22/2/2021

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1191111

Machine Dynamics

Mechansim ⇒ Bodies interconnected by joints ⇒ Produce Energy,
* transmits Power
* trans Portation
⇓
objective
gool الهدف
من الأثر

Bodies
⇓
Rigid Body.

Mechanisms types :-

- four-bar mechansim
- Quick return mechansim
- Slider-crank mechansim
- scotch-Yoke mechansim
- straight line mechansim
- Intermittent motion mechansim
- Toggle mechansim.

Introduction :-

→ mechansim → u-bar, slider-
→ gear trains

اي بستخدم الحركة الى تفاصيل تخلص منه gears.

$$w_1 r_1 = w_2 r_2$$

$$\frac{w_2}{w_1} = \frac{r_1}{r_2}$$

⇒ اي الحركة الى التفصيل في هذه العلاقة وما هي المؤثرات في كل منها ... وظيفة reduction

Introduction:-

↳

↳

↳ **Cams** & follower مثل حركات السيارة في الموتور.

⇐ أي انه فتحة الصمام مدروسة حسب حركة cam أي حسب الزاوية لتتوافق مع الفتح والإغلاق.

⇐ سيتم التعرف على أنواعها وتفاصيلها.

---

* mechansim → Position analysis → Graphical / complex number

أي ايجاد الزوايا والأبعاد عند حالة معينة لـ mechansim

Then

→ Velocity analysis

→ Acceleration analysis.

(Position & velocity & acceleration) analysis

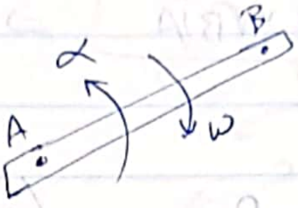
أي كل السابقة لعمل:- ⇒ force analysis

⇐ على سبيل المثال:- معرفة حجم الموتور اللازم للسيارة ما (force analysis).

* $\Sigma \vec{F} = m \vec{a}_G$

* $\Sigma M_G = I_G \alpha$

note:- in Dynamics.



$$\boxed{\vec{a_B} = \vec{a_A} + \vec{\alpha} \times \vec{r_{B/A}} - \omega^2 \vec{r_{B/A}}}$$

اي لا نستطيع عمل (acceleration analysis) الا بعد عمل

(Position analysis ..) , (Velocity analy..)

$$\boxed{\text{Position} \xrightarrow{} \text{Velocity} \xrightarrow{} \text{acceleration} \xrightarrow{} \text{force}}$$

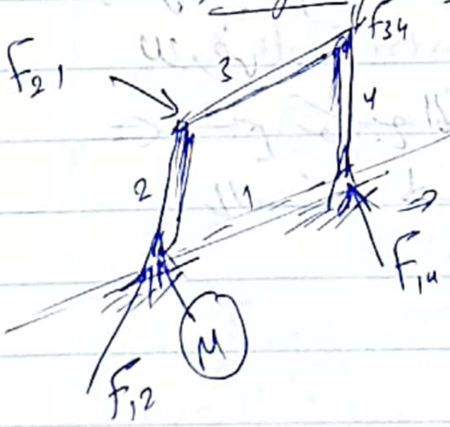
① Position → ② Velocity → ③ acceleration → ④ force

Analysis

Position analysis
Velocity ,,
acceleration ,,
} ⟹ using complex numbers

اي التحليل رتيلو تقريباً عنه Dynamics

وسيتم استخدام Matlab لعمل كل جزء ال Analysis



اي يتم حساب كل القوى عند كل link
لحظة من الدورانة
⇐ ولكي يدور نحتاج الى Ⓜ
⇐ سيتم حساب Torque ايضاً اللازم
عند كل لحظة ...

لاختيار العزم المناسب عند لحظة معينة لعمل Desgine له ايضاً مثلاً.

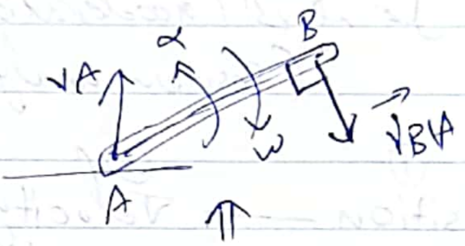
note:-

$\boxed{\vec{V_B} = \vec{V_A} + \vec{\omega} \times \vec{r_{B/A}}}$ ⇒ in Dynamics

But

→ in Machine Dynamics (Using polygons)

$V_B = V_A + \vec{V_{B/A}}$



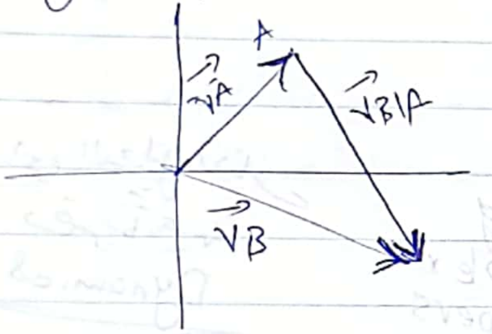
⇑

Points A & B are on the same rigid body

$\boxed{V_{B/A} = \vec{\omega} \times r_{B/A}}$

$V_{B/A} = \omega \, r_{B/A}$

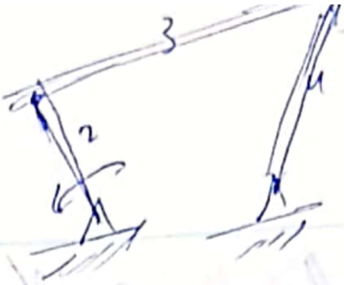
* Using Polygons.



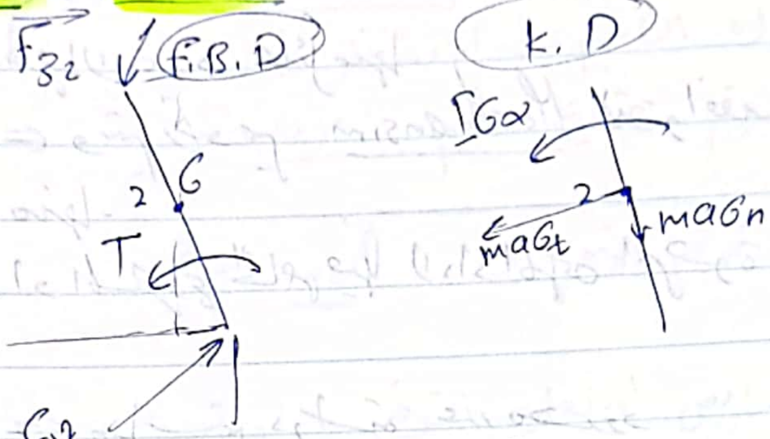
⊟ Instantious center

in Dynamics ⇒ حركة الاجسام بالنسبة للارض ..

⇐ يتم توسيع المفهوم ليتم دراسة حركة جسم بالنسبة لجسم آخر.

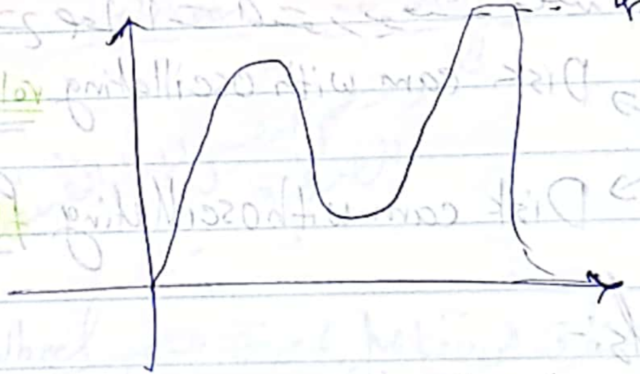


note :- Review



$$\Sigma F_x = \Sigma F_x$$
$$\Sigma F_y = \Sigma F_y$$
$$\Sigma M_G = (\Sigma M_G)_k$$

ونتيجة الحسابات ← يتم حساب Torque.
ليتم رسمها



⇐ اي انه Torque متغير فهو سيذهب الى تحريك الجسم
وهو الذي يأتي بشكل عمودي ، وجسم يذهب لتكبير
Pin-joint .

اي كل هذه التفاصيل ستتم دراستها .
ومعرفة حركة الجسم بناءاً على Torque .
ومجرى سرعة الحركة عند عدة لحظات :

⇐ وهذه Mechansim جزء من الموجودة
فهناك عدد هائل منها.
⇐ ويتم تصميم Mechansim لتتوافق مع العمل المطلوب
منها.
او المشروع الخاص فيها لاداء المهمة المرجوة.

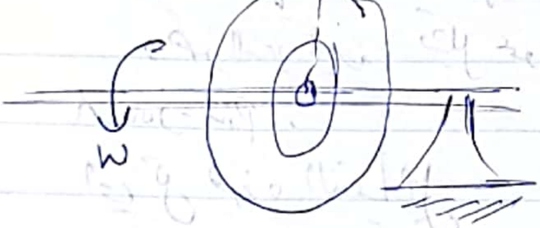
⇐ ما سيتم دراسته عدد محدود
وسيتم دراسة التحليل لهذه Mechansim.

note:

- Numerical Method ≡ complex number
- اهم مواضيع cams في هذا المساق هو متور السيارة
وفتح صمامات المتور بوقت معين.

cams
→ Disk cam with oscillating **roller** follower
→ Disk cam withoscillating **flat-faced** follow

- fly wheels:-



$$Energy = \frac{1}{2} I_G \omega^2$$

$$I_G = \int r^2 dm$$

$$Power = \frac{Energy}{time}$$

خزانة من الطاقة ⇐ يعمل على عملية التنظيم 66 اذا زادت الطاقة
عن المعدل المطلوب تقوم بعملية التخزين واذا قلت تقوم بعملية التزويد . . .

* مثلاً في السيارة لا تنتج بقوة ⇐ Paston

flyWheel في المحرك الناتج عن الانفجار ، وذلك بفعل اي تقوم بعملية تنظيم **Power** .

والاستخدام الآخر لـ **flywheel** :-

⇐ في المكابس اي لعملية التوفير و اعطاء القوة المناسبة لـ المكبس ⇐ لانه المحرك لا يكفي لوحده .

اي يتم اخذ الطاقة من flywheel لعملية القص اللازمة ومن ثم الطاقة المفقودة يتم تعويضها من **motor** .

اي بدل استخدام motor ذو حجم هائل للقيام بالمهمة يتم استخدام **motor** مع **flyWheel** .

⇓ ذو حجم صغير لهذا التطبيق الضخم !!

وسيتم التطرق الى تفاصيل لمعرفة حجم **flyWheel** اللازم - - -

Balance of machinay :-

⇐ مثل عجل السيارة عندما تتعرض لضربة مما يؤدي الى تغيير ⇐ center of mass . وذلك يؤدي الى حدوث اهتزاز ⇐ مما يلزم لعمل **Balancing** ،

⇐ لنجري تعديل center of mass

27/2/2021

Linkages and Mechanisms.

- **Machine** :- a group of mechanisms interconnected together to perform a given operation, or to produce power.

- **Mechanisms** :- A group of Rigid Bodyes, interconnected by joints, to produce a specific designed motion or transmission of Power.

- **Joints** :- Relation between any two connected bodies.

- **Degrees of freedom (DOF)** :- Number of independent parameters, variables, or inputs needed to fully control the joint.

note:-(DOF) ⇒ انواعها موجودة في جدول في الكتاب

Degrees of freedom of a body :-

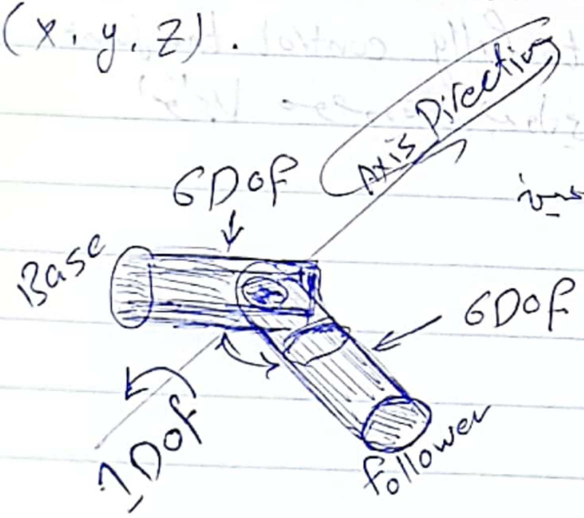
* 2D motion in plane :-
⟶ Translation + Rotation   Dof = 3 for each body.

All bodies are restricted to move on the same plane (x, y), and can rotate about an axis perpendicular to the plan z.

---

* 3D motion in space :
⟹ Translation + Rotation   Dof = 6 for each body.

no restriction about the motion of the body! it can translate in 3-direction (x, y, z) and rotate about (x, y, z).



ولكن عشان نربط الجسمين
مع بعض منع 5 حركات
لكل جسم تركنا الجسم
للجسمين 1Dof
↳ Rotation

إذا بدك 1Dof ← منع 5 حركات
" " " 2Dof ← منع 4 حركات
" " " 3Dof ← منع 3 حركات .
⋮

* 2D motion in plane:- DoF = 3 for each body.

1-DoF joint ⇒ $\overset{\text{تقييد}}{\text{restricts}}$ the motion in 2 direction

2-DoF joint ⇒ 〃 〃 〃 〃 1 direction

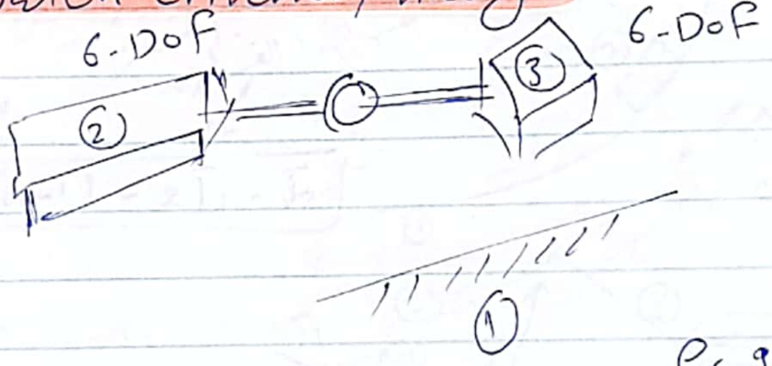
* 3D motion in space:- DoF = 6 for each body

J1 1-DoF joint ⇒ restricts the motion in 5 direction.

J2 2-DoF joint ⇒ 〃 〃 〃 〃 4 〃.

J3 3-DoF joint ⇒ 〃 〃 〃 〃 3 〃.

Kuzbatch Criteria / theory.



ثابتة

for ground

$$\text{mobility} = (m) = 6 \times (\underline{\underline{n}} - \textcircled{1}) \quad ; \underline{\underline{n}}:- \text{number of Body}$$

لإيجاد عدد الحركات اللازمة لعمل حركة بشكل كامل.

$$= 6 \times (3-1)$$

$$= 12 - 3J_3 - \textcircled{5J_1} - 4J_2 - 2J_4 - - -$$

J1 ⇒ restrial the motion in 5 direct.

⟶ $\textcircled{5J_1}$

$J_3 \rightarrow \textcircled{3J_3}$ ⇒ وهكذا ...

في الفكرة حسب الـ $\underline{\text{DoF}}$ والحركة المسموحة.

mobility $= m = 6(n-1) - 5J_1 - 4J_2 - 3J_3 - 4J_2 --$

→ But in 2D motion in plane

$m = 3(n-1) - 2J_1 - 2J_2$ ⇒ * ال course مستغرقة الى motion في plane ،

J_1 ⇒ ال joint ⇒ تمنع 5 حركات للتحرك

اي يجب تحديد النوع ليتم التطبيق

على القانون ← لمعرفة الحركات اللازمة.

Contact Joint (Planar motion) :-

① sliding :- without relative rotations is only permitted.

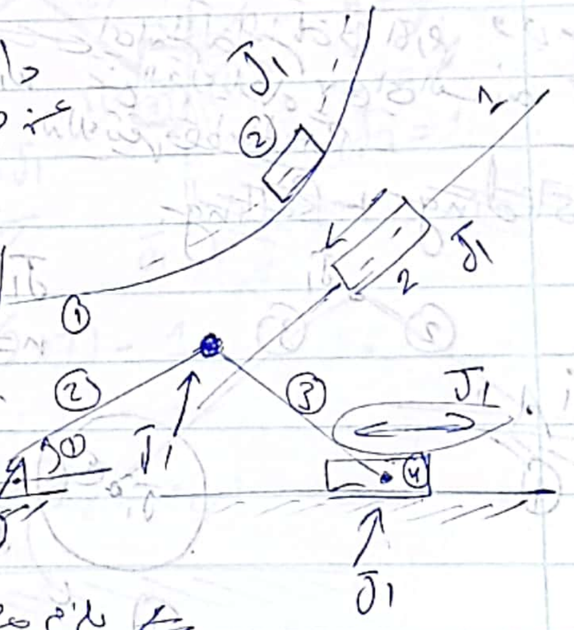
② Rolling :- without relative sliding at the contact point is only permitted.

③ sliding + Rolling are both permitted (Pin. inside a slot).

sliding ⇒ دائماً القدر عند هذا الـ وزنة

Dof = 1

$$\boxed{m = 3(n-1) - 2J_1 - J_2}$$



$n = 4$

$J_1 = 4$

$m = 3(4-1) - 2(4)$

$= 9 - 8$

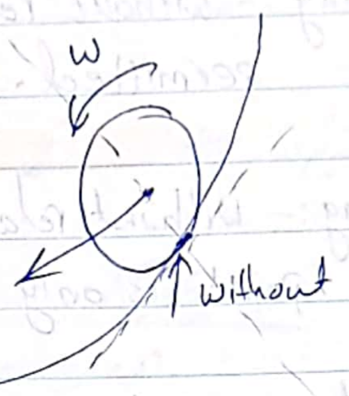
$\boxed{m = 1}$ # يلزم محرك واحد لتحكم بالنظام بشكل كامل. ⇐

J_1 = # of Joints that have 1 - Dof.

J_2 = # of Joints that have 2 - Dof.

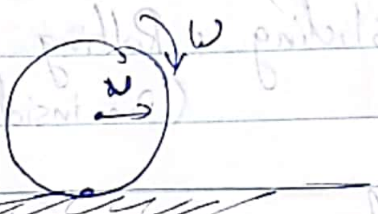
⇒ important

② Rolling without sliding :-



$\underline{\underline{J_1}}$

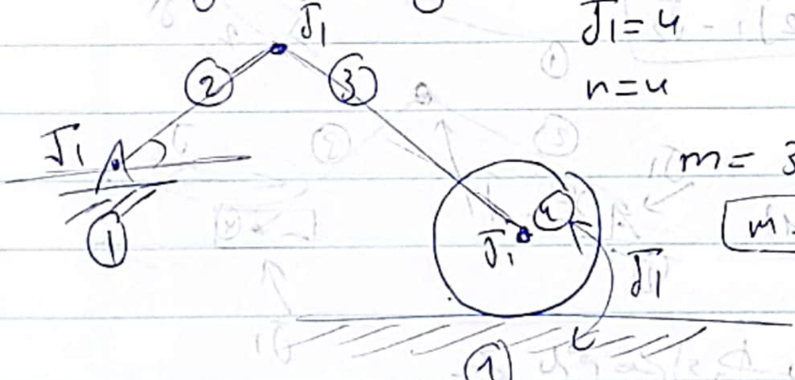
منعت الحركة باتجاه
common tangent
&
common normal
لذلك يتم حركة واحدة فقط ⇐



without slipping

إعاقة الحركة المنزلقة في الارض وفي مجال y
عن الحركة باتجاه x ⇐ اي السينية ممنوع
لذلك يتم فقط Rotation

~~③ sliding + Rolling.~~



$J_1 = 4$

$n = 4$

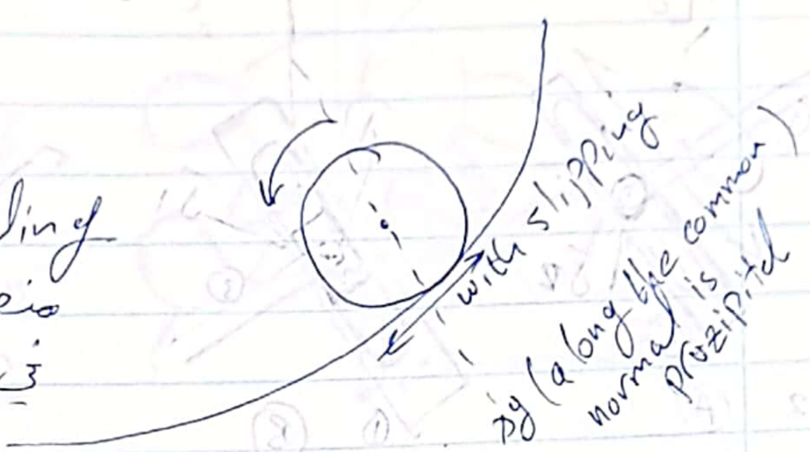
$m = 3(n-1) - 2 \times J_1$

$\boxed{m = 1}$

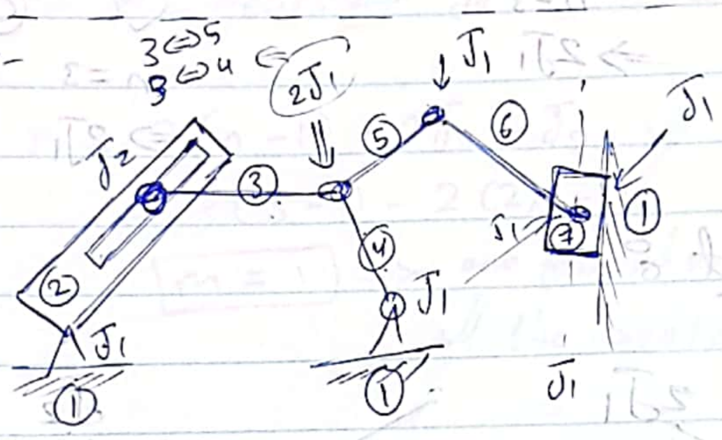
No slipping.

③ sliding + Rolling.

J_2
Rotation + sliding
⇒ منعت حركة
y-axis في مجال



Exp:-



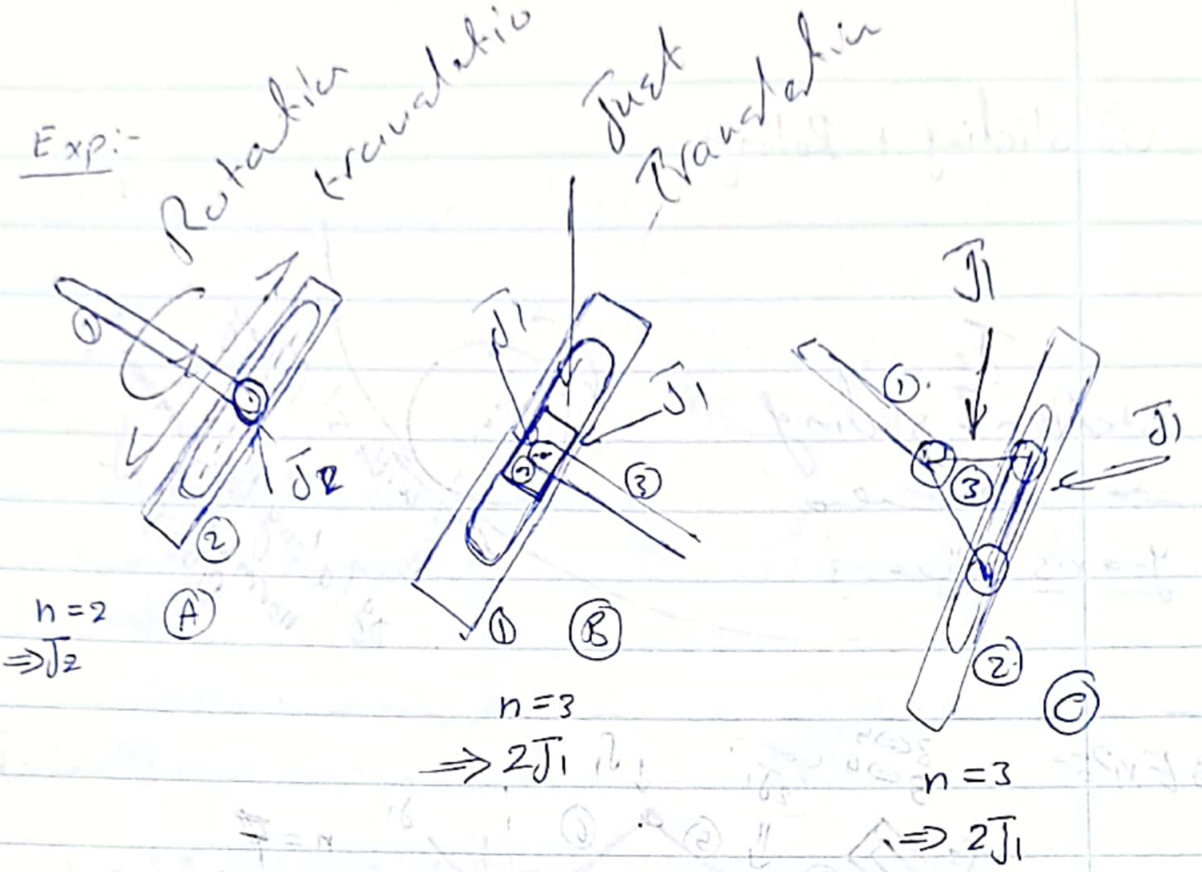
n = 7
$J_1 = 7$
$J_2 = 1$

$m = 3(7-1) - (2\times7) - 1$

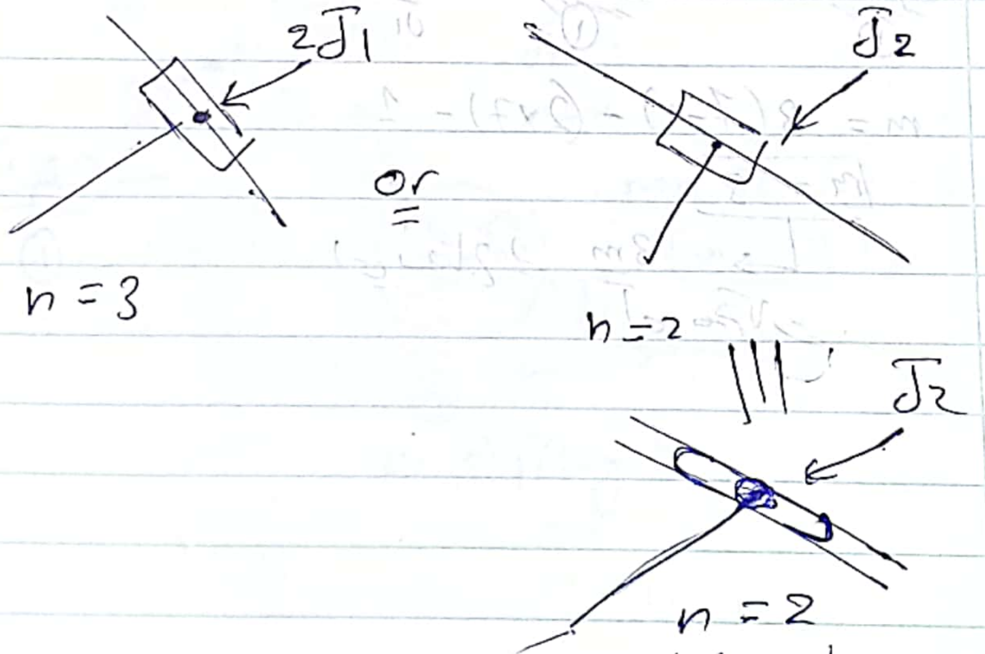
$\boxed{m = 3}$

→ 3m اي نحتاج لـ
ل محركات.

Exp:-



note that :

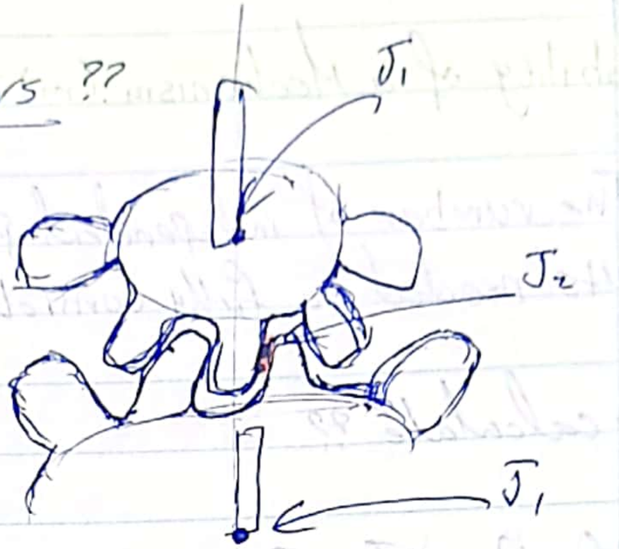


دائماً بعطينا نفس الحركة ##

1/3/2021

Abd Alhkeem

But for Gears ??



⇒ At the contact Point

لانه في هناك منع

of Rotation و translation

⇒ so the gears ⇒ J_2 (at the contact)

$$\Rightarrow m = 3(n-1) - 2J_1 - J_2$$

$$= 3(3-1) - 2(2) - 1$$

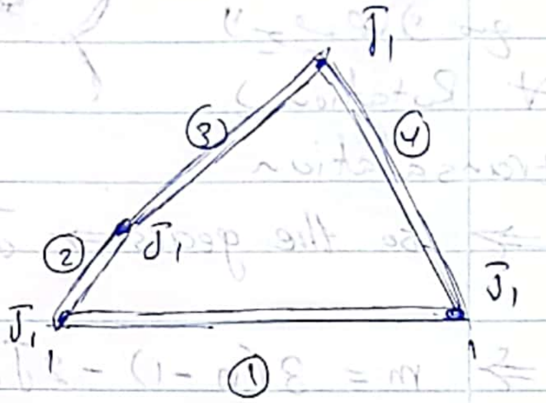
$\boxed{m = 1}$ ⇒ one parameter to control all the system.

Review

Mobility of a Mechanism (m) :-

⇒ The number of independent parameters, or variables, or inputs needed to fully control the mechanism.

How to calculate ??



$m = 3(n-1) - 2J_1 - J_2$

$n = 4$

$J_1 = 4$

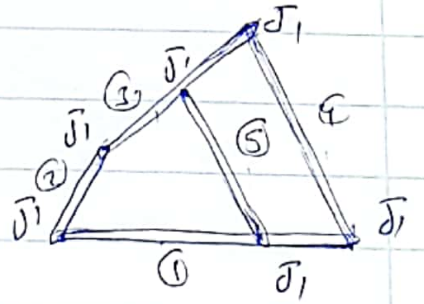
$m = 3(4-1) - 2(4)$

$\boxed{m = 1}$

note that :- n :- number of bodies including the fixed one (ground)

* m > 0 ⇒ motion (# of variables needed).
* m = 0 ⇒ no motion, structure (statically determinate).
* m < 0 ⇒ no motion (statically indeterminate).

but :-

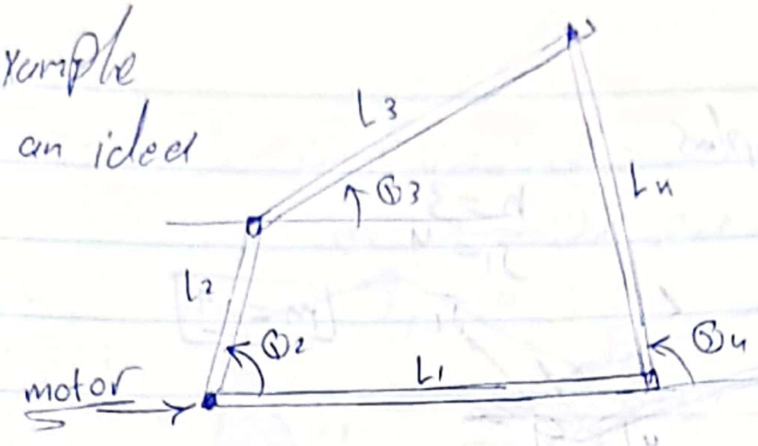


$m = 3(4) - 2(6)$

$\boxed{m = zero}$

↳ no motion

+ like example
to give an idea



given $\theta_2 \Rightarrow$ find θ_3 & θ_4

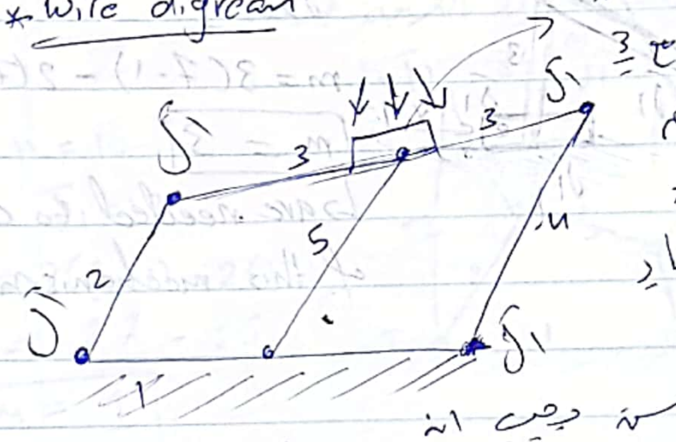
variable $\theta_2, \theta_3, \theta_4$

$m=1 \Rightarrow$ chose one of them to be the independent parameter (input).

---

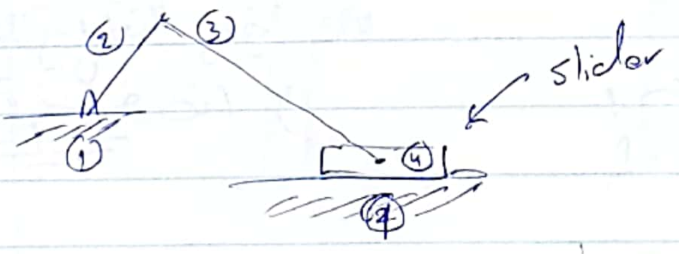
note :-

* wire digream

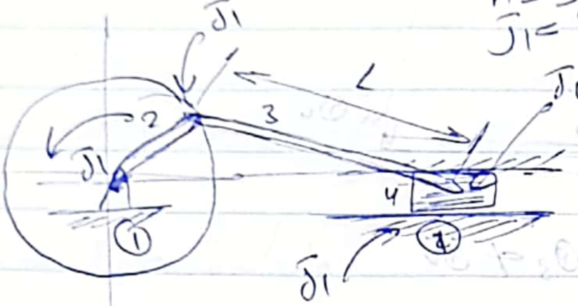


حتى انه كأنه
كقطع 5 مع 3
⇐ 3 بقيت جسم
واحد ولكنه
5 ارتبطت معاه بـ
pin joint

ولكنة يجب انه
slider & link نفرقة بين هدا

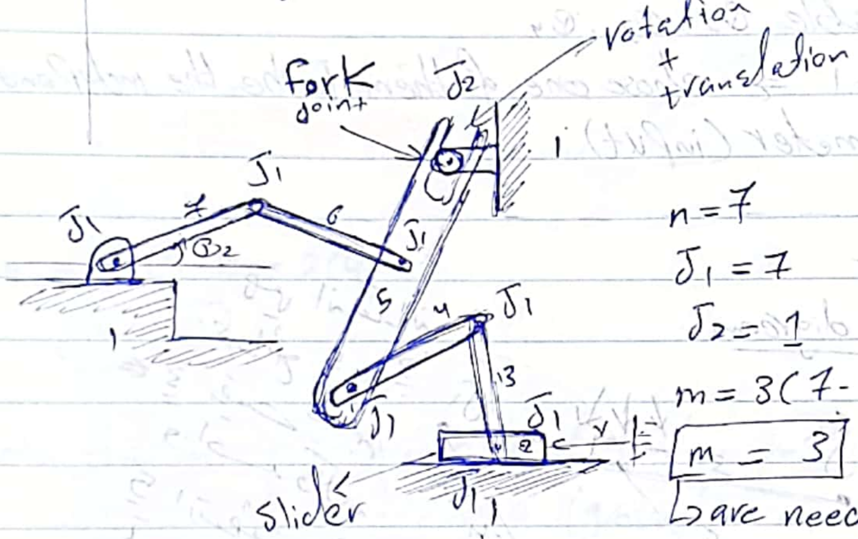


Mobility examples.-



n=3
J1=4

$\boxed{m = 1}$



n=7

$J_1 = 7$

$J_2 = 1$

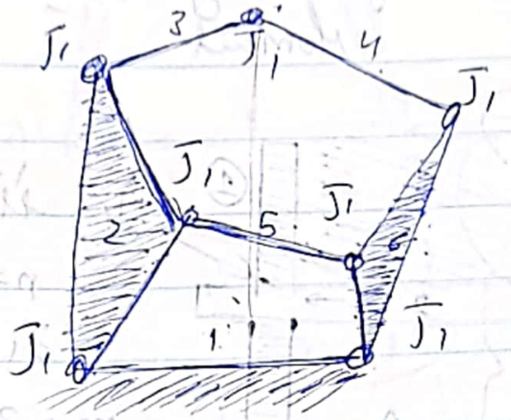
$m = 3(7-1) - 2(7) - 1$

$\boxed{m = 3}$

↳ are needed to contral of this mechanism.

note:- بالنسبة للمثلثات ⇐ يا بنعدها كعضو
او يا بنعدها كثلاث اجسام:

Exp:-

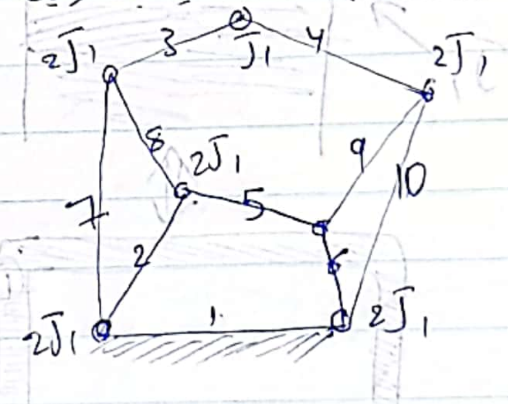


⇐ اعتبار المثلث كجسم واحد

$$\left.\begin{array}{l} n=6 \\ J_1=7 \end{array}\right] \Rightarrow m = 3(5) - 7(2)$$

$$\boxed{m = 1}$$

ولكن سنلاحظ اذا عدنا بالطريقة هذه:



⇐ اعتبار المثلث كثلاثة اجسام

$n = 10$

$J_1 = 13$

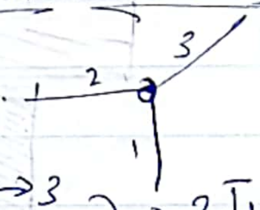
$m = 3(10-1) - 2(13)$

$= 27 - 26$

$$\boxed{m = 1}$$

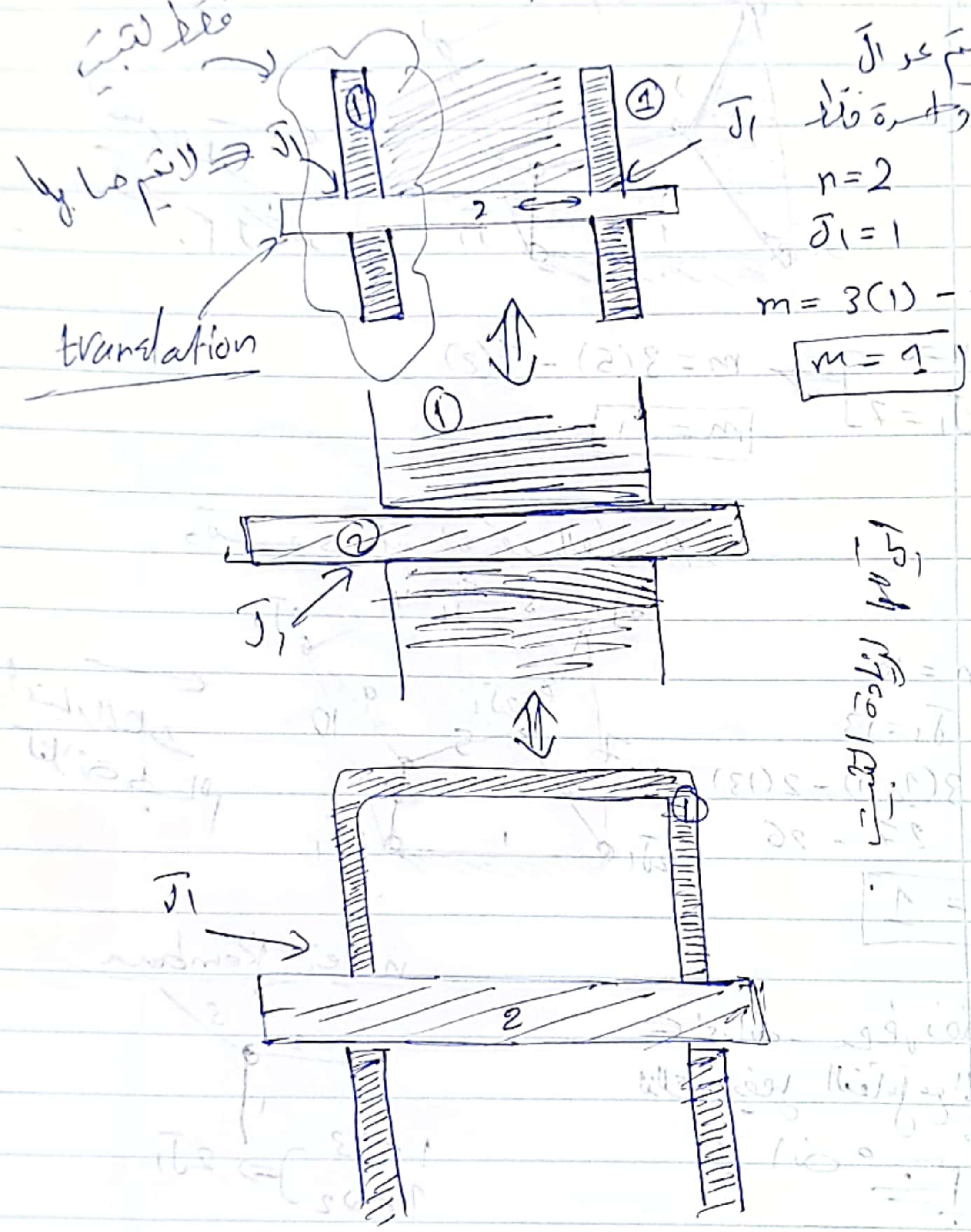
⇐ اي انه سيعطي نفس m
لذلك يُفضل التعامل مع المثلث
على انه جسم واحد.

note:- Rember



$$\left.\begin{array}{l} 1 \rightarrow 3 \\ 1 \rightarrow 2 \end{array}\right\} \Rightarrow 2J_1$$

ملاحظة × في بعض الأحيان يلزم تثبيت الجسم من الطرفين بالأرض حتى لا يحدث high binding Moment ← لذلك يتم حساب ال J1 مرة واحدة



أي يتم عد ال J1 مرة واحدة فقط

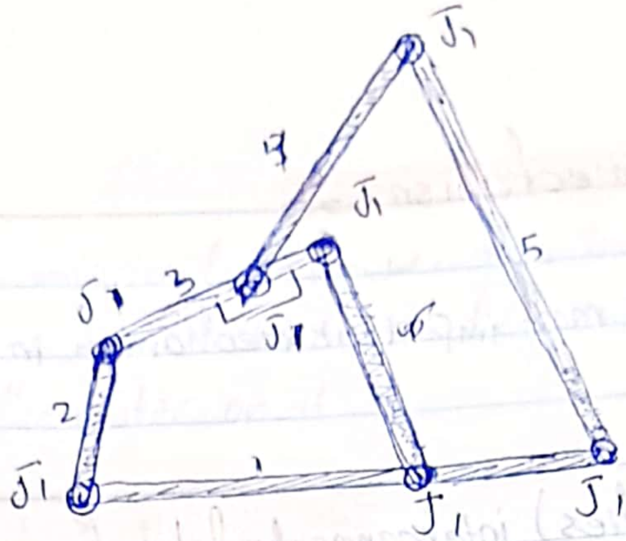
$n=2$

$J_1=1$

$m = 3(1) - 2(1)$

$\boxed{m = 1}$

Ex 2:-



$n = 6$

$J_1 = 7$

$\Rightarrow m = 3(5) - 2(7)$

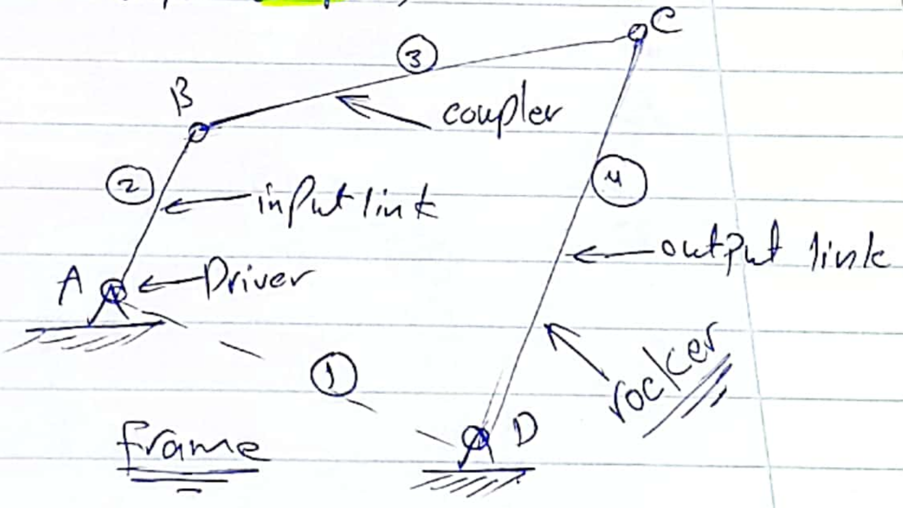
$\boxed{m = 1}$

☐ ⇐ انتهى موضوع Mobility.

