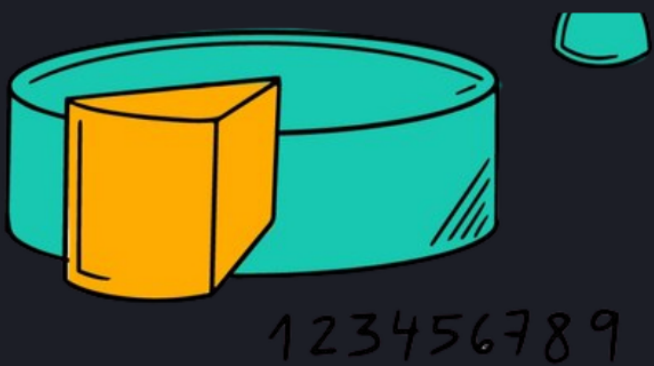
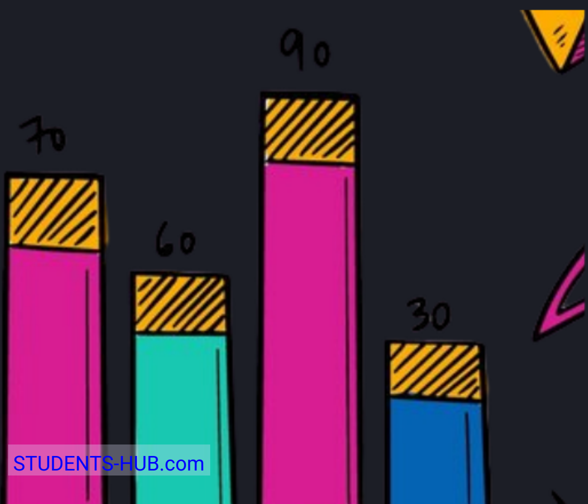
STATISTICS

By Rawan Alfares





شعاره الصبر ، وراحته التعب)

common discrete random variables

I. Binomial Distribution :-

• A Random experiment Consisting of (n) repeated trials, Such that :-

a- the trials are indep. → * التجارب بأثروش على بعض

b- Each trial results in only two possible outcomes, a Success or fail → الاحتمالات فقط بتكون إما Success أو fail

c- the Probability of Success (p) on each trial remains Constant → الاحتمال للنجاح يبقى ثابت

*** the probability mass function :-**

$$p(X=x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & , \ x = 0,1,2,\dots n \\ 0 & , \ o.w \end{cases}$$

⇒ نفهم القانون

$$p(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

n: number of trials

x: number which Successful will occurs.

p^x: probability of Success.

$(1-p)^{n-x}$: probability of fail.

* Mean Value ⇒ $\mu_x = E(x) = \underbrace{n}_{\text{number of trials}}\underbrace{p}_{\text{prob. of Success.}}$

* Variance $= \sigma_x^2 = Var(x) = \underbrace{n}_{\text{number of trials}}\underbrace{p}_{\text{Prob. of Success}}\underbrace{(1-p)}_{\text{prob. of fail.}}$

Example :- Consider the exp. of flipping a Coin 3 times. Assume $p(H) = \frac{1}{4}$, and $p(T) = \frac{3}{4}$

a. determine the prob. of getting a head for 2 times.

b. whats the prob. of getting at least one head.

c. whats the expected number of heads to be observed in the exp.

1st, we want to ensure that this example will be solved in Bionomial.

1. only two outComes.
2. prob. of Success is fixed.
3. the prob. of Success doesn't Affect on the prob of fail.

So, yes its Binomial.

a. $n = 3$, $X = 2$

$$p(X = x) = \begin{cases} \binom{3}{x} \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{3-x} & , n = 0,1,2,3 \\ 0 & , o.w \end{cases}$$

$$p(X = 2) = \binom{3}{2} \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^1$$

$$= \frac{3!}{2!(3-2)!} \cdot \frac{1}{16} \cdot \frac{3}{4} = \frac{9}{64}$$

$$p(X=2) = \begin{cases} \frac{9}{64} & , n = 0,1,2,3 \\ 0 & , o.w \end{cases}$$

* note :-

you determine the Success according to the question,
for example in this question, it requires to find prob. when head Occurs
So, we assume that occuring head is the success.

