> aabcBC --> blocked, so we try another path

S --> a**S**BC --> aab**CB**C --> aa**bB**CC --> aab**bC**C --> aabb**cC**

--> aabbcc $\in$ L(G) <--- A sentence

S --> a**S**BC -->.....-->aaabbbccc $\in$ L(G) <--- A sentence

Therefore, $L(G)=\{a^n b^n c^n | n \geq 1\}$

As another Example, we have these productions

E --> E+T <-- we can write the productions 1 and 2 as a single production E --> E+T | T
E --> T
T --> T*F
T --> F
F --> (E) <-- we can write the productions 5 and 6 as a single production F --> (E) | n
F --> n

Let us follow through some derivations

E --> **T** --> **F** --> n $\in$ L(E)

E --> **E**+T --> T+**T** --> T+**F** --> **T**+n --> **F**+n --> n+n $\in$ L(E)

E --> **E**+T --> **T**+T --> **F**+T --> n+**T** --> n+**F** --> n + (**E**) --> n+(**T**) --> n+(**T***F) --> n+(**F***F) --> n+(n***F**) --> n+(n*n) $\in$ L(E)

Therefore, L(G) = {Any arithmetic expression with * and + operations}, n is an operand here.

Note that, if we add the productions

E --> E+T | E-T | T

T --> T*F | T/F | T%F

We would have a language to express all arithmetic expressions with (* , \ , + , -) operations.

Let us Take another <u>Example</u> (Tokens between double quotes are terminals)

program --> block "#"

block --> "{" stmt-List "}"

stmt-List --> statement ";" stmt-List | λ

statement --> if-stmt | while-stmt | read-stmt | write-stmt | assignment-stmt | block

if-stmt --> "if" condition....

while-stmt --> "while" condition.....

....

....

read-stmt --> "read"

write-stmt --> "write"

V_N = {program, block, stmt-List, statement, if-stmt, while-stmt,

read-stmt, write-stmt, assignment-stmt}

V_T = { "{", "}", "#", ";", "if", "while", "read", "write" }

Let us Follow through some derivations :

Program --> **block** # --> { **stmt-list** } # --> { λ } # --> { } #
Program --> **block** # --> {**stmt-list**} # --> {statement ; **stmt-list**} # -->
{statement ; statement ; **stmt-list**} # -->
{statement; statement ; λ} # --> {**statement** ; statement ;} # --> {**read-stmt** ; statement ;} # -->{read ; **statement** ;} # -->
{read ; write-stmt ;} # -->

{read;write;} #

The language of this language is defined as

L(G) = {Set of all programs that can be written in this language}.

This is only a simple example, of a simple language. For something more complex such as C or Pascal, there are hundreds of productions.

Algorithms for Derivation

There are two derivation techniques:

Def: A Leftmost derivation is a derivation in which we replace the **leftmost** nonterminal in each derivation step.

E→E+T→T+T→F+T→n+T→n+F→n+n

Def: A Rightmost derivation is a derivation in which we replace the **rightmost** nonterminal in each derivation step.

E→E+T→E+F→E+n→T+n→F+n→n+n

For example, given

the grammar

V --> S R $

S --> + | - | λ

R --> .dN | dN.N

N --> dN | λ

V_N = {V,R,S,N}
V_T = {+, - , ., d, $}

Let us follow through on the leftmost derivation

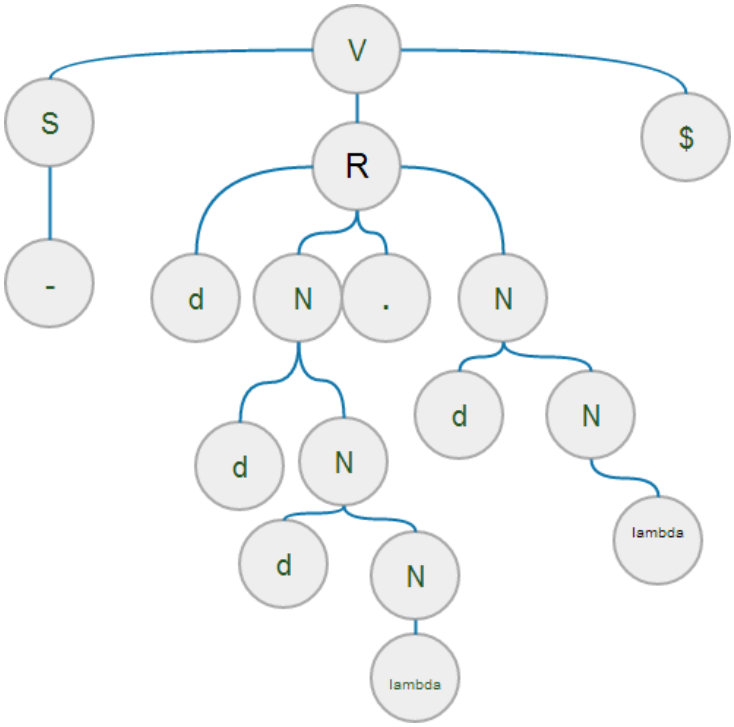
V --> **S**R$ --> -**R**$ --> -d**N**.N$ --> -dd**N**.N$ --> -dddN.N$ --> -ddd.**N**$ --> -ddd.d**N**$ --> -ddd.d$ <-- A sentence.

Let us follow through on the rightmost derivation

V --> S**R**$ --> SdN.**N$** --> SdN.d**N**$ --> Sd**N**.d$ --> sdd**N**.d$ --> sddd**N**.d$ --> **S**ddd.d$ --> -ddd.d$ <-- A sentence.