▤ four bar mechanism ⋙

⇒ one of the most important mechanism in machinery

- ✱ Mobility = 1
- ✱ four links (bodies) interconnected by four revolute joints.
- ✱ Common type, Crank rocker
  - link 1: fixed ground
  - link 2: input, makes complete revolution (crank)
  - link 4: output, oscillates between two dead points (rocker)
  - link 3: transmits the motion form the input to the output (coupler)



▤ crank:- any link connected to the ground and makes complete revolution.

➤ It is important to know: does the 4-bar mechanism contain a link that can make complete revolution to fix the motor on it !!

Grashof theorem:

- s: length of shortest link
- L: length of the longest link
- $p+q$: length of the other two links.

now :-

If $s+L \leq p+q$ ⇒ Grashof's mechanism

⇒ At least one link can make complete revolution.

otherwise ⇒ non-Grashof's mechanism

⇒ no link can make complete Revolution.

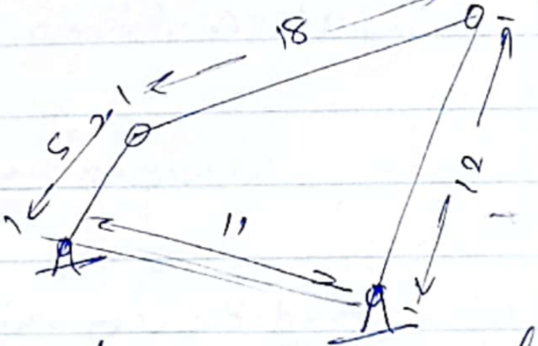
Exp

$s+L = 5+18$

$= 23$

$p+q = 11+12$

$= 23$

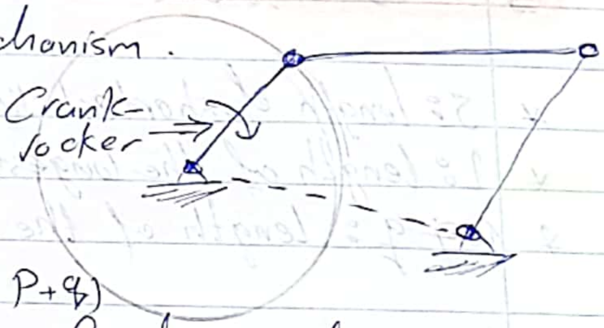


* ⇒ at least one link can make complete revolution.

▤ Grashof's 4-Bar ($S+L < p+q$)

① The shortest link is one of the side links:-

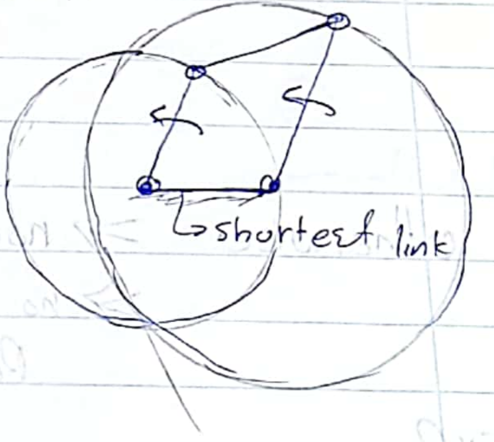
- ☐ ≫ This link will make complete revolution, all other links oscillate.
- ☐ ≫ Crank-rocker mechanism.



▤ Grashof's 4-bar ($S+L < p+q$)

② The shortest link is fixed ground.

- ☐ ≫ All links rotate
- ☐ ≫ Double crank mechanism



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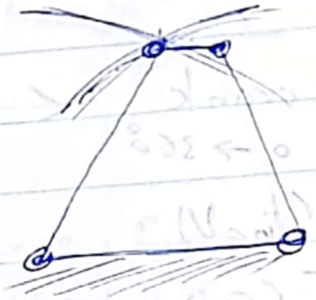
Grashof's 4-Bar ($S+L < P+q$)

③ The smallest link is the coupler

⇒ Both side links oscillate

⇒ Double Rocker mechanism



Grashof's 4-Bar ($S+L \leq q+P$)

④ $S+L = P+q$

⇒ at a given instant all links become straigh line.

see the animation in :- public | Working Model Env.

⇒ change Point mechanism

Grashof's 4-Bar ($S+L \leq q+P$)

⑤ $S+L > P+q$

⇒ No link can make complete revolution.

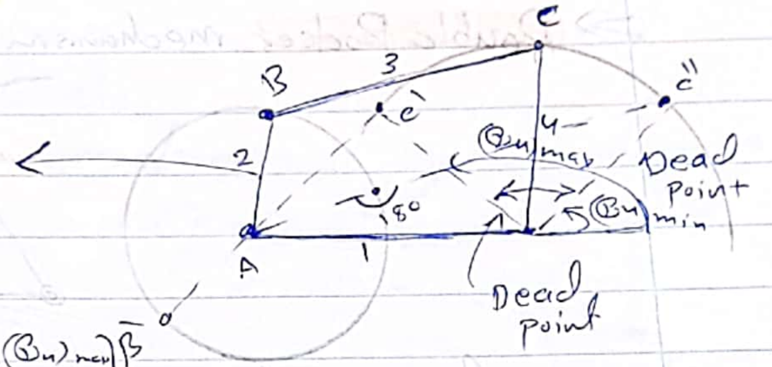
See the animation ......

⇒ Triple rocker mechanism.

اي عند التعامل مع 4-bar ⇐ يجب معرفة ماهية حركة الجزء التالي ملحه وتحليله لاحقاً.

four bar mechanism ; range of motion of each link :-

* Crank rocker



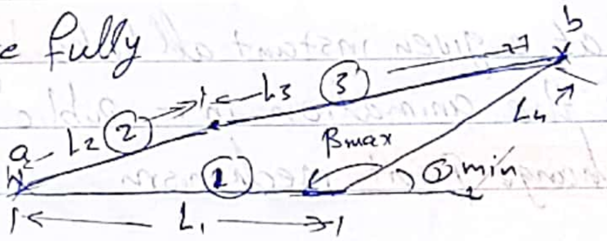
Rocker (line 4)

$\theta_4 \in [(\theta_4)_{min}, (\theta_4)_{max}]\bar{\beta}$

$(\theta_4)_{min} < \theta_4 < (\theta_4)_{max}$

$(\theta_4)_{min}$?? $(\theta_4)_{max}$) ??

link ② & ③ are fully extended (180) stright line.



L_1, L_2, L_3, L_4

قانون الـ cos

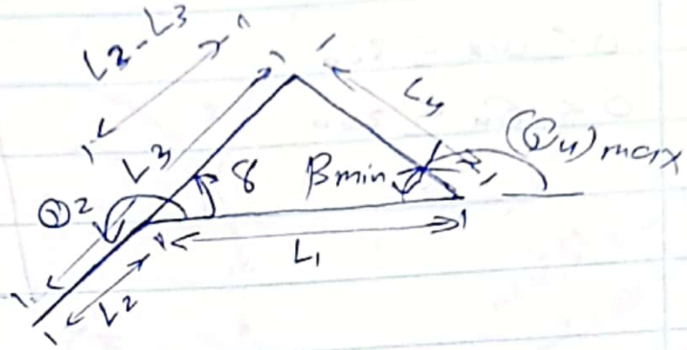
$$(L_2 + L_3)^2 = L_1^2 + L_4^2 - 2L_1L_4 \cos \beta_{max}$$

$$\beta_{max} = \cos^{-1} \frac{L_1^2 + L_4^2 - (L_2 + L_1)^2}{2L_1L_4}$$

$$\Rightarrow \quad \theta_{4\,min} = 180 - \beta_{max}$$

Abd Alhkeem
Barutah

To find $(\theta_4)_{max}$??

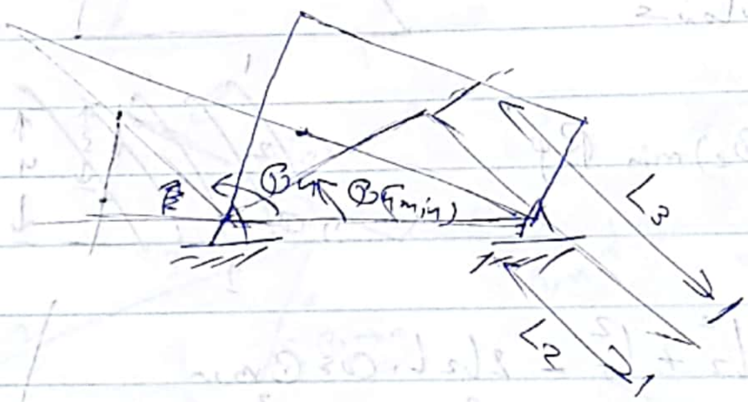


$$(L_3-L_2)^2 = L_1^2 + L_4^2 - 2L_1L_4\cos\beta_{min}$$

$$\beta_{min} = \cos^{-1}\left(\frac{L_1^2 + L_4^2 - (L_3-L_2)^2}{2L_1L_4}\right)$$

$$\theta_{4(max)} = 180 - \beta_{min} \quad \#$$

لي انا Crank Rocker ⇐ اي تحليل بيمشي ل هذه القواعد لإيجاد
(θ4)min و (θ4)max حتى لو كانت معكوسة :-



مثلاً لو طلب منه θ_2 ⇐ $180+\gamma$

To find γ :- $\dfrac{L_4}{\sin\gamma} = \dfrac{L_3-L_2}{\sin(\beta_{min})}$

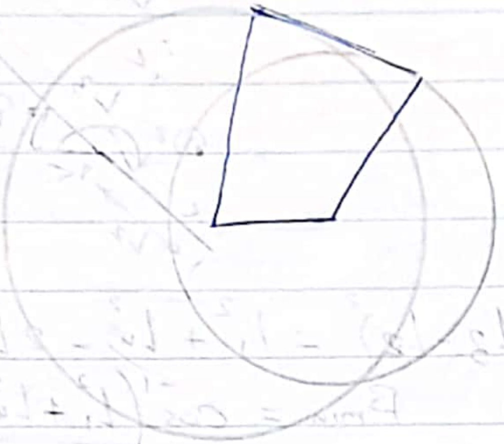
$\Rightarrow$ ✓

$\Rightarrow \boxed{\theta_2 = 180+\gamma} \quad \#$

* Double Crank

$0 \leq \theta_2 \leq 360$

$0 \leq \theta_4 \leq 360$



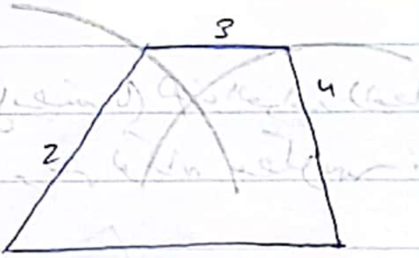
* Double Rocker

⇐ اي انه link 2 اقصر شي

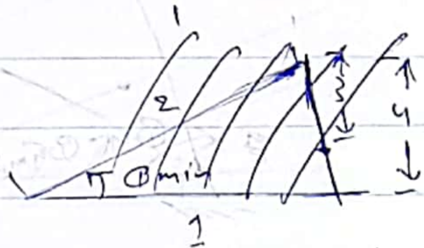
عندما تتطابق 3 & 4

وكذلك بالنسبة لـ link 4

عندما تتطابق 2 & 3

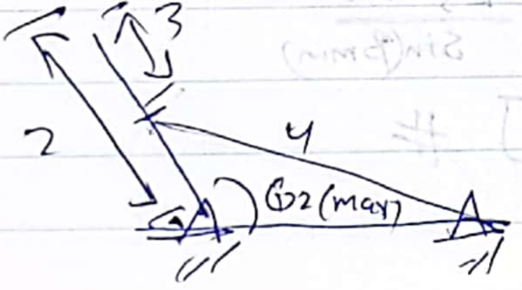


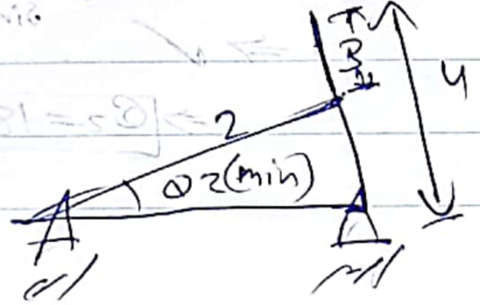
⇒ to find $(\theta_2)_{min}$ By cos law :-



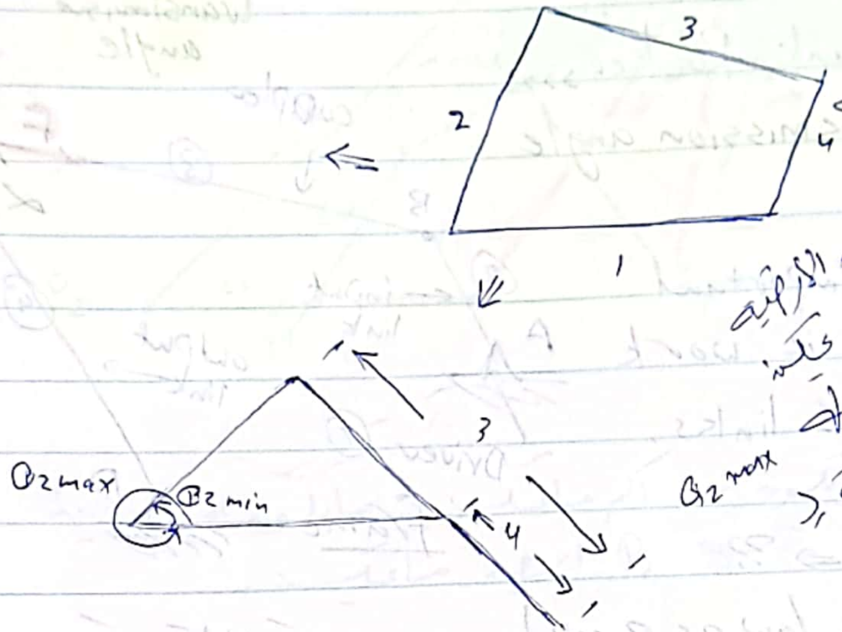
$$(L_4 - l_3)^2 = l_2^2 + l_1^2 - 2 l_2 l_1 \cos\theta_{min}$$

$$\theta_{2\,min} = \cos^{-1} \frac{l_2^2 + l_1^2 - (L_4 - L_3)^2}{2 L_2 L_1}$$





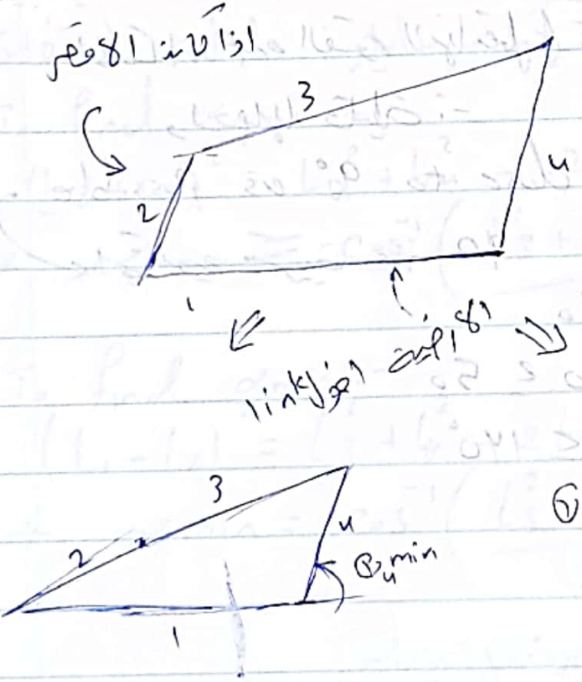
* Triple Rocker



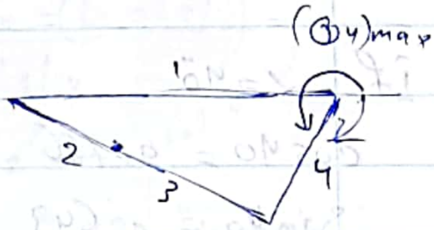
①

عندما الوصلة الأرضية أطول link ⇐ يكون التحرك Ѳ2min & Ѳ2max وكذلك بالنسبة لـ Ѳ4

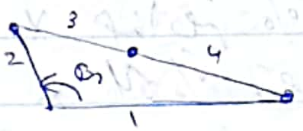
②

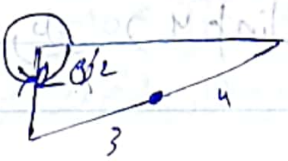


عندما تكون الأرضية أطول link ⇐ لا يوجد مشاكل يعني لابد نحسب max & min لـ Ѳ4 & Ѳ2 ①



* But for Ѳ2:-





$\leq \theta_4 \leq$

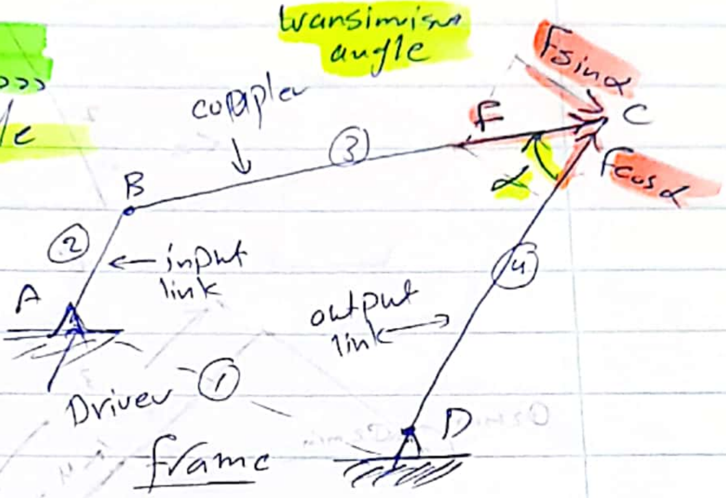
max & min ونحسب اصغرها

$-260 \leq \theta_2 \leq 110$

note :- Important.

▤ for Crank-Rocker ,,,,

▤ transmission angle



Fsinα ⇒ important because it work to rotates links.

* Fcosα ⇒ ??
↳ as low as possible يجب ان تكون قليلة قدر الامكان هذه القوى لانها تعمل على تكسير Pin.
ولجعلها قليلة :-

⇒ keep : α as close to 90° as possible
اي تكون قريبة من 90

$|\alpha - 90| \le 50$

$-50 \le \alpha - 90 \le 50$

$40° \le \alpha \le 140°$

Exp: If $\alpha = 40°$

$\cos 40 = 0.766$

$\sin 40 = 0.643$

* هذا يعني 76% من القوة تعمل على تكسير Pin
* 64% تعمل على تحريك (Rotation for link 4)

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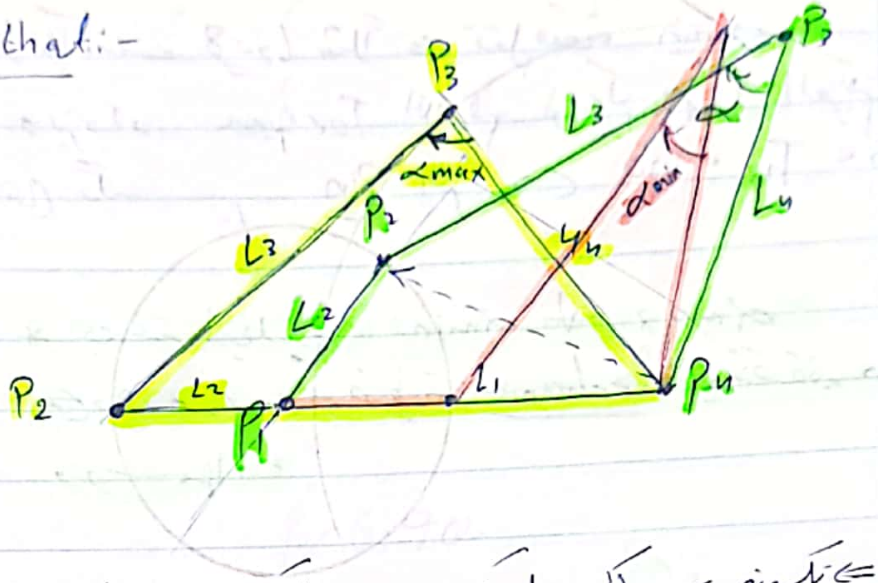
note that:-



📝 ⇐ تكون α أكبر ما يمكن عندما تكون المسافة بين P_2 & P_4 اطول ما يمكن.

📝 ⇐ تكون α اصغر ما يمكن عندما تكون l_1 & L_2 متطابقات

To find αmax:-

$$(L_2+L_1)^2 = L_3^2 + L_4^2 - 2L_3L_4\cos\alpha_{max}$$

$$* \quad \alpha_{max} = \cos^{-1}\left(\frac{L_3^2+L_4^2-(L_2+L_1)^2}{2L_3L_4}\right)$$

To find αmin:-

$$(L_1-L_2)^2 = L_3^2 + L_4^2 - 2L_3L_4\cos\alpha_{min}$$

$$* \quad \alpha_{min} = \cos^{-1}\left(\frac{L_3^2+L_4^2-(L_1-L_2)^2}{2L_3L_4}\right)$$

اذا زادت l_2 ⇐ تقل α_{min}.

⇐ عندما نقلل قيمة l_2 666 ⇐ يقل Torque اللازم لتحريك

اي يقل حجم motor مثلاً عندما كانت $L_2=3$

$L_2=6$ وعندما كانت ⇐ $-10 < T_{12} < 12$

⇐ كانت العزم اللازم لتحريك اكبر (motor اكبر) $-55 \le T_{12} \le 45$

واذا كانت $L_2 = 8$ مثلاً ⇐ تقل قيمة α_{min}
ويزداد Torque اللازم ويترك هذا العزم
في تكسير Pin الشر ⇐ $-155 < T_{12} < 100$

* اي عندما يُطلب α_{max} & α_{min}
⇐ يجب معرفة وضعية mechanism التي تحقق ذلك
ورسمها.

▤ Position analysis, geometrical approach.

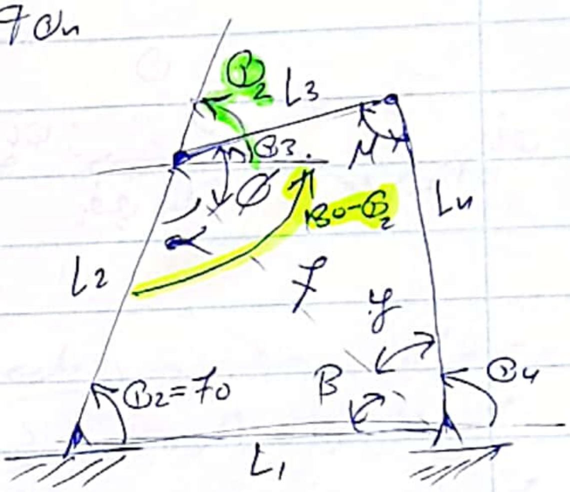
Exp:- Given the requried independent input find the other variables.

variables ≡ $\theta_2, \theta_3, \theta_4$

$m = 1$ ⇒ one independent ⇒ solve for the two others.

$\theta_2 = 70$ ⇒ solve for θ_3 & θ_4

given L_1, L_2, L_3, L_4



$$f^2 = L_1^2 + L_2^2 - 2L_1L_2\cos\theta_2$$

⇒ f ✓

$$\frac{L_2}{\sin\beta} = \frac{f}{\sin\theta_2} \Rightarrow \beta ✓$$

⇒ in the same way find α

$$f^2 = L_3^2 + L_4^2 - 2L_3L_4\cos\mu \Rightarrow \mu ✓$$

γ & ϕ using sin low ⇒ ✓

$$\frac{f}{\sin\mu} = \frac{L_4}{\sin\phi} = \frac{L_3}{\sin\gamma} \Rightarrow ✓$$

$$\Rightarrow \theta_4 = 180 - (\beta + \gamma)$$

⇑

$$\phi + \alpha = (180 - \theta_2) + \theta_3 \Rightarrow \boxed{\theta_3 = \phi + \alpha + \theta_2 - 180}$$

× ⇐ الطريقة السابقة تحتاج الى تركيز لتطبيقها عندما تكون

mechanism ⇐ معقد اكثر.

لذلك يوجد طريقة اسهل وهي :-

Position analysis using complex numbers :-

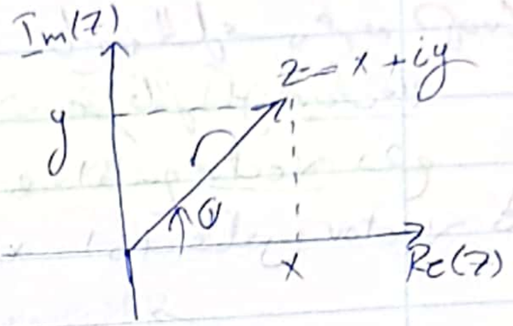
$\Rightarrow$

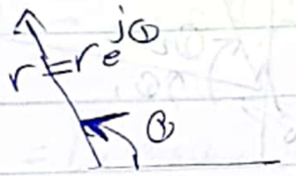
Position analysis using complex numbers:-

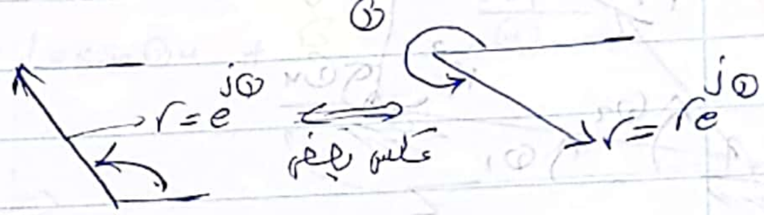
$\vec{r} = re^{j\theta}$

$e^{j\theta} = \cos\theta + j\sin\theta$

$\vec{r} = \underbrace{r\cos\theta}_{x} + \underbrace{jr\sin\theta}_{y}$







⇐ - اي منذ عند بداية vector يتم رسم θ (الزاوية)
مع الاتجاه الموجب لـ x-axis .
⇙ اي دائماً افضل اتجاه vector الاقرب للأرض

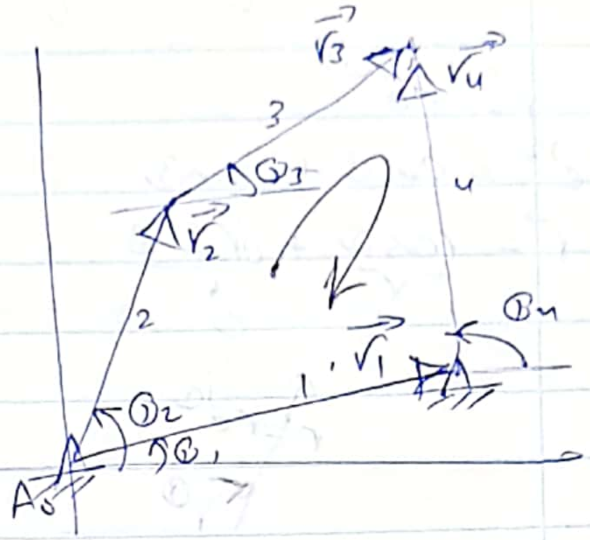
EXP:-

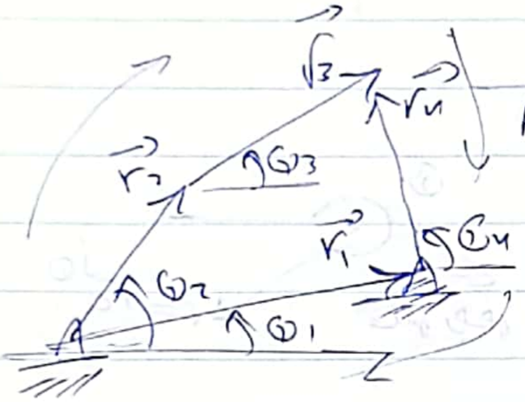
باخد الحل عن طريق loop

تمر خلال كل vector

* اذا مع vector بجمع

* اذا عكس vector بطرح





$$\vec{r_2} + \vec{r_3} - \vec{r_4} - \vec{r_1} = 0$$

$$L_2 e^{j\theta_2} + L_3 e^{j\theta_3} - L_4 e^{j\theta_4} - L_1 e^{j\theta_1} = 0$$

$$L_2[\cos\theta_2 + j\sin\theta_2] + L_3[\cos\theta_3 + j\sin\theta_3] - L_4[\cos\theta_4 + j\sin\theta_4] -$$
$$L_1[\cos\theta_1 + j\sin\theta_1] = 0$$

Real part , Imaginory part

given:- L_1, L_2, L_3, L_4
θ_2, θ_1
unknown
θ_3 & θ_4

Real part :-

① $L_2\cos\theta_2 + L_3\cos\theta_3 - L_4\cos\theta_4 - L_1\cos\theta_1 = 0$

Imag. part :-

② $L_2\sin\theta_2 + L_3\sin\theta_3 - L_4\sin\theta_4 - L_1\sin\theta_1 = 0$

⇒ solve for θ_3 & θ_4 ← to find — Using Matlab

#

⇒ The method to solve this two equation.

$$L_3 \cos 3 = L_4 \cos\theta_4 - L_2 \cos\theta_2 + L_1 \cos\theta_1$$

$$L_3 \sin\theta_3 = L_4 \sin\theta_4 - L_2 \sin\theta_2 + L_1 \sin\theta_1$$

θ_1 & θ_2 ⇒ معروفة =

لذلك نعوض الجزء المعروف من المعادلة بحرف

$$L_3 \cos\theta_3 = L_4 \cos\theta_4 + A \quad — ③$$

$$L_3 \sin\theta_3 = L_4 \sin\theta_4 + B \quad — ④$$

$③^2 + ④^2$ ⇒

$$L_3^2 = \underline{L_4^2 \cos^2\theta_4} + A^2 + 2AL_4\cos\theta_4 + \underline{L_4^2 \sin^2\theta_4} + B^2$$
$$+ 2BL_4 \sin\theta_4$$

$\sin^2 + \cos^2 = 1$ بسط

$$L_3^2 = L_4^2 + A^2 + B^2 + 2AL_4\cos\theta_4 + 2BL_4\sin\theta_4$$

حل ← نعوض بالرموز ثابتة للمعروف لتخفيف.

$$\boxed{C\cos\theta_4 + D\sin\theta_4 = F}$$ ⇒ حل هذه المعادلة عن طريق مثلثات.

note: The equation

$$a\cos\theta + b\sin\theta = c$$

$$\theta = A\tan2(b, a) \pm A\tan2\left(\sqrt{a^2+b^2-c^2}, c\right)$$

⇒ then using equation ③ & ④ solve for $\underline{\theta_3}$.

← ثم نجد θ_3 بعد ايجاد θ_4.

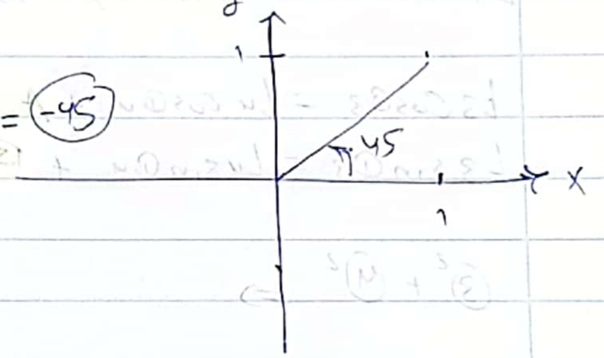
note that:-

atan2(y, x) ⇒ modified atan function

⇓

$\tan^{-1}$

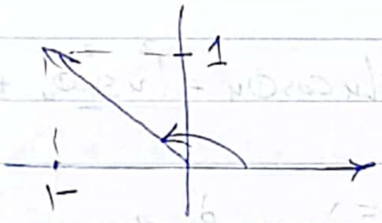
$\tan^{-1}$ ⇒ معرفة في الربع الاول والربع الرابع



$x = -1$
$y = 1$ } ⇒ $\tan^{-1}\frac{y}{x} = \boxed{-45}$

⇓

ولكن هذه خطأ
لأنه:



⇒ $\tan^{-1}$ is defined in the first and the 4th quarter.

ولحل هذه المشكلة في Matlab،
تم تعريف atan2 :-

atan2(x, y)

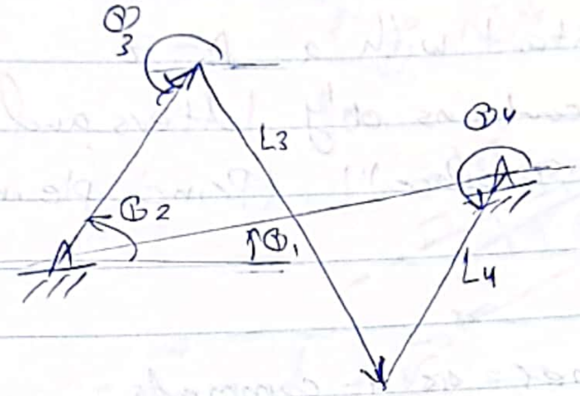
⇒ atan2(y, x) is defined over all quarters.

* اي يتم استخدام هذا function في جميع
لغات البرمجة

15/3/2021

***note that**

للسؤال السابق حركة تشبيك Mechanism :-



نلاحظ في القانون وجود موجب وسالب (±).

$$\theta_4 = Atan2(b, a) \underset{\Uparrow}{\boxed{\pm}} Atan2(\sqrt{a^2+b^2-c^2}, f)$$

آلية استخدام Matlab لحل المعادلات السابقة :-

الملفات المستخدمة :-

Public → Matlab → four bar mechanism → Pos Vel Acc force.

Principle.m ⇒ البرنامج الأساسي.

* Run or F5 ⇒ لتشغيل البرنامج.

▤ matlab file names:-

1- should start with a A-Z.
2- should contains only letters and numbers, "_"
3- Do not use space!!! (Princ ✗ ple.m)

▤ to set all lines as a comments
⇒ CTRL + R after we build it.

▤ to clear the comments
⇒ CTRL + D

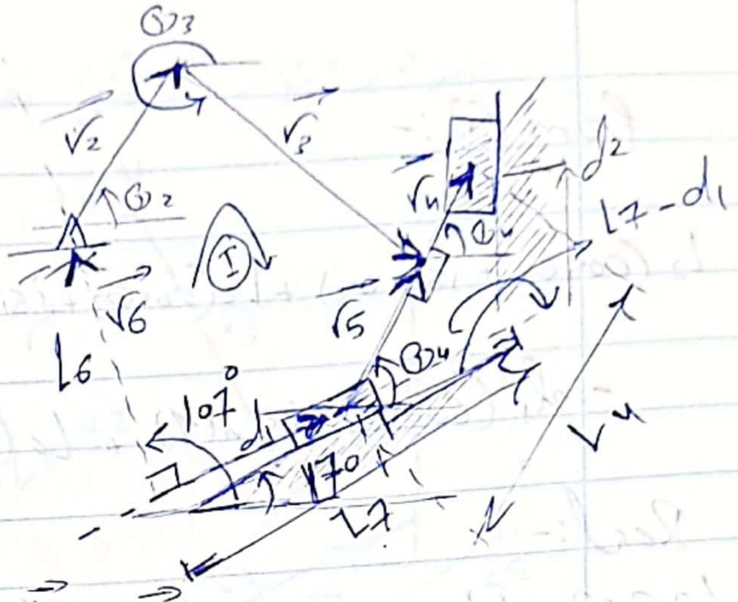
* <u>function</u>

▤ <u>deg2rad</u> ⇒ rad ← deg تحويل من

<<— هذه المحاضرة شرح عن الماتلاب وآلية الاستخدام.