b. $p(X \geq 1) = p(X = 1) + p(X = 2) + p(X = 3)$ ⟶ time consuming.

or instead, $p(X \geq 1) = 1 - p(X = 0)$

$$p(X = 0) = \binom{3}{0} \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^3 = \frac{27}{64}$$

$$p(X \geq 1) = 1 - \frac{27}{64} = 0.578125$$

c. expected number means μ_X = mean or average.

$$\mu_X = np = 3 \cdot \frac{1}{4} = \frac{3}{4}$$

$$\sigma^2 = np(1-p) = 3 \cdot \frac{1}{4} \cdot \frac{3}{4} = \frac{9}{16}$$

Example 2: Consider the exp. of tocing a die, for 3 times.

a. what's the prob. of getting a number less than 2 for 1 time.

b. what's the prob. of getting a number divisible by 3 for 2 times.

$n=3$, $S=\{1,2,3,4,5,6\}$

a. Assume that Success is less than 2, fail greater or equals 2.

Success = $\{1\}$, fail = $\{2,3,4,5,6\}$

$$p(S) = \frac{1}{6}, \quad p(F) = \frac{5}{6}$$

$$p(X=1) = \binom{3}{1}\left(\frac{1}{6}\right)^1 \cdot \left(\frac{5}{6}\right)^2 = \frac{25}{36}$$

b. Assume that Success $S=\{3,6\}$, $F=\{1,2,4,5\}$

$n=3$, $x=2$

$$p(S) = \frac{2}{6} = \frac{1}{3}, \quad p(F) = \frac{4}{6} = \frac{2}{3}$$

$$p(X=2) = \binom{3}{2}\left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^1$$

$$= \frac{3!}{2!(3-2)!} \cdot \frac{1}{9} \cdot \frac{2}{3} = \frac{2}{9}$$

* notes:

note that it has 6 outcomes, which can't be Binomial. until we change the exp into 2 out comes. and that will be determined according to the question as you see above.

II. The Geometric Distribution

- it's a random exp. consists of infinity trials such that:-

a- the trials are indep. ⇒ * التجارب بأثروش على بعض

b- Each trial results in only two possible outcomes, a success or fail → بتكون إما Success أو fail الاحتمالات فقط

c- the probability of success (p) on each trial remains constant → الاحتمال للنجاح يبقى ثابت

※ بالعربي :- بنضل نكرر التجربة حتى تحدث أول مرة نجاح , وال pmf بيحكيلنا احتمال أن يحدث هذا النجاح في المحاولة رقم R .

* the probability mass functions:-

$$p(X=x) = \underbrace{(1-p)}_{\text{prob. of fail}}^{x-1} \underbrace{p}_{\text{prob. of success}} \quad , \quad x = 1,2,3 \ldots$$

* mean Value $= \mu_X = E(X) = \frac{1}{p}$

* Variance $= \sigma_X^2 = Var(X) = \frac{1-p}{p^2}$

EXAMPLE (3-17):
Let the probability of occurrence of a flood of magnitude greater than a critical magnitude in a given year be 0.02. Assuming that floods occur independently, determine the "return period" defined as the average number of years between floods.

μ_X = mean

الفترة بين حصول فيضانات

Sol. :- $p(S) = 0.02$

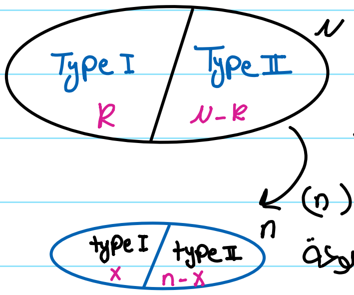
$$\mu_X = \frac{1}{p} = \frac{1}{0.02} = 50$$

⇒ مش معناها إنه كل 50 سنة بصير طوفان وإنما الأرجح إنه كل 50 سنة ممكن يصير طوفان .

III Hyper geometric distribution.

Hyper ⇒ هجين أو خليط

" يعني بتكون منه أكثر من نوع واحد.



أخذنا عينة (n) من المجموعة الكبيرة (N).

