... the integral $\int_0^2 xe^{-x}\,dx$ using

1) Trapezoidal rule
2) Trapezoidal rule with four compositions
3) Simpson's rule
4) Simpson's rule with two compositions
5) Simpson's 3/8 rule
6) Gauss-Legendre two point rule

1) $\int_0^2 xe^{-x}\,dx = \frac{(f_0+f_1)h}{2} = \frac{(f(0)+f(2))(\cancel{2})}{\cancel{2}}$

$h=2$

$0 \quad 2$

$= (0) + 2e^{-2} = 0.27067$

2) $\int_0^2 xe^{-x}\,dx = \frac{(f(0)+f(\frac{1}{2}))(\frac{1}{2})}{2} + \frac{(f(\frac{1}{2})+f(1))(\frac{1}{2})}{2}$

$h=\frac{1}{2}$

$0 \quad \frac{1}{2} \quad 1 \quad 1.5 \quad 2$

$+ \frac{(f(1)+f(1.5))(\frac{1}{2})}{2} + \frac{(f(1.5)+f(2))(\frac{1}{2})}{2}$

$= \frac{(0+0.303265)(\frac{1}{2})}{2} + \frac{(0.303265+0.36788)(\frac{1}{2})}{2} +$

$\frac{(0.36788+0.33469)(\frac{1}{2})}{2} + \frac{(0.33469+0.27067)(\frac{1}{2})}{2}$

$h=$

2) $\int_0 xe^{-x}\,dx = \frac{(f(0)+f(\frac{1}{2}))(\frac{1}{2})}{2} + \frac{(f(\frac{1}{2})+f(1))(\frac{1}{2})}{2}$

$h=\frac{1}{2}$

$0 \quad \frac{1}{2} \quad 1 \quad 1.5 \quad 2$

$+ \frac{(f(1)+f(1.5))(\frac{1}{2})}{2} + \frac{(f(1.5)+f(2))(\frac{1}{2})}{2}$

$= \frac{(0+0.303265)(\frac{1}{2})}{2} + \frac{(0.303265+0.36788)(\frac{1}{2})}{2} +$

$\frac{(0.36788+0.33469)(\frac{1}{2})}{2} + \frac{(0.33469+0.27067)(\frac{1}{2})}{2}$

$= 0.570582$

3) $\int_0^2 xe^{-x}\,dx = \frac{(f_0+4f_1+f_2)h}{3}$

$h=1$

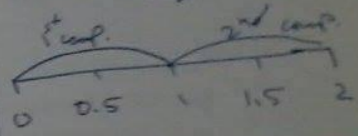
$0 \quad 1 \quad 2$

$= \frac{(f(0)+4(f(1))+f(2))(1)}{3}$

$= \frac{(0+4(0.36788)+0.27067)}{3} = 0.58073$

4) $\int_0^2 x e^{-x}\,dx$ $\cancel{=(f_0+4f_1+f_2)h}$

$h=\frac{b-a}{2M} = \frac{2}{2(2)} = \frac{1}{2}$



$= \frac{(f(0)+4f(\frac{1}{2})+f(1))(\frac{1}{2})}{3} + \frac{(f(1)+4f(1.5)+f(2))(\frac{1}{2})}{3}$

$= \frac{(0+4(0.30326)+0.36788)(\frac{1}{2})}{3}$

$+ \frac{((0.36788)+4(0.334695)+(0.27067))(\frac{1}{2})}{3}$

$= 0.263487 + 0.329555$

$\simeq 0.593042$

5) $\int_0^2 x e^{-x}\,dx = \frac{(f_0+3f_1+3f_2+f_3)3h}{8}$

$= \frac{(0+4(0.30326)+0.36788)(\frac{1}{2})}{3}$

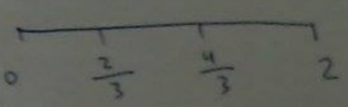
$+ \frac{((0.36788)+4(0.334695)+(0.27067))(\frac{1}{2})}{3}$

$= 0.263487 + 0.329555$

$\simeq 0.593042$

5) $\int_0^2 x e^{-x}\,dx = \frac{(f_0+3f_1+3f_2+f_3)3h}{8}$

$h=\frac{2}{3}$



$\Rightarrow \frac{(f(0)+3f(\frac{2}{3})+3f(\frac{4}{3})+f(2))\cancel{3}(\frac{2}{3})}{\cancel{8}\;4}$

