

Ch.3

Time Value of money

$$FV = PV (1+i)^n \rightarrow \text{Future value (Cumulative)}$$

$i \rightarrow$ Interest rate

$n \rightarrow$ years كل سنة مبدية على التي قبلها

- * annually $i/1$ $2 \times n$ لا يكون مبدية بل مبدية على التي قبلها
- * semiannually $\rightarrow n \times 2$ $i \div 2$
- * quarterly $\rightarrow n \times 4$ $i \div 4$
- * monthly $\rightarrow n \times 12$ $i \div 12$
- * weekly $\rightarrow n \times 52$ $i \div 52$
- * daily $\rightarrow n \times 365$ $i \div 365$
- * continuously $\rightarrow$ طريقة فائدة متصلة

* Example: If Someone Places \$800 in a saving account paying 6% interest rate compounded annually, he wants to know how much money will be in the account at the end of 5 years?

$FV \rightarrow$ المبلغ المستقبلي

$PV \rightarrow$ المبلغ الحاضر

PV هو المبلغ المدفوع

$PV \rightarrow 800$
 interest rate $\rightarrow i \rightarrow 6\%$
 years $\rightarrow n \rightarrow 5$
 $FV \rightarrow ??$

$$FV = PV (1+i)^n$$

$$FV = 800 (1+0.06)^5$$

$$= 1070.58$$

PV is dis

example 2 if someone places a \$1000 in

CD paying 10% interest rate compounded annually for 3 years, what is the future value?

$$PV = 1000$$

$$i = 10\%$$

$$n = 3$$

$$FV = ?$$

$$FV = PV (1+i)^n$$

$$FV = 1000 (1+0.10)^3$$

$$FV = \$1331$$

$$\begin{aligned} * FV &= PV (1+i)^n \\ * PV &= \frac{FV}{(1+i)^n} \end{aligned}$$

example 3

if someone wishes to find the present value of 1700\$ that he will receive 8 years from now and the opportunity cost 8% find P.V

- opportunity cost
- required rate
- Discount rate
- Cost of Capital
- yield to maturity

place in this part
interest rate $\rightarrow i$

$$i = 8\%$$

$$n = 8$$

$$FV = 1700$$

$$PV = \frac{FV}{(1+i)^n}$$

$$PV = \frac{1700}{(1+0.08)^8}$$

$$PV = \$918.46$$

Final PV
interest rate

example 4 Someone has an opportunity to

receive \$300 one year from now if he can earn 6% on his investment in the normal course of events. What is the most he should be now?

وتعني انه يعني لاي يكون
 يعني يتصور $\rightarrow$ $\left[\begin{array}{l} 300 \text{ ف} \\ 300 = FV \\ 1 = n \rightarrow \text{one year from now} \end{array} \right]$
 يعني 300 بعد سنة واه PV

$$FV = \$300$$

$$r = 6\%$$

$$n = 1$$

$$PV = \frac{FV}{(1+i)^n}$$

$$PV = \frac{300}{(1+0.06)^1}$$

$$PV = \$283$$

example 5 Assume that you have \$1000 that you can deposit in any of 3 saving account for 3 years period. First bank compounds interest rate in annual periods, second compounds twice each year and third compound interest rate quarterly. all 3 bank has stated annual interest rate 4%. What is FV ?

يوجد لدينا 3 حسابات

□ First bank $\rightarrow PV = 1000$
 $i = 4\%$ annually
 $n = 3$

$$FV = PV(1+i)^n$$

$$= 1000 [1+0.04]^3$$

$$= \$1124.86$$

second Bank $\rightarrow PV = 1000$

$i = 4\%$ Semi annually (twice)