*** the probability mass function is**

$$p(X=x) = \frac{\binom{R}{x}\binom{N-R}{n-x}}{\binom{N}{n}}$$

R: number of item from type I in N

X: number of items from type I was selected in n

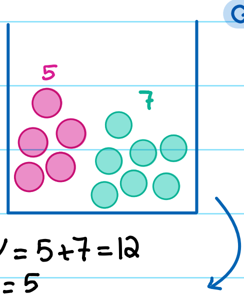
N: number of all outcomes. "العدد الكلي"

n: number of selected items. "عدد العينة"

* μ_X = mean value $= \boxed{\frac{nR}{N} = np}$

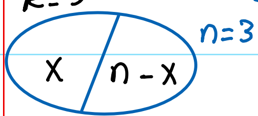
* $\boxed{\sigma_X^2 = np(1-p)\left[\frac{N-n}{N-1}\right]}$

Example 8:-



$N = 5+7 = 12$

$R = 5$



a. what's the prob. of having 2 red balls in 3 selected balls.

$x=2$

$$p(X=x) = \frac{\binom{5}{x}\binom{7}{3-x}}{\binom{12}{3}} = \begin{cases} \dfrac{\binom{5}{2}\binom{7}{1}}{\binom{12}{3}}, & x = 0,1,2,\dots \min(3,5) \\ 0, & \text{o.w} \end{cases}$$

b. what's the prob. of selecting at least one red ball.

$$p(x \geq 1) = p(x=1) + p(x=2) + p(x=3)$$

or $p(x \geq 1) = 1 - p(x=0)$

$$= 1 - \left[\frac{\binom{5}{0}\binom{7}{3}}{\binom{12}{3}}\right] = \boxed{\text{الحسبة}}$$

c. what the prob. of getting at least 4 red balls if 7 balls will be selected.

$$p(X \geq 4) = p(X=4) + p(X=5)$$
$$= \frac{\binom{5}{4}\binom{7}{3}}{\binom{12}{7}} + \frac{\binom{5}{5}\binom{7}{2}}{\binom{12}{7}}$$

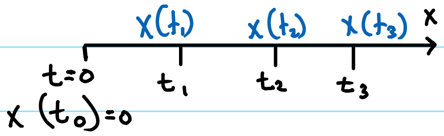
recorded lecture 18-12-2023

IV. Poisson Distribution

→ λ : avarage of occurences per unit
→ T : unite

* Counting Process

1. begin Counting at time $t=0$, $X(0)=0$
2. non over lapping time interval (لايوجد تقاطع بالفترات) and number of occurences are indep
3. prob. of occurences depends only on the length of interval



* مثلاً بدي أعرف عدد الزباين اللي بدخلوا على المحل.

* أو بدي أعرف كم عدد المكالمات عند t_2 (فنجمع كل اللي قبلها)

* the probability mass function (Pdf) :-

$$p(X=x) = e^{-b}\frac{b^x}{x!} , \quad x = 0,1,2,\dots$$, where b is positive number.

$\mu_x = E[x] = b$

$\sigma_x^2 = Var(x) = b$

$b = \lambda T$ → "وحدتها بتكون الإشي اللي بنعده"

خلالها وايحدد

هو

EXAMPLE (3-21):
Messages arrive to a computer server according to a Poisson distribution with a mean rate of 10 messages/hour.

a- What is the probability that 3 messages will arrive in one hour.
b- What is the probability that 6 messages will arrive in 30 minutes.

$\lambda = 10$ mess/hr.

a- $b = \lambda T = 10 \frac{mess}{\cancel{hr}} \times 1 \cancel{hr} = 10$ mess.

$$p(X=3) = e^{-10}\frac{(10)^3}{3!}$$

b- $b = \lambda T = 10 mess/\cancel{hr} \times 0.5 \cancel{hr} = 5$ mess

$$p(X=6) = e^{-5} \times \frac{5^6}{6!}$$

c- at least two messages will arrive in 1 hr

$b = \lambda T = 10 \times 1 = 10$

$p(X \geq 2) = 1 - p(X < 2)$

$= 1 - \left[\frac{e^{-10} \times 10^{0}}{0!} + \frac{e^{-10} \times 10^{1}}{1!} \right]$

شقوق

EXAMPLE (3-22):
The number of cracks in a section of a highway that are significant enough to require repair is assumed to follow a Poisson distribution with a mean of two cracks per mile.

a- What is the probability that there are no cracks in 5 miles of highway?
b- What is the probability that at least one crack requires repair in ½ miles of highway?
c- What is the probability that at least one crack in 5 miles of highway?