Derivation Trees

Def: A Derivation Tree is a Tree that displays the derivation of some sentence in the language. For example, let us look at the tree for the previous example



Note that if we traverse the tree in order, recording **only** the leaves, we obtain the sentence.

Classes of Grammars

According to Chomsky, There are 4 classes of grammars :

1. Unrestricted Grammars : No restrictions whatsoever except the restriction by definition that the left side of the production contains at least one nonterminal from V_N. This grammar is not practical and we cannot work with it.

2. Context-Sensitive Grammars : For each production α --> β, |α| ≤ |β|, i.e. , the **length of alpha(α)** is less than or equal to the **length of Beta(β)**. This means that in this class of grammar, there are no λ productions in the form A --> &lambda, since |λ| = 0 and A ≥ 1.
3. Context-Free Grammar(CFG) : Each production in this grammar class is of the form A --> α , where A $\in$ V_N and α $\in$ $V*$

   that is to say, the left hand side is **only** one nonterminal.

   This is the most important class of grammar. Most programming languages structures are context-free. We will mostly be working with this class of grammar. Most of the examples we have taken are CFG.

4. Regular Grammar (Regular Expressions) : Each production in this grammar class is of the form A -- > aB or A --> a, where A,B $\in V_N$ and a $\in V_T$, **with the exception** of S --> λ

For example, given the grammar:

A --> aA
A --> a

Therefore, we get $L(G)=a^+$

However, adding the production A --> λ

Results in the grammar

G(L)=a*

Parsing Techniques

There are 2 main parsing techniques used by a compiler.

Top-Down Parsing

In Top-Down Parsing, the parser builds the derivation tree from the root(S : the starting symbol) down to the leaves(sentence).

In Simple words, the parser tries to derive the sentence using leftmost derivation.

For example, say we have this grammar :

V --> SR$

S --> + | - | λ

R --> .dN | dN.N

N --> dN | λ

Let us examine if the sentence +dd.d$ if it is derived from this grammar.

V --> **S**R$ --> +**R**$ --> +d**N**.N$ --> +dd**N**.N$

+dd.**N**$ --> +dd.d**N**$ --> +dd.d$

Major Problem. The Parser does not know which production it should select in each derivation step. We will learn how to solve these issues later in the course.

Bottom-Up Parsing

In Bottom-Up Parsing, the parser builds the derivation tree from the leaves(sentence) up to the root(S : Starting Symbol). This type of tree, built from the leaves to the root, is called a B-Tree.

In Simple words, the parser starts with the given sentence, does **reduction**(opposite of derivation) steps, until the starting symbol is reached.

Note that the string λ is present everywhere in the string, and

we can use it wherever we like. Let us follow the reduction

of the example given above.

> +dd.d$ --> +ddλ.d$ --> +ddN.d$ --> +dN.d$ --> +dN.dλ$ -->

+dN.dN$ --> +dN.N$ --> +R$ --> SR$ --> V Which means that the

sentence is in the grammar.

Note that we can run into deadlocks here. say we took this path instead :

> +dd.d$ --> +ddλ.d$ --> +ddN.d$ --> +dN.d$ --> +dN.dλ$ --> +dN.dN$ -->

+dNR$ --> +NR$ --> SNR$ --> Deadlock This technique also has a major

problem : Which substring should we select to reduce in each reduction

step?

how do we solve this?

The FIRST() Function

Given a string $\alpha \in V^*$, then

$FIRST(\alpha) = \{ a \mid \alpha \text{ --*--> } aw, a \in V_T, w \in V^* \}$

in addition, if α --> λ, then we add λ to FIRST(α), that is $\lambda \in FIRST(\alpha)$.

That is to say, FIRST(α) = Set of all terminals that may begin strings derived from α.

For example

α --*--> cBx

α --*--> ayD

α --*--> ab

α -----> ddd

Then

$FIRST(\alpha) = \{c,a,d\}$

Assume as well that

α --*--> λ

then

$FIRST(\alpha) = \{c,a,d,\lambda\}$

That is to say, **λ appears in the FIRST() function**.

The FOLLOW() Function

We define the FOLLOW() function for **only** non-terminals. That is to say

$FOLLOW(A), A \in V_N$, then

$FOLLOW(A) = \{ a \mid S \text{ --*--> } u A \beta, \text{where } a \in FIRST(\beta) \}$

That is, $S \text{ --*--> } uA\beta$, $u \in VT^*$, $A \in V_N$, $\beta \in V^*$ and $FOLLOW(A)=FIRST(\beta)$

That is to say, FOLLOW(A) = The set of all terminals that may appear after A in the any derivation.

S --*--> aaXdd

S --*--> Xa

S --*--> BXc

Then

FOLLOW(X) = {d,a,c}

Rules To Compute FIRST() and FOLLOW() Sets

1. FIRST(λ) = {λ}.
2. FIRST(a) = { a }.
3. FIRST(aα)= {a}.
4. FIRST(XY) = FIRST(FIRST(X).FIRST(Y)) **OR**
   FIRST(X.FIRST(Y)) **OR**
   FIRST(FIRST(X).Y).
5. Given the production A --> αXβ, Then :
   a. FIRST(β) ⊂ FOLLOW(X) if β ≠ λ.
   b. FOLLOW(A) ⊂ FOLLOW(X) if β = λ

Note that the FIRST() and FOLLOW() sets are made of **terminals only**

Notes :

1. λ **may** appear in FIRST() but it doesn't appear in FOLLOW(). We will see this when we define augmented grammars.
2. Generally, we start computing the FIRST() from bottom to top, But FOLLOW() from top to bottom.
3. When we compute FOLLOW(X), we search for X in the right side of any production.