$n = 3$

لما في twice في بجزء n ب 2
وبقسم 1 على 2

$$FV = PV \left[1 + \frac{i}{2} \right]^{n \times 2}$$

$$FV = 1000 \left[1 + \frac{.04}{2} \right]^{3 \times 2}$$

$$FV = 1126.16$$

Third Bank $\rightarrow PV = 1000$

$i = 4\%$ quarterly

$n = 3$

لما في quarterly في بجزء n ب 4
وبقسم 1 على 4

$$FV = PV \left[1 + \frac{i}{4} \right]^{n \times 4}$$

$$FV = 1000 \left[1 + \frac{.04}{4} \right]^{3 \times 4}$$

$$FV = 1126.83$$

* بختيار البنك الذي يعطي أكبر FV

example 6

What is the most you would pay today for promise to repay you \$6000 at the end of 6 years if your opportunity cost is 12% an annually rates?

repay $\rightarrow FV$ لمدى

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$$PV = \frac{FV}{(1+i)^n}$$

$$PV = \frac{6000}{(1+.12)^6}$$

$$PV = 3639.7$$

[2] Annuities

Annuity → equal sequants of payment (سلسلة من الدفعات المتساوية)

ordinary

at the end of each period.
بمنتهى الفترة

due

at the beginning of each period.
لبداية الفترة

* Future Value of annuity: (ordinary)

$$FVA = CF \left[\frac{(1+r)^n - 1}{r} \right]$$

CF → Payment (الدفعات)

n → عدد السنين years

r → Interest Rate

FVA (تدعى)

[example 1] Suppose you want to invest a 1000\$
per year for 3 years at 8% interest rate
What is the FV of annuity

$$FVA = CF \left[\frac{(1+r)^n - 1}{r} \right]$$

$$FVA = 1000 \left[\frac{[1 + .08]^3 - 1}{0.08} \right]$$

$$FVA = \$3246.4$$

example 2

assume that you are planning to invest of \$5000 each year for 6 year at 10% interest rate. determine the FV of annuity?

$$CF = \$5000$$

$$n = 6$$

$$r = 10\%$$

$$FVA = CF \left[\frac{(1+r)^n - 1}{r} \right]$$

$$FVA = 5000 \left[\frac{(1+0.10)^6 - 1}{0.10} \right]$$

$$FVA = \$38578.05$$

* Future value annuity (due)

$$FVA(\text{due}) = CF \left[\frac{(1+r)^n - 1}{r} \right] * (1+r)$$

example 1

Calculate the Future Value of annuity due of each \$5000 each year for 8 years deposit at 6%?

$$CF = 5000$$

$$n = 8$$

$$r = 6\%$$

$$FVA \text{ due} = CF \left[\frac{(1+r)^n - 1}{r} \right] * (1+r)$$

$$FVA_{\text{due}} = 5000 \left[\frac{(1+0.06)^8 - 1}{0.06} \right] * (1+0.06)$$

$$FVA_{\text{due}} = \$52456.58$$

* Present Value Annuity: (ordinary)

$$PVA = \frac{CF}{r} \left[1 - \frac{1}{(1+r)^n} \right]$$

example 1 ^{PVA _{cash}} assume that we will receive a \$1000 per year beginning one year from now, of 3 years and 8% compounded interest rate?

$$PVA = \frac{1000}{0.08} \left[1 - \frac{1}{(1+0.08)^3} \right]$$
$$= 2577.126$$

example 2 What is the PV of a loan that close the payment of \$500 for 6 years in the discount rate 10% and the first payment will be make one year from now?

$$CF = 500$$
$$r = 10\%$$
$$n = 6$$

$$PVA = \frac{CF}{r} \left[1 - \frac{1}{(1+r)^n} \right]$$
$$PVA = \frac{500}{0.10} \left[1 - \frac{1}{(1+0.10)^6} \right]$$
$$PVA = 2177.63$$

* Present Value of annuity (due)

$$PVA \text{ (due)} = \frac{CF}{r} \left[1 - \frac{1}{(1+r)^n} \right] * (1+r)$$

example 1 Calculate the PV of annuity due of each 3900 each years for 4 years, assuming and the opportunity cost 10%?