$\lambda = 2 \frac{\text{cracks}}{\text{mile}}$,

a) $T = 5$ mile , $b = \lambda T = 2 \times 5 = 10$ cracks

$p(X=0) = \frac{e^{-10} \times (10)^{0}}{0!} = e^{-10}$

b) $b = 2 \frac{\text{cracks}}{\cancel{\text{mile}}} \times 0.5 \cancel{\text{mile}} = 1$ crack

$P(X \geq 1) = 1 - p(X < 1)$

$= 1 - \left[\frac{e^{-1} \times 1^{0}}{0!} \right] = 1 - \frac{1}{e}$

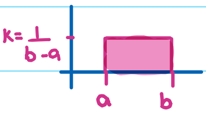
c) $b = 2 \times 5 = 10$

$p(X \geq 1) = 1 - p(X < 1)$

$= 1 - \left[\frac{e^{-10} \times (10)^{0}}{0!} \right] = 1 - e^{-10}$

Common Continuous Random Variables:

I. uniform distribution :-

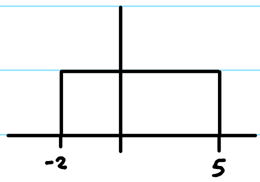


$$f_X(x) = \begin{cases} \frac{1}{b-a} & , a \leq x \leq b \\ 0 & , \text{o.w} \end{cases} \quad , \quad \mu_X = \frac{a+b}{2} \quad , \quad \sigma_X^2 = \frac{(b-a)^2}{12}$$

example :- let X be Random variable that follows uniform distribution, in the interval $[-2, 5]$.

a. write and plot pdf.

$$* \ \text{pdf} = \begin{cases} \frac{1}{5--2} = \frac{1}{7} & , -2 \leq X \leq 5 \\ 0 & , \text{o.w} \end{cases}$$



b. what the prob. that X is less than zero.

$$P(X \leq 0) = \int_{-2}^{0} \frac{1}{7} dx = \frac{1}{7}(2) = \frac{2}{7}$$

c. find μ_X and σ_X^2

$$\mu_X = \frac{b+a}{2} = \frac{5-2}{2} = \frac{3}{2}$$

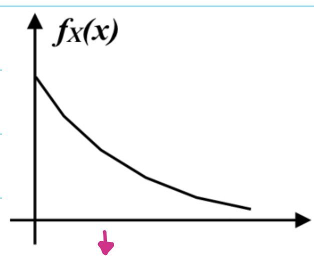
$$\sigma_X^2 = \frac{(b-a)^2}{12} = \frac{(5--2)^2}{12} = \frac{49}{12}$$

recorded Lecture
20-12-2023

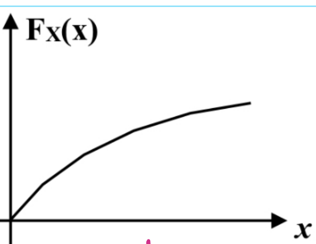
II. exponential Distribution :-

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & , x \geq 0 \\ 0 & , \text{o.w} \end{cases} \qquad \mu_X = \frac{1}{\lambda} \qquad \sigma_X^2 = \frac{1}{\lambda^2}$$

موجبة عشان تكون bounded



bounded
(exponential decaying)



un bounded
(exponential growing)

Important note!

⚠ λ in poisson distribution means average occurence per unit
However in exp. dist. it's a constant. (doesn't equal average).

