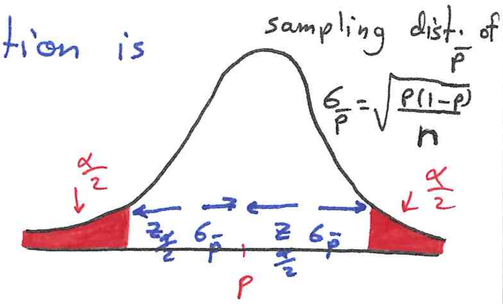
8.4 Population Proportion

- Recall that: the sample proportion $\bar{p}$ is the point estimator for the population proportion p.
- $\bar{p}$ can not be expected to provide the exact estimate of p
- An interval estimate of a population proportion is

Point estimate $\pm$ Margin of error

$$\bar{p} \pm z_{\alpha/2} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$



where • $(1-\alpha)$ is the confidence coefficient

• $z_{\alpha/2}$ is the z value providing an area of $\frac{\alpha}{2}$ in the upper tail of the standard normal distribution.

- Recall that the sampling distribution $\bar{p}$ can be approximated by a normal distribution if $np \geq 5$ and $n(1-p) \geq 5$.

Example (ϕ31 page 316) A simple random sample of 400 individuals provides 100 Yes responses.

[a] what is the point estimate of the proportion of the population that would provide Yes responses?

$$\bar{p} = \frac{100}{400} = 0.25$$

[b] What is your estimate of the standard error of the proportion?

$$\sigma_{\bar{p}} = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = \sqrt{\frac{0.25 \times 0.75}{400}} = 0.0217$$

[c] Compute the 95% confidence interval for the population proportion?

$$\bar{p} \pm z_{0.025}\, \sigma_{\bar{p}} = 0.25 \pm (1.96)(0.0217) = 0.25 \pm 0.0424$$
$$= [0.2076, 0.2924]$$

How Large the sample size should be to obtain an estimate of a population proportion for a desired margin of error?

- Recall that the margin of error $= z_{\alpha/2} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

$$E = z_{\alpha/2} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \iff \boxed{n = \frac{z_{\alpha/2}^2\, p^*(1-p^*)}{E^2}}$$

, where p^* is the planning value for $\bar{p}$ that can be determine in the following ways:

1) use $p^* = \bar{p}$ the sample proportion as the planning value for p when data are available in sample.

2) Select a sample and estimate $\bar{p} = p^*$ as the planning value for p when there is no sample.

3) Use judgment or "best guess" for p^*

4) If 1), 2), 3) don't apply or if data are not available, then use planning value $p^* = 0.5$

Example (Q33 page 317) In a survey, the planning value for the population is $p^* = 0.35$. How Large a sample should be taken to provide a 95% confidence interval with a margin of error 0.05?

$$n = \frac{Z^2_{\alpha/2}\, p^* (1-p^*)}{E^2} = \frac{(1.96)^2 (0.35)(0.65)}{(0.05)^2} = 349.6 \approx 350$$

Example How large a sample should be taken to provid a 99% confidence interval with a margin of error 0.03 if the past data are not available for developing a planning value for p^*?

$Z_{\alpha/2} = Z_{0.005} = 2.576$

$p^* = 0.5$

$$n = \frac{Z^2_{\alpha/2}\, p^* (1-p^*)}{E^2} = \frac{(2.576)^2 (0.5)(0.5)}{(0.03)^2} = 1067.1 \approx 1068$$

The desired margin of error for estimating a population proportion is almost always ≤ 0.1

$\Rightarrow$ This provides a sample that is large enough to satisfy $np \geq 5$ and $n(1-p) \geq 5$ so that we use the normal distribution to approximate the sampling distribution of $\bar{p}$.