20/3/2021

Exp.



first loop Ⓘ :-

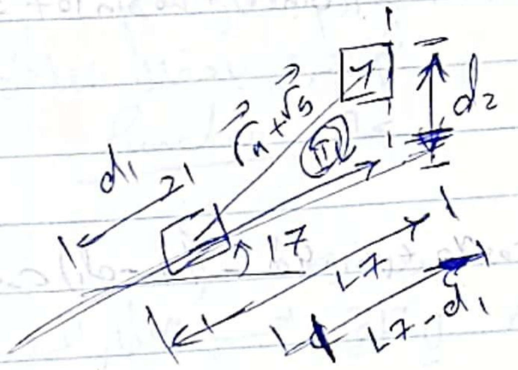
$$\vec{r_2} + \vec{r_3} - \vec{r_5} - \vec{d_1} + \vec{r_6} = 0$$

$$L_2 e^{j\theta_2} + L_3 e^{j\theta_3} - \frac{L_4}{2} e^{j\theta_4} - d_1 e^{j17} + L_6 e^{j107} = 0 \quad — ①$$

variables:-

$\theta_2, \theta_3, \theta_4, d_1, d_2$

الأطوال رح نستخدم بغرها



θ_2 : input

$\theta_3, \theta_4, d_1, d_2$: output

4 unknow

second loop:-

$$\vec{r_4} + \vec{r_5} - d_2 - (L_7 - d_1) = 0 \quad \text{4 - equation}$$

$$L_4 e^{j\theta_4} - d_2 e^{j90} - (L_7 - d_1) e^{j17} = 0 \quad — ②$$

4-equation ⟸ كل معادلة ستصير معادلتين

from ① :-

$$l_2(\cos\theta_2 + j\sin\theta_2) + l_3(\cos\theta_3 + j\sin\theta_3) - \frac{L_4}{2}(\cos\theta_4 + j\sin\theta_4)$$

$$- d_1(\cos 17 + j\sin 17) + l_6(\cos 107 + j\sin 107) = 0$$

Real:-

$$L_2\cos\theta_2 + l_3\cos\theta_3 - \frac{L_4}{2}\cos\theta_4 - d_1\cos 17 + L_6\cos 107 = 0 \quad \text{—} ①$$

Imgin:-

$$l_2\sin\theta_2 + l_3\sin\theta_3 - \frac{L_4}{2}\sin\theta_4 - d_1\sin 17 + L_6\sin 107 = 0 \quad \text{—} ②$$

from ② :-

$$L_4(\cos\theta_4 + j\sin\theta_4) - d_2(\cos 90 + j\sin 90) - (L_7 - d_1)\cos 17 + j\sin 17 = 0$$

Real:-

$$L_4\cos\theta_4 - d_2\cos 90 - (L_7 - d_1)\cos 17 = 0 \quad \text{—} ③$$

Imgin:-

$$L_4\sin\theta_4 - d_2\sin 90 - (L_7 - d_1)\sin 17 = 0 \quad \text{—} ④$$

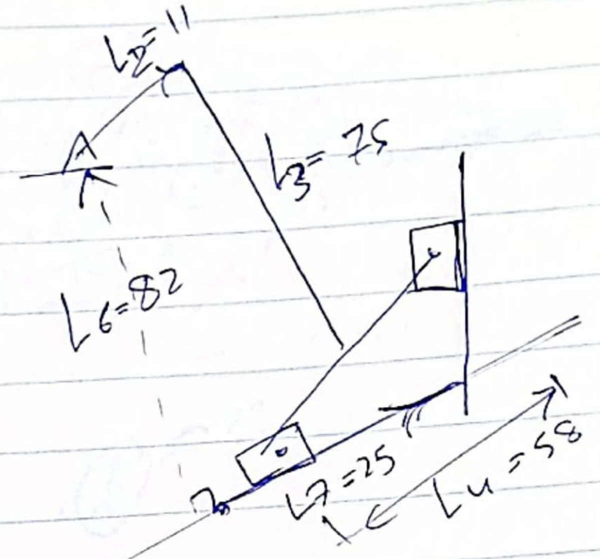
⇐ ثم نذهب الى matlab لادخال هذه المعادلات .

for variable :-

input	output
θ_2	θ_3 , θ_4 , d_1 , d_2

$$\theta_3 \to X(1) \quad \theta_4 \to X(2) \quad d_1 \to X(3) \quad d_2 \to X(4)$$

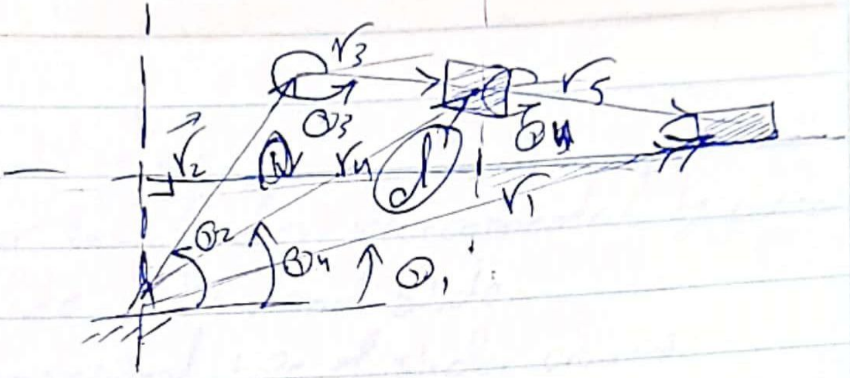
يتم أخذ القياسات بشكل تقريبي :-



أي نقوم بإدخال القيم
ثم نقوم بإدخال المعادلة =
على function .

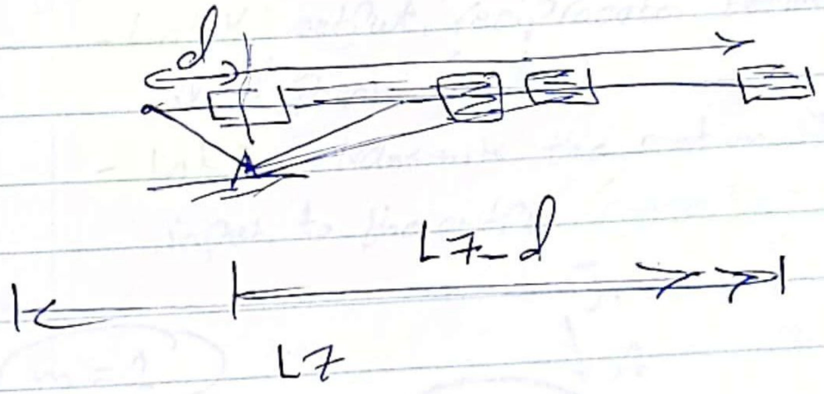
⇐ هنالك أيضاً ما يسمى حدود للرسمة.
× عند عمل run لكي تأتي داخل الإطار المحدد

$\theta_5 = \theta_3$



θ_2 : input

θ_3 , θ_1 , θ_4 , d → ??



22/3/2021

slider crank mechanism, description:-

* mobility = 1

* four links (bodies) interconnected by three revolute joints, and slider

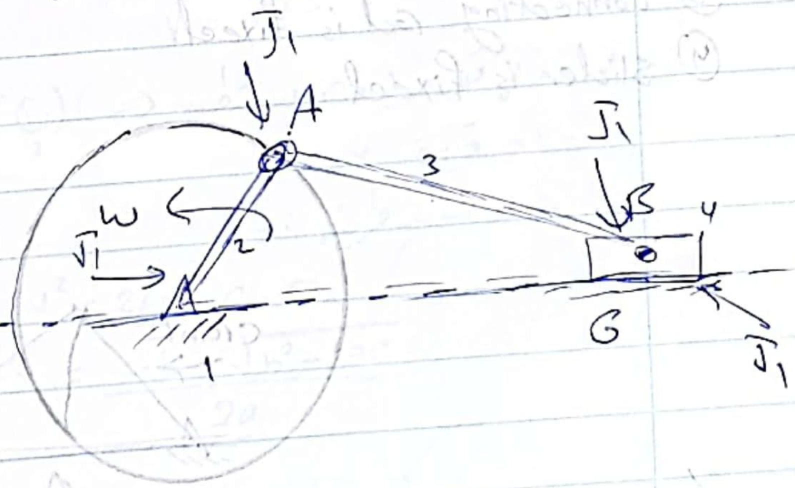
☒ conventional type of slider crank:-

- link 1: fixed ground.
- Link 2: input, makes complete revolution (crank)
- Link 4: output, reciprocates between two dead points (slider)
- link 3:- transmits the motion from the input to the output (connecting rod).

$\boxed{m = 1}$

$m = 3(4-1) - 2J_1(4)$

$m = 1$ #



* واكثر تطبيقه على غرف الاحتراق الداخلي في
محرك السيارة .

⇐ هذا نفس four bar ولكن لا تمتلك أبعاد
نفس four bar .

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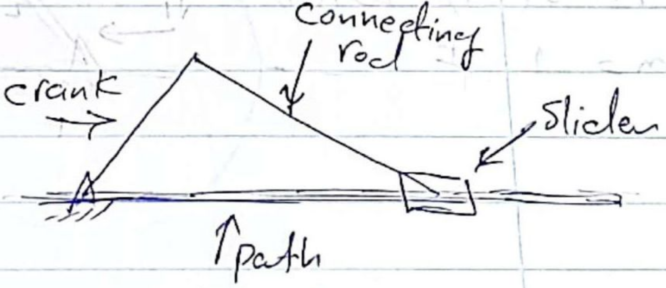
☰ slider crank mechanism , inversion :-

- * ground with the path fixed
- * link 2 fixed
- * link 3 fixed
- * slider fixed
- * offset slider crank mechanism : Model Example

⇒ see working Model Example folder\working

≪ اي شكل نثبت اي جزء بـ slide سيتم الحركة مختلفة

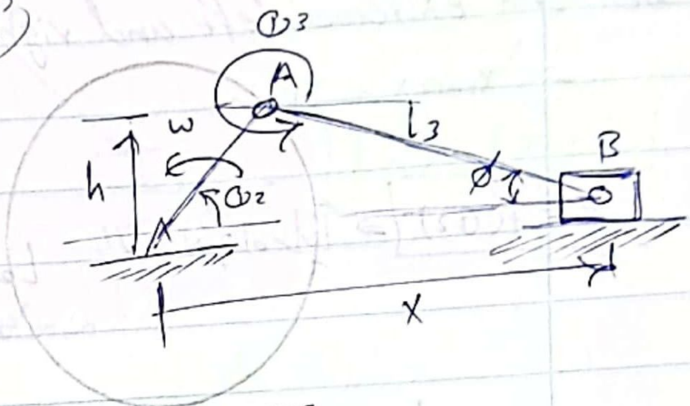
① path is fixed.
② crank is fixed.
③ connecting rod is fixed
④ slider is fixed



slider crank mechanism , position analysis :-

⇒ * Graphical method :-

$x = L_2 \cos\theta_2 + L_3 \cos\phi$



$h = L_2 \sin\theta_2 = L_3 \sin\phi$

$\sin\phi = \frac{L_2}{L_3} \sin\theta_2$

$\sin^2\phi + \cos^2\phi = 1$

$\cos\phi = \sqrt{1 - \sin^2\phi} = \sqrt{1 - \left(\frac{L_2}{L_3}\sin\theta_2\right)^2}$

$\Rightarrow x = L_2 \cos\theta_2 + L_3 \sqrt{1 - \left(\frac{L_2}{L_3}\sin\theta_2\right)^2}$

$\Rightarrow x = f(\theta_2)$ ⇒ اي عند كل قيمة لِ θ_2 نستطيع

ايجاد قيمة x .

⇐ او عن طريق :-

$L_3^2 = L_2^2 + x^2 - 2L_2 x \cos\theta_2$

$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

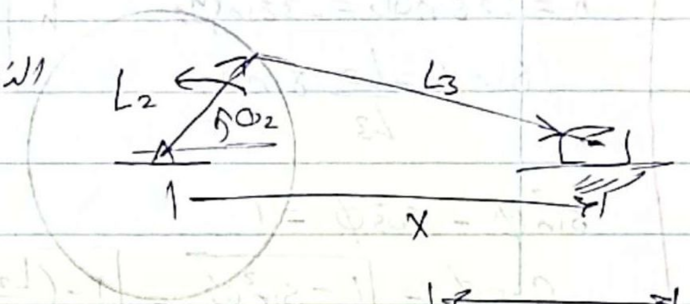
انتهينا درس slider و crank.

* slider crank mechanism, **stroke** :-

Total distance traveled by the slider = distance between extreme left and right position of the slider

$= x_{max} - x_{min}$

$\boxed{x = f(\theta_2)}$ ⇒ الذي اخرجناه سابقاً

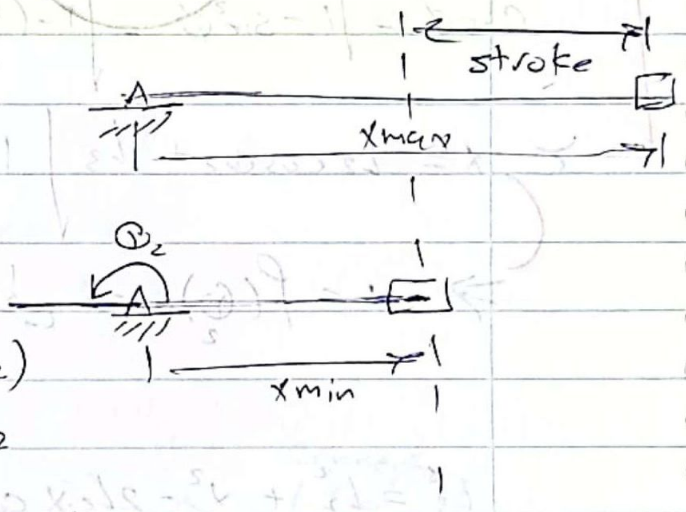


xtreme right position:-

$\boxed{x_{max} = L_2 + L_3}$

xtreme left position:-

$\boxed{x_{min} = L_3 - L_2}$



$stroke = L_2 + L_3 - (L_3 - L_2)$

$= L_2 + L_3 - L_3 + L_2$

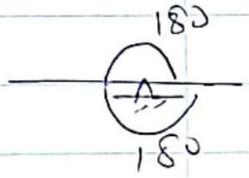
$\boxed{stroke = 2L_2}$

$\hookrightarrow$ double of the crank

θ_R = crank angle when the slider at extrem right position = zero

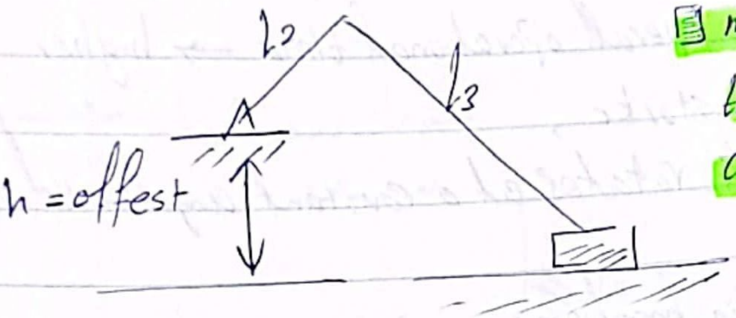
θ_L = crank angle when the slider at extrem left position = 180

* لذلك الوقت المستغرق من extra right ← extra left متساوي ∈ لذلك تكون سرعة الذهاب والاياب متساوي لذلك يعتبر slider ∈ وليس Quick slider



Abd Alhkeem Bawatneh

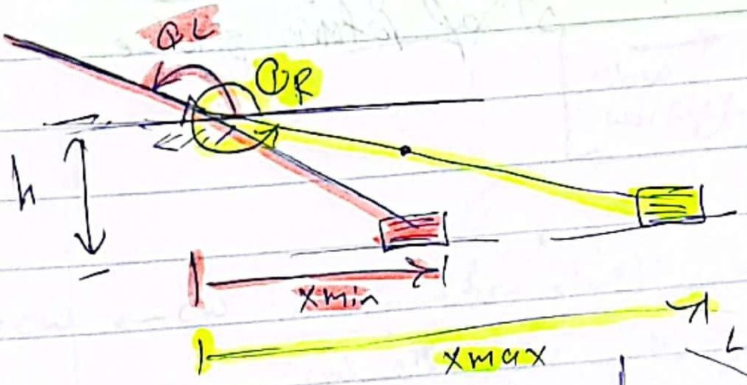
▤ offset slider crank mechanism.



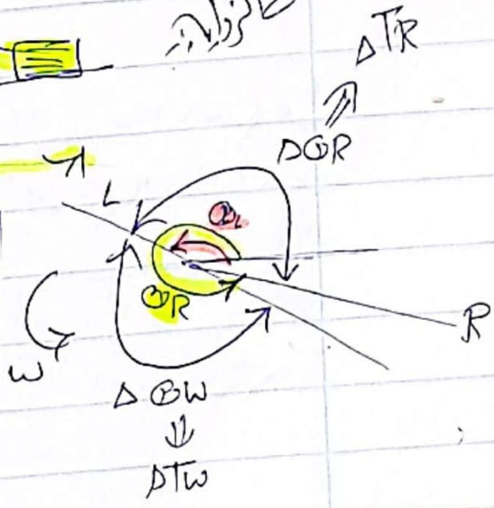
▤ note: Condition for the offset slider crank mechanism

$$\boxed{L_3 > L_2 + h}$$

* stroke = Xmax − Xmin



في حالة الزاوية



* $X_{max} = \sqrt{(L_2+L_3)^2 - h}$

* $X_{min} = \sqrt{(L_3-L_2)^2 - h}$

▤ Stroke = Xmax − Xmin

$$\boxed{\text{Time Ratio} = \frac{\Delta T_W}{\Delta T_R} = \frac{\Delta \theta W}{D\theta R}}$$

* اي الوقت المستغرق من L الى R

البرمة الوقت المستغرقه من R الى L لانه

الزاوية (DθW) البرمة (DθR)

(Quick Return mechanism) اي تكون

* عندما يوجد فرق بزاوية working وزاوية return

Quick return mechanisms.

* speed in working is limited ∵ surface finish, temperature...
* To minimize the overall operational time → higher speed in return storke,
* The input crank rotates at a constant angular velocity.

≫ Quick return mechanism.

$$\text{Time Ratio} = \frac{\Delta T \text{ of working stroke}}{\Delta T \text{ of return stroke}}$$

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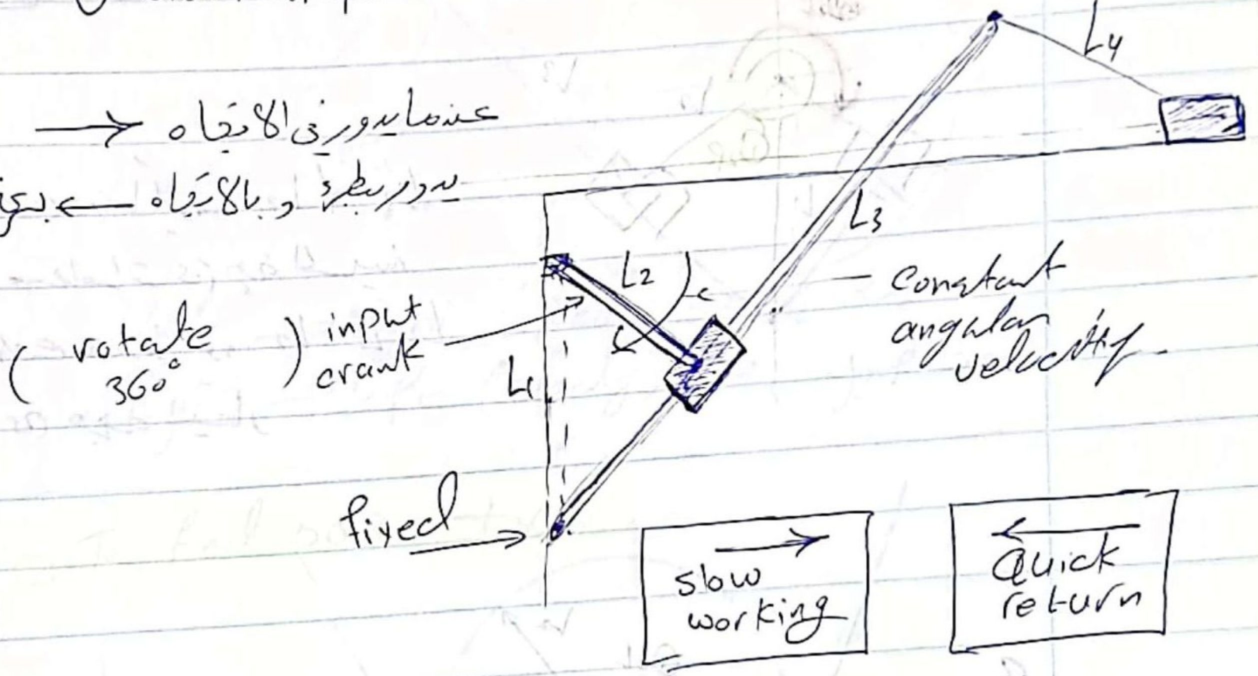
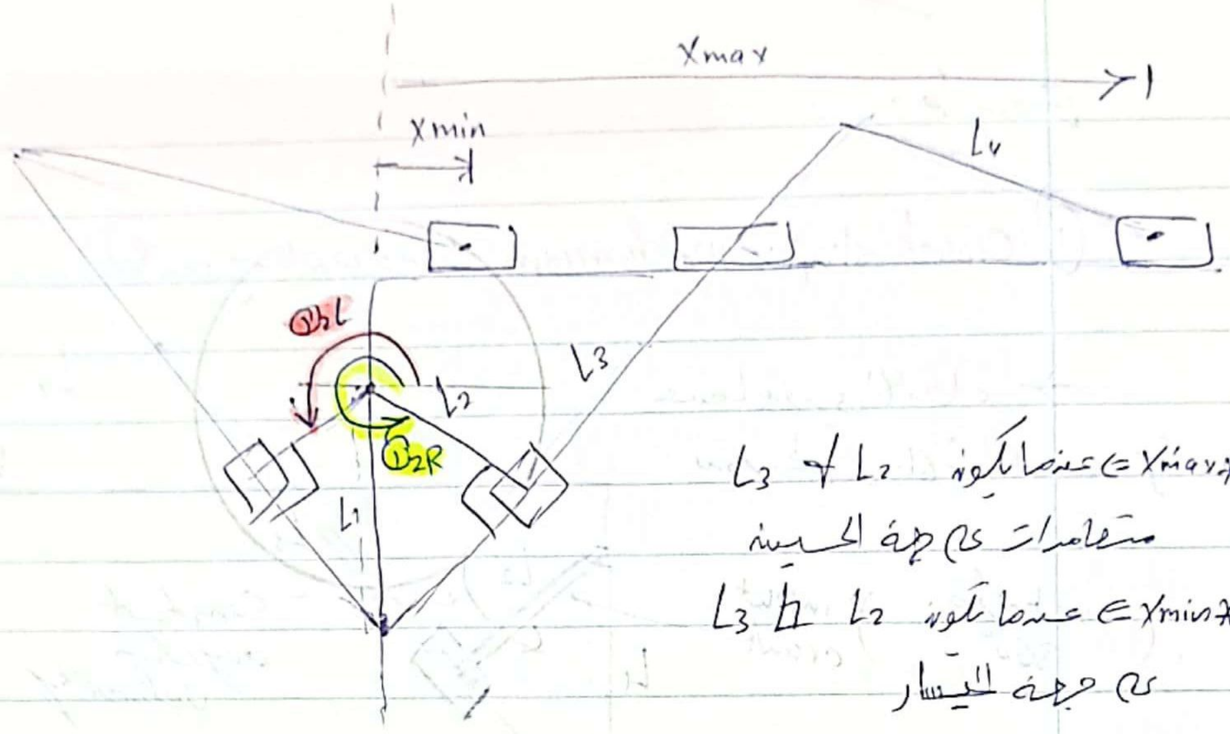
Example :-

① crank shaper mechanism. (see working model).

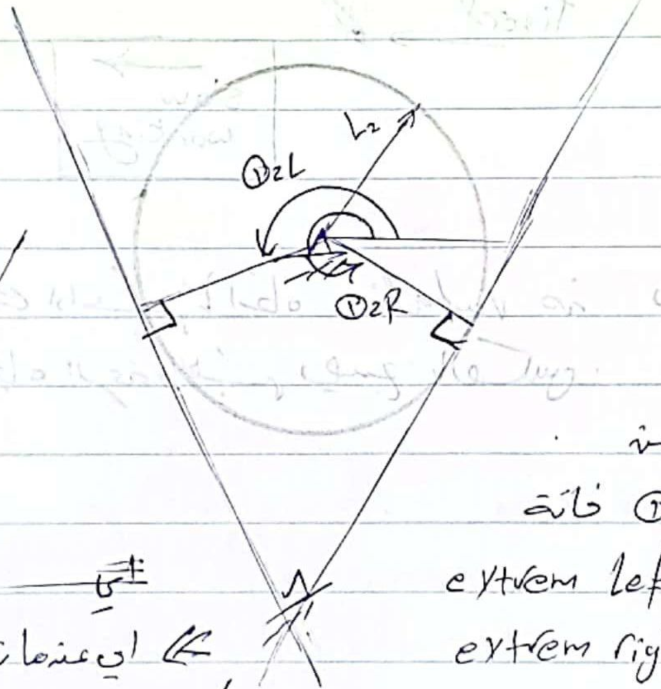
→ عندما يدور في الاتجاه
يدور ببطء و بالاتجاه ← بسرعة

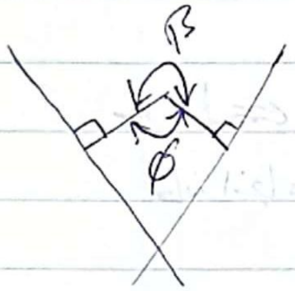


يلاحظ اذا غيرنا اتجاه rotation من cw ← ccw
فانه اتجاه الرجعة والقشر يصبح بالعكس.



* Xmax ∈ عندما يكون L3 ⊀ L2
متعامدات في جهة اليمين
* Xmin ∈ عندما تكون L3 ⊥ L2
في جهة اليسار

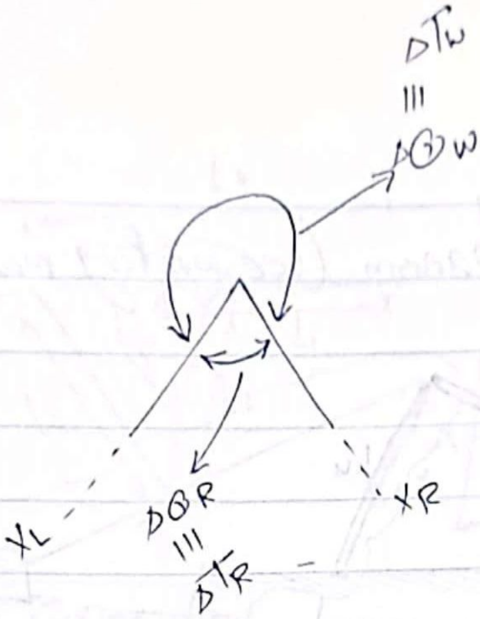




* اي عندما يدور من θ_{2L} الى θ_{2R} فانه
يتحرك من extrem left
الى extrem right position

* وعندما يدور من θ_{2R} الى θ_{2L}
فانه يتحرك من extrem right الى
extrem left . . . .

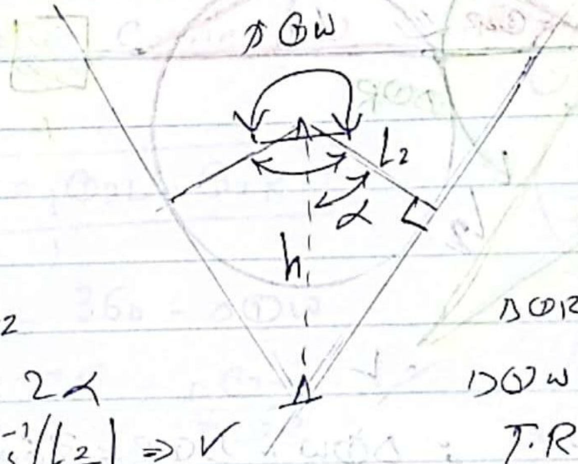
~~θ_{2L}~~ ~~اي~~
⇐ اي عندما تتحرك من
extrem left الى extrem right
فانه الزمن المستغرق اكبر
اذا كان ccw
وزمن اقل في حالة من
extrem left ← right.
$\beta > \phi$
لهذا السبب سريع تجاه ← ∈(ccw)
وبطيء تجاه ←



* left → Right (speed) ⇒ وقت قصير

* Right → left (slowly) ⇒ وقت أطويل

To find DΘR + DΘW.



Exp:-

if h = 5.7 cm

L_2 = 2.8 cm

$D\Theta R = 2\cos^{-1}\left(\frac{2.8}{5.7}\right) = 122°$

$D\Theta W = 360 - 122 = 238°$

$T.R = \frac{D\Theta W}{D\Theta R} = \frac{238}{122} = \underline{\underline{1.95}}$

⇓

أي حركة working

نصف حركة return.

أي الوقت المستغرق في

working أول (ضعفين).

given h + L_2

$\Rightarrow D\Theta R = 2\alpha$

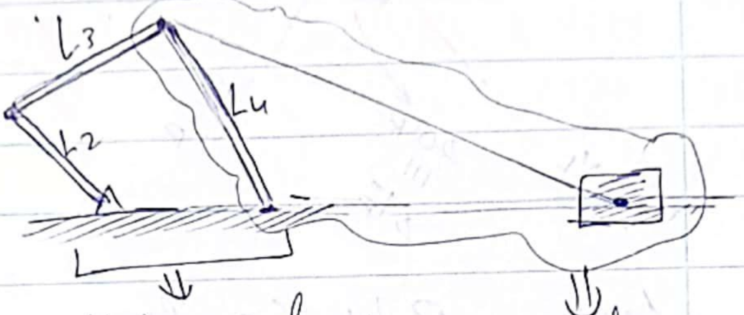
$= 2\cos^{-1}\left(\frac{L_2}{h}\right) \Rightarrow \checkmark$

$D\Theta W = 360 - D\Theta R$

$T.R = \frac{D\Theta W}{D\Theta R}$ Time Ratio.

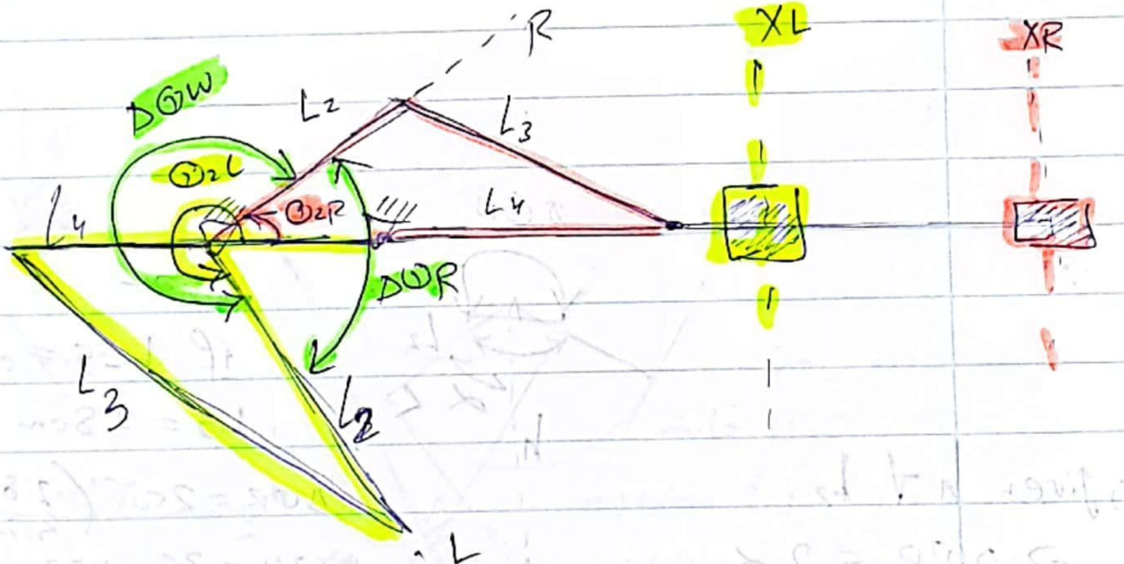
- working ⇒ بطيئة ⇒ DΘW أكبر
- Return ⇒ سريعة ⇒ DΘR أقل.

② Drag link mechanism (see working model example).



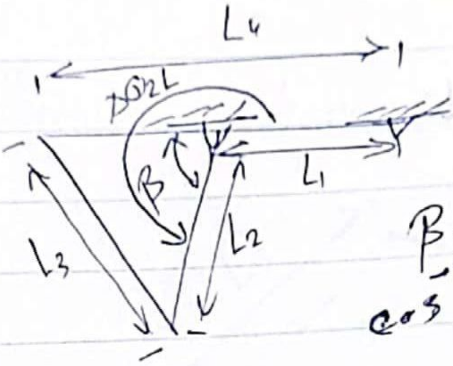
4-bar mechanism
Double crank
$S+L < P+Q$
ground ≡ S

Slider
crank mechanism



$$T.R = \frac{\Delta\theta w}{\Delta\theta R} \quad ; \quad \Delta\theta w + \Delta\theta R = 360^\circ$$

Extrem left

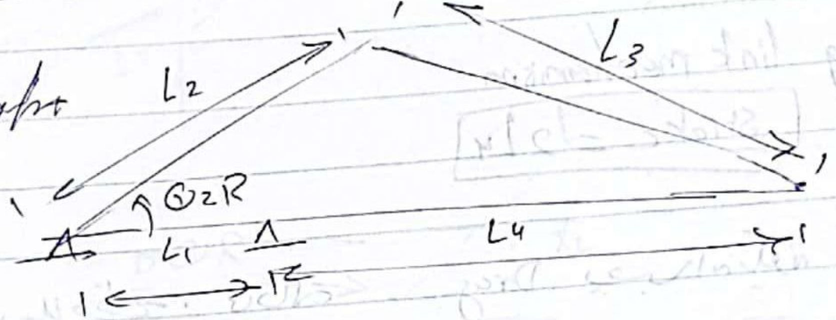


$$\beta = \cos^{-1}\left[\frac{(L_4-L_1)^2 + L_2^2 - L_3^2}{2(L_4-L_1)\,L_2}\right]$$

β = Cosine law

$$\boxed{\theta_{2L} = 180 + \beta}$$

Extrem Right



⇒ θ_{2R} = Cosine law

$$\boxed{\Delta\theta_w = \theta_{2L} - \theta_{2R}}$$

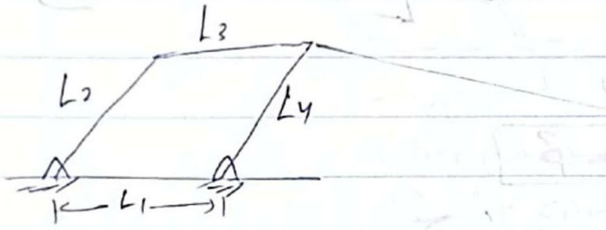
$\Delta\theta_R = 360 - \Delta\theta_w$

⇒ $T.R = \dfrac{\Delta\theta_w}{\Delta\theta_R}$ ✓

$|\theta_{2R} - \theta_{2L}| < 180$
$\Downarrow$
$\Delta\theta_R$

$|\theta_{2R} - \theta_{2L}| > 180$
$\Downarrow$
$\Delta\theta_w$

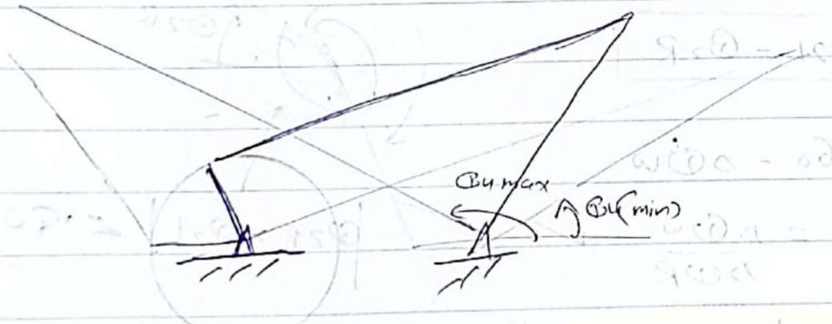
given $L_2 . L_3 . L_4 . L_1 . L_5$.
⇒ find T.R
& The stroke



Drag link mechanism
$\boxed{\text{stroke} = 2L_4}$

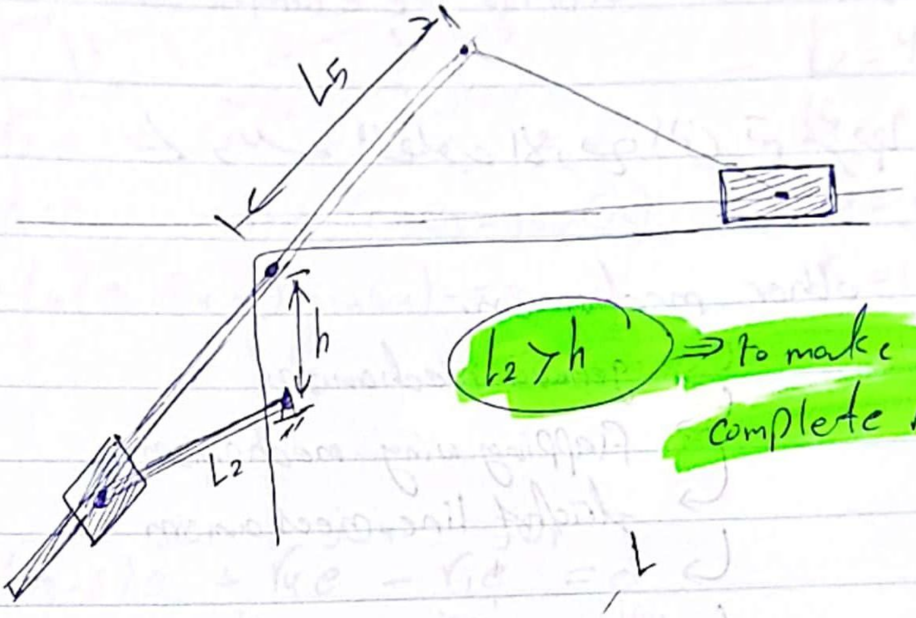
---

*ملاحظة: في حالة ... Drag يجب ان يكون
هنالك Double crank.
ولكن هنالك rock ⇐ اذا كان هنالك crank rocker.

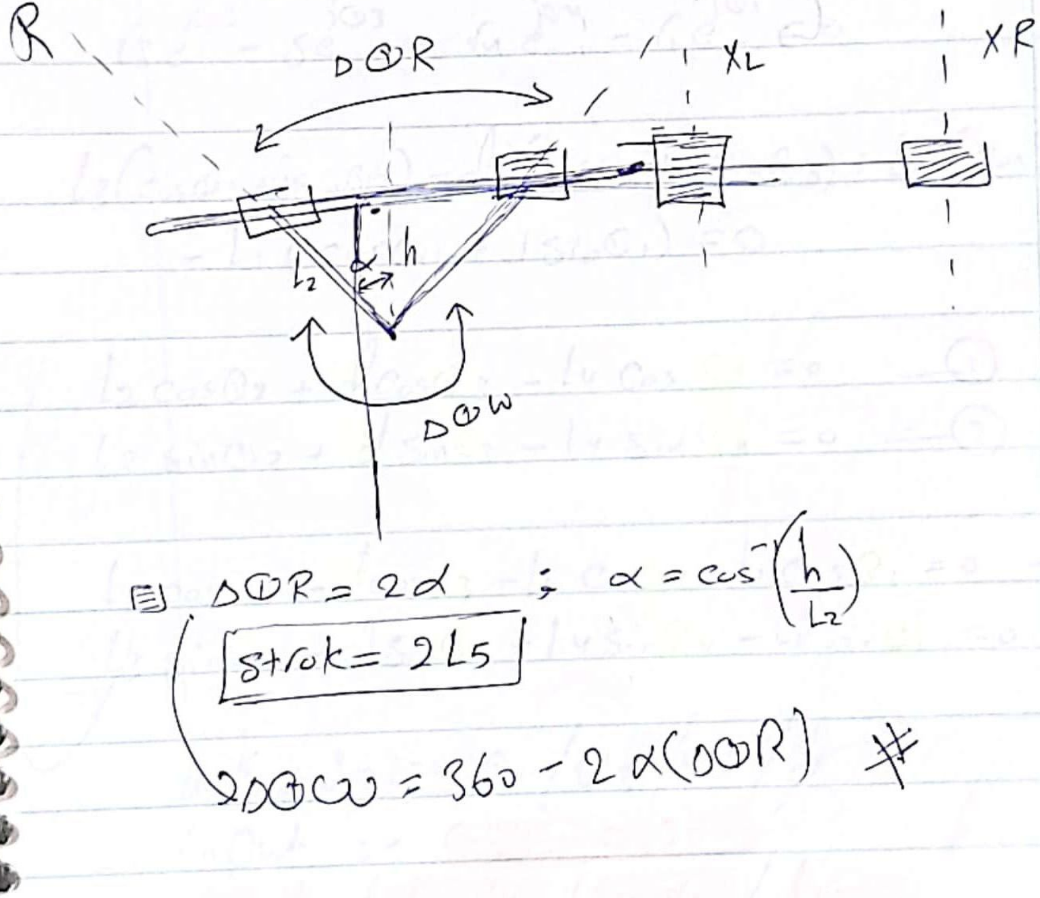


* ان نعمل تحليل مختلف هنا اذا كان L_4 لا يعمل
complete rev ⇐ وبناءً عليه نقوم بعملية التحليل.

③ Whitworth mechanism (see working model example).



$L_2 > h$ ⇒ to make complete revolution



$\therefore DOR = 2\alpha$; $\alpha = \cos^{-1}\left(\frac{h}{L_2}\right)$

$\boxed{\text{strok} = 2L_5}$

$DOW = 360 - 2\alpha (DOR)$ #

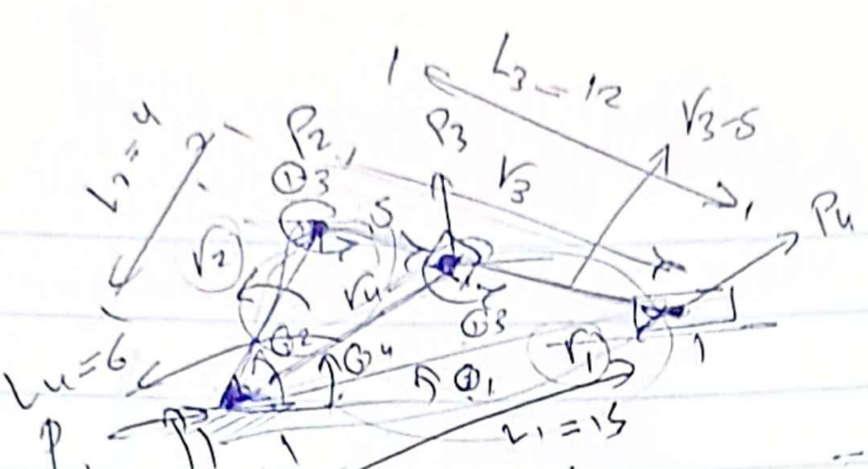
>> other mechanisms:- see example in public folder.