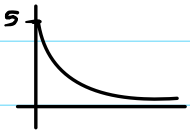
example 8- let X be an exponential random variable with a mean of 0.2

$\mu_x = 0.2$

$\lambda = \frac{1}{\mu_x} = \frac{1}{0.2} = 5$

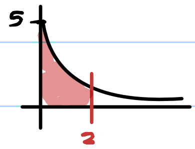
a) write and plot the pdf of X.

$$f_X(x) = \begin{cases} 5e^{-5x} , & x > 0 \\ 0 , & \text{o.w} \end{cases}$$



b) $P(X \leq 2)$

$$\int_0^2 5e^{-5x}\,dx = -e^{-x}\Big|_0^2 = 1 - e^{-2}$$



c) find σ_x^2

$$= \frac{1}{\lambda^2} = \frac{1}{(5)^2} = 0.04$$

EXAMPLE (3-24):

Suppose that the depth of water, measured in meters, behind a dam (سد) is described by an exponential random variable with pdf:

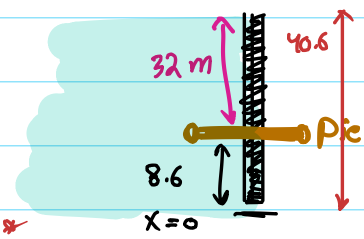
$$f_X(x) = \begin{cases} \frac{1}{13.5} e^{\frac{-x}{13.5}} & x > 0 \\ 0 & \text{o.w} \end{cases} \quad \text{random variable.}$$

There is an emergency overflow at the top of the dam that prevents the depth from exceeding 40.6 m. There is a pipe placed 32.0 m below the overflow that feeds water to a hydroelectric generator (turbine).

a- What is the probability that water is wasted though emergency overflow? → * احتمالية المياه تنسكب من فوق الجدار

b- What is the probability that water will be too low to produce power? ← يعني المياه مش راح تطلع من الماسورة.

c- Given that water is not wasted in overflow, what is the probability that the generator will have water to derive it?



a) $P(X \geq 40.6) = \int_{40.6}^{\infty} f_X(x)\,dx = -e^{\frac{-x}{13.5}}\Big|_{40.6}^{\infty}$

$$= 0 + e^{-\frac{40.6}{13.5}} = e^{-3}$$

b) $P(X \leq 8.6) = \int_0^{8.6} f_X(x)\,dx = -e^{\frac{-x}{13.5}}\Big|_0^{8.6} = 1 - e^{-5}$

c) $P(X > 8.6 / X \leq 40.6) = \frac{P(X > 8.6 \cap X < 40.6)}{P(X \leq 40.6)} = \frac{\int_{8.6}^{40.6} \frac{1}{13.5} e^{\frac{-x}{13.5}}\,dx}{\int_0^{40.6} \frac{1}{13.5} e^{\frac{-x}{13.5}}\,dx}$

$$= \frac{-e^{-3} + e^{-0.6}}{1 - e^{-3}} = 0.525$$

~> إذا ما قرّبت ولا منزلة بيطلع معك 0.504

not important ←

IV. Rayleigh Distribution :-

$f_X(x) = \frac{2}{b} x e^{\frac{-x^2}{b}} \; ; \; x \geq 0$, $F_X(x) = 1 - e^{\frac{-x^2}{b}} \; , \; x \geq 0$

$\mu_x = E(X) = \sqrt{\frac{\pi b}{4}}$, $\sigma_X^2 = Var(X) = \frac{b(4-\pi)}{4}$

not important ←

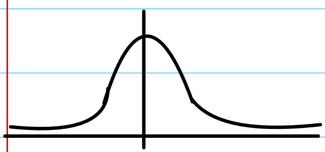
V. Cauchy Random Variable

$f_X(x) = \frac{\frac{\alpha}{\pi}}{x^2 + \alpha^2}$

$F_X(x) = \frac{1}{2} + \frac{1}{\pi} \tan^{-1}\left(\frac{x}{a}\right)$

recorded lecture 23-12-2023

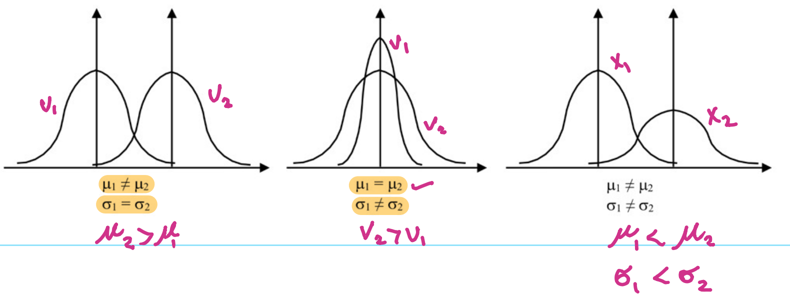
Gaussian (Normal) Distribution



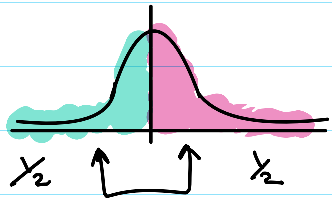
$$f_X(x) = \frac{1}{\sqrt{2\pi \sigma_x^2}} e^{\frac{-(x-\mu_x)^2}{2\sigma_x^2}} \quad , \; -\infty < x < \infty$$