Augmented Grammars

Given the grammar G=(V_N,V_T,S,P), then the augmented grammar G`=($V_N$`,V_T`,S`,P`) can be obtained from G as follows:

1. V_N` = $V_N$ ∪ {S`}.
2. V_T` = V_T ∪ { $ }.
3. S` = new starting point.
4. P = P U {S' -->S$}

For example :

E --> E + T | T

T --> T * F | F

F --> (E) | a

Becomes :

G --> E$

E --> E + T | T

T --> T * F | F

F --> (E) | a

This is because we want to create a FOLLOW() set for S.

Example 1 :

S` --> S$

S --> AB

A --> a | λ

B --> b | λ

Let us compute the FIRST() sets for this grammar :

FIRST(A) = {a,λ}

FIRST(B) = {b,λ}

FIRST(S) = FIRST(AB) = FIRST(FIRST(A).FIRST(B))

= FIRST({a,λ}.{b,λ})

= FIRST({ab,a,b,λ})

= {a,b,λ}

FIRST(S`) = FIRST(S$) = FIRST(FIRST(S).FIRST($))

= FIRST({a,b,λ}.$)= FIRST(a$,b$,$)

= {a,b,$}

Now Let us compute the FOLLOW() sets for this grammar :

FOLLOW(S) = {$}

FOLLOW(A) = {b,$}

FOLLOW(B) = {$}

Example 2 :

S` --> S$

S --> aAcb

S --> Abc

A --> b | c | λ

Let us take the FIRST() for this grammar :

FIRST(A) = {b,c,λ}

FIRST(S) = FIRST(aAcb)∪FIRST(Abc) = {a,} ∪{b,c}

= {a,b,c}

FIRST(S`) = FIRST(S$) = FIRST(FIRST(S).FIRST($))

= FIRST({a,b,c}.{$})

= {a,b,c}

Now let us take the FOLLOW() :

FOLLOW(S) = {$}

FOLLOW(A = {c,b}

Example 3:

G --> E$

E --> E + T | T

T --> T * F | F

F --> (E) | a

Let us calculate FIRST() :

FIRST(F) = {(,a}

FIRST(T) = FIRST(T* *F)∪FIRST(F) = FIRST(T** F)∪{(,a}

= {(,a} (Because every T will eventually become an F)

FIRST(E) = FIRST(E + T) ∪ FIRST(T) = {(,a} ∪ {(,a}

= {(,a}

FIRST(G) = FIRST(E$) = {(,a}

Now let us Calculate FOLLOW() :

FOLLOW(E) = {$,+,)}

FOLLOW(T) = FOLLOW(E) ∪ {*} = {$,+,*,)}

FOLLOW(F) = FOLLOW(T) = {$,+,*,)}

But what makes all this so important?

Well, All of the parsing techniques we are going to depend will heavily on FIRST() and FOLLOW().

Extended BNF Notation

So far, we have been using **BNF Notation**(Production rules) to express languages.

However, there is another form to

Express a language, which is **Extended BNF Notation**

if there is repetition in the grammar, say in the example of the grammar

E --> E + T | T

T --> T * F | F

F --> (E) | a

which can give us a derivation in the form of

E --> **E** + T --> **E** + T + T --> **E** + T + T + T --> T + T + T + T....+T

or in the same line,

T --> T* *F* --> T* F* F --> T* F*F* F --> T* F*F*F.... F

We can express this grammar as :

E --> T { + T }

T --> F { * F }

F --> (E) | a

We know that [x] means that we take x 0 or 1 time only.
However, { x } means we take x zero or any number of times. This is equivalent to (x)*
We can also express this grammar as:

E --> T (+ T)*
T --> F (* F)*
F --> (E) | a

Syntax Diagrams

Another way to express languages are **Syntax Diagrams**. These are used only with Extended-BNF notation.
A square shape represents a nonterminal and an oval shape represents a terminal.

DRAW

Recursive Descent Parsing

Recursive Descent Parsing is very simple. It works like this :

Divide the grammar into primitive/simple components

1- For the token "a" :

```
If(token == "a")
        get-next()
else
        report-error()
```

2- For $X = \alpha_1 \alpha_2 \ldots \alpha_n$:

Code(X):

```
{       Code(α1);
        Code(α2);
        .
        .
        Code(αn);
}
```

3- For $X = \alpha_1 \mid \alpha_2 \mid \ldots \mid \alpha_n$, If none of the α_i's = λ

Code(X):

```
{       If (token ϵ FIRST(α1))
            Code(α1);
        Else
                If (token ϵ FIRST(α2))
                    Code(α2);
                Else
        .
        .
                Else
                        If (token ϵ FIRST(αn))
                                Code(αn);
                        Else
                                Report-error();
}
```

4- For $X = \alpha_1 \mid \alpha_2 \mid \ldots \mid \alpha_n = \lambda$, If one of the α_i's = λ, say $\alpha_n = \lambda$

Code(X):

```
{       If (token ϵ FIRST(α1))
            Code(α1);
        Else
                If (token ϵ FIRST(α2))
                    Code(α2);
                Else
        .
        .
                Else
                        If (token ϵ FIRST(αn-1))
                                Code(αn-1);
                        Else
                                If (token  is not ϵ FOLLOW(X))
                                        Report-error();
}
```

5- For $X = \alpha^*$

Code(X):

```
While (token ϵ FIRST(α))
        Code(α);
```

Notes :

1. Every nonterminal has a code(a function).
2. S` in augmented grammar is represented by the function "main".
3. We only start with calling "get-token" in function "main".