~~$= 0 + 1.54025 +$~~

$= \frac{(0+0.342278+0.351463+0.27067)}{4}$

$$\Longrightarrow \frac{\left(f(0)+3f\left(\tfrac{2}{3}\right)+3f\left(\tfrac{4}{3}\right)+f(2)\right)\cancel{3}\left(\tfrac{2}{\cancel{3}}\right)}{\cancel{8}\ 4}$$

$$= \cancel{0\ \times 1.5402537}$$

$$= \frac{(0 + 0.342278 + 0.351463 + 0.27067)}{4}$$

$$= 0.2411027$$

6) $\displaystyle\int_0^2 x e^{-x}\,dx = f\left(\frac{a+b}{2} - \frac{b-a}{2}\left(\frac{1}{\sqrt{3}}\right)\right) + f\left(\frac{b+a}{2} + \frac{b-a}{2}\left(\frac{1}{\sqrt{3}}\right)\right)$

$$= f\left(1 - \frac{1}{\sqrt{3}}\right) + f\left(1 + \frac{1}{\sqrt{3}}\right)$$

$$= f(0.42265) + f(1.57735)$$

$$= 0.2769659 + 0.3257569$$

$$= 0.6027228$$

$= e^{-x}(x-2)$

$E_T(f,h) = \left| -\frac{h^2 f''(c)(4-2)}{12} \right| \le 5 \times 10^{-9}$

$= \frac{h^2 (2)}{12} |f''(c)| \le 5 \times 10^{-9}$

Max $|f''(c)|$ is at $x=0 \Longrightarrow |e^{-0}(0-2)| = |2| = 2$

$\Longrightarrow E_T(f,h) = \frac{h^2 (\cancel{2})(\cancel{2})}{\cancel{12}\,3} \le 5 \times 10^{-9}$

$= \frac{h^2}{3} \le 5 \times 10^{-9}$

$\sqrt{h^2} \le \sqrt{15 \times 10^{-9}}$

$\boxed{h \le 1.225 \times 10^{-4}}$

$h = \frac{b-a}{m} = \frac{2}{m}$ $\Longrightarrow \frac{M}{2} \ge \frac{1}{1.225 \times 10^{-4}}$

$\Longrightarrow \boxed{M \ge 16326.53}$ $\Longrightarrow$ followed.

$f^{(4)} = e^{-x}(-1 + x - 2 - 1)$

$f^{(4)} = e^{-x}(x - 4)$

$$E_s(f,h) = \left| \frac{-h^4 (b-a) f^{(4)}(c)}{180} \right| \le 5 \times 10^{-9}$$

$$\frac{h^4 (2)}{\cancel{180}\ 90} |f^{(4)}(c)| \le 5 \times 10^{-9}$$

$$\Rightarrow \frac{h^4 (4)}{90} \le 5 \times 10^{-9}$$

$$\sqrt[4]{h^4} \le \sqrt[4]{\frac{90 * 5}{4} \times 10^{-9}}$$

$$\boxed{h \le 1.83 \times 10^{-2}}$$

$$h = \frac{b-a}{2M} = \frac{2}{2M} = \frac{1}{M}$$

$$\Rightarrow \frac{1}{M} \le 1.83 \times 10^{-2}$$

$$\boxed{M \geq 54.645}$$

How many compositions M and what bandwidth h do we need to get an ac[...] for approximating $\int_0^2 xe^{-x}\,dx$ using

1) Composite Trapezoidal Rule 2) Composite Simpson's Rule

$\boxed{1.}$ $f(x) = xe^{-x}$

$f'(x) = x(-e^{-x}) + e^{-x}(1)$

$= -xe^{-x} + e^{-x}$

$f''(x) = -x(-e^{-x}) + e^{-x}(-1) + -e^{-x}$

$= xe^{-x} - e^{-x} - e^{-x}$

$= e^{-x}(x-2)$

$$E_T(f,h) = \left| \frac{-h^2 f^{2}(c)(b-a)}{12} \right| \leq 5\times10^{-9}$$

$$= \frac{h^2(2)|f^{2}(c)|}{12} \leq 5\times10^{-9}$$

$|-e^{0}(-2)| = |2| = 2$

2) $f(x) = xe^{-x}$

$f' = -xe^{-x} + e^{-x}$

$f'' = e^{-x}(x-2)$

$f^{3} = e^{-x}(1) + (x-2)(-e^{-x})$

$f^{4} = -e^{-x} + (x-2)(e^{-x}) + (-e^{-x})(1)$

$= -e^{-x} + (x-2)(e^{-x}) - e^{-x}$

$= e^{-x}(-1 + x - 2 - 1)$

$f^{4} = e^{-x}(x-4)$

$$E(f,h) = \left| \frac{-h^4(b-a)f^{4}(c)}{180} \right| \leq 5\times10^{-9}$$

$h^4(2)\,|f^{4}(c)| \leq 5\times10^{-9}$

$f^{4} = e^{-x}(x-4)$

max $|f^{4}(x)|$ at $x = 0$

$\Rightarrow |-4| = 4$