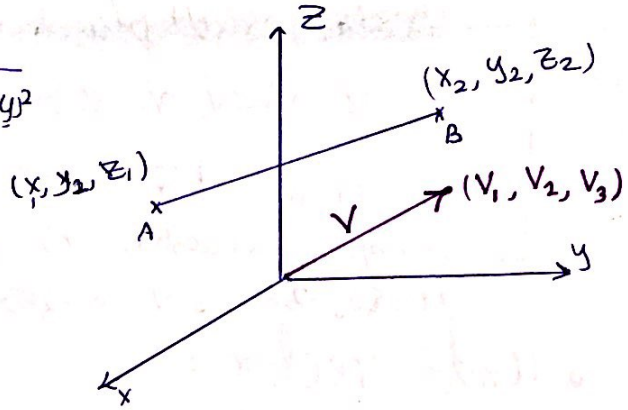
12.2 vectors

- $\overrightarrow{AB} \equiv V$ is a vector
- length = $|\overrightarrow{AB}| = \sqrt{(x_2-x_1)^2+(z_2-z_1)^2+(y_2-y_1)^2}$
  - A: initial point
  - B: terminal point



- standared position of $\vec{v}$
  - initial point at $(0,0,0)$
  - terminal point at (V_1, V_2, V_3)

- How to write vectors :-
  - Component form: $< V_1, V_2, V_3 >$
  - length of the vector: $\sqrt{(V_1)^2+(V_2)^2+(V_3)^2}$

- Midpoint M of $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is :- $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2} \right)$

- proprieties

1- **Equality of vectors** :-

$\vec{v} = \vec{w} \iff$ same length and Direction

2- **Zero vector** :-

$\vec{0} \Rightarrow |\vec{0}| = 0$ has no direction

$\Rightarrow <0, 0, 0>$

3-
- $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
- $\vec{u} + 0 = \vec{u}$
- $a(\vec{u}+\vec{v}) = a\vec{u} + a\vec{v}$
- $(a+b)\vec{w} = a\vec{w} + b\vec{w}$

- where a and b are Constants

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• Operations on vectors

→ **Addition**

if $\vec{v} = \langle v_1, v_2, v_3 \rangle$, $\vec{u} = \langle u_1, u_2, u_3 \rangle$ Then

$\vec{v} \pm \vec{u} = \langle v_1 \pm u_1 , v_2 \pm u_2 , v_3 \pm u_3 \rangle$

→ **Scalar Multiplication**

if $\vec{v} = \langle v_1, v_2, v_3 \rangle$, t is a constant (scalar)

Then $t\vec{v} = \langle tv_1, tv_2, tv_3 \rangle$

Example: Question 5 page #16:

$u = \langle 3, -2 \rangle$, $v = \langle -3, 5 \rangle \Rightarrow 2u - 3v = \langle 2\times3 - 3\times(-3), 2\times-2 - 3\times5 \rangle$

$= \langle 12, 6 \rangle$

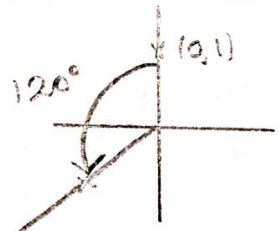
• Unit vector :-

- Def: a vector that has length of 1
- if $\vec{u}$ is a vector Then it's unit vector is $\frac{\vec{u}}{|\vec{u}|}$ ← has the same direction as $\vec{u}$

Example :- Question 15 page #16 (outline)

$(0,1) \xrightarrow[\text{counterclockwise}]{120^\circ}$

$|V| = 1$



- The new vector is :-

$\left(\frac{|V|\cos 30}{|V|}, \frac{-|V|\sin 30}{|V|}\right)$ or $(|V|\cos 210, |V|\sin 210)$

The unit vector is $= \left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Note:- You can express A vector $\vec{v}$ by it's length & direction using unit vector

$\vec{v} = \frac{\vec{v}}{|\vec{v}|} |\vec{v}|$

direction ← → length

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