Example:

G --> E$
E --> T(+ T)*
T --> F(* F)*
F --> (E) | a

```
main(){      //represents G
get-token;
call E();
if(token!="$")
        Error;
else
        Successful parsing;
}

function E(){                // E → T (+ T)*
call T ();
while(token == "+"){
get-token();
call T ()
}
}

function T(){       //T--> F (* F)*
call F ();
while(token == "*"){
get-token();
call F ();
}
}

function F(){       //F--> ( E )  |  a
if(token == "(")
{
        get-token();
        call E();
        if(token == ")")
                get-token();
        else
                ERROR;
}
Else
        if(token=="a")
                get-token();
        else
                ERROR;
}
```

Note that ERROR is a function we should write.

Example:

Given the grammar :
Program --> body .
body --> Begin stmt (; stmt)* End
stmt --> Read | Write | body | λ

and
V_N = { Program, body, stmt}
V_T = { ., Begin, ;, End, Read, Write}

examples of programs of this language would be:

```
Begin
        Read;
        Write;
        Read;
        Write;
End.
```

Or

```
Begin
        Read;
End.
```

Or

```
Begin
        Read;
        Begin
                Read;
                Write;
        End.
        Write;
End.
```

Or

```
Begin;
;
```

;
;
End.

Let us write the recursive descent code for this programming language.

```
main(){
get-token();
call body();
if(token != "."){
        ERROR;
}
Else   {
        SUCCESS;
        }
}

function body()
{ if(token == "Begin")
        {get-token();
        call stmt();
        while(token ==";")
                { get-token();
                  call stmt();
                }
        if(token == "End")
                get-token();
        else
                ERROR;
        }
else
        ERROR;
}

function stmt()
{
if(token == "Read")
        get-token();
else  if (token == "Write")
                get-token();
        else if(token == "Begin")
                        call body();
                else
                        if(token != ";" || token != "End" )
                                ERROR();
}
```

LL(1) Parsing

This Parsing method is a **table-driven** parsing method. The LL(1) parsing table selects which production to choose for the next derivation step.

Formal Definition of LL(1)

The Formal definition of LL(1) grammars is given by :

DEF: Given the Productions :

$A \rightarrow \alpha_1$

$A \rightarrow \alpha_2$

$A \rightarrow \alpha_3$

.

.

$A \rightarrow \alpha_n$

then the grammar is LL(1) if :

1. $FIRST(\alpha_i) \cap FIRST(\alpha_J) = \emptyset$ for all i,j if none of the $\alpha_i = \lambda$
2. if one of α_i is λ, $\alpha_n = \lambda$, in addition to 1

   $FIRST(\alpha_i) \cap follow(A) = \emptyset$, for , $\forall$ i < n

For example, Given the grammar :

S' --> S$

S --> aABC

A --> a | bbD

B --> a | λ

C --> b | λ

D --> c | λ

let us see if it is LL(1)

$FIRST(a) \cap FIRST(bbD) = \emptyset$

$FIRST(a) \cap FOLLOW(B) = \emptyset$

$FIRST(b) \cap FOLLOW(C) = \emptyset$

$FIRST(c) \cap FOLLOW(D) = \emptyset$

Then this grammar is LL(1).

Given another Grammar :

S` --> S$

S --> aAa | λ

A --> abS | λ

FIRST(aAa)∩FOLLOW(S) = {a}∩{$,a} = {a} ≠ ∅

This grammar is **not** LL(1).

LL(1) Parsing Table Building Algorithm

Let us assume that we have a grammar that is LL(1). How do we build the LL(1) parsing table?

1. For each production A --> α in the grammar G,
   Add to the table entry T[A,a] the production A --> α, where a ∈ FIRST(α)
   If λ ∈ FIRST(α), Add to the table entry T[A,b] the production A --> α ,
   ∀ b ∈ FOLLOW(A).
2. All Remaining Entries are Error Entries.

For **example**, given the grammar :

V --> SR $$_1$$

S --> +$$_2$$ | -$$_3$$ | λ$$_4$$

R --> dN.N$$_5$$ | .dN$$_6$$

N --> dN$$_7$$| λ$$_8$$

note that the superscript denotes the production number.

FIRST(SR $) = {+,-,d,.}

FIRST(+) = {+}

FOLLOW(S) = {d,.}

FIRST(R) = {d, .}

FIRST(d) = { d }

FOLLOW(N) = {d, $}

V_N \ V_T	+	-	d	.	$
V	1	1	1	1	
S	2	3	4	4	
R			5	6	
N			7	8	8

There should be no conflict(multiple entries) in the LL(1) table.
L(G) of this grammar = all floating point numbers.

The parser works like this

Stack	Remaining Input	Action
V	-dd.d$	Production 1
SR$	-dd.d$	Production 3
-R$	-dd.d$	Pop & advance input
R$	dd.d$	Production 5
dN.N$	dd.d$	Pop & advance input
N.N$	d.d$	Production 7
dN.N$	d.d$	Pop & advance input
N.N$	.d$	Production 8
.N$	.d$	Pop & advance input
N$	d$	Production 7
dN$	d$	Pop & advance
N$	$	Production 8
$	$	Pop and Advance
λ	λ	Accept

If at any point the parser reaches a place where the input and the stack have 2 different terminal symbols, it throws a syntax error.