⇐ وبس المطلوب الاربع اللي تم شرحها.

other mechanism:-

- ↳ geneva mechanism
- ↳ flapping wing mechanism.
- ↳ stricht line mechanism
- ↳
- ↳

Project 5



θ_2; 0:360

$r_2=4$

$r_3=12$

$r_4=6$

$r_1=13$

$$r_2 e^{j\theta_2} + s e^{j\theta_3} - r_4 e^{j\theta_4} = 0$$

$$L_2(\cos\theta_2 + j\sin\theta_2) + d(\cos\theta_3 + j\sin\theta_3)$$
$$- L_4(\cos\theta_4 + j\sin\theta_4) = 0$$

$$(r_3 - s)e^{j\theta_3} + r_4 e^{j\theta_4} - r_1 e^{j\theta_1} = 0$$
$$r_3 e^{j\theta_3} - s e^{j\theta_3} + r_4 e^{j\theta_4} - r_1 e^{j\theta_1} = 0$$

$$L_3(\cos\theta_3 + j\sin\theta_3) - d(\cos\theta_3 + j\sin\theta_3) + L_4(\cos\theta_4 + j\sin\theta_4)$$
$$- L_1(\cos\theta_1 + j\sin\theta_1) = 0$$

$L_2\cos\theta_2 + d\cos\theta_3 - L_4\cos\theta_4 = 0$ — ①

$L_2\sin\theta_2 + d\sin\theta_3 - L_4\sin\theta_4 = 0$ — ②

$L_3\cos\theta_3 - d\cos\theta_3 + L_4\cos\theta_4 - L_1\cos\theta_1 = 0$ — ③

$L_3\sin\theta_3 - d\sin\theta_3 + L_4\sin\theta_4 - L_1\sin\theta_1 = 0$ — ④

unknown :- θ_1 / θ_3 / θ_4 / d

input :- $\theta_2 \Rightarrow 0:360$

$L_2=4$ / $L_3=12$ / $L_4=6$ / $L_1=13$

Loop 1:-

$$r_2 e^{j\theta_2} + d_1 e^{j\theta_3} - r_4 e^{j\theta_4} = 0$$

$$r_2(\cos\theta_4 + j\sin\theta_4) + d_1(\cos\theta_3 + j\sin\theta_3) - r_4(\cos\theta_4 + j\sin\theta_4) = 0$$

Real.P $r_2\cos\theta_4 + d_1\cos\theta_3 - r_4\cos\theta_4 = 0$ —— ①

Imag.P $r_2\sin\theta_4 + d_1\sin\theta_3 - r_4\sin\theta_4 = 0$ —— ②

Loop 2:-

$$r_4 e^{j\theta_4} + (r_3 - d_1)e^{j\theta_3} - r_1 e^{j90} - d_2 e^{j\,zero} = 0$$

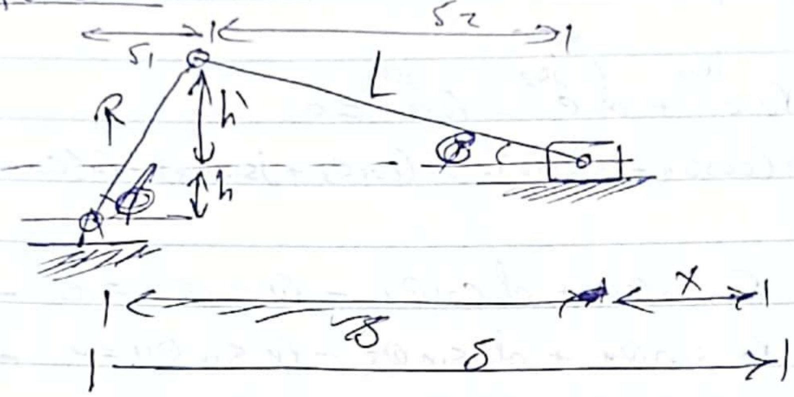
$$r_4(\cos\theta_4 + j\sin\theta_4) + (r_3 - d_1)(\cos\theta_3 + j\sin\theta_3) - r_1(\cos 90 + j\sin 90)$$
$$- d_2(\cos 0 + j\sin 0) = 0$$

Real.P $r_4\cos\theta_4 + (r_3 - d_1)\cos\theta_3 - d_2 = 0$ —— ③

Imag.P $r_4\sin\theta_4 + (r_3 - d_1)\sin\theta_3 - r_1 = 0$ —— ④

* note that :- Graphical Method $x = f(\theta)$

shift slider -



$$x = \delta - [\delta_1 + \delta_2]$$

$$\delta = \sqrt{(R+L)^2 - h^2}$$

$$\delta_1 = R\cos\theta = R\sqrt{1-\sin^2\theta}$$

$$\delta_2 = L\cos\phi$$

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Velocity & Acceleration Analysis

(A) ⇒ Using Polygons.

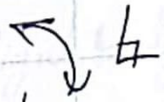
$\boxed{\vec{v}_B = \vec{v}_A + \vec{v}_{B|A}}$ Any two points in space

If point A & B are on the same rigid body

$\Rightarrow \boxed{\vec{v}_B = \vec{v}_A + \vec{w} \times \vec{r}_{B|A}}$

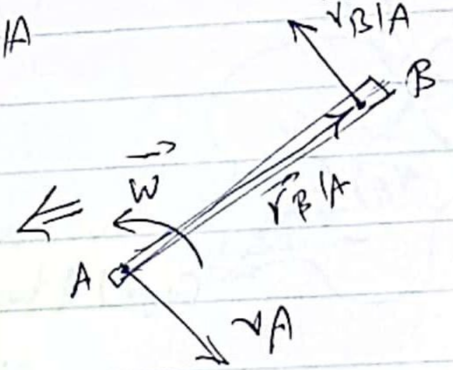
* $\vec{v}_{B|A} = \vec{w} \times \vec{r}_{B|A}$

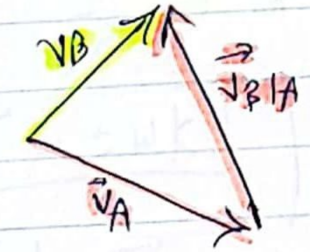
for planer mechanisms $\left[\begin{array}{l}\vec{w} = \pm w\hat{k} \\ \vec{r}_{B|A} = \text{in the plane.}\end{array}\right.$



بالمقادير $v_{B|A} = \omega r_{B|A}$

Polygons ⇐





$\vec{a}_B = \vec{a}_A + \vec{a}^{\,t}_{B|A} + \vec{a}^{\,n}_{B|A}$

$a^t_{B|A} = \alpha \times \vec{r}_{B|A}$

$a^n_{B|A} = -w^2 r_{B|A}$

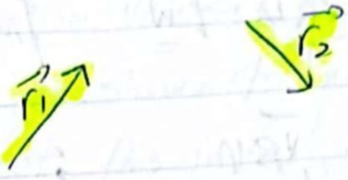
Ⓑ Using instant Centers for the velocity analysis.

Ⓒ Using Complex numbers (velocity & acceleration) analysis

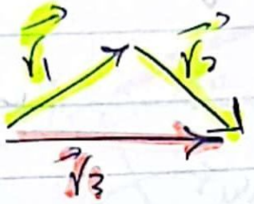
29/3/2021

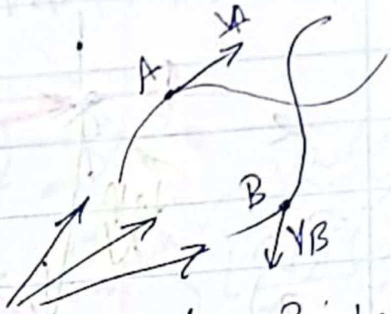
Velocity analysis using polygons.

$\vec{r}_3 = \vec{r}_1 + \vec{r}_2$



⇓





⇒ The relative velocity between any two Points:-

$\vec{V}_{B/A} = \vec{V}_B - \vec{V}_A$; $\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$

⇒ If points A anB are on the same Rigid body:-

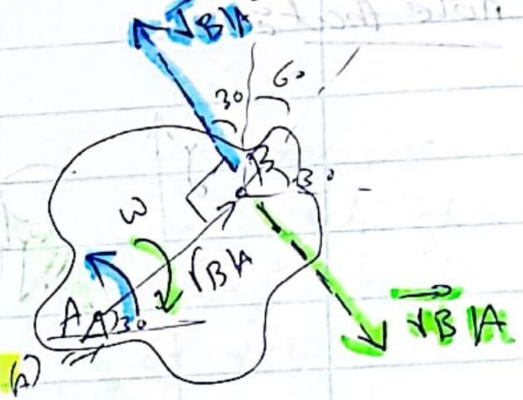
$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$

$\vec{V}_B = \vec{V}_A + w \times \vec{r}_{B/A}$

$\vec{V}_{B/A} = w \times \vec{r}_{B/A}$

= Translation as A ($\vec{V}_A$) + Rotation about A ($\vec{V}_{B/A}$)

= $\vec{V}_A + \vec{V}_{B/A}$



$\boxed{\vec{w} = \pm w k}$ $\boxed{\vec{r}_{B/A} = x_{B/A}\, \hat{i} + y_{B/A}\, \hat{j}}$

$\vec{w} \perp \vec{r}_{B/A}$

→ $|\vec{V}_{B/A}| = \boxed{V_{B/A} = w r_{B/A}}$

$\vec{V}_{B/A} = \vec{w} \times \vec{r}_{B/A}$

→ $\vec{V}_{B/A} \perp \vec{w}$

→ $\vec{V}_{B/A} \perp r_{B/A}$

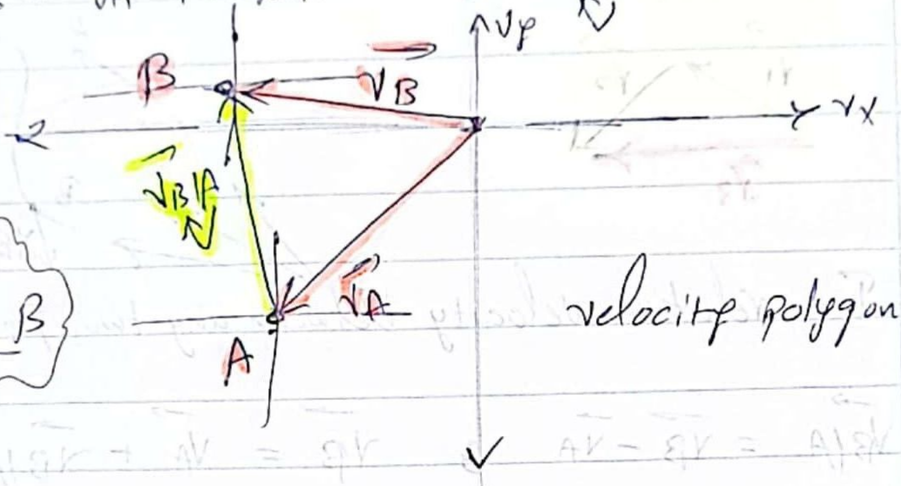
* $\vec{V}_{B/A}$ in the plane

$\overset{+}{CW}$/CCW ← * In the direction of $\vec{w}$

* given $\vec{V_A}$, $\vec{V_{B|A}}$ & $\vec{w}$ find $\vec{V_B}$??

$\vec{V_B} = \vec{V_A} + \vec{V_{B|A}}$; $\vec{V_{B|A}} = \vec{\omega} \times \vec{r_{B|A}}$

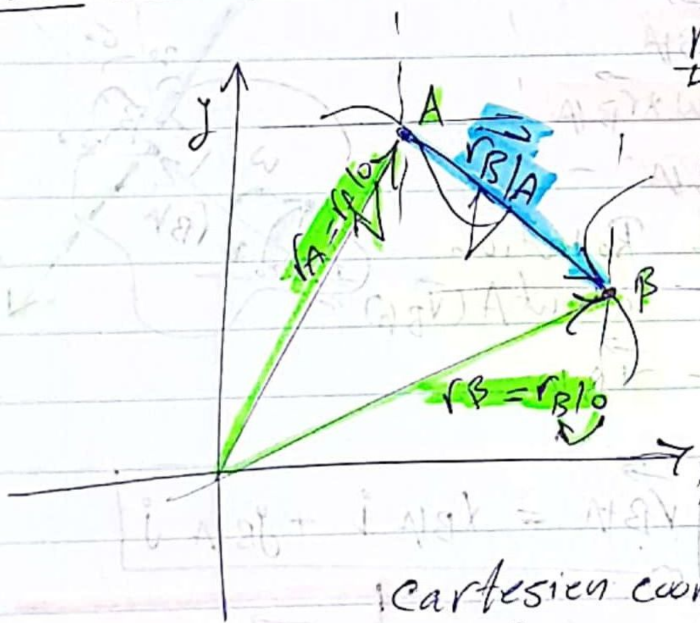
* $V_{B|A}$ from A to B



velocity polygons

---

note that:-



$$\underbrace{\vec{r_B} = \vec{r_A}} + \vec{r_{B|A}}$$

Absolute vectors (from the origin)

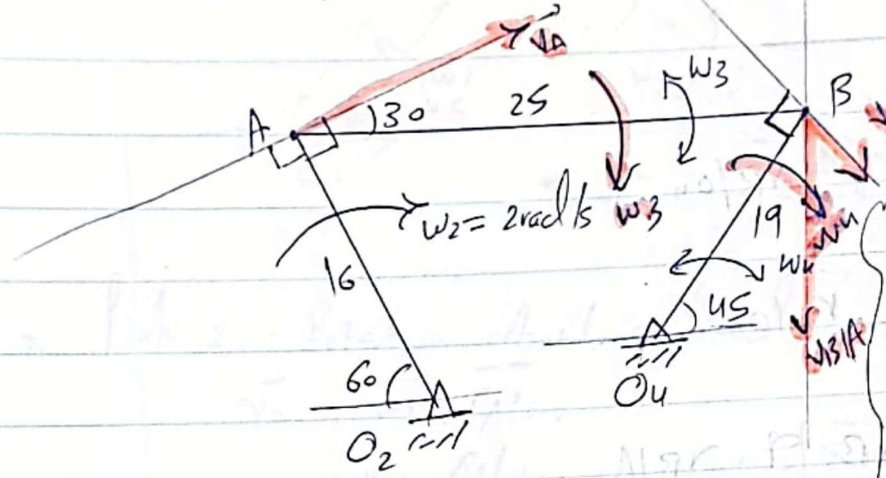
↳ Relative vectors (from A to B)

وكذلك بالنسبة velocity

cartesien coordinat
Position polygons

Example:- 4-bar mechanism velocity analysis using Polygons.

⇒ given $w_2 = 2\,rad/s$ as shown & link length, determine w_3 & w_4 at this instant.



note that:-

$\boxed{\overline{O_2A}}$ ⇒ link 2

$\vec{V}_A = \vec{V}_{O_2} + \vec{V}_{A/O_2}$

V_{O_2} = zero

$V_{A/O_2} = w_2 \times \vec{r}_{A/O_2}$

$V_{A/O_2} = w_2 r_{A/O_2}$

$\vec{V}_{A/O_2} \perp r_{A/O_2}$

Solution:-

link 2 ⇒ Rotation about a fixed axis.

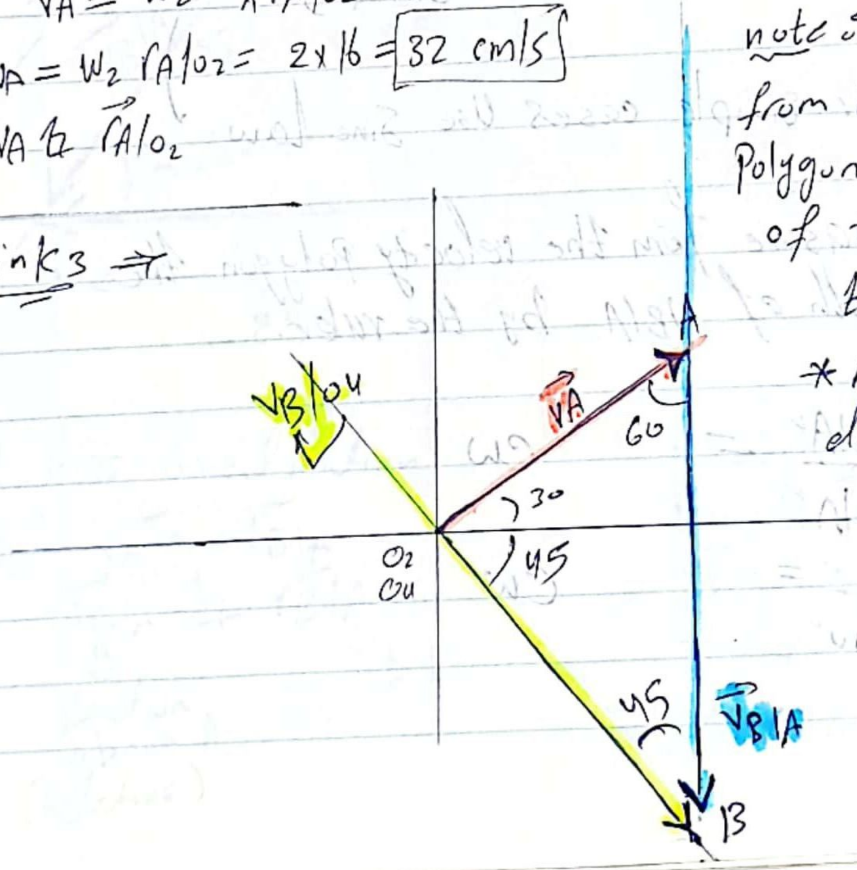
$V_A = \vec{w}_2 \times \vec{r}_{A/O_2}$

$V_A = w_2 r_{A/O_2} = 2 \times 16 = \boxed{32\ cm/s}$

$V_A \perp \vec{r}_{A/O_2}$

link 3 ⇒

↳



note:- measure from the velocity Polygon the length of $V_{B/A}$ by the ruler.

* And also it's direction the same for $\vec{V}_B$

link 3

$\vec{v_B} = \vec{v_A} + \vec{v_{B/A}}$

$\vec{v}_{B/A} \perp \vec{r}_{B/A}$

---

link 4..

$\vec{v_B} = \cancel{v_{O4}} + \vec{v}_{B/O4}$

$\vec{v}_{B/O4} \perp \vec{r}_{B/O4}$

* To find v_B & $v_{B/A}$

sin Law:-

$$\frac{v_A}{\sin 45} = \frac{v_B}{\sin 60} = \frac{v_{B/A}}{\sin 75}$$

OR

① for simple cases Use Sine Law.

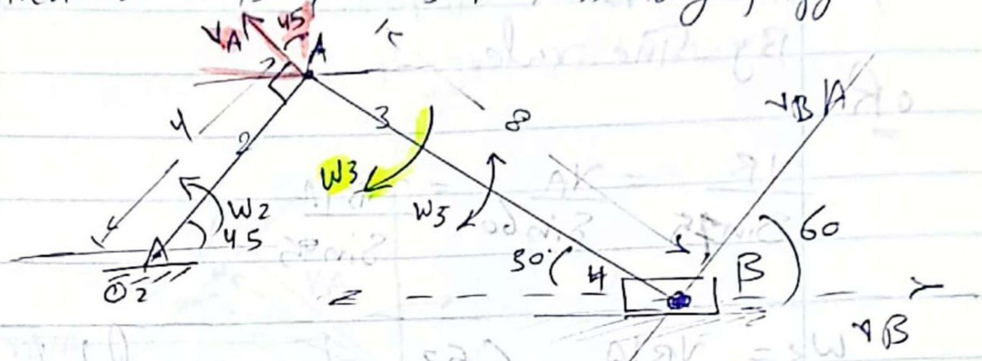
② → Measure from the velocity Polygon the length of $v_{B/A}$ by the ruler.

$\omega_3 = \frac{v_{B/A}✓}{r_{B/A}✓} =$ CW

$\omega_4 = \frac{v_B✓}{r_{B/O4}✓} =$ CW

3/4/2021.

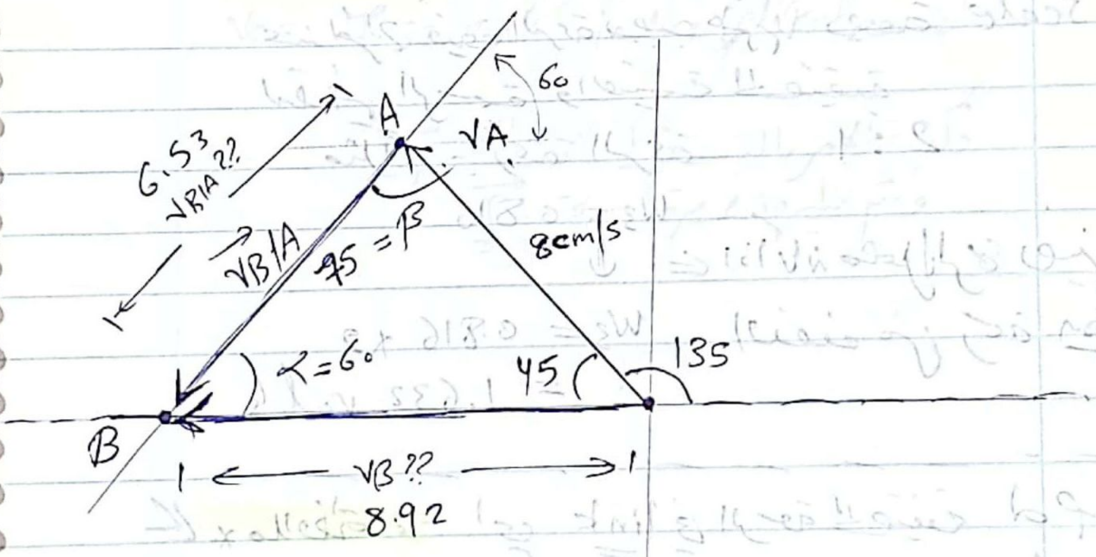
Exp. given $w_2 = 2 rad/s$ find w_3 + w_4 ?? Using Polygons.



* link 2:- Rotation about a fixed axis

$$\vec{v_A} = \vec{w_2} \times \vec{r_{A/O_2}}$$

$$= w_2 \, r_{A/O_2} = 2 \times 4 = \boxed{8 \, cm/s}$$



* link 3:- General motion

$$\vec{v_B} = \vec{v_A} + \vec{v_{B/A}}$$

$\vec{v_{B/A}} \perp r_{B/A}$

⇓

Rotation about A

(Relative)

* Slider:

Translational motion

$\vec{v_B}$ = along the path

⇓ of the slider

(absolute)

To find V_B & $V_{B/A}$:
By The ruler

OR

$$\frac{V_B}{\sin 75} = \frac{V_A}{\sin 60} = \frac{V_{B/A}}{\sin 45}$$

$$\omega_3 = \frac{V_{B/A}}{r_{B/A}} = \frac{6.53}{8} = \boxed{0.816 \text{ rad/s}} \quad \text{c.w}$$

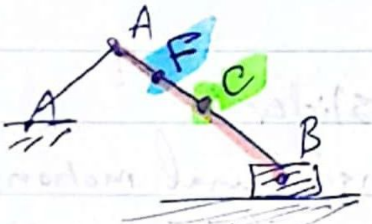
* ملاحظة :- اذا كان معمول للرسمة scale
عند اخراج قيمة السرعة يجب ضربها بقيمة scale
لتصغير الرسمة والقيمة الحقيقية
مثلاً :- رسمة السرعة 2 : 1

$\omega_3 = 0.816$
$\Downarrow$

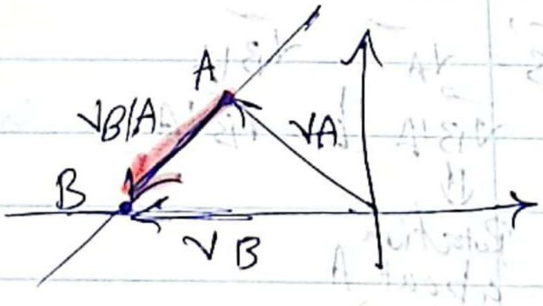
اذا كان معمول الرسمة بتصغير بمقدار ⇐
النصف من رسمة position

$\omega_3 = 0.816 \times 2$
$\overset{\text{لـ}}{=} 1.632$ rad/s

⇐ * ملاحظة :- اي link في الرسمة الحقيقية له صورة
في velocity polygon ..

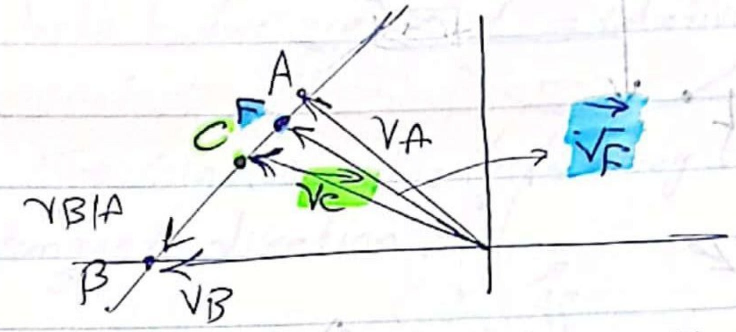


⟹



اي ان link ∈ AB
له صورة في velocity
polygon.

← مثلاً اذا كان هنالك نقطة C على link لـ AB :-
لايجادها على polygon :-

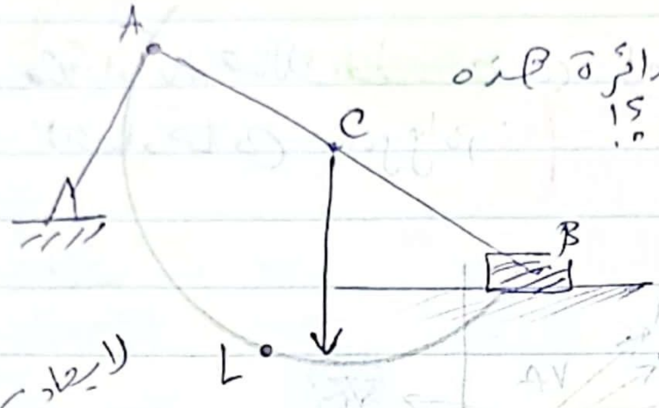


$\vec{V_c}$ ⇒ origin to point C on the polygon.

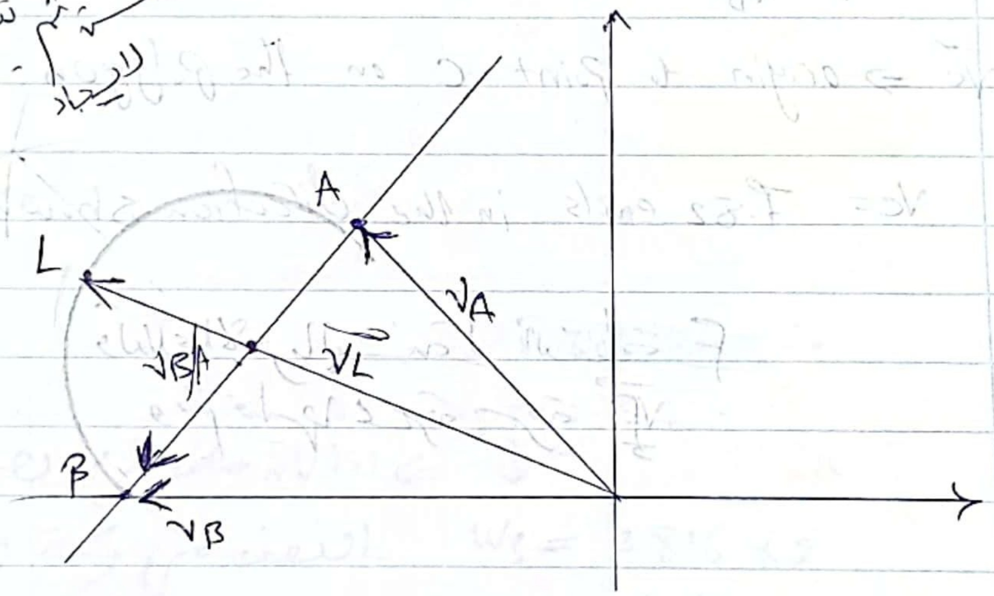
Vc = 7.82 cm/s in the direction shown.

وكذلك الامر بالنسبة لِ النقطة F
ويتم عليها معرفة سرعة $\vec{V_f}$.

مثلاً : اين تقع نصف الدائرة هذه
ضمن velocity Polyg ؟!



لايجاد سرعة النقطة L
⇓
وسـ ثم تم قياسها لايجاد
$\vec{V_L}$:



Abd Alkreem
Bawatneh

At the contact point :-

- contact without slipping/sliding:
  The velocities of the contacting points on both bodies are equal ≡ relative velocity = zero.
- contact with slipping:
  The relative velocity is along the common tangent direction.

* Contact without slipping:
$\vec{V}_A = \vec{V}_B$
$V_{B/A} = V_B - V_A = 0$

* β on the body
* A on the ground (contacting point)

$\boxed{\vec{V}_c = \vec{V}_B + \vec{V}_{c/\beta}} \rightarrow = \text{Zero} + \vec{V}_{c/\beta}$

$\boxed{V_c = \vec{V}_{c/\beta}}$

$\vec{V}_{c/\beta} = \vec{w} \times r_{c/\beta}$

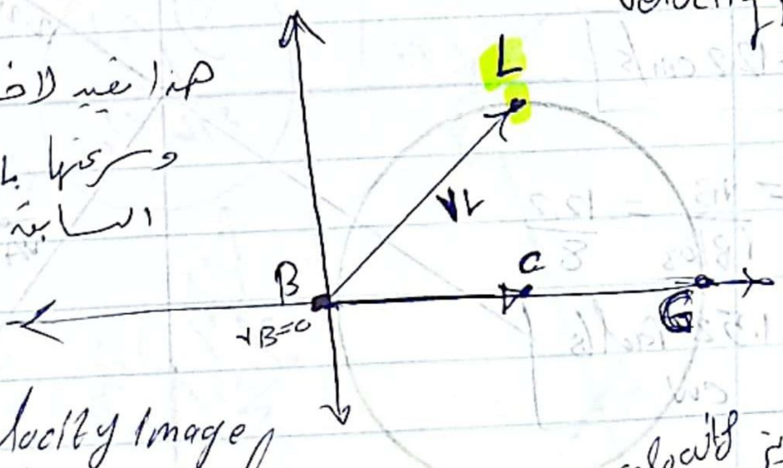
$V_{c/\beta} \Rightarrow \perp r_{c/\beta}$

$V_{c/\beta} = w\, r_{c/\beta} \Rightarrow ✓$

* $\boxed{V_f = W\, r_{f/B}}$

هنا نفيد الافراج اي نقطة
ونرسمها بالقياس . لا
السابقة

velocity polygon



velocity Image of the Wheel :-

* حل كويز velocity polygon ع public

5/4/2021

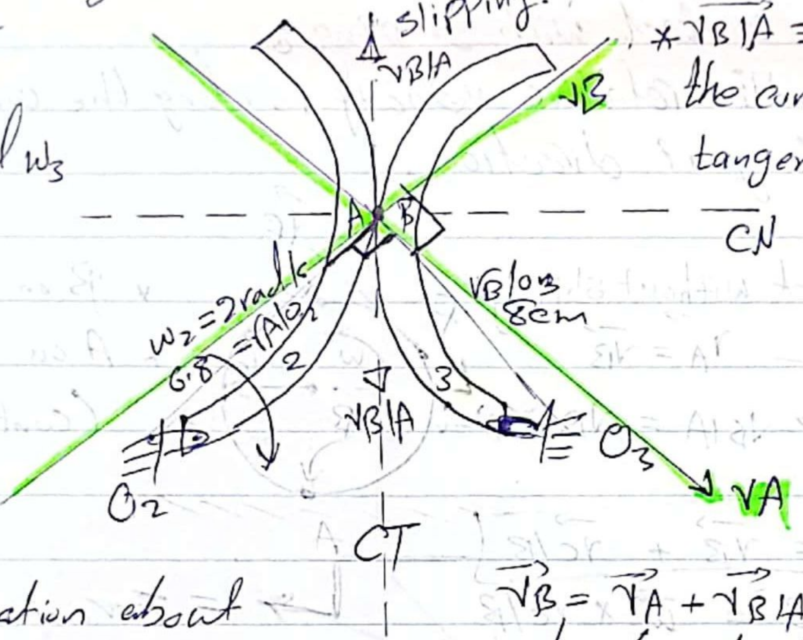
- contact with slipping ⇒ The relative velocity is along the common tangent direction.

⇒ The relative velocity of the contacting points in the direction of the common tangent direction.

Example :-

*given w_2 as shown ⇒ find w_3

* $\vec{V}_{B|A}$ ≡ along the common tangent direction



solution:-

link 2: Rotation about a fixed axis.

$V_A = w_2 \, r_{A/O_2}$

$= 2 \times 6.8$

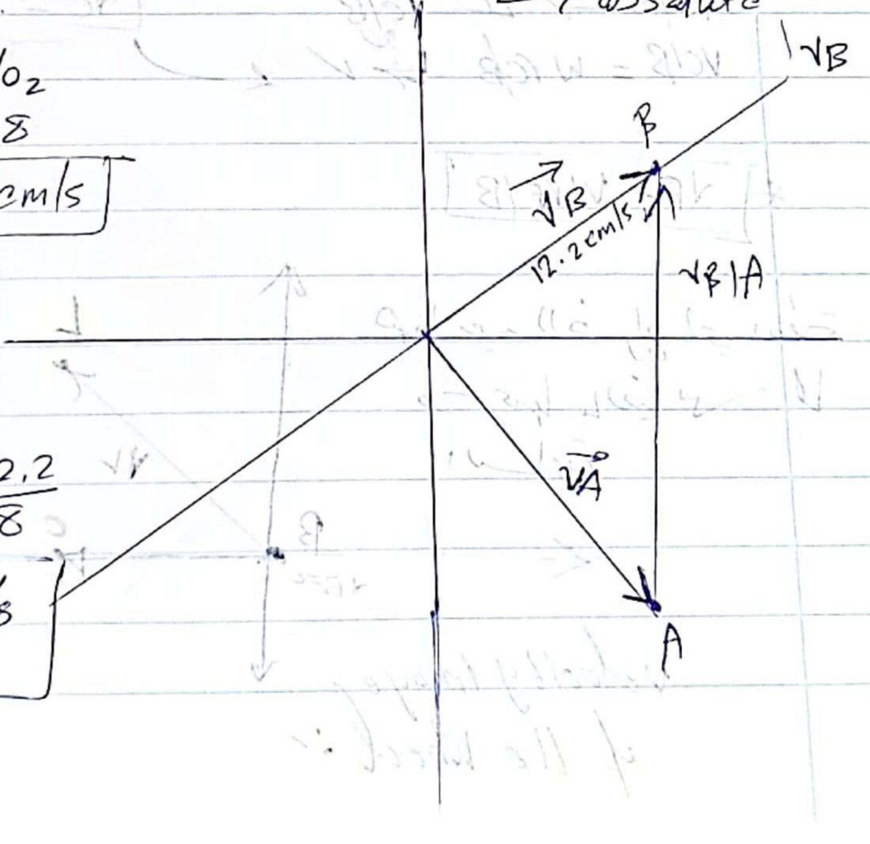
$\boxed{V_A = 13.6 \text{ cm/s}}$

$$\underbrace{\vec{V}_B}_{\text{absolute}} = \vec{V}_A + \underbrace{\vec{V}_{B|A}}_{\text{Relative}}$$

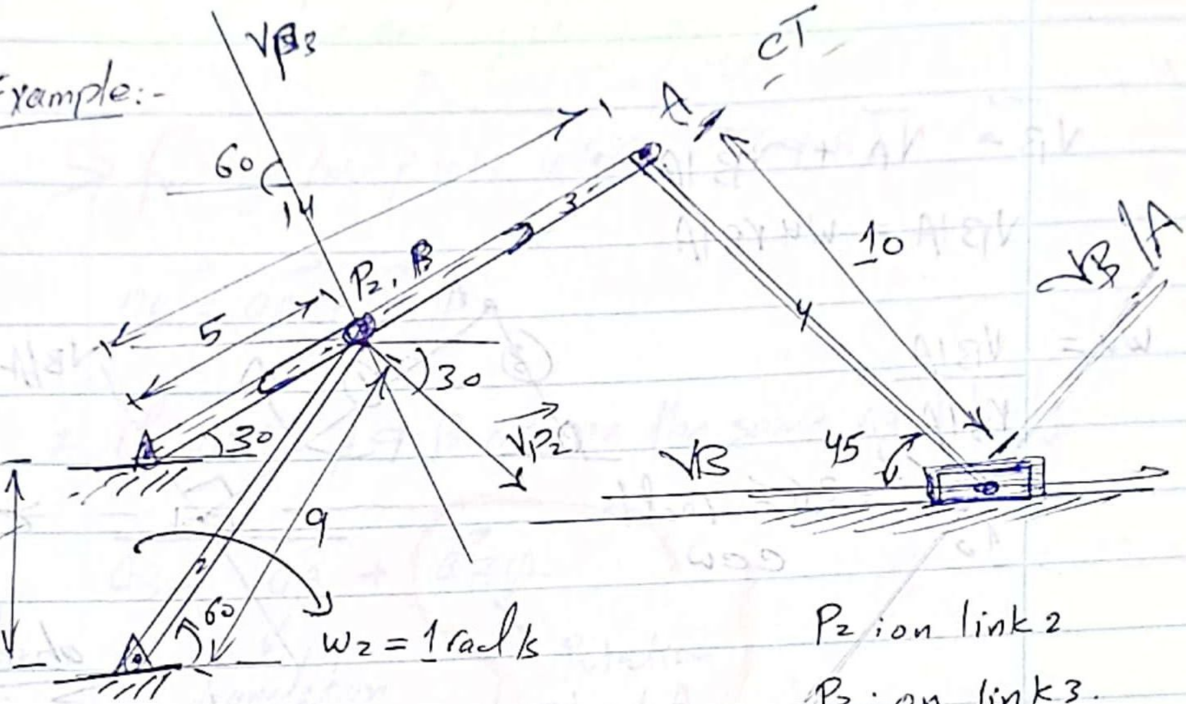
$\boxed{V_B = 12.2 \text{ cm/s}}$

$$w_3 = \frac{V_B}{r_{B/O_3}} = \frac{12.2}{8}$$

$\boxed{w_3 = 1.52 \text{ rad/s CW}}$



Example:-



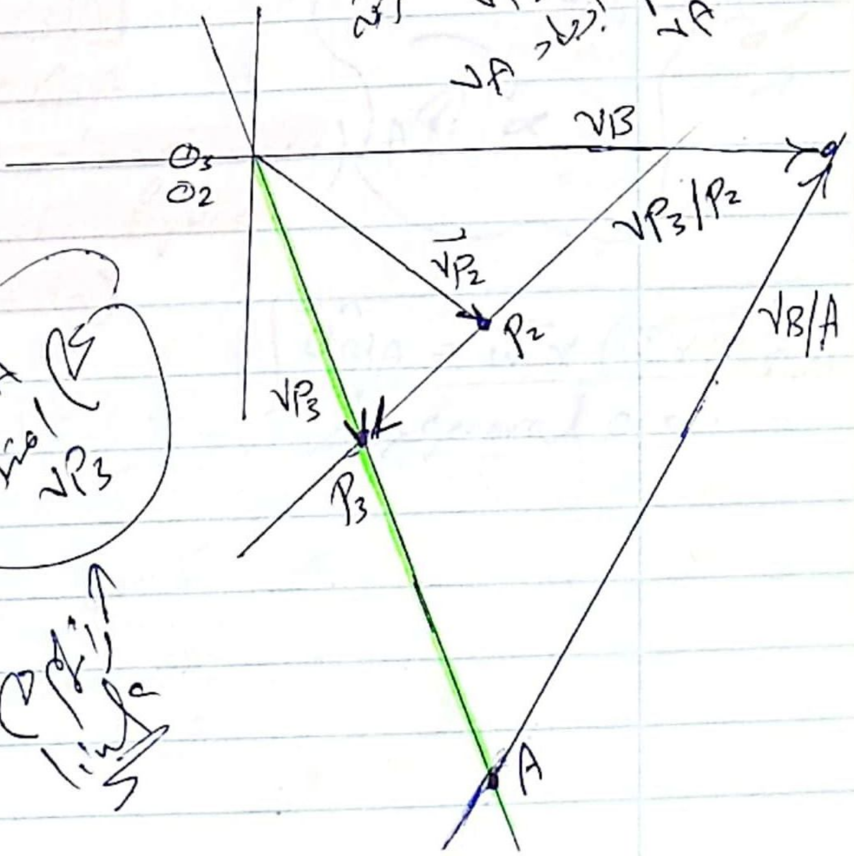
P2 : on link 2

P3 : on link3.