$E(X) = \mu_x$, $Var = \sigma_x^2$ → تشتت البيانات حول ال mean

نقطة الارتكاز



* gausian dist. is Symmetric



جزئين متساويين كل جزء احتماليته نصف

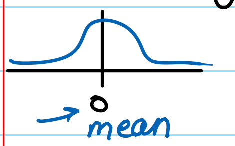
فيصير احتماليته كله = 1 ← valid pdf

Example 8- let X be a normal Random Variable with zero mean and unity Variance $[\mu_X = 0,\ \sigma_X^2 = 1]$

a) write and plot the distribution of x.

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma_X^2}} e^{-\frac{(x-\mu_X)^2}{2\sigma_X^2}} = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad , -\infty < x < \infty$$

mean, var, var



b) $P(X \leq 0) = \int_{-\infty}^{0} f_X(x)\,dx = \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} = \Phi(0)$

c) $P(X \leq 1.12)$

$$\int_{-\infty}^{1.12} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} = \Phi(1.12) = 0.8686$$

من الجدول

أول خانتين من العمود z (1.1)

ثم من الصف 0.02

Q.
* Why we don't integrated?
A. it's a numerical integration, "إشي ما أخذناه بده طريقه الخاصة"
ارتّح حالك وخذ الجاهز من الجدول ت

* بدك تنتبه عزيزي الطالب إنه الجدول فقط بيعطيك من $-\infty$ إلى z يعني $p(Z \leq z)$ فإذا أجاك سؤال $p(Z \geq z)$ طوّل بالك وتستعجلش ت

only for ~~Standard~~ normal Distribution

mean = 0, Var = 1

Important Note:-

$$\Phi(-a) = 1 - \Phi(a)$$

d) $p(X \geq 3.12) = 1 - p(X \leq 3.12)$

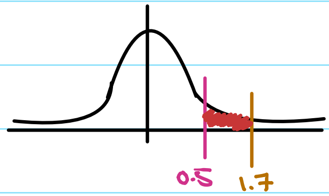
$= 1 - \phi(3.12)$

$= 1 - 0.9991 = 9 \times 10^{-4}$

e) $p(0.5 \leq X \leq 1.7)$

$= \phi(1.7) - \phi(0.5)$

$= 0.9554 -$



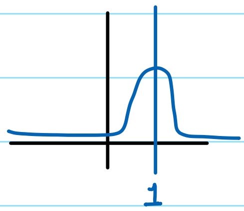
f) $p(X \leq -1.4) = 1 - \phi(1.4)$

$= 1 - 0.9192 = 0.0808$

Examples- let X be normal random variable with $\mu_x = 1$, $\sigma_x^2 = 9$

a) write and plot the pdf of x.

$$f_x(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{\frac{-(x-\mu_x)^2}{2\sigma_x^2}} = \frac{1}{3\sqrt{2\pi}} e^{\frac{-(x-1)^2}{(2)(9)}}$$



b) $p(X \leq -2) = \phi\left(\frac{-2-1}{3}\right) = \phi(-1)$

$= 1 - \phi(1)$

* السؤال هنا مابيقع نجيب الجواب
من الجدول لإنه not standard
فبالتالي لازم نحوله لـ standard
ومن ثم نكمل