Let us Take another example. Let the Grammar be :

program --> block $ 1

block --> { decls stmts } 2

decls --> **D** ; decls 3 | λ 4

stmts --> statement ; stmts 5 | λ 6

statement --> **if** 7 **| while** 8 **| ass** 9 **| scan** 10 **| print**11 | block 12 | λ13

V_T = {$,{,},D,;,if,while,ass,scan,print}

$V_N \backslash V_T$	if	while	ass	scan	print	{	}	D	;	$
Program						1				
block						2				
decls	4	4	4	4	4	4	4	3	4	
stmts	5	5	5	5	5	5	6		5	
statement	7	8	9	10	11	12			13	

Another example is the If..else statement with a delimiter. the grammar looks like this :

S` --> S$
S --> iCSE
E --> eS | λ
S --> a
C --> c

$V_N \mid V_T$	i	a	e	c	$
S `	1	1			
S	2	5			
E			3,4		4
C				6	

There is a conflict. To solve this, we can add a delimiter.

S` --> S$

S --> iCSEd

E --> eS | λ

S --> a

C --> c

$V_N \backslash V_T$	i	a	e	c	d	$
S `	**1**	**1**				
S	**2**	**5**				
E			**3**		**4**	
C				**6**		

The grammar is now unambiguous.

Alternatively, we can just remove out the production 4 in the conflict entry from the LL(1) table.

The New table is:

$V_N \backslash V_T$	i	a	e	c	d	$
S `	**1**	**1**				
S	**2**	**5**				
E			**3**		**4**	
C				**6**		

Note:

- if a grammar is LL(1), then it is unambiguous. However, the opposite is not necessarily true.
- Another thing to note is that in Top-Down parsing, we should avoid a grammar that is not LL(1).

Problems with Topdown parsing;

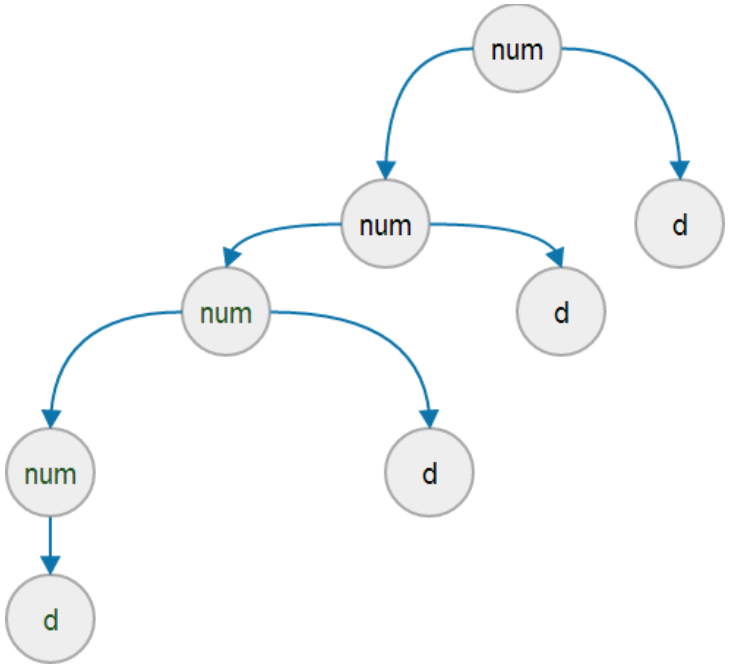
Ambiguity

Given the following grammar :

num --> num d

num --> d

Let us draw the derivation tree for the sentence **dddd**



Question : is there another derivation tree that represents the sentence? The answer is **no**.

If there is only one derivation tree representing the sentence, this means there is only one way to derive the sentence. Based on this, we can say that :

Def: A Grammar G is said to be ambiguous if there is at least one sentence with more than one derivation tree. That is, there are more than one way to derive the sentence.

This means that our algorithm is **non-deterministic**.

EX: given the grammar:

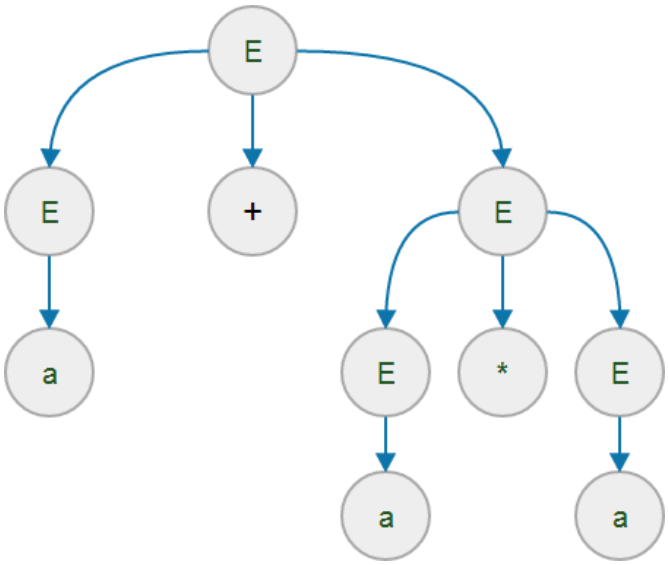
E --> E + E

E --> E * E

E --> (E) | a

Take the sentence : a + a * a

Let us draw the derivation tree



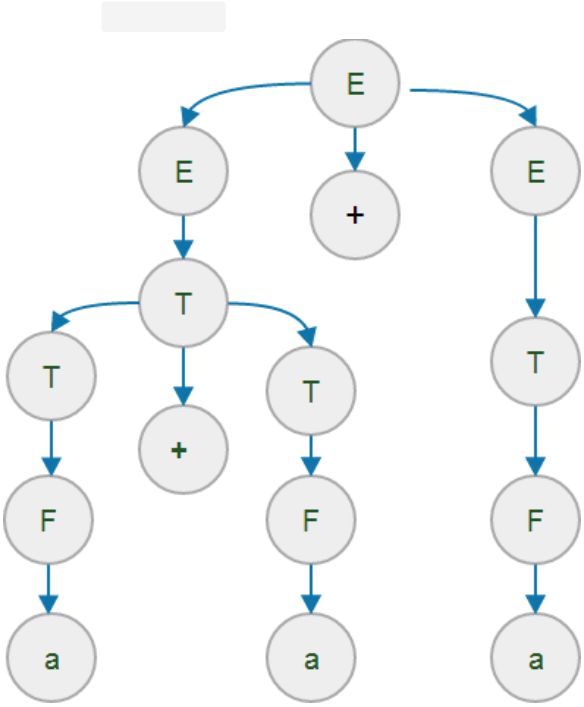
Due to the fact that we have 2 trees that give the same result, we can say that this grammar is ambiguous. In this case, to enforce the associativity rule, this grammar can be re-written as :