$V_{P_2} = w_2 \, r_{P_2/O_2} = 1 \times 9 = \boxed{9 \, cm/s}$

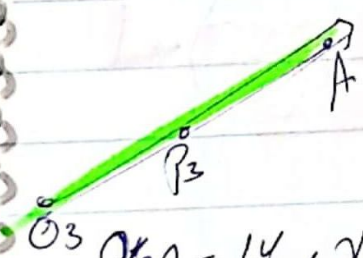
ايجاد w3 عن طريق V_{P_2} / P_3 ومنه ثم ايجاد VA

$V_A = r_{A/O_3} \, w_3$



لتحليل link3 : نسبة وتناسب

$$\frac{O_3A}{O_3P_3} = \frac{V_{A/O_3}}{V_{P_3/O_3}}$$



VA هي امتداد VP3

$\Rightarrow V_{O_3A} = \frac{14}{5} \times V_{P_3/O_3}$

$= \frac{14}{5} \times 7.8$

$V_{O_3A} = 21.8 \, cm/s$

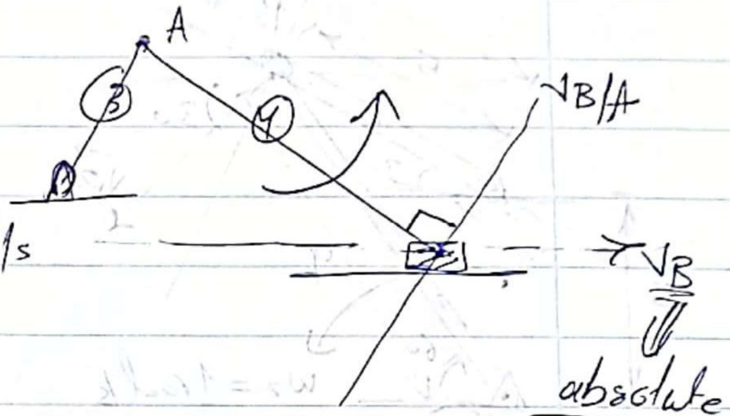
نعم لأنها link3

$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$

$v_{B/A} = w_4 r_{B/A}$

$$w_4 = \frac{v_{B/A}}{r_{B/A}} = \frac{26.7}{10} = 2.67 \text{ rad/s}$$

ccw



$\Rightarrow \underline{v_{B/A} \perp r_{B/A}}$

Acceleration analysis Using Polygons.

⇒ for any two points in space (A & B)

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

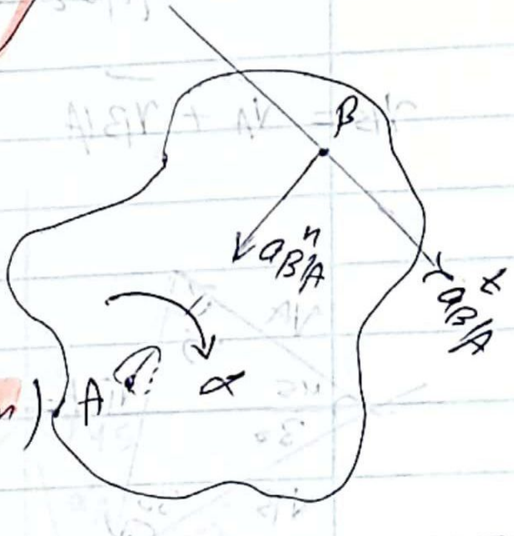
* If points A & B are on the same rigid body.

$$\vec{a}_B = \underbrace{\vec{a}_A}_{\text{translation as A}} + \underbrace{\vec{a}_{B/A}}_{\text{Rotation about A}}$$

$$\vec{a}_{B/A} = \vec{a}^{\,n}_{B/A} + \vec{a}^{\,t}_{B/A}$$

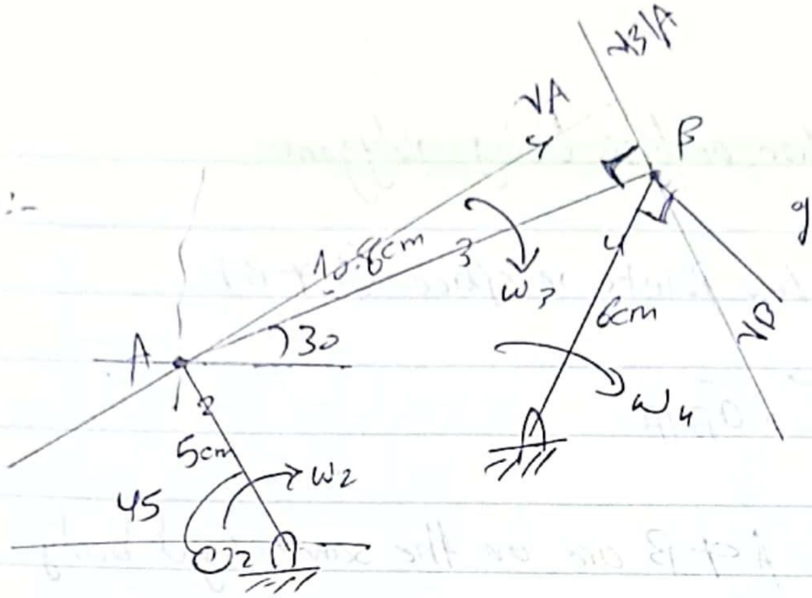
$$\boxed{\vec{a}^{\,n}_{B/A} = w^2 r_{B/A}}$$ → (planar case)(Direction shown in the figure)

$$a^t_{B/A} = \alpha\, r_{B/A}$$ → (Direction shown in the figure)



$$\boxed{\vec{a}^{\,n}_{B/A} = \vec{w} \times (\vec{w} \times \vec{r}_{B/A})}$$ → general case

Exp:-



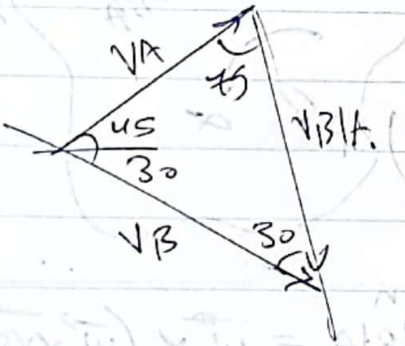
given $w_2 = 1 rad/s$ constant & link's length, determine α_3 & α_4.

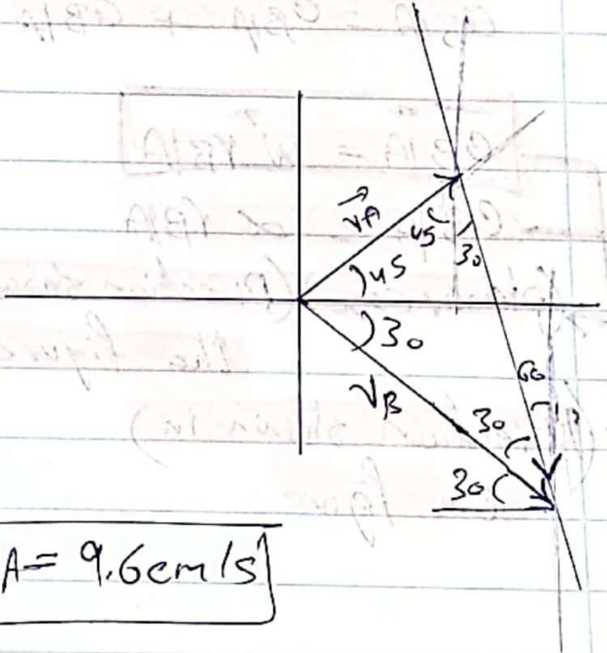
* start with velocity analysis :-

لاتستطيع الذهاب الى acceleration analysis الا بعد الانتهاء من velocity analysis

$$VA = W_2 \times r_{A/O_2} = 1 \times 5 = \boxed{5\cancel{rad}cm/s}$$

$$VB = \vec{V_A} + \vec{VB/A}$$





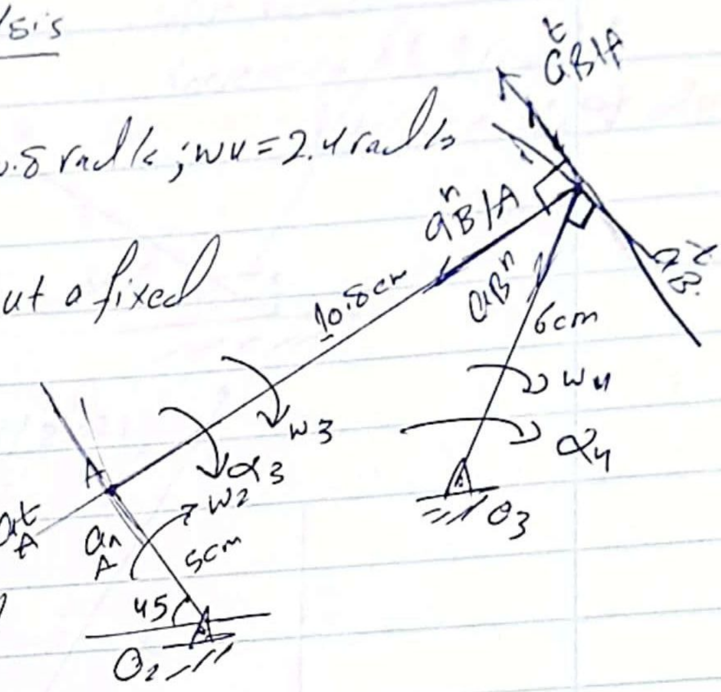
$$\frac{VB/A}{\sin 75} = \frac{VA}{\sin 30} \Rightarrow \boxed{VB/A = 9.6 cm/s}$$

$$\omega_3 = \frac{UB/A}{rB/A} = \frac{9.6}{10.8} = \boxed{0.8 rad/s}$$

$$\omega_4 = \frac{VB}{rB/O_4} = \frac{9.6}{4} = \boxed{2.4 rad/s}$$

* acceleration analysis

$w_2 = 1$ rad/s ; $w_3 = 0.8$ rad/s ; $w_4 = 2.4$ rad/s



link 2:- rotation about a fixed axis

$\vec{a}_A = \vec{a}_A^n + \vec{a}_A^t$

$a_A^n = w_2^2 \, r_{A/O_2}$

$= 5(1)^2 = \boxed{5 cm/s^2}$

$a_A^t = \alpha_2 \, r_{A/O_2}$

$=$ zero $\times 5 = \boxed{\text{zero}}$

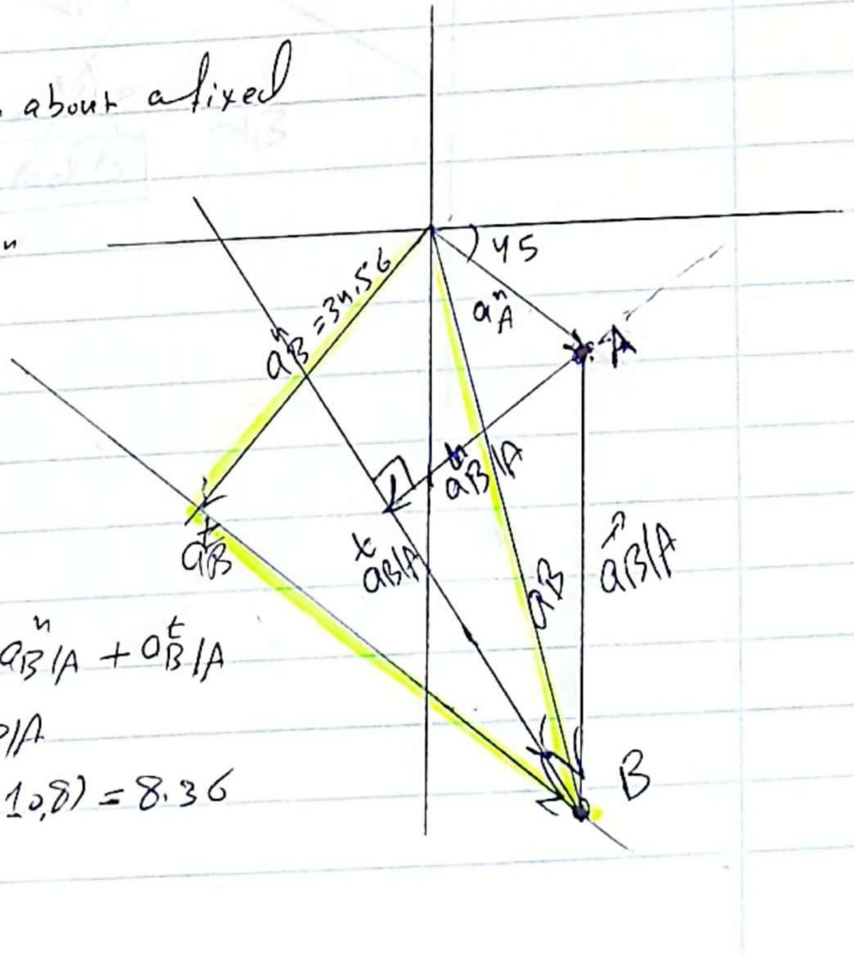
link 4:- Rotation about a fixed axis

$a_B^n = w_4^2 \, r_{B/O_4}$

$= (2.4)^2(6)$

$= 34.56$

$a_B = \vec{a}_B^n + \vec{a}_B^t$



link 3.-

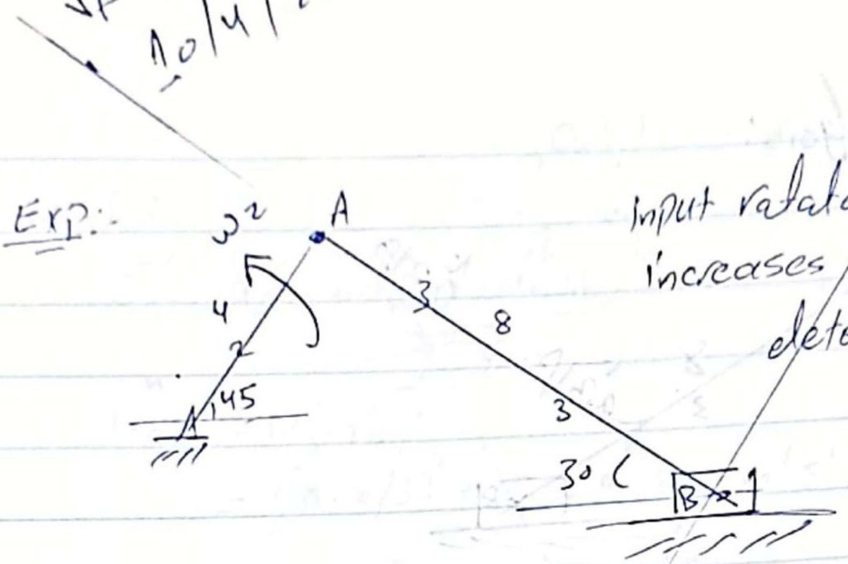
$\vec{a}_B = \vec{a}_A + a^n_{B/A} + a^t_{B/A}$

$a^n_{B/A} = w_3^2 \times r_{B/A}$

$= (0.8)^2(10.8) = 8.36$

VP 10/4/2021

Exp:-



Input rotates at 2rad/s but increases at 1 rad/s²
determine α_3 & α_B

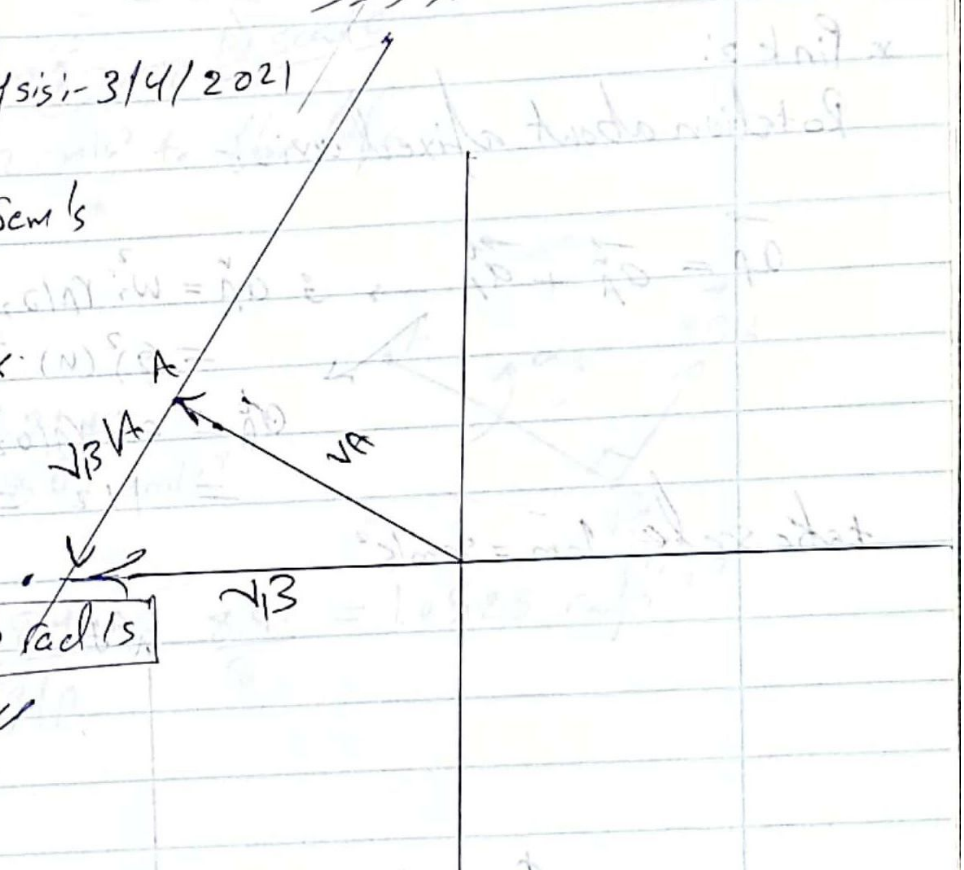
Velocity Analysis:- 3/4/2021

$V_A = (2)(4) = 8cm/s$

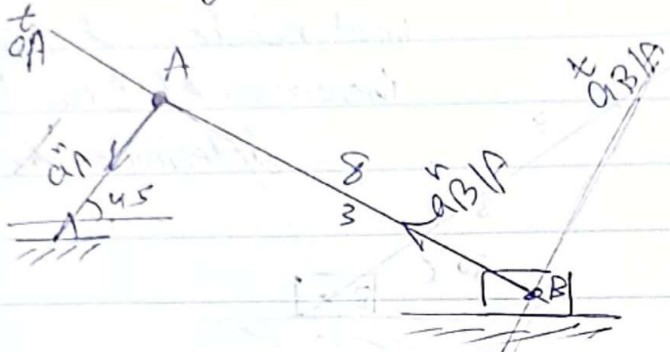
$V_{B/A} = 5cm/s$

$\omega_3 = \frac{V_{B/A}}{r_{B/A}}$

$\omega_3 = \frac{5}{8} = \boxed{0.625 \text{ rad/s}}$



⊟ acceleration analysis:-



* link 2:

Rotation about a fixed axis:-

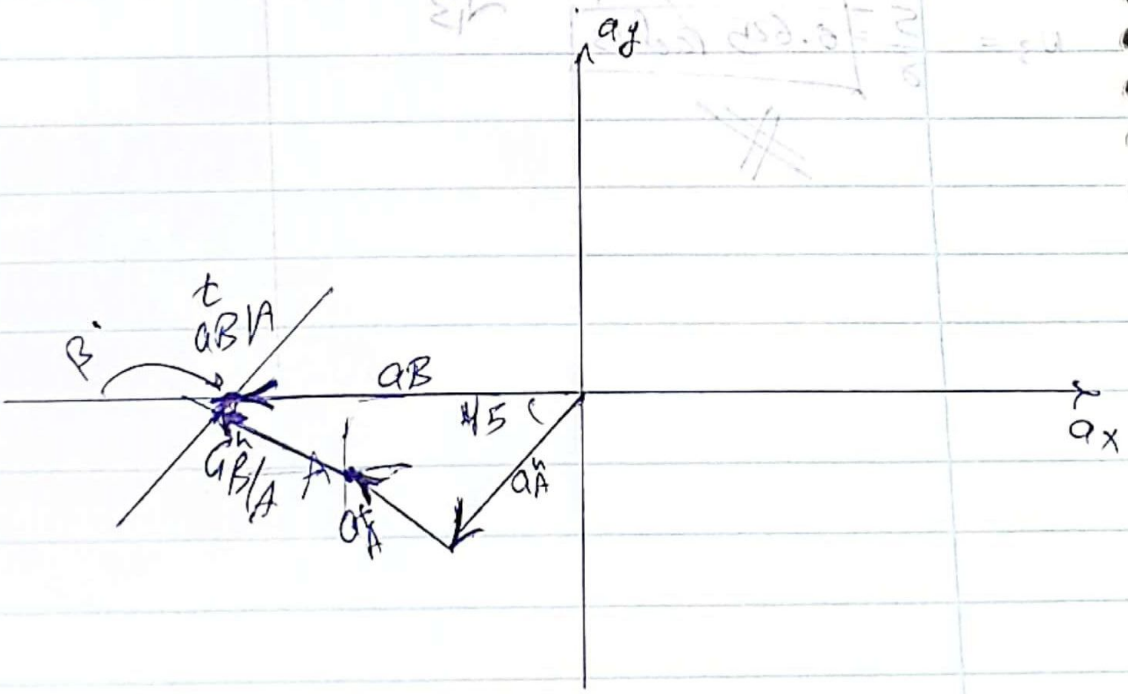
$$\vec{a}_A = \vec{a}_A^n + \vec{a}_A^t \longrightarrow ; \; a_A^n = w_2^2 \, r_{A/O_2}$$

$$= (2)^2(4) = \boxed{16 \, cm/s^2}$$

$$a_A^t = \alpha_2 \, r_{A/O_2}$$

$$= 1 \times 2 = \boxed{2 \, cm/s^2}$$

take scale $1cm = 20m/s^2$



*link 3:-

$$\vec{a}_B = \vec{a}_A + \underbrace{\vec{a}^{\,n}_{B/A} + \vec{a}^{\,t}_{B/A}}_{a_{B/A}}$$

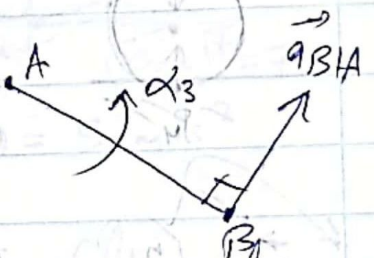
$$a^n_{B/A} = w_3^2 \, \vec{r}_{B/A}$$

$$= (6.8/6)^2 \times 8 = 5.32 \, cm/s^2 \equiv \boxed{2.66 \text{ cm on graph}}$$

$a_B = 6.51 \times \boxed{2} \rightarrow$ to scale

$= 13.2 \, cm/s^2$ to the left

from graph



$$a^t_{B/A} = 4.21 \times 2$$

$$= 8.42 \, cm/s^2$$

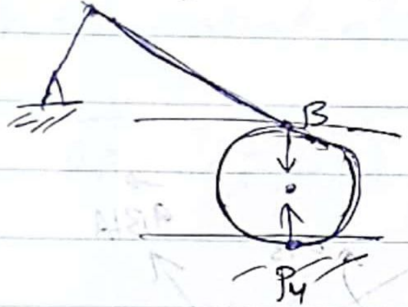
$$\alpha_3 = \frac{8.42}{r_{B/A}} = \frac{8.42}{8} = 1.0525 \, rad/s^2$$

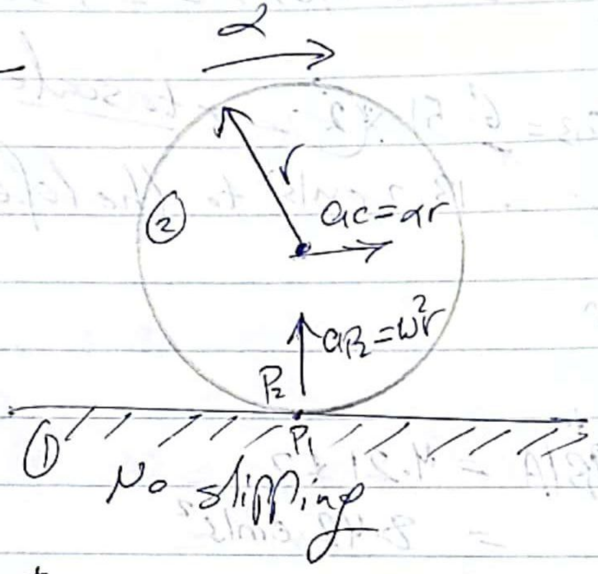
☐ Remark

① contact without slipping the componet of the acceleration of the contacting points along the common tangent direction are equal.

سؤال سيشبه لنفس الفكرة

Public ني





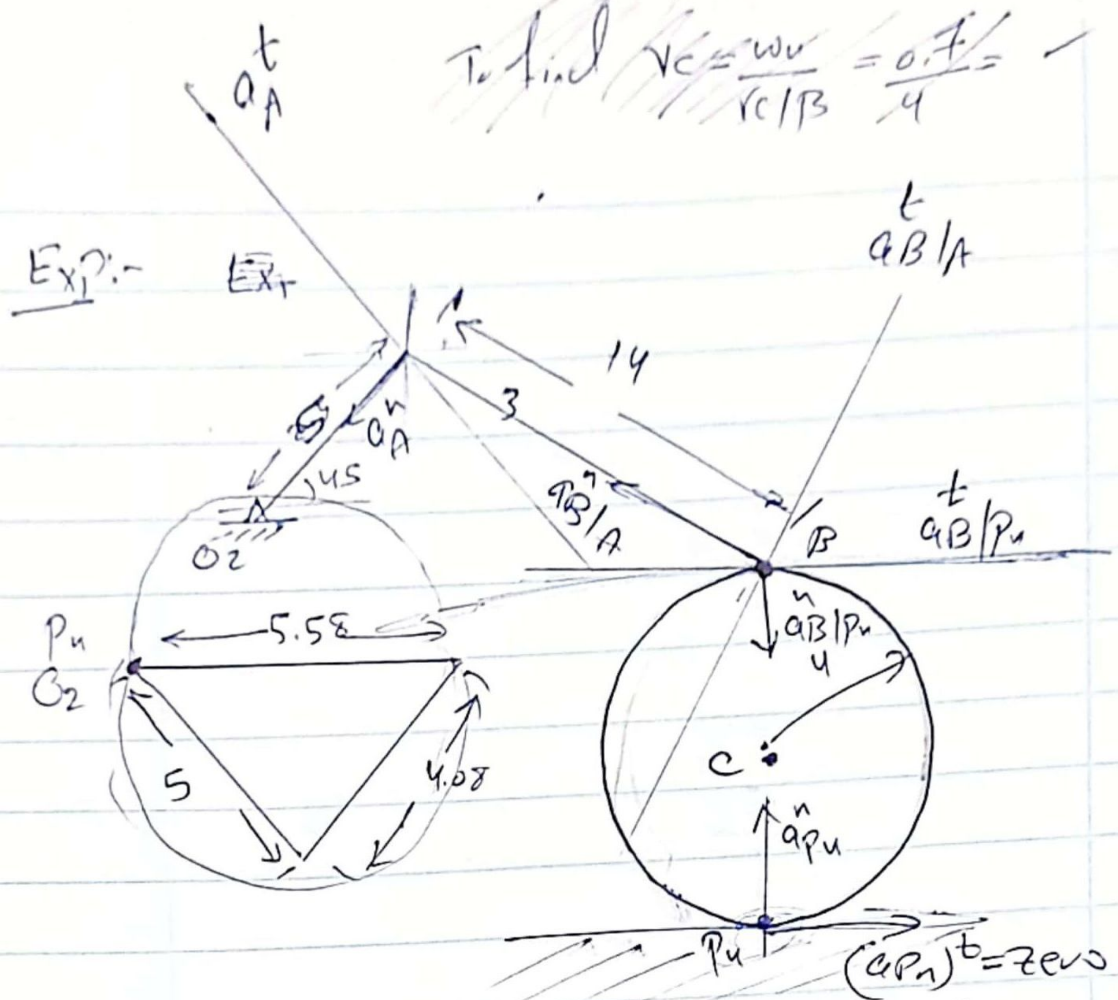
$$\vec{a}_B = \vec{a}_{P_4} + \underbrace{\overset{n}{a}_{B/P_4}}_{\omega^2_{Disk}\, r_{B/P_4}} + \overset{t}{a}_{B/P_4}$$

اي التقاطع يذهب الى النقطة B ؟

⟹ Solution

To find $V_C = \frac{\omega_4}{r_{C|B}} = \frac{0.7}{4} = -$

Ex1:- Ex,



$(a_A)_n = \frac{\omega_2^2}{\cancel{r_{A/O_2}}} (r_{A/O_2}) = 1(5) = 5\ cm/s^2$

$\omega_3 = \frac{V_{B/A}}{r_{B/A}} = \frac{4.08}{14} = 0.291\ rad/s$

$(a_{B/A})_n = \omega_3\ r_{B/A} = (0.291)^2 (14) = 1.19\ cm/s^2$

$\omega_4 = \frac{V_{B/P_4}}{r_{B/P_4}} = \frac{5.58}{8} = 0.7\ rad/s$ $\left[\rightarrow \omega_4 = \frac{V_C}{r_{Disk}} \right.$ هاي الطريقة (ما بنحتاجها) لأن V_C غير معروفة $\left. \right]$

$(a_{P_4})_n = \omega_4^2 (r_{Disk}) = (0.4)^2 (4) = 1.95\ cm/s^2$

$(a_{B/P_4})_n = \omega_4^2 (r_{B/P_4}) = (0.7)^2 (8) = 3.9\ cm/s^2$

$\alpha_3 = \frac{a_{B/A}^t}{r_{B/A}} =$, $\alpha_4 = \frac{a_{B/P_4}^t}{r_{B/P_4}} =$

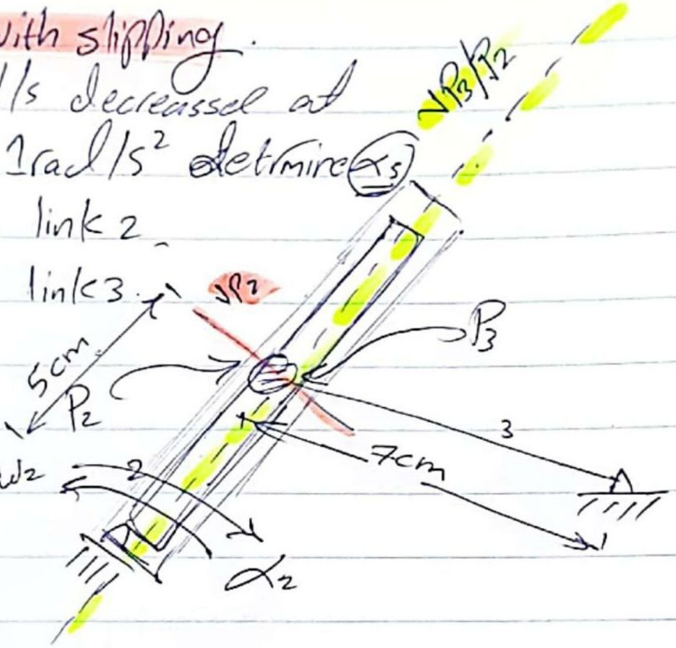
② contact with slipping.

given $w_2 = 2rad/s$ decreassel at

↳ solution:- $1rad/s^2$ determine α_3

Assume P_2 on link 2.

P_3 on link 3.



* velocity analysis.

$$\vec{V_{P_3}} = \vec{V_{P_2}} + \vec{V_{P_3/P_2}}$$

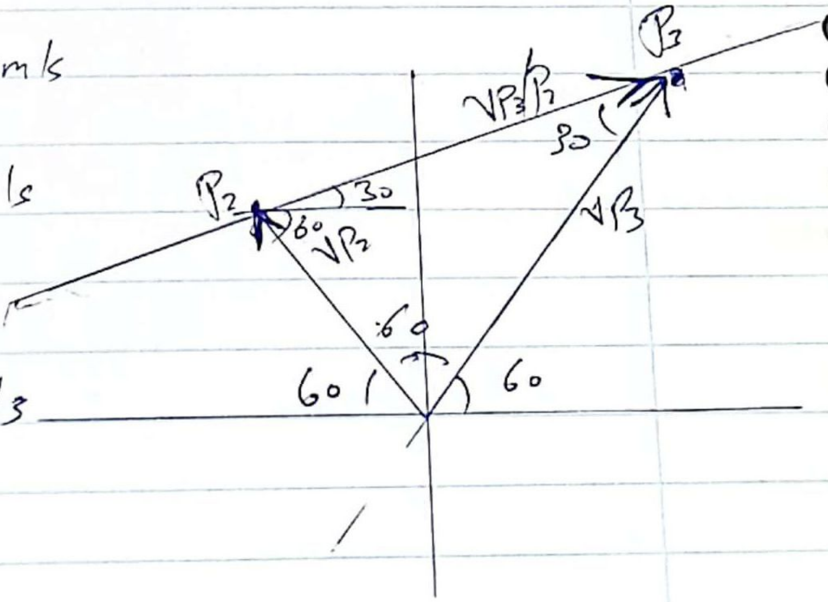
$$\vec{V_{P_2}} = W_2 \, P_2 W_2$$

$$= 2 \times 5 = 10cm/s$$

$$* \, V_{P_3} = \frac{10}{\cos 60} = 20cm/s$$

$$* \, V_{P_3/P_2} = 20\cos 30$$

$$= 17.3cm/3$$



⑤ Remark

$$\Rightarrow a_{P_3} = \vec{a}_{P_2} + \vec{a}^{\,rel}_{P_3/P_2} + 2\vec{\omega}_2 + \vec{V}_{P_3/P_2}$$

in Dynamics:-

$$\vec{a}_B = \vec{a}_A + \alpha \times r_{B/A} - \omega^2 r_{B/A} + \boxed{2\vec{\omega} \times (V_{B/A})_{xyz} + (\vec{a}_{B/A})_{xyz}}$$

هذا ال term الذي زضافيه

note:-

$$a_{P_3} = \vec{a}_{P_2} + \vec{a}_{P_3/P_2} \Rightarrow \text{absolute}$$

ولكن بالنسبة لـ link 2 وللموجودين عليه ويدورو معها:-

$$\Rightarrow a_{P_3/P_2}$$ relative

$$a_{P_3} = \vec{a}_{P_2} + \underbrace{\vec{a}^{\,rel}_{P_3/P_2} + 2\vec{\omega}_2 \times \vec{V}_{P_3/P_2}}$$

$$\vec{a}_{P_3} = \vec{a}_{P_2} + \vec{a}_{P_3/P_2}$$

⇓ Coriolis acceleration

$$\vec{a}_{P_2} = \vec{a}_{P_3} + \vec{a}^{\,rel}_{P_2/P_3} + 2\vec{\omega}_3 \times V_{P_2/P_3}$$

$\Rightarrow a^{rel}_{P_3/P_2} \equiv$ the acceleration of point P_3 as seen by an observer located on body 2 and rotates with it

$\Longleftrightarrow$

$a^{rel}_{P_2/P_3}$ ≡ the acceleration of point P_2 as seen by an observer located on body 3 and rotates with it.

* ⇐ اذا كنا نقف على link2 ونتحرك معه فانه P_2 ستتحرك بالنسبة لنا straight line motion.

اي من السهل معرفة حركة P_2.

* ⇐ ولكن اذا كان هنالك شخص يقف على link3. فليس من السهل معرفة حركة P_2 الواقعة على link2

اي من الصعب الشيء بالنسبة

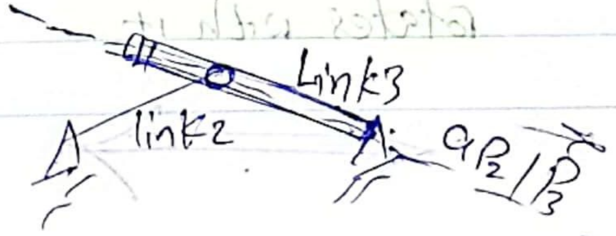
⇐ so

* $a^{rel}_{P_3/P_2}$ ⇒ easy for this example
* $a^{rel}_{P_2/P_3}$ ⇒ not easy to obtain in this Example. !!

اي لذلك سيكون مرجعنا للمجرى

⇐ اذا كان المجرى موجود على link2 نستخدم $\vec{a}_{P_3/P_2}$

واذا كان المجرى موجود على link3 نستخدم a_{P_2/P_3}



17/10/2021

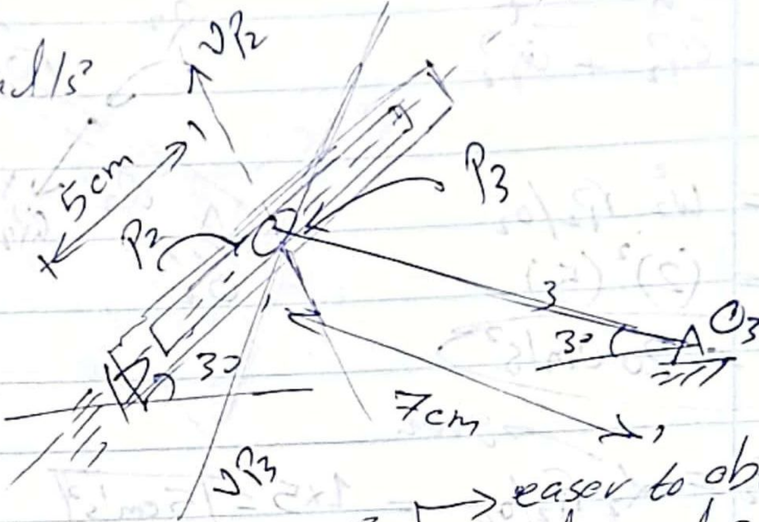
given $w_2 = 2\,rad/s$

deceases at $1\,rad/s^2$

determine α_2.

Assume P_2 on link 2

P_3 on link 3.



$\overrightarrow{V_{P_3/P_2}}$ → easer to obtain

$\overrightarrow{V_{P_3/P_2}} \equiv$ velocity of P_3 as seen by body 2.

$\overline{V_{P_2/P_3}} \equiv$ velocity of P_2 as seen by body 3.

velocity analysis:-

$V_{P_3} = V_{P_2} + V_{P_3/P_2}$

$V_{P_2} = W_2\, r_{P_2/O_2}$

$= 2 \times 5 = 10\,cm/s$

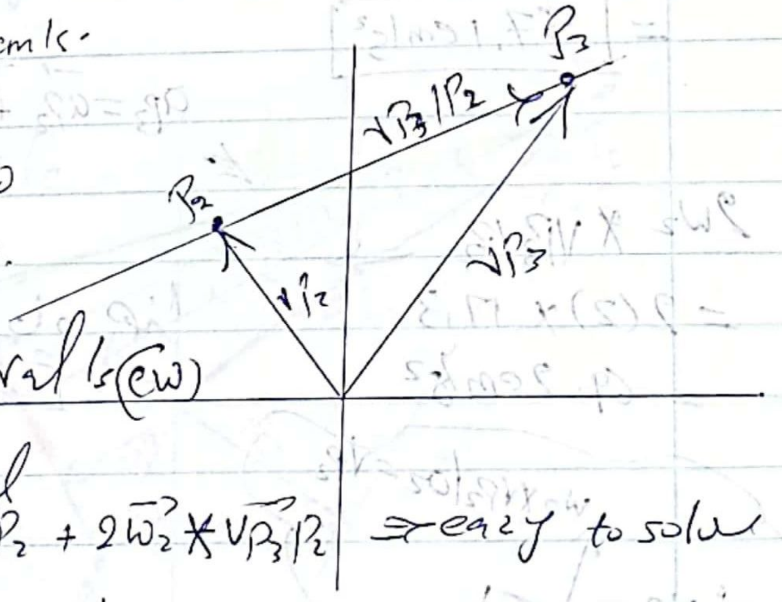
$V_{P_3} = \frac{10}{\cos 60} = 20\,cm/s$.

$V_{P_3/P_2} = 20 \cos 30$

$= 17.3\,cm/s$.