$$\begin{aligned} CF &= 3900 \\ n &= 4 \\ r &= 10\% \end{aligned}$$

$$PVA \text{ due} = \frac{CF}{r} \left[1 - \frac{1}{(1+r)^n} \right] * (1+r)$$

$$PVA \text{ due} = \frac{3900}{0.10} \left[1 - \frac{1}{(1+.10)^4} \right] * (1+.10)$$

$$PVA \text{ due} = 13598.72$$

* Finding The Present Value Perpetuity

$$PV = \frac{CF}{r} \quad n \rightarrow \infty$$

example 1 Calculate the present value of each perpetuity

	(CF)	(r)
A	\$1000	6%
B	\$2000	8%

$$PV(A) = \frac{CF}{r} = \frac{1000}{0.06} = 16666.67$$

$$PV(B) = \frac{CF}{r} = \frac{2000}{0.08} = 25000$$

$$FV = P[1+r]^n$$

* Future Value of the Mixed Stream ^{في مختل}

Example 1

during her 4 year at College Someone receives the following amounts of money at the end of each year

CF ← $\begin{cases} \text{Year 1} = 100 \\ \text{Year 2} = 200 \\ \text{Year 3} = 300 \\ \text{Year 4} = 400 \end{cases}$

she deposit her money as saving account paying 6% rate of interest. How much money will she has at graduation.

$$r = 6\%$$

$$FV = PV(1+r)^n$$

$$= 100(1+0.06)^3 + 200(1+0.06)^2 + 300(1+0.06)^1 + 400(1+0.06)^0$$

$$FV = 1061.02$$

[1-3] ← بعض القوة [بعض اول سنة] كذا السن
[4-1] = 3 mixed FV Stream

$$3-1=2 \text{ سنة}$$

$$2-1=1 \text{ سنة}$$

$$1-1=0 \text{ سنة}$$

* Present Value of mixed stream

$$PV = \frac{FV}{(1+r)^n}$$

Example 1

Calculate the PV of \$5800 \$
receive at the end of year one,
\$6400 \$ receive at the end of year two,
\$8700 receive at the end of year three
opportunity cost 8%.

Year 1 = \$5800
Year 2 = \$6400
Year 3 = \$8700

$$r = 8\%$$

$$n_1 = 0$$

$$n_2 = 1$$

$$n_3 = 2$$

المزيج
فيكون عدد السنين
هكذا نبدأ من السنة
الاولى 6 واهل السنة
التيه وهكذا

$$PV = \frac{FV}{(1+r)^n}$$

$$= \frac{5800}{(1+0.08)^0} + \frac{6400}{(1+0.08)^1} + \frac{8700}{(1+0.08)^2}$$

$$= \$19184.78$$

Continuous Compounding:

$$FV = PV e^{r \cdot n}$$

$n \rightarrow$ year

$r \rightarrow$ interest rate

$e \rightarrow 2.7183$

Example 11

What is the future value at the end of 2 years of 100 \$ deposit in a saving account paying 8% annual interest rate compounded continuously?

$n \rightarrow 2$

$PV \rightarrow 100$

$e \rightarrow 2.7183$

$r = 8\%$

$$FV = PV e^{r \cdot n}$$

$$FV = 100 \cdot 2.7183^{[0.8 \cdot 2]}$$

$$FV = \$117.35$$

* Annual Percentage Rate: APR

$$[APR = r \times m \rightarrow m = \frac{APR}{r}]$$

* Effective Annual rates: EAR

$$[EAR = (1 + \frac{r}{m})^m - 1]$$

(m) $\rightarrow$ monthly [12] $m = 12$

quarterly $m = 4$

annually $m = 1$

semi-annually $m = 2$

weekly $m = 52$

daily $m = 365$

[21]

Example 1 you have recently seen a credit card advertisement stating that the annual percent rate that is 16%. If the credit card require monthly payment what is the EAR?

$$r = 16\%$$

$m \rightarrow$ monthly [12]

$$EAR = \left[1 + \frac{r}{m} \right]^m - 1$$

$$EAR = \left[1 + \frac{0.16}{12} \right]^{12} - 1$$

$$EAR = 17\%$$