E --> E + E | T

T --> T*T | F

F--> (E) | a

Now, Take the sentence `a + a * a` and find the derivation tree now.



There is only possible derivation trees now. This solves the associativity problem with + and * of the grammar before with the operations.

But Let us say we have the sentence : a + a + a

Let us try to find the derivation tree and any alternative trees.

We can see here that there is more than 1 derivation tree, and the language is still ambiguous.

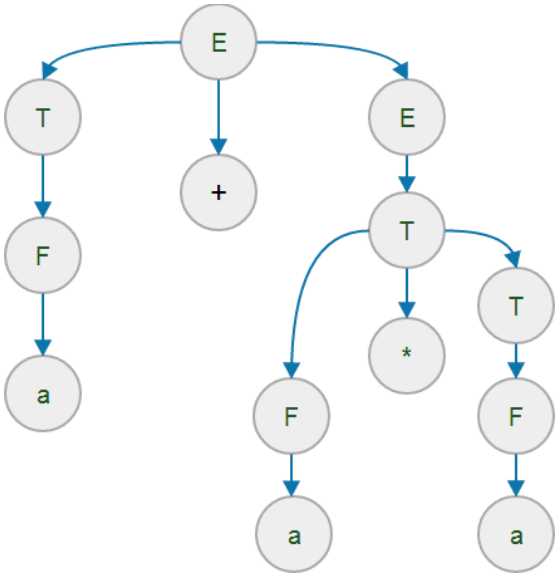
We can solve this if we rewrite the grammar with the **left-associative rule**

E --> E + T | T
T --> T * F | F
F --> (E) | a

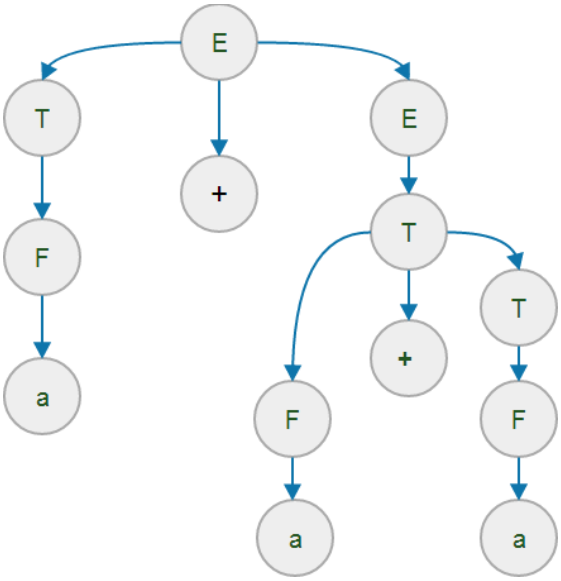
The grammar now is left-associative. This grammar solves the problems of :

- ambiguity.
- precedence.
- associativity.

Let us try rewriting it with the **right-associative rule**
E --> T + E | T
T --> F * T | F
F --> (E) | a
Let us try creating the derivation tree of a + a * a



Now let us check the derivation tree of a + a + a



This new grammar is not ambiguous, however, however it does not solve the fact That associativity issue according to our standard.

left-recursive Grammar

This causes problems when it comes to Top-down parsing techniques(see why later).

Def: A grammar is said to be left recursive if there is a production of the form:

$A \rightarrow A\alpha$

Conversely, a grammar is right-recursive if there is a production of the form:

$A \rightarrow \alpha A$

which causes no problems in top-down parsing.

The solution is to **transform** the grammar to a grammar which is not left-recursive.

Algorithm.

Given that:

$A \rightarrow A\alpha_1 \mid A\alpha_2 \mid \ldots\ldots \mid A\alpha_n$

$A \rightarrow \beta_1 \mid \beta_2 \mid \ldots\ldots \mid \beta_m$

To do this, we must introduce a new non-terminal, say A`.

The grammar now becomes :

$A \rightarrow \beta_1 A` \mid \beta_2 A` \mid \ldots\ldots \mid \beta_m A`$

$A` \rightarrow \alpha_1 A` \mid \alpha_2 A` \mid \ldots\ldots \mid \alpha_n A` \mid \lambda$

For example, say we have

$A \rightarrow Ab$

$A \rightarrow a$

$L(G) = ab^*$

Then according to the above

$A \rightarrow aA`$

$A` \rightarrow bA` \mid \lambda$

which results in the same grammar.

Let us apply this to the grammar :

E --> E + T | T

T --> T * F | F

F --> (E) | a

Then the new grammar :

E --> T E`

E` --> + T E` | λ

T --> F T`

T` --> * F T` | λ

F --> (E) | a

This grammar is now **perfect**. It solves all our ambiguity issues, and this is a grammar we can use to construct the production rules for our programming language.