$W_3 = V_{P_3} / r_{P_3/O_3}$

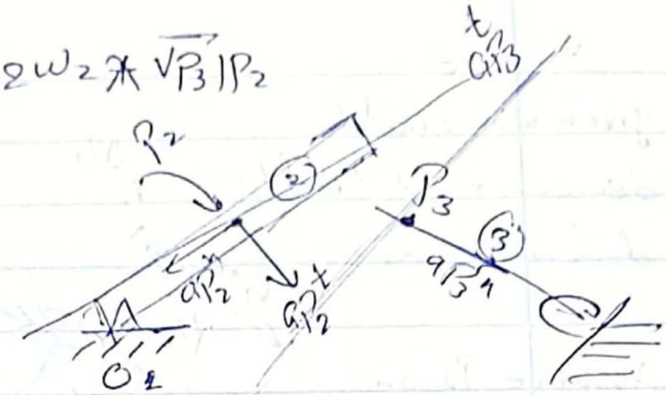
$= 20/7 = 2.86\,rad/s$ (CW)



accaleration:-

$\overrightarrow{a_{P_3}} = \overrightarrow{a_{P_2}} + a^{rel}_{P_3/P_2} + 2\overrightarrow{W_2} \times \overrightarrow{V_{P_3/P_2}}$ ⇒ eazy to solve

$$\vec{a_{P_3}} = \vec{a_{P_2}} + a^{rel}_{P_3/P_2} + 2\omega_2 \times \vec{V_{P_3/P_2}}$$



$$a_{P_2} = \vec{a}^{n}_{P_2} + \vec{a}^{t}_{P_2}$$

$$a^{n}_{P_2} = \omega_2^2 \, r_{P_2/O_2}$$
$$= (2)^2 (5)$$
$$a^{n}_{P_2} = 20 \text{ cm/s}^2$$

$$a^{t}_{P_2} = \overset{\text{given}}{\alpha_2} \, r_{P_2/O_2} = 1 \times 5 = \boxed{5 \text{ cm/s}^2}$$

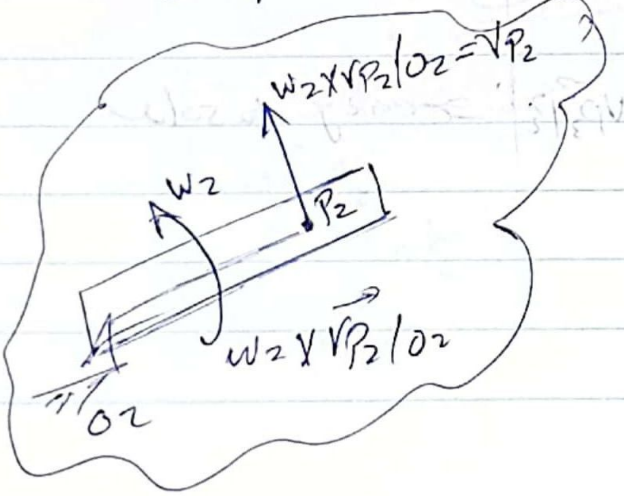
$$\vec{a_{P_3}} = \vec{a}^{n}_{P_3} + \vec{a}^{t}_{P_3}$$

$$a^{n}_{P_3} = \omega_3^2 \, r_{P_3/O_3}$$
$$= (2.86)^2 (7)$$
$$= \boxed{57.1 \text{ cm/s}^2}$$

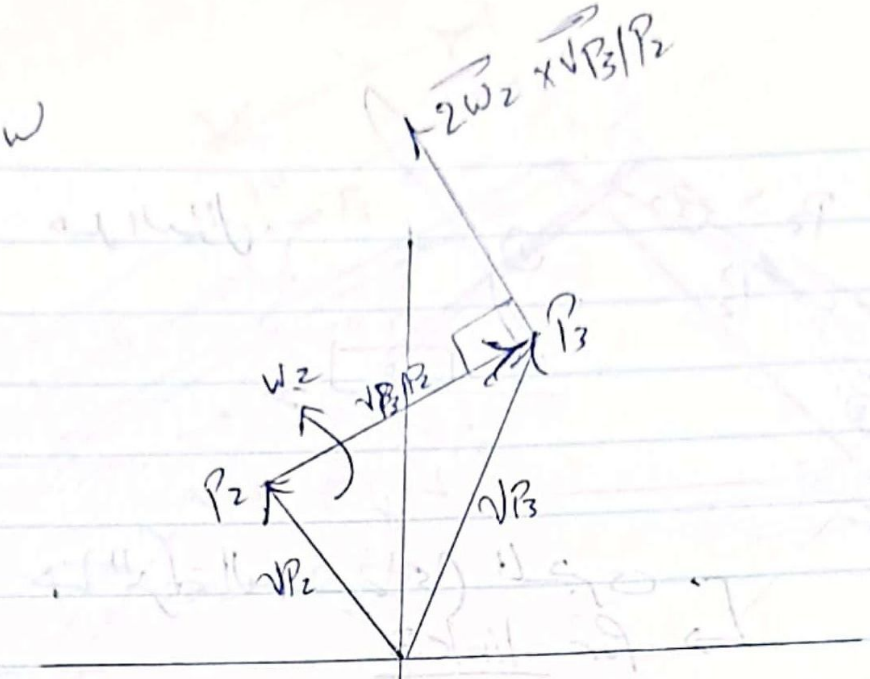
$$a_{P_3} = \vec{a}_{P_2} + \boxed{2\vec{W_2} \times V_{P_3/P_2}} + \vec{a}_{P_3/P_2}$$

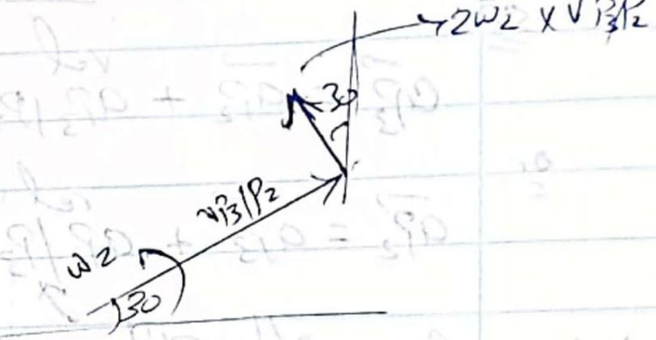
$$= 2W_2 \times V_{P_3/P_2} \Rightarrow$$
$$= 2(2) \times 17.3$$
$$= 69.2 \text{ cm/s}^2$$

لا دجاج هنا
E دجاج



$w_2 = ccw$





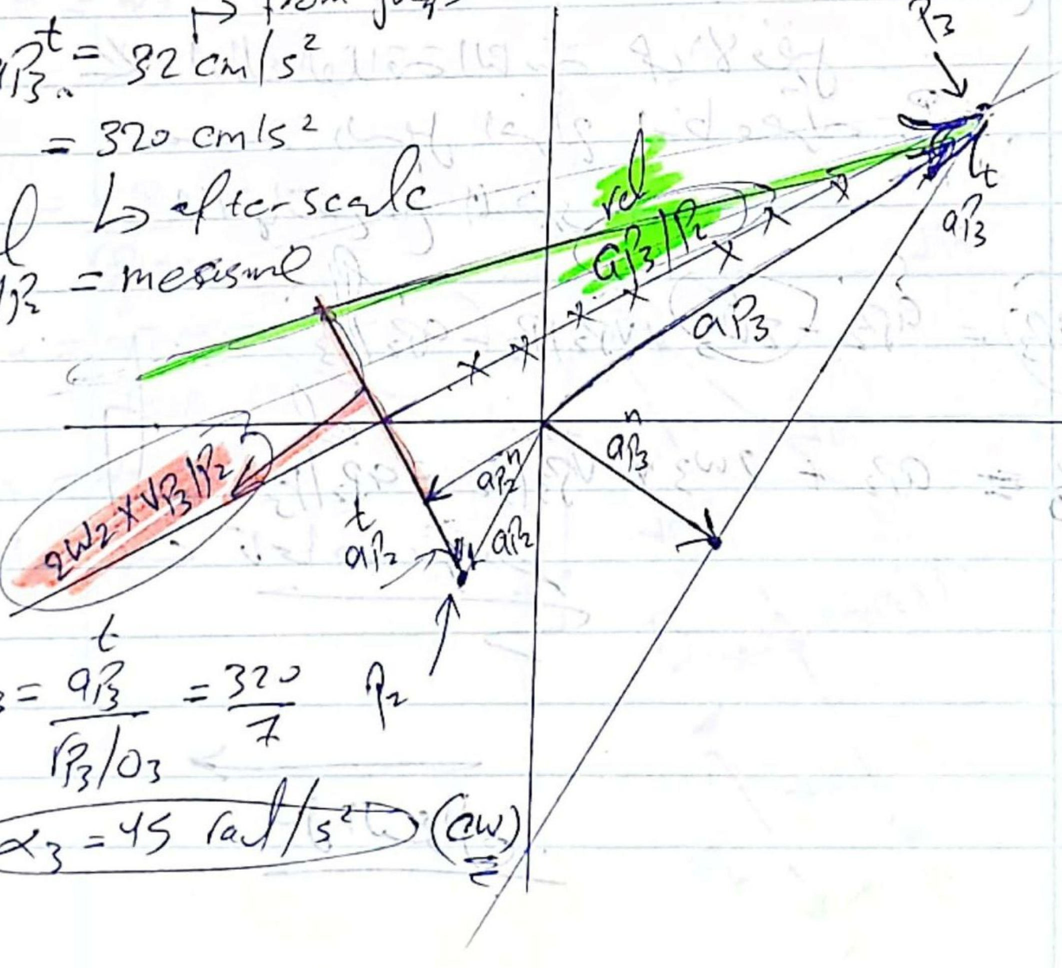
scale 1cm = 10cm

$a^t_{P_3} = 32 \text{ cm/s}^2$ ↦ from graph

$= 320 \text{ cm/s}^2$

↳ after scale

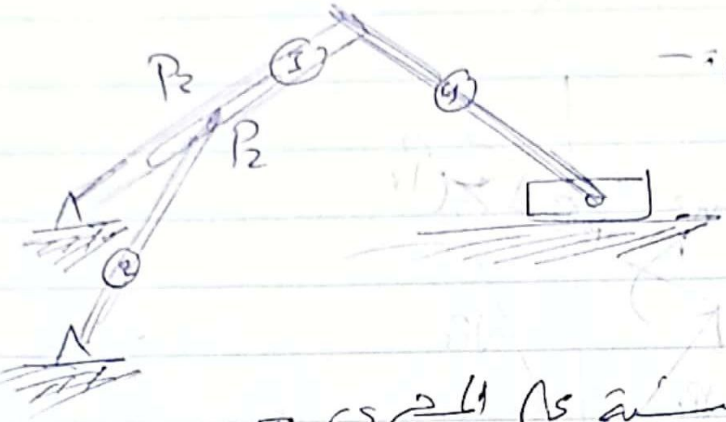
$a^{rel}_{P_3/P_2}$ = measure



$$\alpha_3 = \frac{a^t_{P_3}}{r_{P_3/O_3}} = \frac{320}{7}$$

$$\boxed{\alpha_3 = 45 \text{ rad/s}^2} \text{ (cw)}$$

حل السؤال :-



هنا الحركة النسبية على المجرى

$\hookrightarrow$ for <u>link 3</u>

so

$$\vec{a}_{P_3} = \vec{a}_{P_2} + \overset{rel}{a_{P_3/P_2}} + 2w_2 \times V_{P_3/P_2}$$

or

$$\vec{a}_{P_2} = \vec{a}_{P_3} + \overset{rel}{a_{P_2/P_3}} + 2w_3 \times V_{P_2/P_3} \quad \checkmark$$

لانه المجرى على link3

لذلك العلاقة الثانية هي الأفضل.

ومن السهل اخراج تسارع مركبة P3 ،

لانها تقع على المجرى

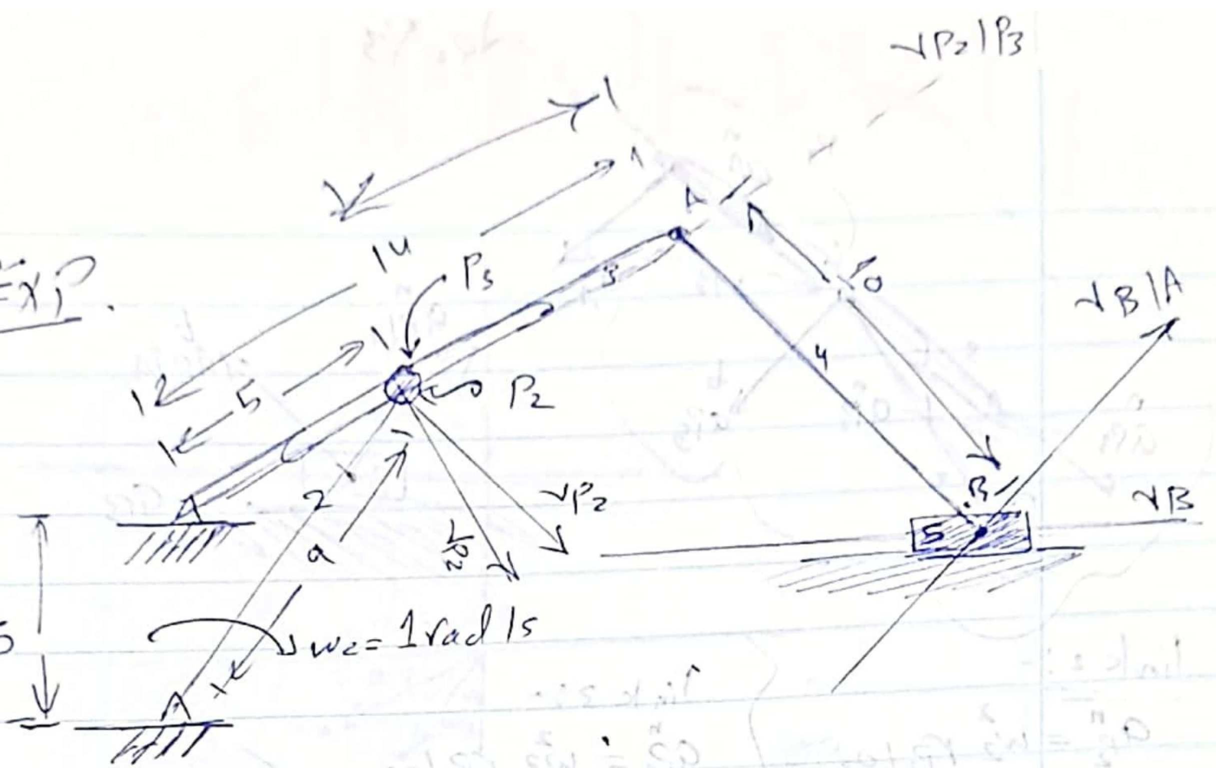
$$\overset{a^t_{P_3},\ a^n_{P_3}}{\boxed{a_{P_3}}} = a_{P_2} - 2w_3 \times V_{P_2/P_3} - \overset{rel}{a_{P_2/P_3}}$$

$$a_{P_3} = a_{P_2} + 2w_3 \times \vec{V}_{P_3/P_2} - \overset{rel}{a_{P_2/P_3}}$$

تأخذه سالب

حل السؤال :

Ex1.



$v_{P_2} = w_2\, r_{P_2|O}$
$= 1(9)$

$\boxed{v_{P_2} = 9 cm/s}$

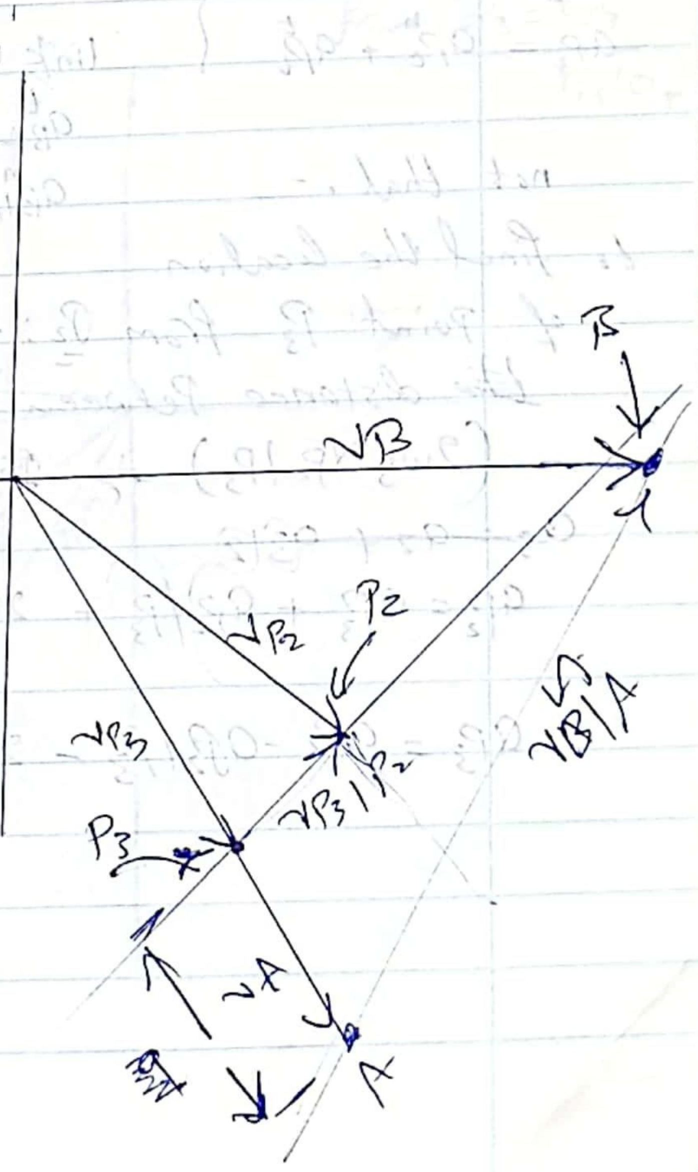
$v_A = w_2 (14)$
$= 14$

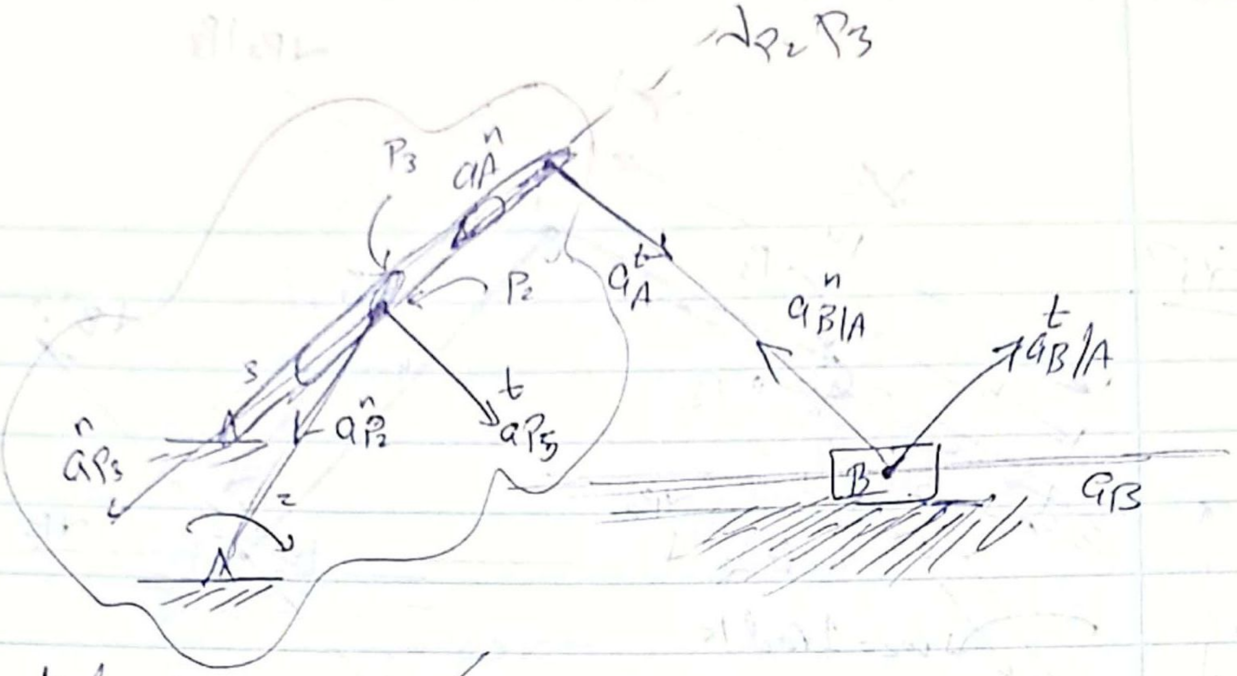
$w_3 = \frac{v_{A_3}}{r_{A_3|O_3}} =$ ✓

$w_4 = \frac{v_{B|A}}{r_{B|A}} =$ ✓

$v_A = w_3\, r_{A|O_3}$

$v_A =$ ✓





link 2:-

$a^n_{P_2} = w_2^2 \, r_{P_2/O_2}$

= ✓

$a^t_{P_2}$ = zero

$a_{P_2} = a^n_{P_t} + a^t_{P_t}$

link 3:-

$a^n_{P_3} = w_3^2 \, r_{P_3/O_3}$ = ✓

$a^t_{P_3} = \alpha_3 \, r_{P_3/O_3}$ = !!

link 4:-

$a^t_{BA} = \alpha_4 \, r_{B|A}$ = !!

$a^n_{B|A} = w_4^2 \, r_{B|A}$ = ✓

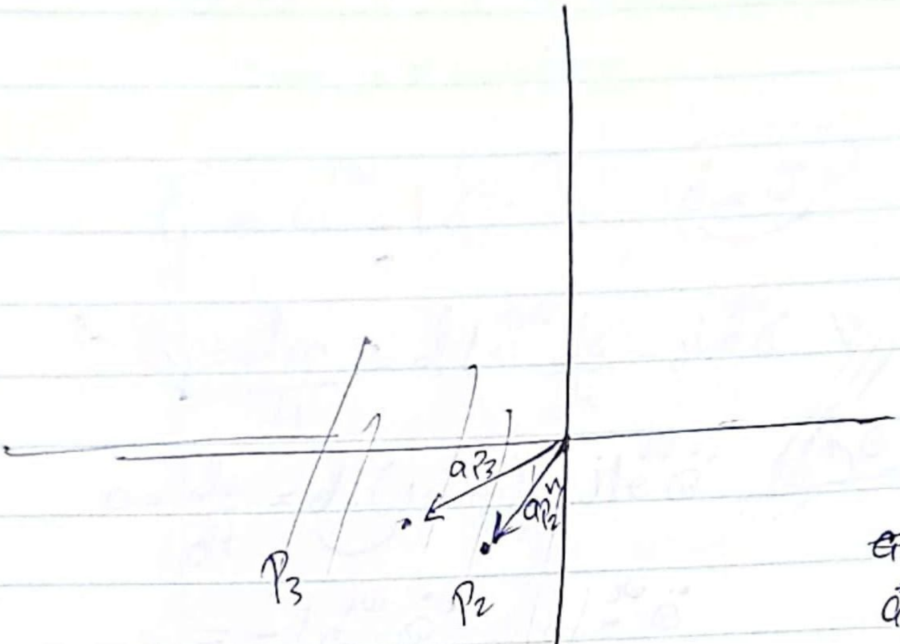
not that :-

to find the location of point P_3 from P_2 :- By the ~~the~~ distance Petween P_2 + P_3 $(2w_3 V_{P_2|P_3})$ ⇒ B from the relation :-

~~$a_3 = a_2 + a_{3|2}$~~

$a_{P_2} = a_{P_3} + a_{P_2/P_3} + 2w_2 V_{P_2|P_3}$

∴ $a_{P_3} = a_{P_2} - a_{P_2|P_3} - 2w_2 V_{P_2|P_3}$ ✓

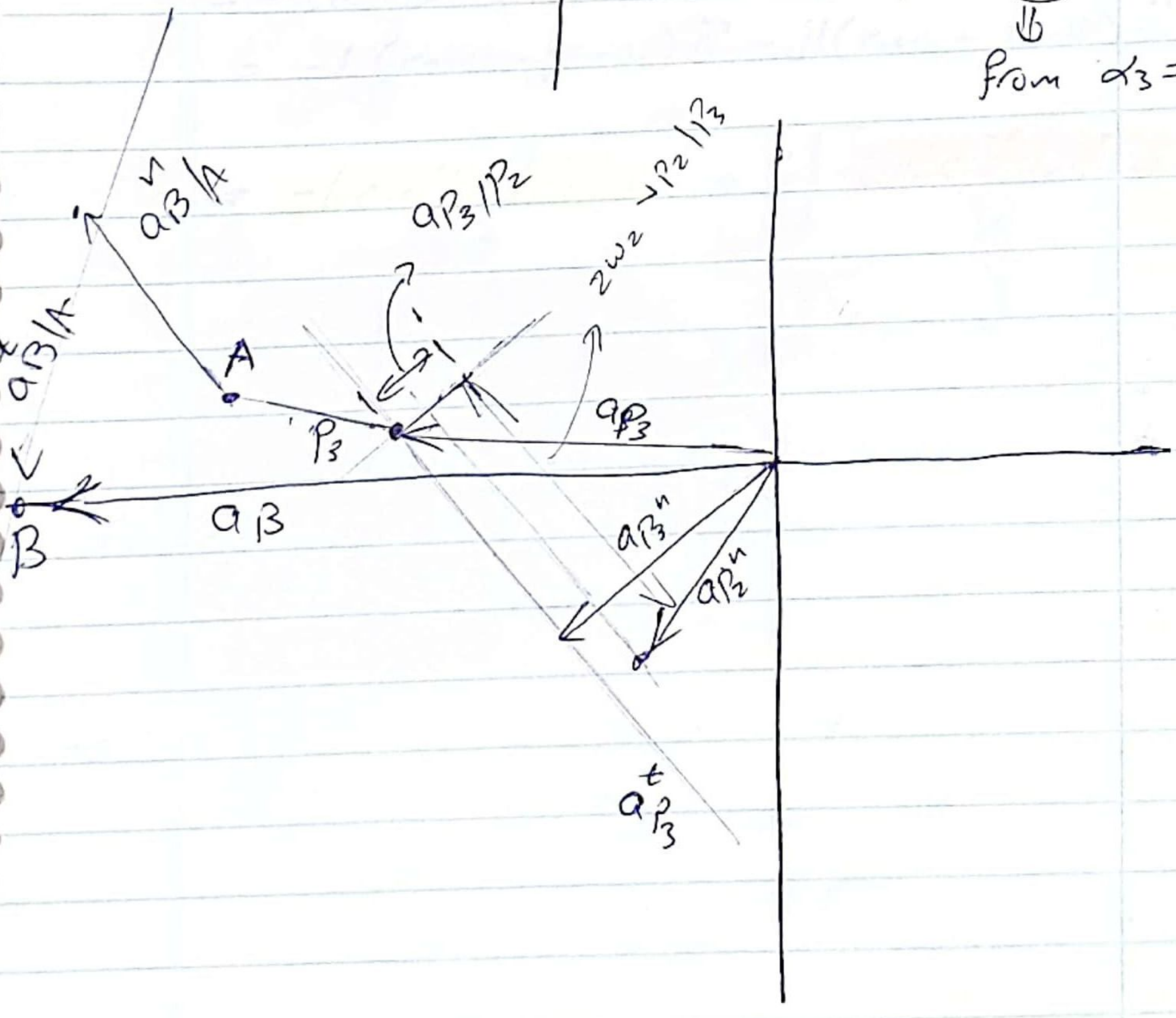


~~GP~~

$$a_A^t = \boxed{\alpha_3} r_{A/O_3}$$

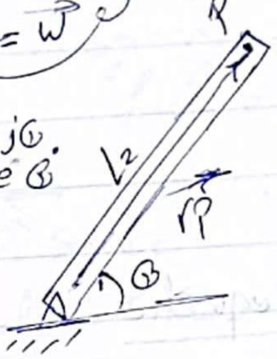
$\Downarrow$

$$\text{from } \alpha_3 = \frac{a_{P_3}^t}{r_{P_3/O_3}}$$



Velocity and acceleration analysis using complex number.

$\vec{r_p} = re^{j\theta} = Le^{j\theta}$ $\boxed{\dot\theta = \omega}$



$$v = \frac{dr_p}{dt} = dLe^{j\theta}\frac{d\theta}{dt} = jLe^{j\theta}\dot\theta$$

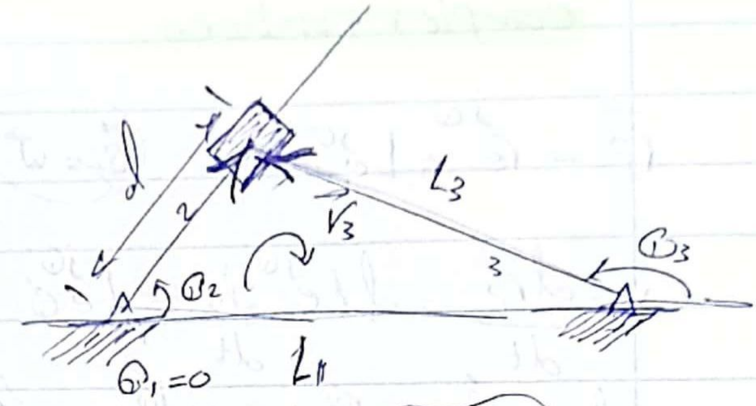
$$a = \frac{dv}{dt} = jl(je^{j\theta}\dot\theta) + jle^{j\theta}\ddot\theta$$

$$= -Le^{j\theta}\dot\theta^2 + jLe^{j\theta}\ddot\theta$$

$$= -L(\cos\theta + j\sin\theta)\dot\theta^2 + jL(\cos\theta + j\sin\theta)\ddot\theta$$

$$= \underset{x}{\underset{\Uparrow}{-L\cos\dot\theta^2 - L\ddot\theta\sin\theta}} + j\Big[\underset{y}{\underset{\Uparrow}{-L\sin\theta\dot\theta^2 + L\cos\ddot\theta}}\Big]$$

Exp:-



Position loop equation:-

$$\vec{D} - \vec{r_3} - \vec{L_1} = 0$$

$\frac{d}{dt}$ $$de^{j\theta_2} - L_3 e^{j\theta_3} - L_1 e^{j\theta_1} = 0$$

$$y = ae^{bx}$$
$$\frac{dy}{dt} = abe^{bx}\dot{x}$$

velocity analysis :- ⇒ $\frac{d}{dt}$ (Position)

$$\dot{d}e^{j\theta_2} + dje^{j\theta_2}\dot{\theta}_2 - L_3 je^{j\theta_3}\dot{\theta}_3 - 0 = \text{Zero}$$

$$\dot{d}[\cos\theta_2 + j\sin\theta_2] + dj\dot{\theta}_2[\cos\theta_2 + \overset{-1}{j}\sin\theta_2]$$
$$- L_3 j\dot{\theta}_3[\cos\theta_3 + \overset{-1}{j}\sin\theta_3] = 0$$

$$\dot{d}\cos\theta_2 - d\dot{\theta}_2\sin\theta_2 + L_3\dot{\theta}_3\sin\theta_3$$
$$+ j[\dot{d}\sin\theta_2 + d\dot{\theta}_2\cos\theta_2 - L_3\dot{\theta}_3\cos\theta_3] = 0 \quad \boxed{j \times j = -1}$$

Real Part:-

$$\dot{d}\cos\theta_2 - d\dot{\theta}_2\sin\theta_2 + L_3\dot{\theta}_3\sin\theta_3 = 0$$

Imagin's Part:-

$$\dot{d}\sin\theta_2 + d\dot{\theta}_2\cos\theta_2 - L_3\dot{\theta}_3\cos\theta_3 = 0$$

→ given $\theta_2 \Rightarrow d$ & $\dot{\theta}_3$

⇒ variable θ_2, d, θ_3 (Position)

$\dot{\theta}_2, \dot{d}, \dot{\theta}_3$ (velocity)

↳ given $w_2 = 1$ rad/s ccw find $\dot{d}$ & w_3.

$$\left\{\begin{aligned} \dot{d}\cos\theta_2 + L_3\dot{\theta}_3\sin\theta_3 &= \boxed{d\dot{\theta}_2\sin\theta_2} \\ \dot{d}\sin\theta_2 - L_3\dot{\theta}_3\cos\theta_3 &= \boxed{-d\dot{\theta}_2\cos\theta_2} \end{aligned}\right. \Rightarrow \underline{\text{known}}$$

↳ velocity and acceleration equations are linear with relative to respect (w.r.t) to the unknown.

$$\begin{bmatrix} \cos\theta_3 & L_3\sin\theta_3 \\ \sin\theta_2 & -L_3\cos\theta_3 \end{bmatrix}_{2\times2} \cdot \begin{bmatrix} \dot{d} \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} d\dot{\theta}_2\sin\theta_2 \\ -d\dot{\theta}_2\cos\theta_2 \end{bmatrix}$$

$$A\vec{x} = \vec{b}$$

→ Depend on the position only.

$$\boxed{A^{-1}A}\,\vec{x} = A^{-1}\vec{b} \Rightarrow \boxed{\underset{2\times1}{d\vec{x}} = \underset{2\times2}{A^{-1}}\,\underset{2\times1}{\vec{b}}} \Rightarrow \begin{aligned} \dot{d} &= x(1) \\ \dot{\theta}_3 &= x(2) \end{aligned}$$

* in matlab

→ dx = inverse(A) * b

or ⇒ dx = A\b

→ Gausian elimination with backward substituts.

لتقليل عدد العمليات الحسابية

في عملية الضرب الترتيب مهم ،

$$\begin{aligned} 4/2 &= 2 \\ 2/4 &= 2 \end{aligned}$$

⇒ في الماتلاب فيهما نفس الشيء

CTRL + R ⇒ set the line as comment

CTRL + T ⇒ Remove the commond

$$\text{Acceleration} = \frac{d}{dt}\vec{v}$$

$$v \Rightarrow \dot{d}e^{j\theta_2} + dje^{j\theta_2}\dot{\theta}_2 - L_3je^{j\theta_3}\dot{\theta}_3 - 0 = 0$$

$$\Rightarrow \boxed{\ddot{d}}e^{j\theta_2} + \dot{d}je^{j\theta_2}\dot{\theta}_2 + \cancel{\dot{d}je^{j\theta_2}\dot{\theta}_2} + d(-1)e^{j\theta_1}\dot{\theta}_2\dot{\theta}_2$$
$$+ dje^{j\theta_2}\boxed{\ddot{\theta}_2} - L_3(-1)e^{j\theta_3}\dot{\theta}_3\dot{\theta}_3 - L_3e^{j\theta_3}\boxed{\ddot{\theta}_3} = 0$$

Unknowns: $\Rightarrow \boxed{\ddot{d}}, \boxed{\ddot{\theta}_3}$

⇒ Real Part + Imegany part :-

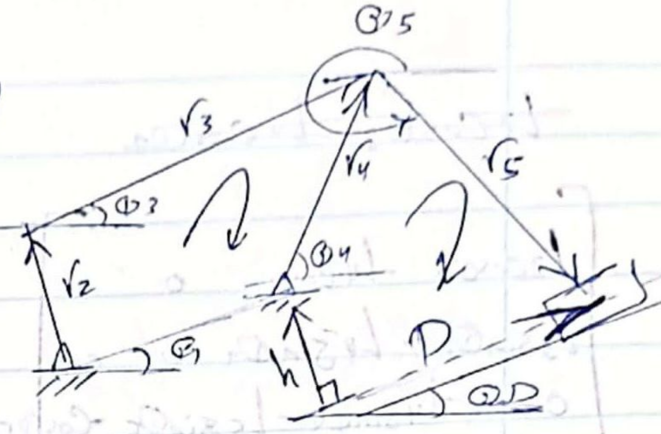
$$\Rightarrow \begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix} \underset{\overset{\Uparrow}{ddx}}{\begin{bmatrix} \ddot{d} \\ \ddot{\theta}_3 \end{bmatrix}} = \begin{bmatrix} \\ F \\ \\ \end{bmatrix}$$

$$\vec{ddx} = A^{-1}\vec{F}.$$

$$\ddot{d} = ddx(1) \quad \text{———} ①$$
$$\ddot{\theta}_3 = ddx(2) \quad \text{———} ②$$

Exp:- given $\dot{\theta}_2$ ⇒ find

$\dot{\theta}_3, \dot{\theta}_4, \dot{\theta}_5, \dot{D}$??



Position loop equation.

$$\vec{r}_2 + \vec{r}_3 - \vec{r}_4 - \vec{r}_1 = 0$$

$$r_4 + r_5 - D + h = 0$$

$$L_4 e^{j\theta_4} + l_5 e^{j\theta_5} - D e^{j\theta_D} + h e^{j(\theta_D + 90)} = 0$$

$$jL_4 e^{j\theta_4} \dot{\theta}_4 + jL_5 e^{j\theta_5} \dot{\theta}_5 - \dot{D} e^{j\theta_D} + 0 = 0$$

$$jL_4 \dot{\theta}_4 [\cos\theta_4 + j\sin\theta_4] + jL_5 \dot{\theta}_5 [\cos\theta_5 + j\sin\theta_5] - \dot{D}[\cos\theta_D + j\sin\theta_D] = 0$$

R; $-L_4 \dot{\theta}_4 \sin\theta_4 - L_5 \theta_5 \sin\theta_5 - \dot{D}\cos\theta_D = 0$

I; $L_4 \dot{\theta}_4 \cos\theta_4 + L_5 \dot{\theta}_5 \cos\theta_5 - \dot{D}\sin\theta_D = 0$

$$L_2 e^{j\theta_2} + L_3 e^{j\theta_3} - L_4 e^{j\theta_4} - L_1 e^{j\theta_1} = 0$$

$$\cancel{j} L_2 e^{j\theta_2} \boxed{\dot{\theta}_2} + \cancel{j} L_3 e^{j\theta_3} \boxed{\dot{\theta}_3} - \cancel{j} L_4 e^{j\theta_4} \boxed{\dot{\theta}_4} - 0 = 0$$

$$L_2 \dot{\theta}_2 [\cos\theta_2 + j\sin\theta_2] + L_3 \dot{\theta}_3 [\cos\theta_3 + j\sin\theta_3] - L_4 \dot{\theta}_4 [\cos\theta_4 + j\sin\theta_4] = 0$$

R; $\boxed{L_2 \dot{\theta}_2 \cos\theta_2} + L_3 \dot{\theta}_3 \cos\theta_3 - L_4 \dot{\theta}_4 \cos\theta_4 = 0$

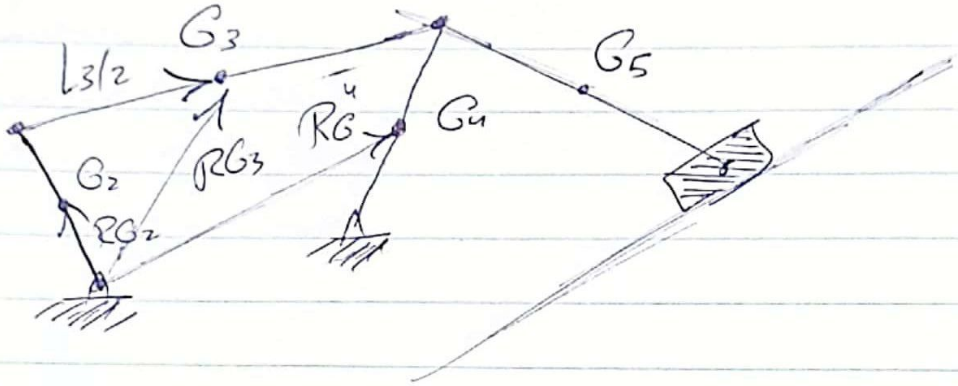
I; $\boxed{L_2 \dot{\theta}_2 \sin\theta_2} + L_3 \dot{\theta}_3 \sin\theta_3 - L_4 \dot{\theta}_4 \sin\theta_4 = 0$

~~$L_3\cos\theta_3 - L_4\cos\theta_4$~~

$$\begin{bmatrix} L_3\cos\theta_3 & -L_4\cos\theta_4 & 0 & 0 \\ L_3\sin\theta_3 & -L_4\sin\theta_4 & 0 & 0 \\ 0 & -L_4\sin\theta_4 & -L_5\sin\theta_5 & -\cos\theta_D \\ 0 & L_4\cos\theta_4 & L_5\cos\theta_5 & -\sin\theta_D \end{bmatrix} \begin{bmatrix} \dot\theta_3 \\ \dot\theta_4 \\ \dot\theta_5 \\ \dot D \end{bmatrix} = \begin{bmatrix} -L_2\dot\theta_2\cos\theta_2 \\ -L_2\dot\theta_2\sin\theta_2 \\ 0 \\ 0 \end{bmatrix}$$

<u>for acceleration analysis</u> :-

☐ acceleration of center of gravity of each link:-



$$\overrightarrow{RG_3} = L_2 e^{j\theta_2} + \frac{L_3}{2} e^{j\theta_3}$$

$$d\overrightarrow{RG_3} = L_2 j e^{j\theta_2} \dot{\theta}_2 + \frac{L_3}{2} j e^{j\theta_3} \dot{\theta}_3$$

$$dd\overrightarrow{RG_3} = L_2(-1) e^{j\theta_2} \dot{\theta}_2^2 + L_2 j e^{j\theta_2} \ddot{\theta}_2 + \frac{L_3}{2}(-1) e^{j\theta_3} \dot{\theta}_3^2$$
$$+ \frac{L_3}{2} j e^{j\theta_3} \ddot{\theta}_3$$

$$= -L_2 \dot{\theta}_2 e^{j\theta_2} + L_2 \ddot{\theta}_2 j e^{j\theta_2} - \frac{L_3}{2} \dot{\theta}_3^2 e^{j\theta_3} + \frac{L_3}{2} \ddot{\theta}_3 j e^{j\theta_3}$$

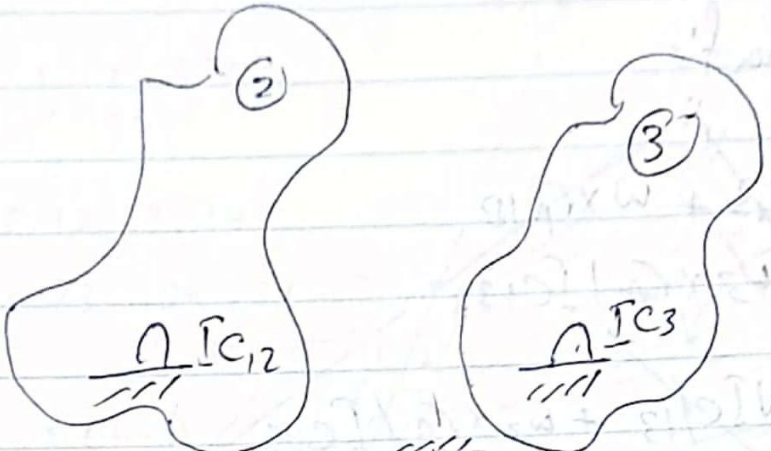
$$= -L_2 \dot{\theta}_2 [\cos\theta_2 + j\sin\theta_2] + L_2 \ddot{\theta}_2 j [\cos\theta_2 + j\sin\theta_2]$$
$$- \frac{L_3}{2} \dot{\theta}_3^2 [\cos\theta_3 + j\sin\theta_3] + \frac{L_3}{2} \ddot{\theta}_3 j [\cos\theta_3 + j\sin\theta_3]$$

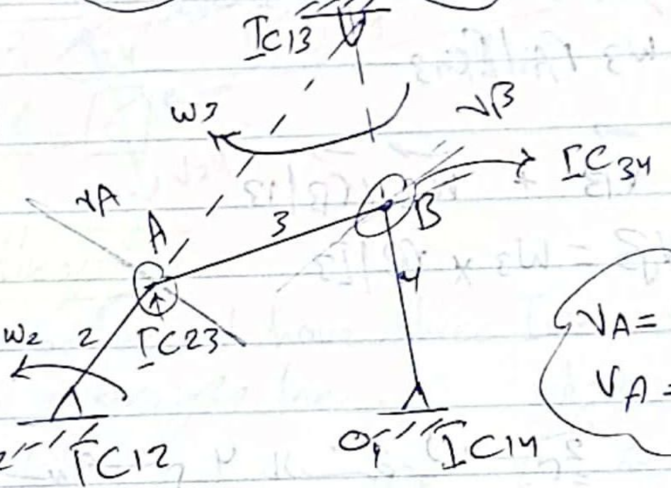
☐ $A_{g3x}(i) = -L_2 \dot{\theta}_2 \cos\theta_2 - L_2 \ddot{\theta}_2 \sin\theta_2 - \frac{L_3}{2} \dot{\theta}_3^2 \cos\theta_3$
$- \frac{L_3}{2} \ddot{\theta}_3 \sin\theta_3$

☐ $A_{g3y}(i) = -L_2 \dot{\theta}_2 \sin\theta_2 + L_2 \ddot{\theta}_2 \cos\theta_2 - \frac{L_3}{2} \dot{\theta}_3^2 \sin\theta_3$
$+ \frac{L_3}{2} \ddot{\theta}_3 \cos\theta_3$

وكذلك الامر بالنسبة لباقي : cetntel

⇒ Velocity analysis Instal centers





ان النقطة A التي تقع على جسم 2 نفس سرعة النقطة التي تقع على جسم 3.