$\phi\left(\frac{X-\mu_x}{\sigma_x}\right)$

d) $p(-2.3 \leq X \leq 5.3)$

$= \phi\left(\frac{5.3-1}{3}\right) - \phi\left(\frac{-2.3-1}{3}\right)$

$= \phi(1.43) - \phi(-1.1)$

$= \phi(1.43) - (1 - \phi(1.1))$

e) $F_x(-2.3) = p(X \leq -2.3)$

$= \phi\left(\frac{-2.3-1}{3}\right) = \phi(-1.1)$

$= 1 - \phi(1.1)$

f) $P = 0.6628 \rightsquigarrow 0.42 = \frac{X-1}{3}$

$X = (3)(0.42) + 1$

Normal Approximation of the Binomial and Poisson Distribution:

EXAMPLE (3-28):

Consider a binomial experiment with n = 1000 and p = 0.2. if X is the number of successes, find the probability that $X \leq 240$.

$$P(X=x) = \binom{1000}{x} (0.2)^{x} (0.8)^{1000-x}$$

$$P(X \leq 240) = \sum_{0}^{240} \binom{1000}{x} (0.2)^{x} (0.8)^{1000-x}$$

↳ we cant do this, so we will approximate it using normal dist.

$$\mu_x = np = (1000)(0.2) = 200.$$

$$\sigma_x^2 = np(1-p) = (1000)(0.2)(0.8) = 160$$

$$\Phi\left(\frac{240-200}{\sqrt{160}}\right) = \Phi(3.162)$$

EXAMPLE (3-29):

Assume the number of asbestos particles in a cm^3 of dust follow a Poisson distribution with a mean of 1000. If a cm^3 of dust is analyzed, what is the probability that less than 950 particles are found in 1 cm^3?

$$P(X=x) = \frac{e^{-b} b^{x}}{x!} \quad , \quad \mu_x = \sigma_x^2 = b = 1000 \frac{\text{abs.}}{\cancel{cm^3}} \times \cancel{cm^3} = 1000$$

$$P(X=x) = \frac{e^{-1000} (1000)^{x}}{x!}$$

$$P(X \leq 950) = \sum_{x=0}^{950} \frac{e^{-1000} (1000)^{x}}{x!}$$

$$\Phi\left(\frac{950-1000}{\sqrt{1000}}\right) = \Phi(-1.581)$$

$$= 1 - \Phi(1.581)$$

Transformation of Random Variables:

I. Discreat

EXAMPLE (3-30):

Let (X) be a binomial r.v with parameters (n = 3) and (p = 0.75). Let Y = g(x) = 2X + 3
P(Y = y) = P(X = x)

$$P(X=x) = \begin{cases} \binom{3}{x}(0.75)^{x}(0.25)^{3-x} & , x=0,1,2,3 \\ 0 & , o.w \end{cases}$$

x	$P(X=x)$	$y=2x+3$	$P(Y=y)$
0	$\binom{3}{0}(0.75)^{0}(0.25)^{3}=\frac{1}{64}$	3	$\frac{1}{64}$
1	$\binom{3}{1}(0.75)^{1}(0.25)^{2}=\frac{9}{64}$	5	$\frac{9}{64}$
2	$\binom{3}{2}(0.75)^{2}(0.25)^{1}=\frac{27}{64}$	7	$\frac{27}{64}$
3	$\binom{3}{3}(0.75)^{3}(0.25)^{0}=\frac{27}{64}$	9	$\frac{27}{64}$

$$P(Y=y) = \begin{cases} \frac{1}{64} & , y=3 \\ \frac{9}{64} & , y=5 \\ \frac{27}{64} & , y=7 \\ \frac{27}{64} & , y=9 \\ 0 & , o.w \end{cases}$$

EXAMPLE (3-31):

Let (X) has the distribution $P\{X = x\} = \frac{1}{6}$; x = -3 , -2 , -1 , 0 , 1 , 2

Define $Y = g(x) = X^2$. Find the pdf of the random variable Y.

$Y = X^2$

x	$P(X=x)$	$Y=X^2$	$P(Y=y)$
-3	1/6	9	1/6
-2	1/6	4	1/6
-1	1/6	1	1/6
0	1/6	0	1/6
1	1/6	1	1/6
2	1/6	4	1/6

* $E[Y] = E[x^2]$

* $\sigma_x^2 = E[x^2] - \mu_x^2$

$$P(Y=y) = \begin{cases} 1/6 & , y=9 \\ 1/6 + 1/6 & , y=4 \\ 1/6 + 1/6 & , y=1 \\ 1/6 & , y=0 \\ 0 & , o.w \end{cases}$$

II. Continuoes

$$f_y(y) = \frac{f_x(x)}{\left|\frac{dy}{dx}\right|}$$

EXAMPLE (3-32): μ_x σ_x^2

Let (X) be a Gaussian r.v with mean (0) variance (1).