Another ambiguity in programming languages is the **if...else** statement.

stmt --> if-stmt | while-stmt | ....
if-stmt --> IF condition stmt
if-stmt --> IF condition stmt ELSE stmt
condition --> C
stmt --> S

This grammar is **ambiguous**.

Let us take the following nested i**f...else** statement :

IF C
IF C
S
ELSE
S

This statement results in 2 derivations trees.

Draw the two derivation Trees

The first results in the ELSE belonging to the first IF , while the second results in the ELSE belonging to the second IF .
The second tree is the correct one since we know that the ELSE statement follows the nearest IF .

But how can the compiler behaves in this case?
There are a bunch of solutions to this problem:

1. Add a delimiter to the IF statement, such as ENDIF or END or FI to the end of the statement, resulting in these
productions :

if-stmt --> IF condition stmt **ENDIF**
if-stmt --> IF condition stmt ELSE stmt **ENDIF**
Resulting in this statement :
```
IF C
   IF C
      S
   ELSE
      S
   ENDIF
ENDIF
```
The grammar is now unambiguous, since we have to clearly state whenan ```IF``` statement ends.
However, this is not a pretty solution,
and is extra work for both the programmer and compiler, and result in less readable code.

2. Make the compiler **always** prefers to shift the ELSE when it sees the ELSE in the source code.

Left Factoring

Consider the productions :

A --> αβ

A --> αγ

Note how the **first part** of the productions is the same. This grammar can be transformed by introducing a new non-terminal B,

So what happens now is:

A --> αB

B --> βγ

For our grammar, this results in

if-stmt --> IF condition stmt

if-stmt --> IF condition stmt ELSE stmt

becomes:

if-stmt --> IF conditon stmt else-part

else-part --> ELSE stmt | λ

Does this solve the ambiguity? No, but it helps in removing choices, since the if-stmt is now one production. If we look at the

statement :

IF C
  IF C
    S
  ELSE
    S

It still has 2 derivation trees

Bottom-Up Parsing

Recall that in Bottom-Up parsing, the parser starts from the given sentence, applying reductions until it reaches the starting symbol of the grammar or a deadlock.

The **major problem** with Bottom-Up parsing is which substring we should select in each reduction step.

The answer to the above question is :
In each reduction step, we select what is called **the handle**.

DEF : The Handle is obtained by a **rightmost** derivation **in reverse.**

For **example**, Given the grammar :

V --> S R $
S --> +|-|λ
R --> .dN | dN.N
N --> dN | λ

and the sentence
-dd.d$

First, we derive the sentence rightmost.
V --rm--> SR$ --rm--> SdN.N$ --rm--> SdN.dN$ --rm--> SdN.d$ --rm--> SddN.d$ --rm--> Sdd.d$ --rm--> -dd.d$

So our handles would be :

V <-- **SR$** <-- S**dN.N**$<-- SdN.**dN**$ <-- SdN.d**λ**$ <-- Sd**dN**.d$ <-- Sdd**λ**.d$ <-- **-**dd.d$

But Compilers do not work like this. We already derived the sentence, why would we go back and do it again?

We could not build a Bottom-Up parser for every Context-Free Grammar. However, we are fortunate enough that there exist subsets of the Context-Free Grammar for which we can build a deterministic Bottom-Up parser i.e. the parser can determine/decide precisely where the handle is in each reduction step.
some of these subsets are:

LR Parsers :

- SLR(Simple-LR).
- LALR(Look-Ahead LR).
- LR.

Operator Precedence.

We will only be talking about the LR parsers, just to get an idea of how Bottom-Up parsing works.

SLR Parsing

SLR parsing, and LR parsing in general, is a table driven parsing method.
All LL(1) grammars are a subset of SLR grammars.

All LR parsers contains :

1. A parsing table.
2. A stack.
3. The input string.

As a reminder, the LL(1) parser contains :
1. A parsing table.
2. A stack.
3. The input string.

However, the way we build it is different.

Building the SLR Parsing Table

Def: An LR(0) item of a grammar G is a production in G with a dot(.) at some position in the right side.

For example, the production

A --> aBY

This production generates the following LR(0) items :

A --> .aBY

A --> a.BY

A --> aB.Y

A --> aBY. --> complete item

Note that for A --> λ, this generates only A --> λ. .

Generally speaking, if the right side of the production is of length n, then there are n+1 LR(0) items.

The LR(0) item

A --> aB.Y

Means that the parser has scanned on the input a string derived from aB and expects to see a string derived from Y .

We need to define the following 2 functions.

The CLOSURE function

```
function CLOSURE(I)      //  I is a set of LR(0) items
{       Repeat
           For (every LR(0) item A-->α.Bβ in I, and for every production B--> δ in G,
                Add the LR(0) item B-->.δ to I)  //  B belong to V_N , that is B in noneterminal
        Until no more items to be added;
}
```

Let us apply this to our grammar :

(1) E --> E+T

(2) E --> T

(3) T --> T*F

(4) T --> F

(5) F --> (E)

(6) F --> a

This grammar is not LL(1) because

FIRST(T*F)∩FIRST(F) = {(,a} ≠ ∅

We will need to build the **LR(0) sets of items**.

we start with : I_0: E` --> .E

CLOSURE(I_0)

I_0: E`-->.E

E-->.E+T

E-->.T

T--> .T*F

T-->.F

F-->.(E)

F-->.a

The GOTO function

function GOTO(I,X) =

CLOSURE(all items A-->αX.β Where A-->α.Xβ in I)

Let us apply this to the grammar above. groups:

I1 : E-->.E, E-->.E+T

I2 : E-->.T, T--> .T*F

I3 : T-->.F

I4 : F-->.(E)

I5 :F-->.a

and take the CLOSURE for all these sets. The resultant is :