ولذلك بالنسبة لـ 3 & 1 وغيرها من النقاط

$$V_A = w_2 \, r_{A/O_2}$$

$$V_A = w_3 \, r_{A/IC_{13}}$$

Instant center:-

① ≡ A point a bout which body 3 is rotating relative to body 2.

② ≡ Concider points on both bodies that have the same velocity.

* velocity of IC_{12} & IC_{13} & IC_{14} = zero

[← لانها نقاط مشتركة بين الاطار واجسام 2 & 3 & 4.

* note that:-

$$v_A = \cancelto{0}{v_B} + \omega \times r_{A/B}$$
$$\to v_A = \omega_3 \times r_{A/IC_{13}}$$

$$v_A = \cancelto{0}{v_{IC_{13}}} + \omega_3 \times r_{A/IC_{13}}$$
$$\to v_A = \omega_3\, r_{A/IC_{13}}$$

$$v_B = \vec{v}_B + \vec{\omega}_3 \times \vec{r}_{B/13}$$
$$v_B = \omega_3 \times r_{B/I3}$$

⇐ ولكن الجسم 4 بالنسبة لجسم 2
حول أي نقطة يدور ؟!

$$v_A = v_B + \omega_3 \times r_{B/A}$$
$$v_{23} = v_{34} + \omega_3 \times r_{23/34}$$ ⇒ لا يمكننا
$$\text{zero} = v_{3/4} \neq v_{23}$$ اختصار بأنه

لذلك سيتم ايجاد IC بين
هذه الاجسام

24 / 4 / 2021

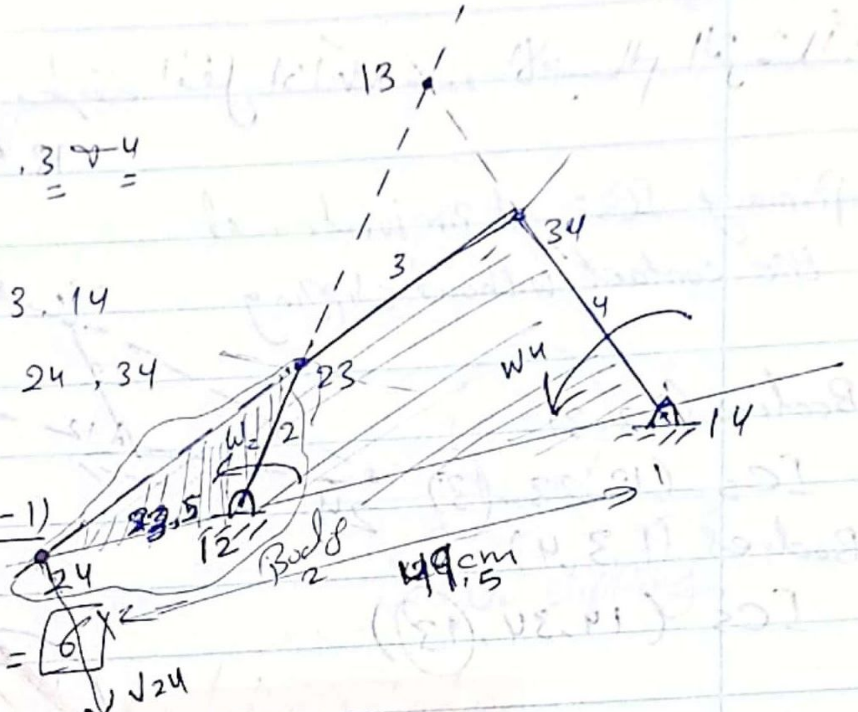
Ex2:-

Bodies 1, 2, 3 & 4

All ICs ≡ 12, 13, 14
23, 24, 34

$$\# \; ICs = \frac{n(n-1)}{2}$$

$$= \frac{4 \times 3}{2} = \boxed{6}$$



Kennedy's theorem :-

⟹ Any three bodies will have three ICs that lay on a straight line.

هي التي تحقق الشرط بين العلاقتين

* for example:-
- Bodies (1.2.3) ⇒ IC ≡ (12) (23), $\boxed{13}$ an a straight line.
- Bodies (1.3.4) ⇒ IC ≡ $\boxed{13}$ (14) (34) should lay on a stright line.
- Bodies (1.2.4) ⇒ ICs ≡ (12), (14), $\boxed{24}$
- Bodies (2.3.4) ⇒ ICs ≡ (23), (34), $\boxed{24}$

24 ⇒ output & input هي علاقة مباشرة بين

Given $w_2 = 2$ rad/s in the direction shown, find w_4

IC 24 on link 2

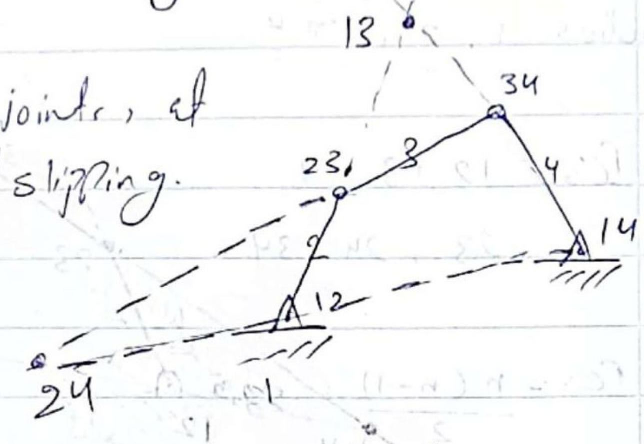
$v_{24} = \cancel{v_{12}} + w_2 \times r_{24/12} \Rightarrow v_{24} = w_2 \, r_{24/12} = 2(23.5) = 47$

$v_{24} = \cancel{v_{14}} + w_4 \, r_{24/14} \Rightarrow 47 = w_4 \, (49) \Rightarrow \boxed{w_4 = 0.9 \text{ rad/s}}$

IC 24 on link 4

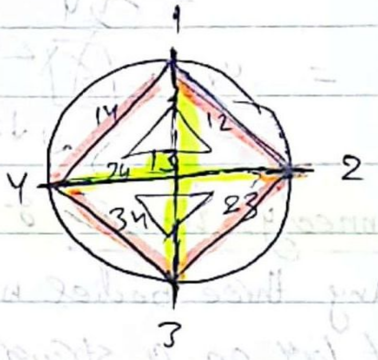
≪ لايجاد IC بطريقة افضل اذا كانت عدد الاجسام اكثر مثلاً،

Primary ICs :- at pin joints, at the contact without slipping.



* Bodies (1, 2, 3)
ICs (12, 23, ⑬)

* Bodies (1, 3, 4)
ICs (14, 34, ⑬)

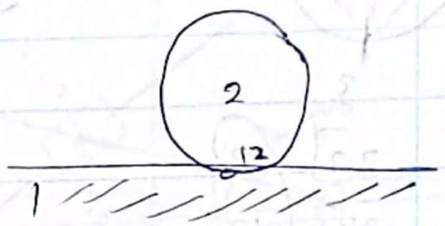


* Bodies (2, 3, 4)
ICs (23, 34, ㉔)

* Bodies (1, 2, 4)
ICs (12, 14, ㉔)

▭ Contact types :-

① - Rolling without slipping ⇒ IC ≡ at the contact Point.

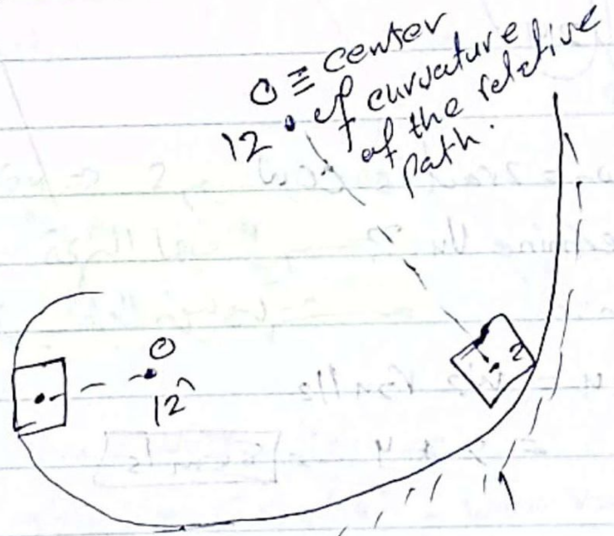


No slipping

② - slipping without relative rotation.

أي عند اللحظة يقع فيها الجسم ع curve

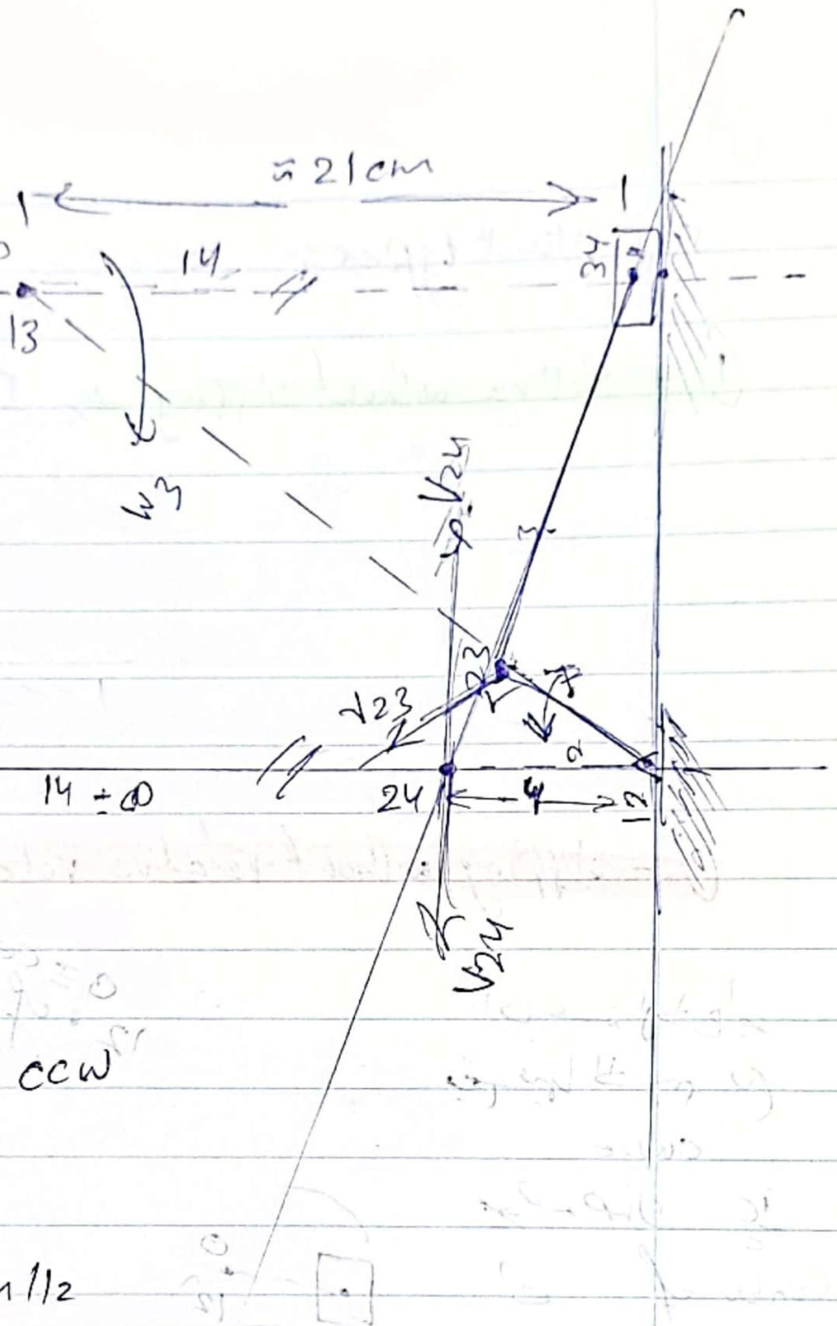
يوجد هناك IC = center of ي curvature.

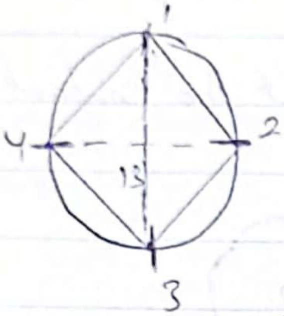


⇐ ولكن عند اللحظة التي يسير فيها الجسم stright line ⇐ تكون $\boxed{IC \Rightarrow \pm \infty}$

↳ $w_2 = w_1$; $w_1 = 0 \Rightarrow w_2 = 0$

Example 14 I∞





12 . 23 . 13
14 . 34 . 13

12 . 14 . 24
23 . 34 . 24

* given $w_2 = 2 rad/s$ ccw
Determine V_4 ??

solution:-

$V_{24} = W_2 \, r_{24/12}$
$= 2 * 4 = \boxed{8 cm/s}$

as IC 14 I∞ ⇒ $w_4 = w_1 = zero$ ⇒ body 4 make translation motion only.

⇒ $V_{slider} = V_{24}$ # $= V_{34}$

$V_{34} = 8 cm/s$

$V_{34} = W_3 \, r_{34/13}$ ⇒ $W_3 = 8/21 = 0.38 rad/s$ #

* $V_{23} = W_2 \, r_{23/12}$
$V_{23} = W_3 \, V_{23/13}$ ⇒ اذا كان الرسم دقيق ستكون نفسها

26/4/2021.

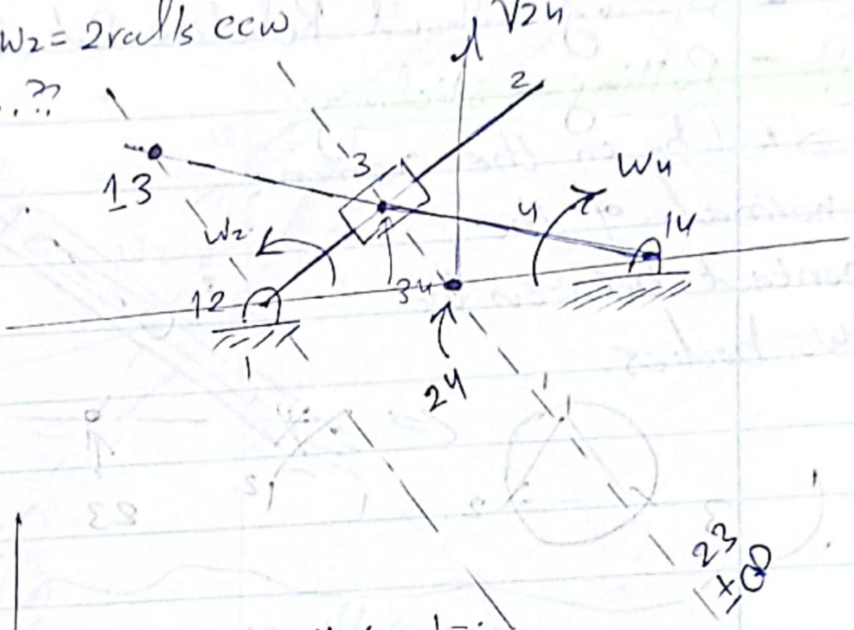
☐ **Remark**:- If point A lay on body 2

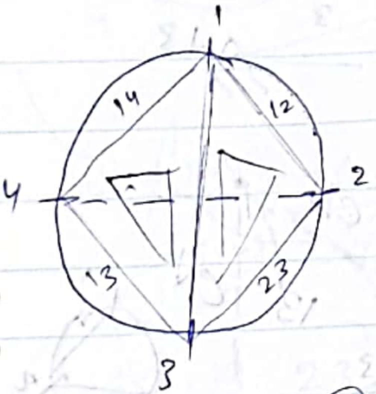
$\Rightarrow V_A = w_2 \, r_{A/12}$

Example given $w_2 = 2$ rad/s ccw

find ICs & w_4 ??

Primary ICs:-





$r_{24} \Rightarrow$ نقطة في الجسم 2
والجسم 4 فيها الحركة
مشتركة بين الجسمين

12 , 23 , 13
14 , 34 , 13

12 . 14 , 24
23 , 43 , 24

$V_{24} = w_{②} \, r_{24/⑫} = ✓$

$V_{24} = w_{④} \cdot r_{24/⑭} \Rightarrow w_4 = V_{24} / r_{24}$ ✓

$w_3 = ??$ IC23 ±∞ $\Rightarrow w_2 = w_3$

* $V_{34} = w_3 \, r_{34/13}$
* $V_{34} = w_4 \, r_{34/14}$

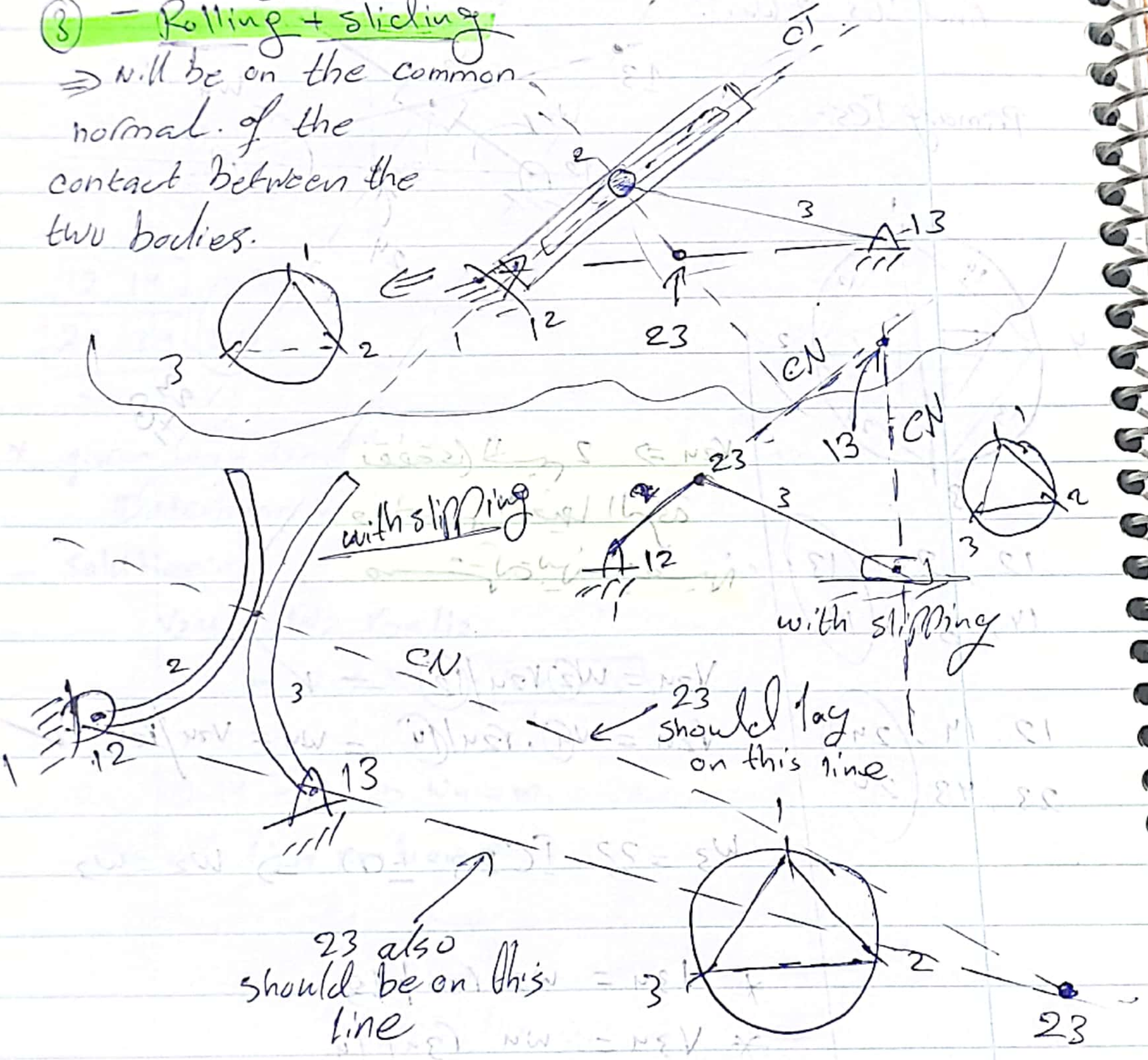
③ Cotacts :-

① - Rolling without J1

② - sliding without Relative Rotation J1

③ - Rolling + sliding

⇒ will be on the common normal of the contact between the two bodies.

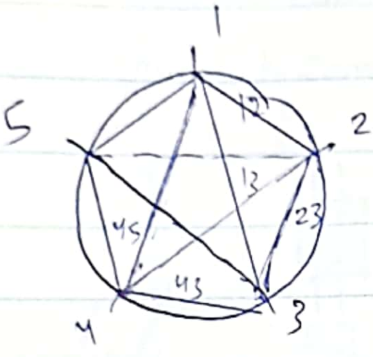


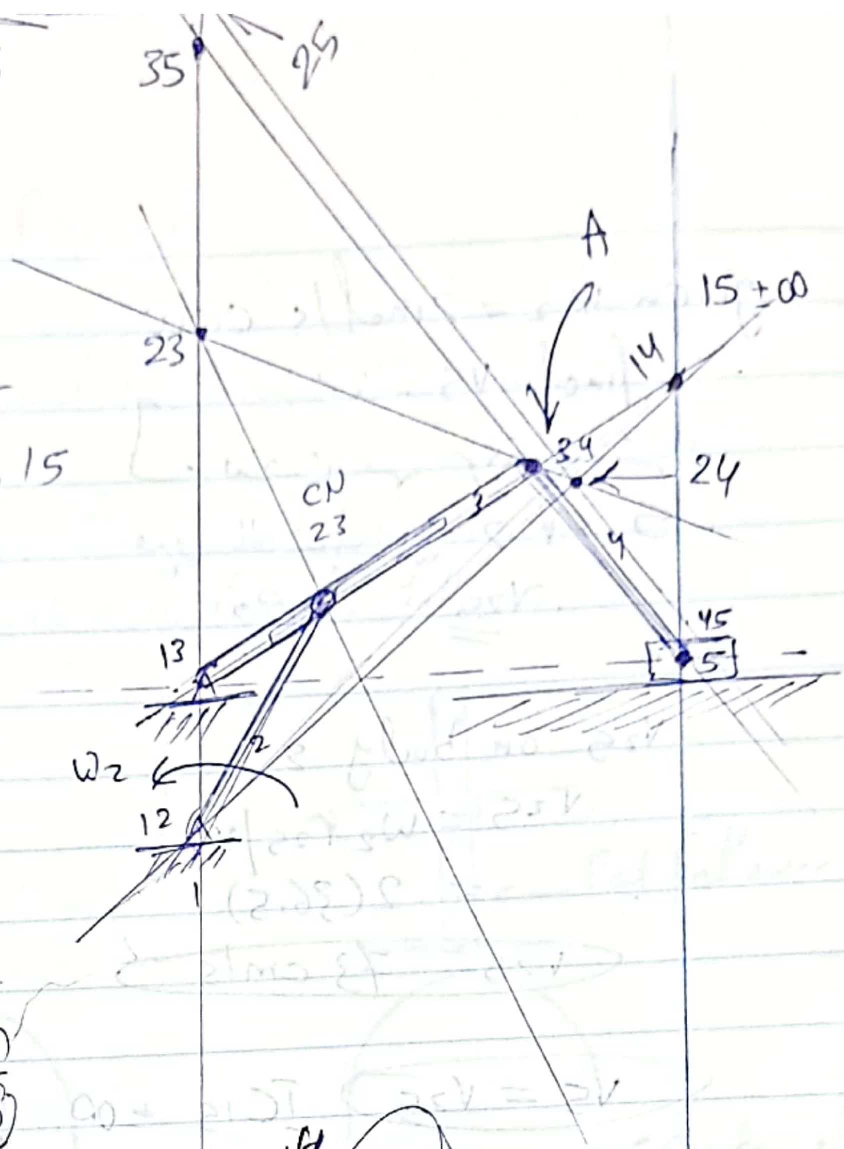
سؤال يشمل كل التفاصيل

Exp:-

Primary IC:-

12, 13, 34, 45, 15





# of ICs $= \frac{n(n-1)}{2}$

$= \frac{5(4)}{2} = \boxed{10}$

12, 13, $\boxed{23}$

شبه معلوم ⇒ $\boxed{23}$

CN (دائما)

13, 15, $\boxed{35}$

54, 34, $\boxed{35}$

12, 15, $\boxed{25}$

23, 35, $\boxed{25}$ } ⇒ لايهم النقطة نفس الخط

أي ليس بالضرورة دائماً يغير موقع 25

12, 14 $\boxed{24}$

23, 34, $\boxed{24}$

15, 54, $\boxed{14}$

13, 43, $\boxed{14}$

24, 45, $\boxed{25}$

12, 15, $\boxed{25}$

given $w_2 = 2 \text{rad/s}$ ccw

find $V5 = ??$

بنبحث عن نقطة بتشترك فيها الجسمين 2 و 5 وهي $\underline{\underline{V25}}$

V_{25} on body 2

$$V_{25} = w_2 \, r_{25/12}$$

$$= 2(36.5)$$

$$\boxed{V_{25} = 73 \text{ cm/s}}$$

$\boxed{V5 = V25}$ IC_{15} ±∞

---

body 2 → body 3 → body 4 → body 5

To find $V5$!!

$V23 = w_2 \, r23/12 \Rightarrow ✓$

$V23 = w_3 \, r23/13 \Rightarrow w_3 = ✓$

$V34 = w_3 \, r34/13 \Rightarrow ✓$

$V34 = w_4 \, r34/14 \Rightarrow w_4 = ✓$

$V45 = w_4 \, r45/14$

$V_{15} = \pm\infty \Rightarrow$ body moving translation/infinite

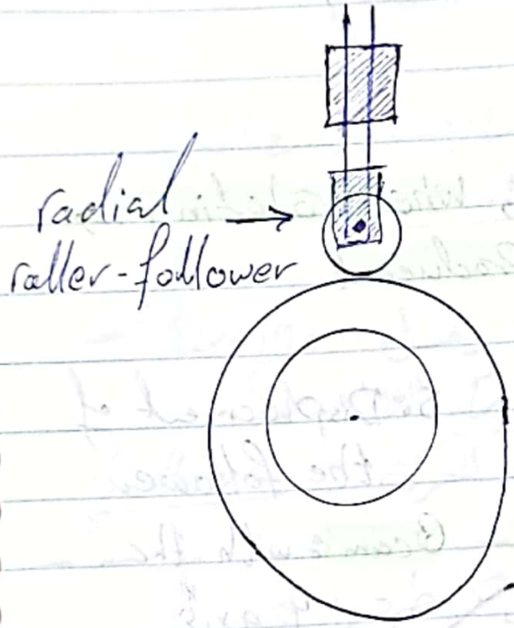
$\boxed{V5 = V45}$ #

22/5/2021

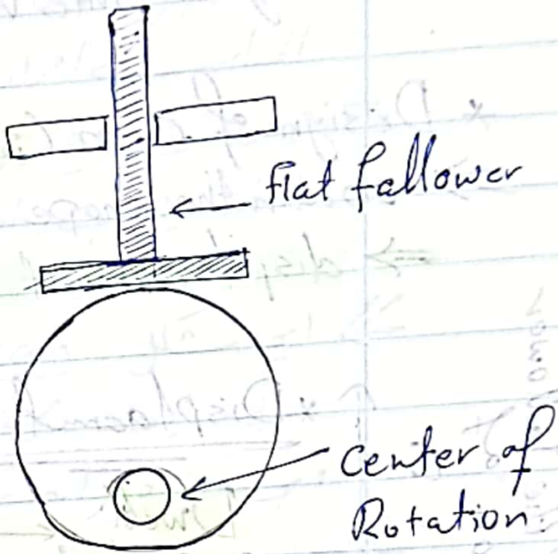
▤ Cam and follower

⇒ Types of cam and followers [in this course] 8

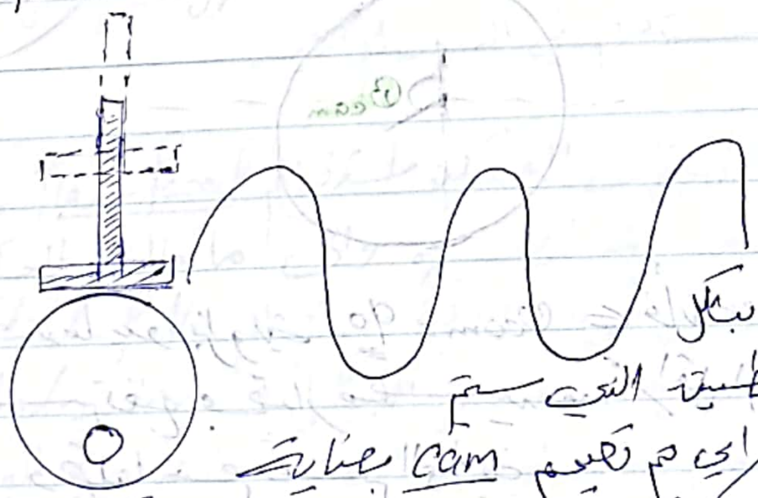
① Disk cam with flat-faced follower.
② Disk cam with radial roller follower.



Disk cam with Radial raller follower.



Disk cam with flat-faced follower.



* هي تم دراسة
حركة follower بشكل
دقيق حسب التطبيق التي سيتم
استخدامها فيه . أي تم تصميم cam بعناية
لان follower سيكون حسب زاوية cam حركته .

▣ Design of cam and follower:-

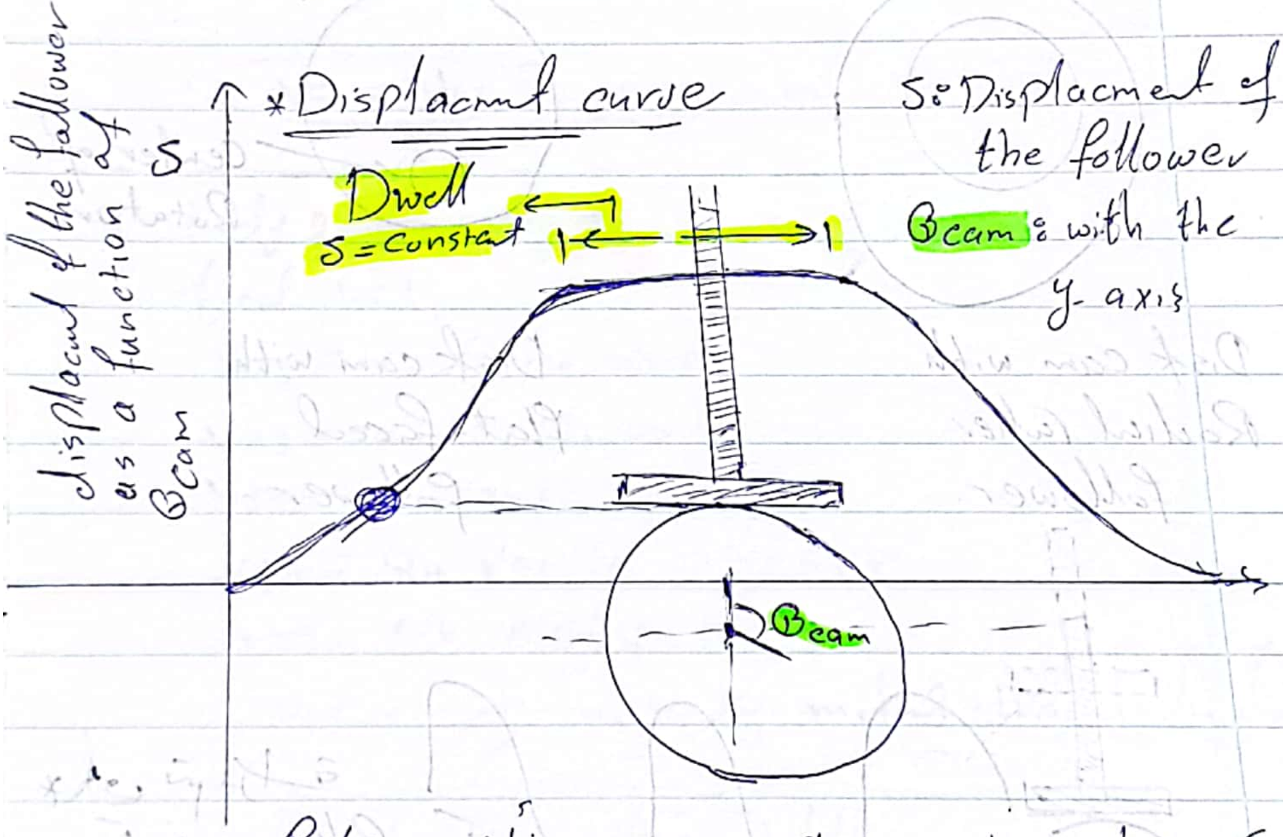
* Design of displacement curves:

⇒ Design the pretended motion of the follower taking into account the specific requirements for some given intervals.

* Design of cam (shape):

⇒ obtain the shape of the cam, when rotating ⇒

⇒ displacement curve is produced



S: Displacment of the follower

θcam: with the y-axis

مثلاً عندما يدور بزاوية $90°$ ⇐ θcam = 90 فإنه follower
سيرتفع بمقدار معين.... / أقل ارتفاع ⇐ θcam = zero
وهكذا مع تغير الزاوية

⇐ اي سيتم عمل Design لـ cam بحيث عندما يُفتح و يُغلق ⇐ تكون continous الـ امتداد Displacment & velocity & Acceleration

* اي الوقت اللازم لانغلاق الصمام في محرك السيارة ⇐ ليعطي كمية الوقود اللازمة و المناسبة لعملية الاحتراق.

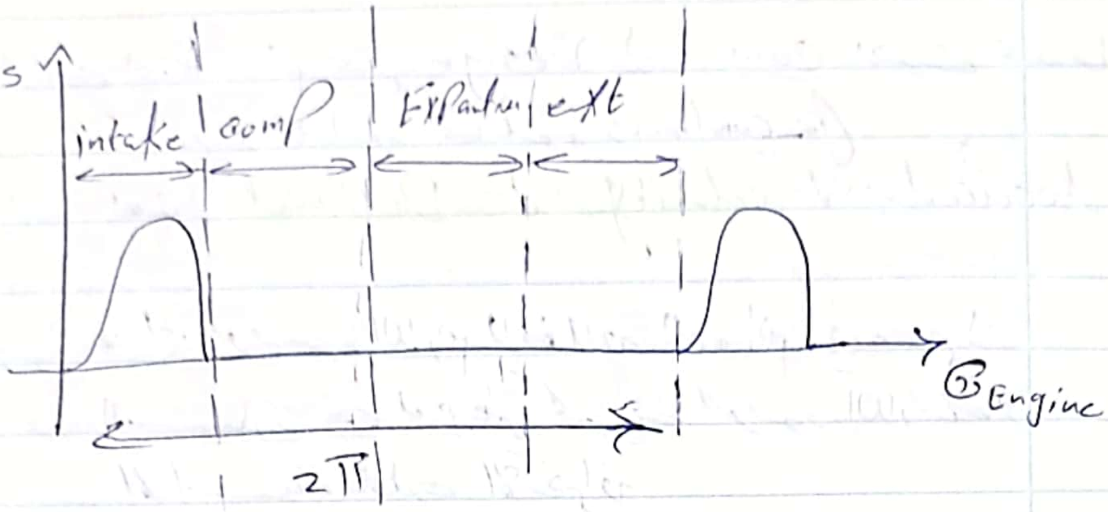
⇐ وحتى لا يصدر صوت عندما يفتح وينغلق لذلك يجب انه يكون continous كما

⇐ اي شكل cam تم ايجادة وعمل Design له بعد ايجاد Displacment curve اللازم. اي عند دورانه سيعطي هذه الحركة لـ fallower.

كيف يتم اختيار بين الانواع التي تم طرحها ؟!

⇐ مثلاً flat-faced ⇐ فيها friction عالي لذلك تحتاج الى زيت مثلاً ⇐ لتسهيل عملية الحركة فهذا يستخدم في السيارات

⇐ مصانع الطباعات لا يمكن استخدام flat-faced. كون الورق يتم تغييره والى ما ذلك لا يمكن ادخال الزيت ، لذلك يتم استخدام radial-fallo مع امكانية تغيير البيل لـ radial-roller بدلاً من ادخال الزيت الى عملية الطباعة في الورق.

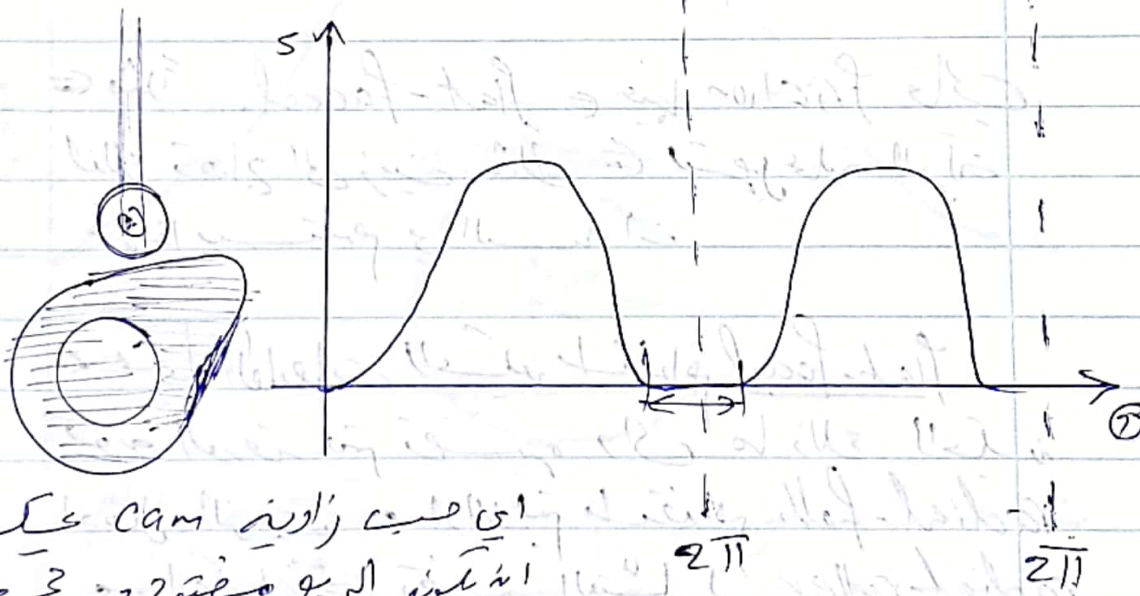


⇐ على سبيل المثال فتحة الصمام تفتح في مرحلة intake / ومن ثم تغلق لباقي الدورة

ثم يحدث الضغط ← الانفجار ← طرد

⇐ اي الدورة الكاملة لهذا المحرك 2π .