Let Y = X^2. Find $f_Y(y)$

SOLUTION:

1) $f_x(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

2) $y' = 2x$

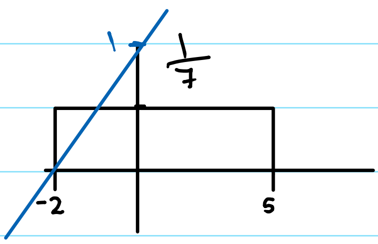
3) $x = \mp\sqrt{y}$

4) $f_y(y) = \cancel{2}\frac{f_x(x)}{\cancel{2}x} = \frac{1}{x\sqrt{2\pi}} e^{-\frac{x^2}{2}} = \frac{1}{\sqrt{2\pi y}} e^{-\frac{y}{2}}$

Cause its gaussian

---

example :- let x be a Random variable that follow a uniform Distribution over the interval [-2,5] , $y = 2x+1$, Find pdf of y ?

1 

$f_x(x) = \begin{cases} 1/7 , & -2 \leq x \leq 5 \\ 0 , & o.w \end{cases}$

2 $f_y(y) = \frac{f_x(x)}{\left|\frac{dy}{dx}\right|} = \frac{1/7}{2} = \frac{1}{14}$

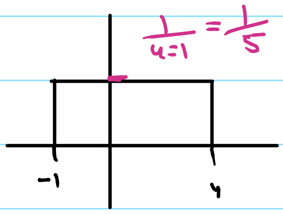
$x = \frac{y-1}{2}$

3 $\left|\frac{dy}{dx}\right| = 2$

4 $f_y(y) = \begin{cases} \frac{1}{14} , & -3 \leq x \leq 11 \\ 0 , & o.w \end{cases}$

EXAMPLE (3-33):

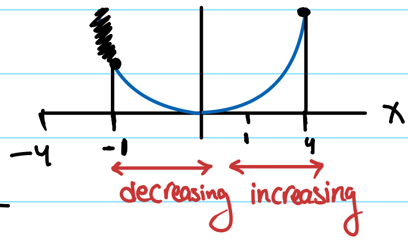
Let (X) be a uniform r.v in the interval (-1 , 4). If $Y = X^2$. Find $f_Y(y)$

① 

$$f_X(x) = \begin{cases} 1/5 & , -1 < x < 4 \\ 0 & , \text{o.w} \end{cases}$$

② $\frac{dy}{dx} = 2x$, $\rightarrow x = \mp\sqrt{y}$

③ $x = \sqrt{y}$ or $x = -\sqrt{y}$



④ $f_y(y) = \frac{f_x(x)}{\left|\frac{dy}{dx}\right|} = \frac{f(x)}{2x}$

* Case 1: $x < -4$ and $x > 4 \rightarrow f_y(y) = 0$

* Case 2: $-1 < x < 0 \rightarrow 0 < y < 1 \rightarrow f_y(y) = \left.\frac{1/5}{|2x|}\right|_{x=\sqrt{y}} + \left.\frac{1/5}{|2x|}\right|_{x=-\sqrt{y}} = \frac{2}{10\sqrt{y}} = \frac{1}{5\sqrt{y}}$

* Case 3: $1 < x < 4 \rightarrow 1 < y < 16 \rightarrow f_y(y) = \frac{1/5}{|2x|} = \frac{1}{10\sqrt{y}}$

$$f_y(y) = \begin{cases} \frac{1}{5\sqrt{y}} & , 0 \leq y \leq 1 \\ \frac{1}{10\sqrt{y}} & , 1 < y < 16 \\ 0 & , \text{o.w} \end{cases}$$

Notes

Y is Gaussian with mean ($\mu_Y = a\mu_X + b$) and variance ($\sigma_Y^2 = a^2\sigma_X^2$)

(شعاره الصبر ، وراحته التعب)

ربنا تقبل منا إنك
أنت السميع العليم ..