I_0:	E`-->.E	I1
	E-->.E+T	I1
	E-->.T	I2
	T--> .T*F	I2
	T-->.F	I3
	F-->.(E)	I4
	F-->.a	I5
I_1 :	E'-->E.	C
	E-->E.+T	I6
I_2 :	E-->T.	C
	T--> T.*F	I7
I_3 :	T-->F.	C
I_4 :	F-->(.E)	I8
	E --> .E+T	I8
	E-->.T	I2
	T-->.T*F	I2
	T-->.F	I3
	F-->.(E)	I4
	F-->.a	I5
I_5 :	F-->a.	C
I_6 :	E-->E+.T	I9
	T-->.T*F	I9
	T-->.F	I3
	F-->.(E)	I4
	F-->.a	I5
I_7 :	T-->T*.F	I10
	F-->.(E)	I4
	F-->.a	I5
I_8 :	F-->(E.)	I11
	E-->E.+T	I6
I_9 :	E --> E+T.	C
	T --> T.*F	I7
I_{10} :	T --> T*F.	C
I_{11} :	F --> (E).	C

Constructing the SLR table

Input : LR(0) sets of items

Output : SLR(1) parsing table

1. For every item A-->α.aB in I_i, a ∈ V_T , and GOTO(I_i,a)=I_j, then set: ACTION[i,a]=Sj(shift and push j on top stack).
2. For item A-->α.(complete item) in I_i, ACTION[i,b]=Reduce by A-->α FOR ALL b ∈ FOLLOW(A).
3. For S` --> S. in Ii, ACTION[i,$] = Accept.
4. If GOTO(Ii,A) = Ij then set, GOTO(i,A) = j.
5. All remaining entries are error entries.

Let us apply this to the example above and generate the table

	ACTION						GOTO		
State/ token	**a**	**+**	*****	**(**	**)**	**$**	**E**	**T**	**F**
0	S5			S4			1	2	3
1		S6				A			
2		R2	S7		R2	R2			
3		R4	R4		R4	R4			
4	S5			S4			8	2	3
5		R6	R6		R6	R6			
6	S5			S4				9	3
7	S5			S4					10
8		S6			S11				
9		R1		S7	R1	R1			
10		R3	R3		R3	R3			
11		R5	R5		R5	R5			

No conflict --> SLR(1) grammar

Parsing The SLR Table

E --> E+T
E --> T
T --> T*F
T --> F
F --> (E)
F --> a
Let us examine the sentence a + a $

Stack	Remaining	Action
0	a + a $	S5
0 a 5	+ a $	R6
0 F 3	+ a $	R4
0 T 2	+ a $	R2
0 E 1	+ a $	S6
0 E 1 + 6	a $	S5
0 E 1 + 6 a 5	$	R6
0 E 1 + 6 F 3	$	R4
0 E 1 + 6 T 9	$	R1
0 E 1	$	Accept

LR Parsing Techniques

The main difference between LR and SLR is the CLOSURE function,

function CLOSURE(I) //I is a set of LR(1)items
{ Repeat
for(every LR(1) item [A-->α.Bβ, a] in I, and for every production B--> δ in G,
Add the LR(1)item [B-->. δ, b] where b belongs to FIRST(βa)to I)
Until no more items to be added;
}

Where An LR(1) item is an LR(0) item with a **Look-ahead Symbol**.

For example [A --> α.B , a] where a is the look-ahead. The look-ahead symbol "a" has no effect whatsoever on an item **[A --> α. β,a] β ≠ λ.**

(not complete item) However, if the item is a complete [A --> α.,a], this means we reduce by the production A --> α on token "a".

For example, Give the grammar :

S' --> S
(1) S --> CC
(2) C --> cC
(3) C --> d

I_0:

S` --> .S	$	I_1
S --> .CC	$	I_2
C --> .cC	c,d	I_3
C --> .d	c,d	I_4

I1:

S` --> S.	$	Accept

I2:

S --> C.C	$	I_5
C --> .cC	$	I_6
C --> .d	$	I_7

I3:

C --> c.C	c,d	I_8
C --> .cC	c,d	I_3
C --> .d	c,d	I_4

I4:

	C --> d.	c,d	Complete

I5 :

	S --> CC.	$	Complete

I6 :

	C --> c.C	$	I_9
	C --> .cC	$	I_6
	C --> .d	$	I_7

I7 :

	C --> d.	$	Complete

I8 :

	C --> cC.	c,d	Complete

I9 :

	C --> cC.	$	Complete

The above are the LR(1) sets of items

	ACTION			GOTO	
V	**c**	**d**	**$**	**S**	**C**
0	**S3**	**S4**		**1**	**2**
1			**A**		
2	**S6**	**S7**			**5**
3	**S3**	**S4**			**8**
4	**R3**	**R3**			
5			**R1**		
6	**S6**	**S7**			**9**
7			**R3**		
8	**R2**	**R2**			
9			**R2**		

No conflict, the grammar is an LR grammar.

If we look at the above example, we can see that some sets of items have the same core items(LR(0) items), but the look-ahead is different.
For example (I7 , I4), (I3 , I6), (I8 , I9). Let us say we merge the states.

	ACTION			GOTO	
V	**c**	**d**	**$**	**S**	**C**
0	**S3**	**S4**		**1**	**2**
1			**A**		
2	**S3**	**S4**			**5**
3	**S3**	**S4**			**8**
4	**R3**	**R3**	**R3**		
5			**R1**		
8	**R2**	**R2**	**R2**		

This is now a simplified table. if the parsing table after merging has no conflicts(like in the above example), then the grammar is an LALR(1) Grammar.