* for Example => radial roller follower



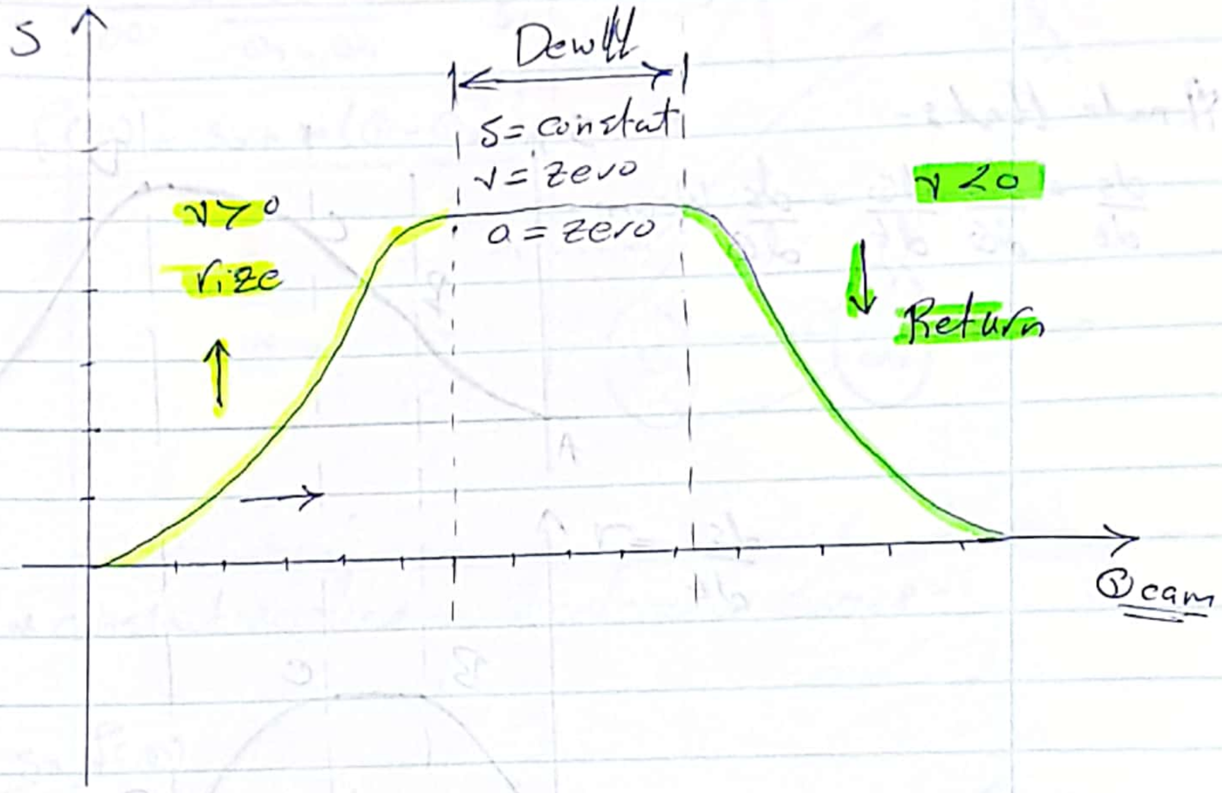
اي حسب زاوية cam يمكنه

انه يكون الربع مفتوح و $\frac{3}{4}$ مغلقة

⇐ اي حسب Design .

* Displacemat curve Design:-

>> Rise - dwell - Return



⊟ Displacement curve Design:-

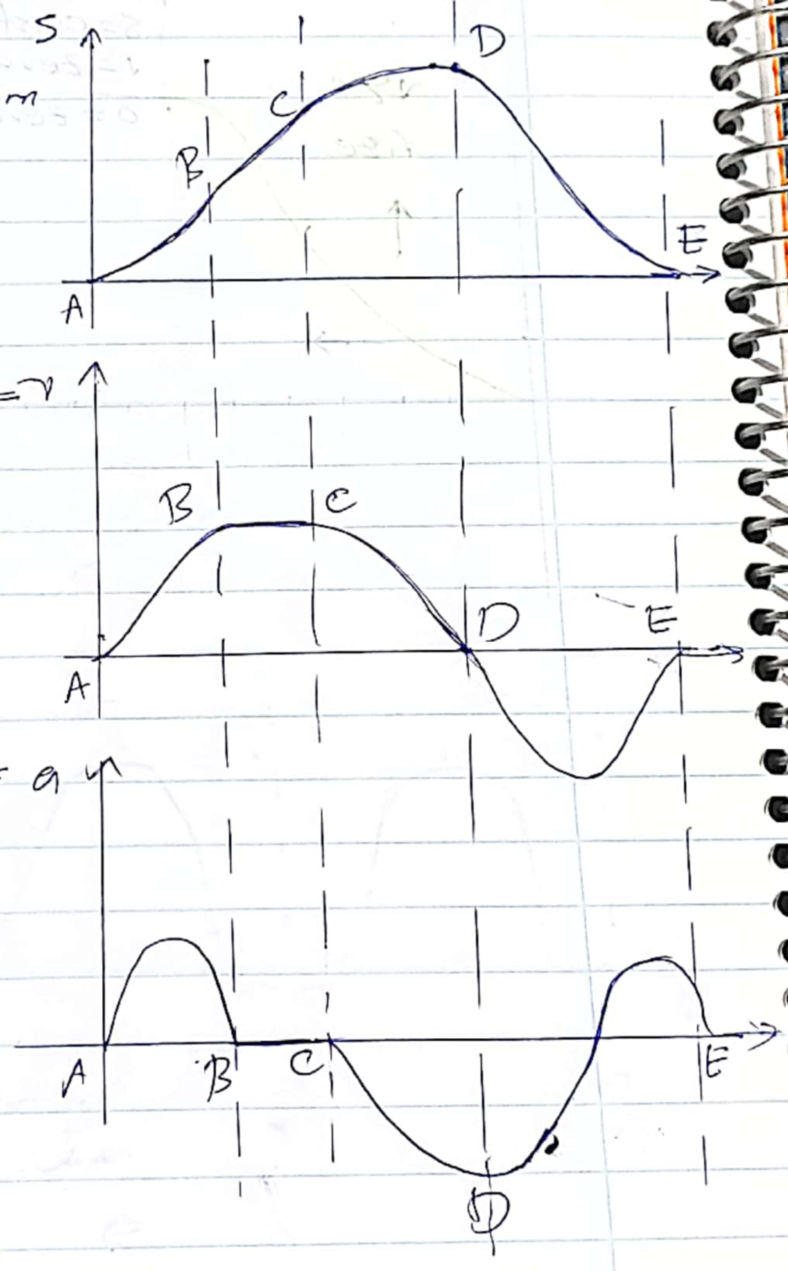
- Position or displacement (s)
- velocity (v)
- Acceleration (a)

note that:-

$$\frac{ds}{dt} = \frac{ds}{d\theta}\,\frac{d\theta}{dt} = \frac{ds}{d\theta}\,W_{cam}$$

$$\frac{ds}{dt} = v$$

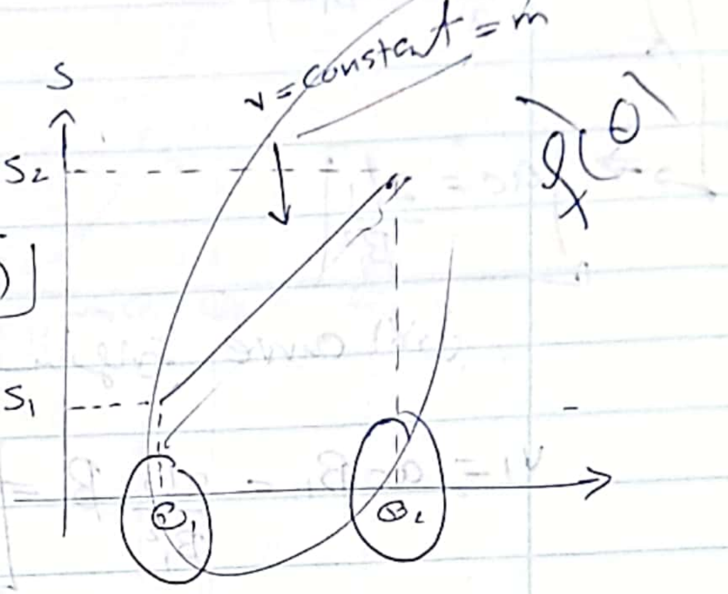
$$\frac{ds^2}{dt} = \frac{dv}{dt} = a$$



* constant velocity displacement curves:-

$$m = \frac{\Delta S}{\Delta \theta} = \frac{S_2 - S_1}{\theta_2 - \theta_1}$$

$$\boxed{S = f(\theta) = S_0 + m(\theta - \theta_0)}$$



* constant acclration deceleration curves:-

$S = f(\theta)$

$S = S_0 + V_0 \theta + \frac{1}{2} a_c \theta^2$

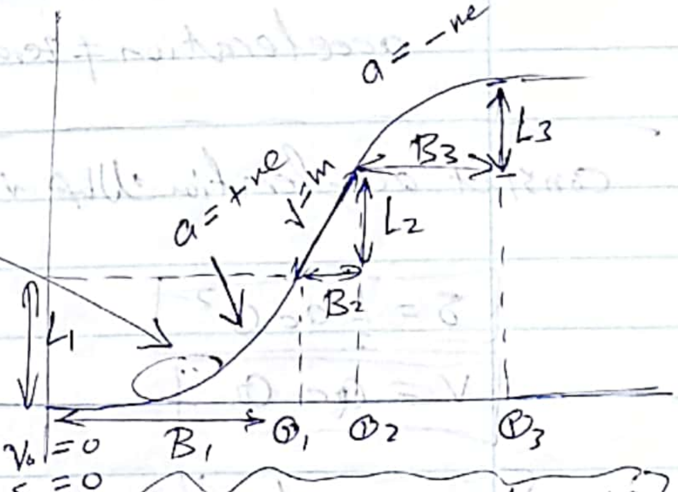
$$\boxed{S = \frac{1}{2} a_c \theta^2}$$

$V = V_0 + a_c \theta = a_c \theta$

$\boxed{V = a_c \theta}$ $\quad 0 \le \theta \le \theta_1$

$V_1 = a_c \theta_1$

$$\boxed{V_1 = \frac{2L_1}{\beta_1^2} \beta_1 = \frac{2L_1}{\beta_1}}$$



$$\boxed{\begin{aligned} S &= S_0 + V_0 t + \frac{1}{2} a_c t^2 \\ V &= V_0 + a_c t \end{aligned}}$$

علاقات معروفة

at $\theta = \beta_1$ → $S = L_1$ ※ a_c :- constant accelation

$$\boxed{L_1 = \frac{1}{2} a_c \beta_1^2}$$

$$\Rightarrow \boxed{a_c = \frac{2 \cdot L_1}{\beta_1^2}}$$

ايجاد الـV عند نهاية ال curve الاول

$$V_1 = a_c \beta_1 = \frac{2L_1}{\beta_1^2} \beta_1 = \boxed{\frac{2L_1}{\beta_1}} = m$$

⇐ لانستطيع استخدام constat a ومن ثم constant v

لان عند ~~[illegible]~~ بداية const velocity curve

← تكون acceleration = zero

وعند نهاية Const acaleration curve

← acceleration ≠ zero

اي بالتفصيل :- اذا كان هنالك constit acceleration

$$\boxed{S = \frac{1}{2} a_c \theta^2}$$

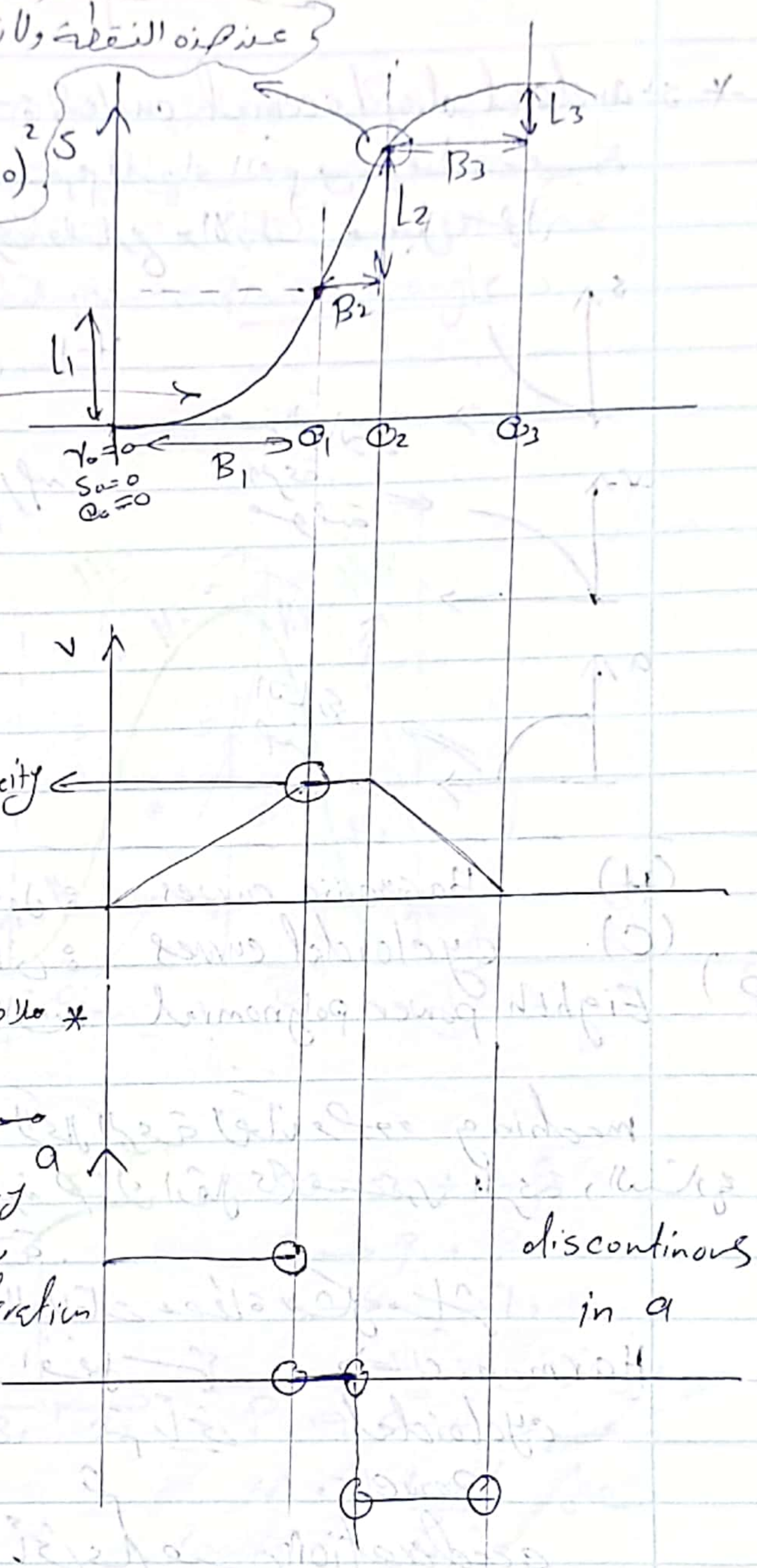
$$\boxed{V = a_c \theta}$$

$$\boxed{a_c = \frac{2L_1}{\beta_1^2}}$$ وعلاقة

عند هذه النقطة ولابد اعمل Shift

$$s = s_0 + v_0(\theta - \theta_0) + \frac{1}{2} a_c (\theta - \theta_0)^2$$

$$s = \frac{1}{2} a_c \theta^2$$



v_1 continuity velocity

$$\boxed{\frac{2L_1}{\beta_1}} = m = \frac{L_2}{\beta_2}$$

* ملاحظة: لتدقيق

continuity

مبوجب استخدام

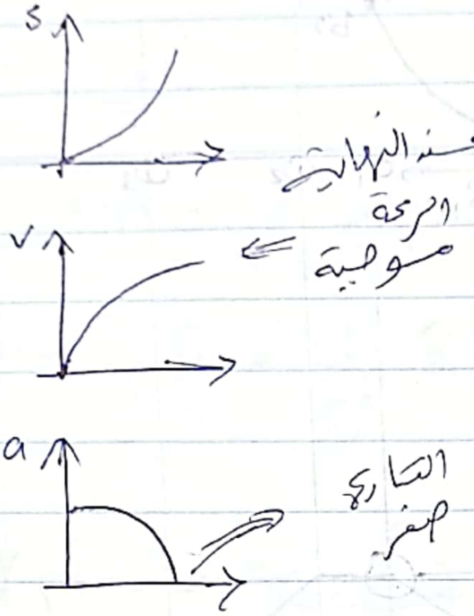
constant velocity

بعد استخدام

constant acceleration

* standard displacemat curves :-

لـ تستخدم لايجاد القيم في مسائل معينة]
قيم الحركة والتسارع والازاحة بشكل سهل.



مثلاً في H-1

عند النهاية الحركة موجبة ⇐

وهكذا القيم الحكم

التسارع صفر

---

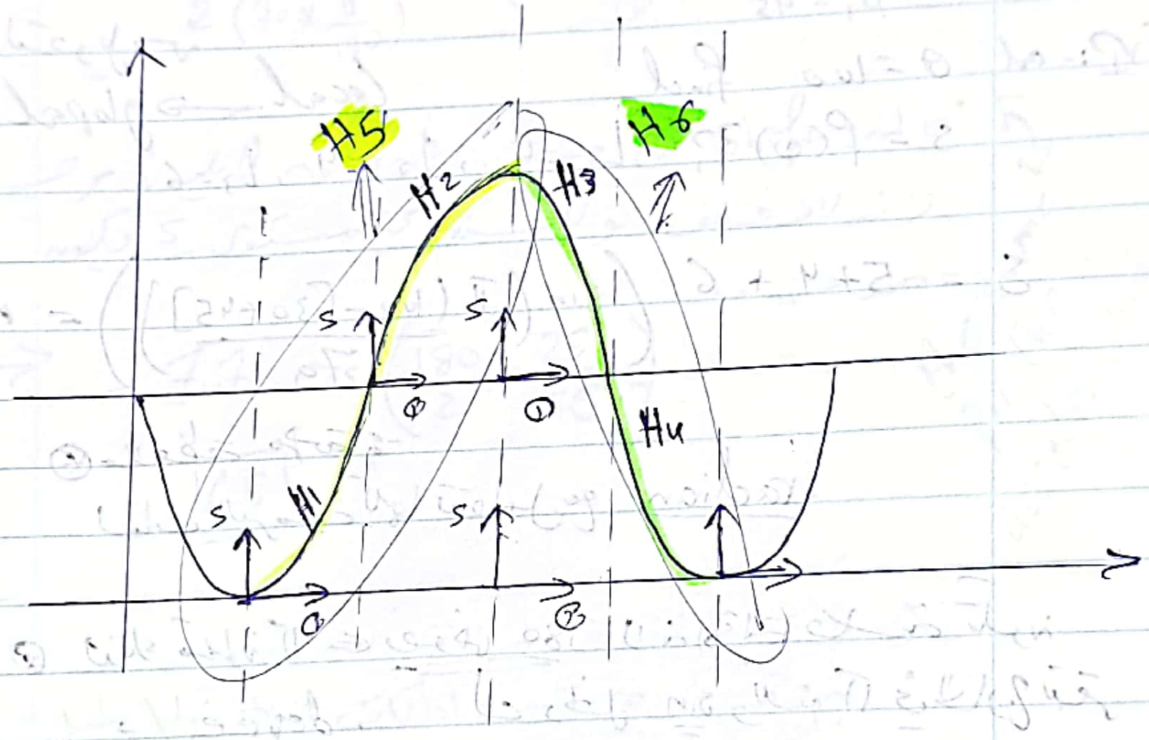
* النوع الاول : Harmonic curves (H)
* النوع الثاني : cycloidal curves (C)
* النوع الثالث :- Eighth power polynomval (P)

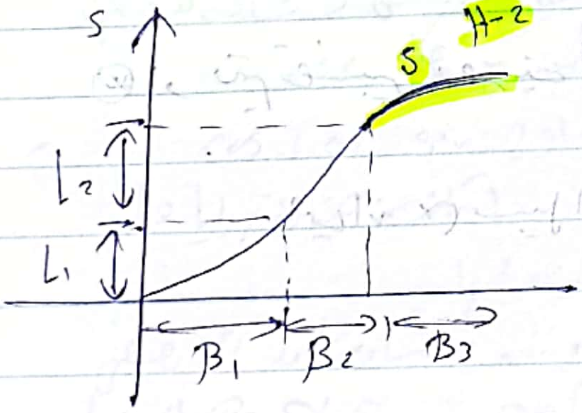
هذه تتشابه لأشكال الحركة لضمان وجود maching
⇐ حتى تكون هناك اتصال في مستوى الحركة والتسارع والازاحة.

* وتكون العلاقات معطاة بشكل مباشر.

Harmonic ⇒ افضلها
cydoidal ثم يأتي
Power ثم
⇐ التفضيل يأتي على حسب accderation.

* ← اي الذي يعطي تسارع اقل من الثاني يكون
التوجه للذي يعطي تسارع اقل منه لانه يعطي
force اقل من الثاني وهذا ما يسمى
بـ Pressure angle ← سيتم التطرق لها.





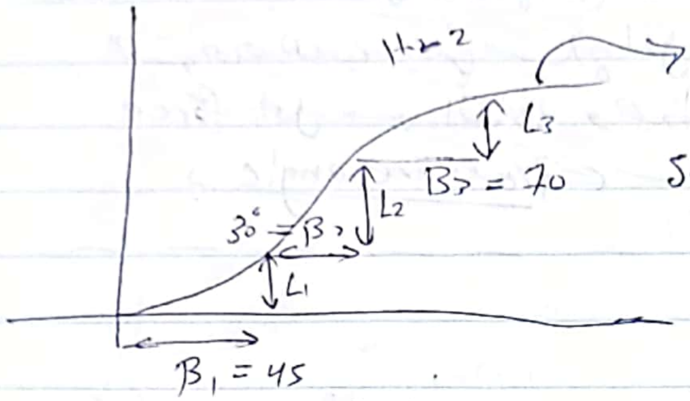
$SG = SL + shift$

$\theta \rightarrow \beta_1 + \beta_2$

$S \rightarrow L_1 + L_2$

اي اذا في H
ونيزها فيه C و P
يعطي local اي انه
S تكون locally

22/5/2021 part-2



$S_L = L_3 \left(\sin \frac{\pi\theta}{2\beta_3}\right)$

$S_G = L_1 + L_2 + L_3 \left(\sin \pi \left(\frac{\theta - [\beta_1 + \beta_2]}{2\beta_3} \right) \right)$

اي للتحويل من
local → global

Exp:- at $\theta = 100$ find $s = f(\theta)$?? $L_1 = 5$, $L_2 = 4$, $L_3 = 6$

$S = 5 + 4 + 6 \left(\sin \left(\pi \frac{100 - [30 + 45]}{70} \right) \right) =$ ✓

ملاحظة مهمة :-
لغات البرمجة كلها تتعامل مع radian.

لذلك مكان π ⇐ نعوض 180 لان الالة حاسبة تكون
بـ degree. لـ داخل sin ولكنه π في الخارج تبقى .

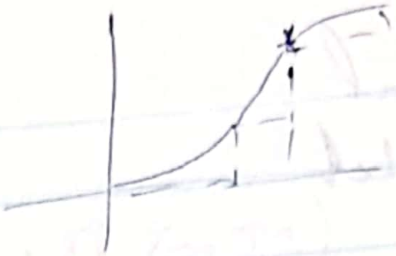
او يتم تغيير تعريف الالة حاسبة الى radian.

اما انه م سبيل المثال :-

$V = \frac{\boxed{\pi} L}{2\beta} \left(\cos \frac{\boxed{\pi}\theta}{2\beta} \right)$

تبقى كما هي

يتم تعويض
مكانها 180



v at $\theta = 100$:

$$v\Big|_{\theta=100} = \frac{\pi l_3}{2\beta_3}\left(\cos\frac{\pi}{2}\frac{\theta}{\beta_3}\right) = \frac{\pi 6}{2\left(70 * \frac{\pi}{180}\right)} \cos\left(\frac{\pi}{2}\frac{\overset{100}{\theta} - (\overset{30}{\beta_1} + \overset{45}{\beta_2})}{\beta_3}\right)$$

$$= \frac{\pi 6}{2\left(70 * \frac{\pi}{180}\right)} \cos\left(\frac{\pi}{2}\frac{25}{70}\right)$$

اي انه shift لل مستوى velocity لكونه موجود
لانه لا يختلف ولكنه لل مستوى في تكونه
موجب ..

$$= 7.7 \cos\left(\frac{180}{2}\frac{25}{70}\right)$$

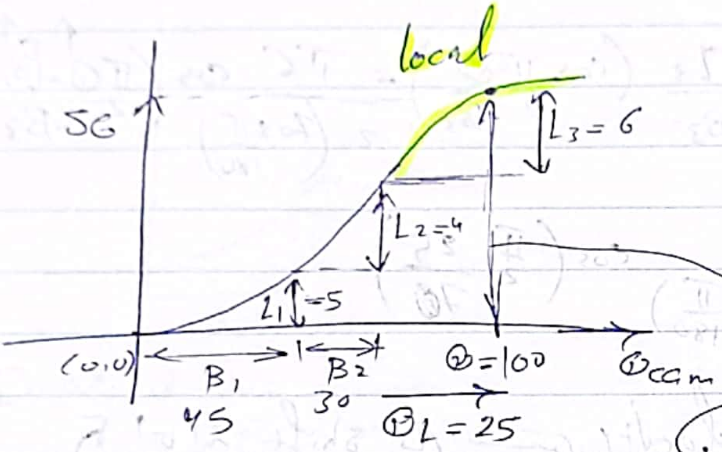
$$= \boxed{6.5}$$

the same for the acceleration

اي π عندما تكونه داخل cos or sin
يتم تعويضه 180 ⇐ واذا كانت هناك زوايا داخل
sin or cos معرفة لل بعض لا حاجة لتحويلها
لانه التحويل سيلغي ع بعض ...
واذا كانت π خارج sin or cos .
تعبر بحالها ... واذا كانت هناك زوايا خارج
sin or cos يتم تحويلها
واذا كانت هناك زاويتان متشابهتان لل بعض
مثل $\cos\left(\frac{\pi}{2}\frac{\theta}{\beta_3}\right)$ ⇐ θ & β_3 لا حاجة
لتحويلها اما اذا كانت $\cos\left(\frac{\pi}{2}\theta\right)$ ⇐ يتم تحويل θ
$= \left(\theta * \frac{\pi}{180}\right)$
⇐ كما وضحنا ما مستوى السابق.

$S_L \equiv S_{local}$:
$S_G \equiv S_{Global}$

⇐ * بالحل shift :-



$S_G = S_L + shift$
$\theta_{local} = 100 - (45 + 25)$
$\theta_L = 25°$

$$S_G = 9 + 6 \sin\left(\frac{180}{2} \frac{25}{70}\right) \Rightarrow \boxed{S_G = 12.19 \text{ unit}}$$

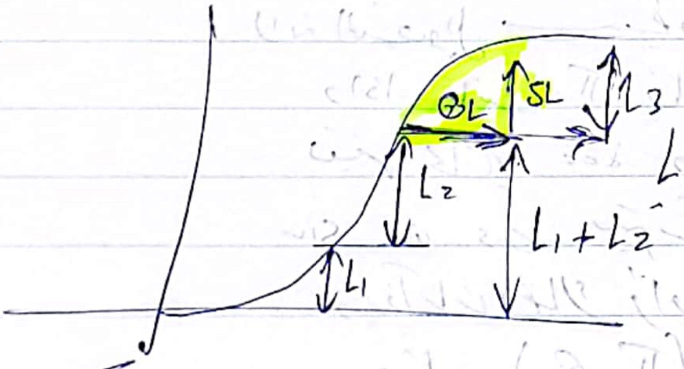
* without shift

$$\theta_L = 100 - 75 = 25°$$

$$S_L = 6 \sin\left(\frac{180}{2} \frac{25}{70}\right) = 3.19$$

$$S_G = L_1 + L_2 + S_L$$
$$= 5 + 4 + 3.19$$
$$\boxed{S_G = 12.19}$$

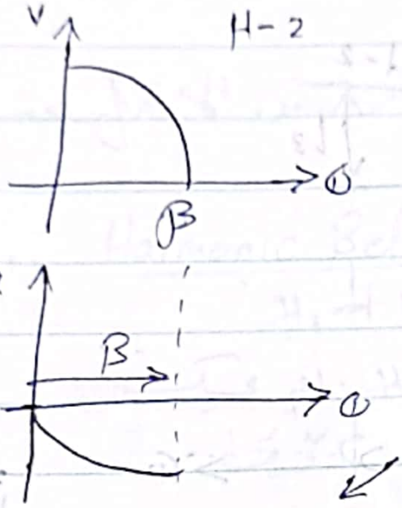
*ملاحظة - عا مستوى
الحركة والشارع لا يوجد هناك
shift فقط نوجد θ_L
ونطبقه [illegible] في
الإزاحة كي نوجد مقدار θ_L
ونحل الإزاحة



اي ايجاد S_L + θ_L
وبعدها نجمعها $L_1 + L_2$
لايجاد S_G

* وهان مستوى الحركة
نفس الشيء ⇐ $\nabla_L = \nabla_G$ ⇐ لانه الميل نفسه يكون

▤ note that:



H-2

$v = \frac{\pi l}{2\beta} (\cos \frac{\pi \theta}{2\beta})$

عند نهاية curve

$v = zero$ ⇐

⇐ عند بداية $a = zero$

وعند نهاية curve

$a = \frac{-\pi^2 L}{4\beta^2} (\sin \frac{\pi \theta}{2\beta})$

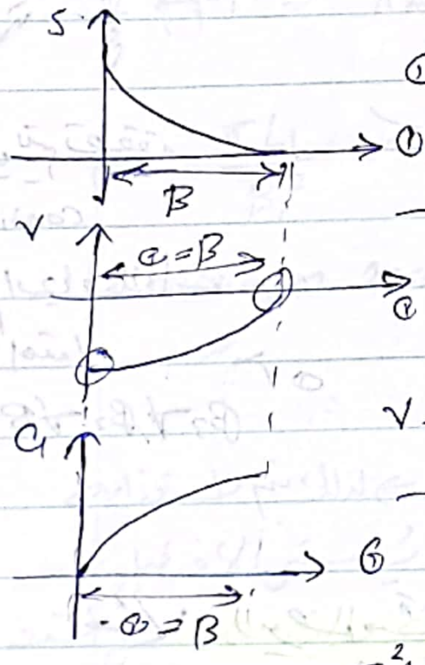
$$\boxed{a\Big|_{\theta=\beta} = \frac{-\pi^2 L}{4\beta^2}}$$

*دايمًا انتبهوا curves لو 2

بحولة min و max

من الحركة.

Exp: H-4



$s = L (1 - \sin \frac{\pi \theta}{2\beta})$

* عند البداية $\theta = zero \Rightarrow \boxed{s = L}$

* عند النهاية $\theta = \beta \Rightarrow \boxed{s = zero}$

$v = \frac{-\pi l}{2\beta} (\cos \frac{\pi \theta}{2\beta})$

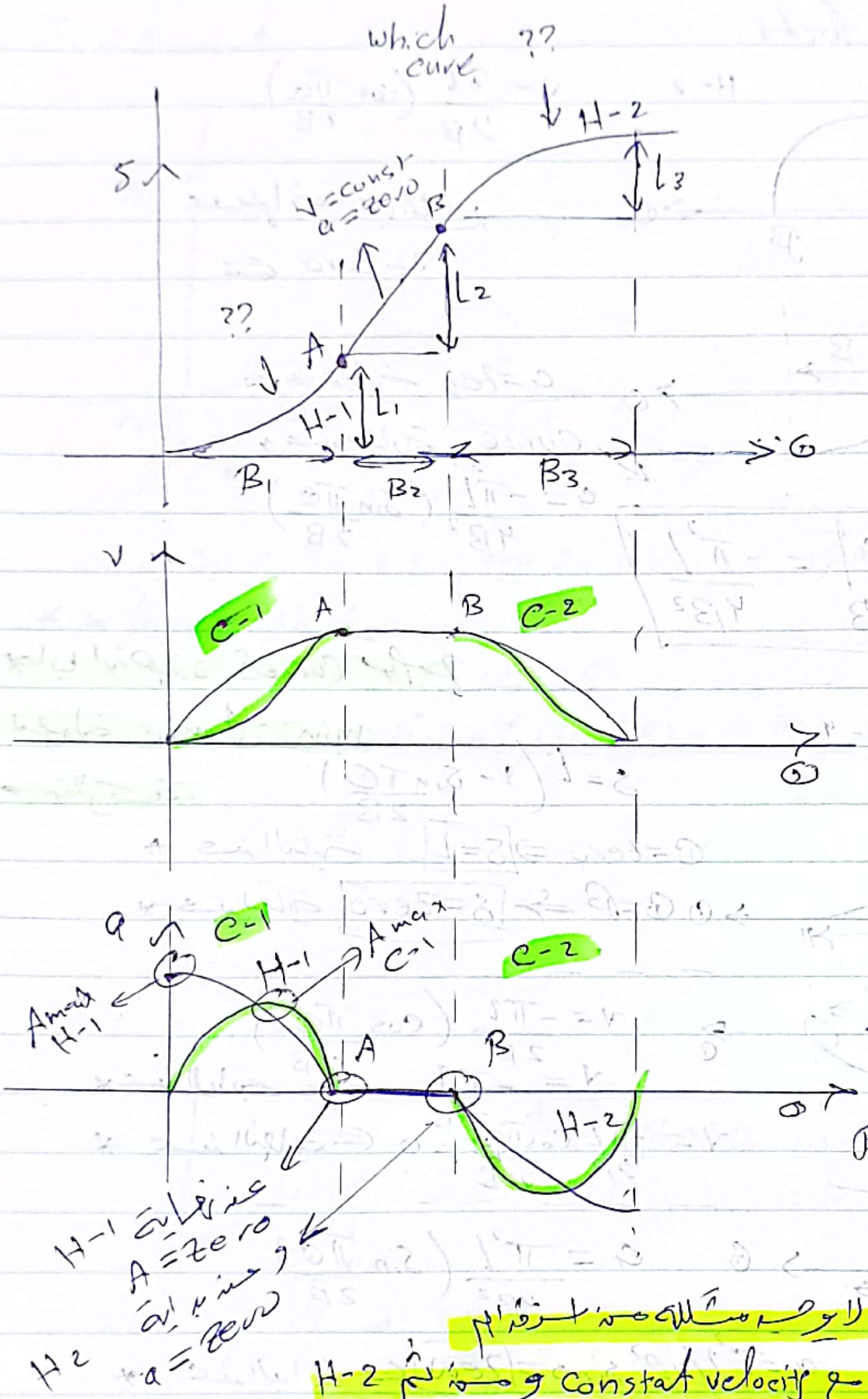
* عند البداية ⇐ $v = -\frac{\pi . L}{2\beta}$

* عند النهاية ⇐ $v = \frac{-\pi L}{2\beta} \cos \frac{\pi \beta}{2\beta} = 0$

$a = \frac{\pi^2 L}{4\beta^2} (\sin \frac{\pi \theta}{2\beta})$

* عند البداية ⇐ $a = \pi^2 L / 4\beta^2 \sin 0 = \boxed{zero}$

* عند النهاية ⇐ $a = \frac{\pi^2 L}{4\beta^2} \sin \frac{\pi \beta}{2\beta} = \boxed{\frac{\pi^2 L}{4\beta^2}}$



تم التحقق من الـ continuity ع امتداد السرعة والتسارع

لكي يتم تحقيق continuity يتم ايجاد علاقات ع امتداد $B_3 + B_2 + B_1$

⟸ اي لا يوجد مشكلة من استخدام H-1 مع constant velocity و من ثم H-2

⟸ لأنه هناك continuity لذلك H-1 + H-2 مناسب.

≪ في حال تم استخدام C-1 + C-2

اي لا يوجد مشاكل انه سيفقد الاتصال عن A + B .

≤ ولكنه **Harmonic Better**

$H_1 + H_2$

* اي يتم استخدام Harmonic دائما عكسه
الا اذا تطلب استخدام C فقط

لهذا يوجد افضل ؟

for acceleration :-

* C-1 ⇒ $A_{max} = \frac{\pi L_1}{\beta_1^2}$

* H-1 ⇒ $A_{max} = \frac{\pi^2 L_1}{4\beta_1^2}$

$$\frac{\pi L_1}{\beta_1^2} > \frac{\pi^2 L_1}{4\beta_1^2}$$

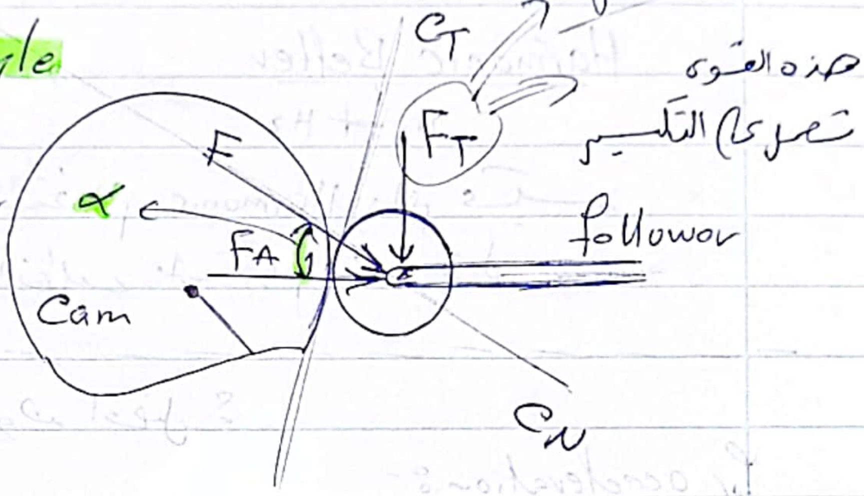
↳ maximum acceleration for C-1 biggar than
" " for H-1 .

⊛ Harmonic ⇐ يعني اقل تسارع ⇐ لذلك يتم استخدامه
على سرعات اعلى ... لذلك يعتبر الافضل.
لانه cam عندما يدور بسرعة عالية ⇐ يفضل استخدامه

⊗ الأقل تسارع ⇐ تكون Pressure angle - صغيرة ،

- ▤ pressure Angle :

α : pressure angle
Between CN
& axis of follower



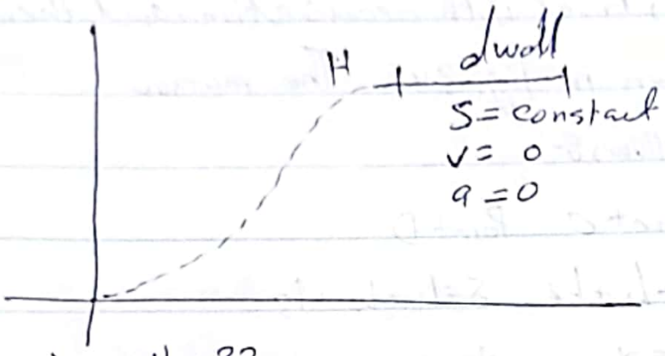
≪ أي أنه Harmonic ⇐ يعطي Pressure angle أقل صغيرة
لذلك يُفضل استخدامه ..

⇒ Keep α as law as Possible.

▤ Harmonic motion curves :-

- lowest peak acceleration.
- Lowest peak pressure angle.
- Used in high speed cams
- Can't be matched with dwell ?? ⇒
- Preferable When possible.

* can't be matched with dwell



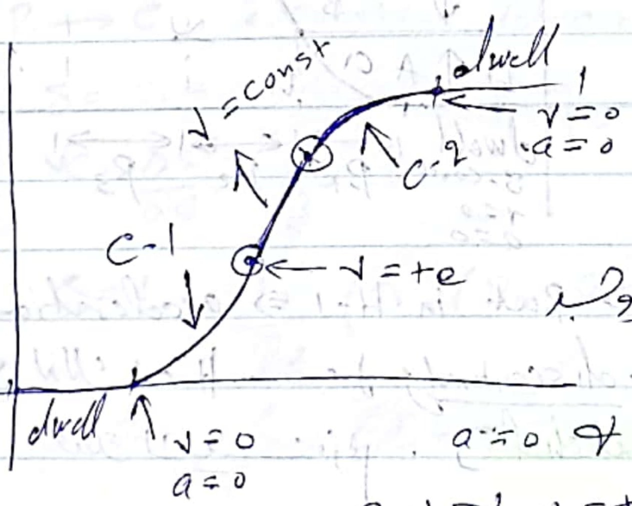
* لا يمكن ربطه مع dwell.

⇒ why??

* لا يوجد نوع من انواع H
يعني v=0 & a=0 عند بداية او نهاية.
اي لا يوجد امكانية لايجاد عند نهاية v=0 & a=0
والاخرى عند البداية.

Cycloidal motion curves :-

- High Peak acceleration
- High Peak Pressure angle
- Used in low speed cams
- can be matched with dwell !!



اي انه C-1 عند بداية
v=0 & a=0 لذلك
تمكن استخدامه لانه يوجد
dwell ، وكذلك C-2
عند نهايته ⇐ v=0 & a=0
وكذلك عند البداية ⇐ v=+ & a=+

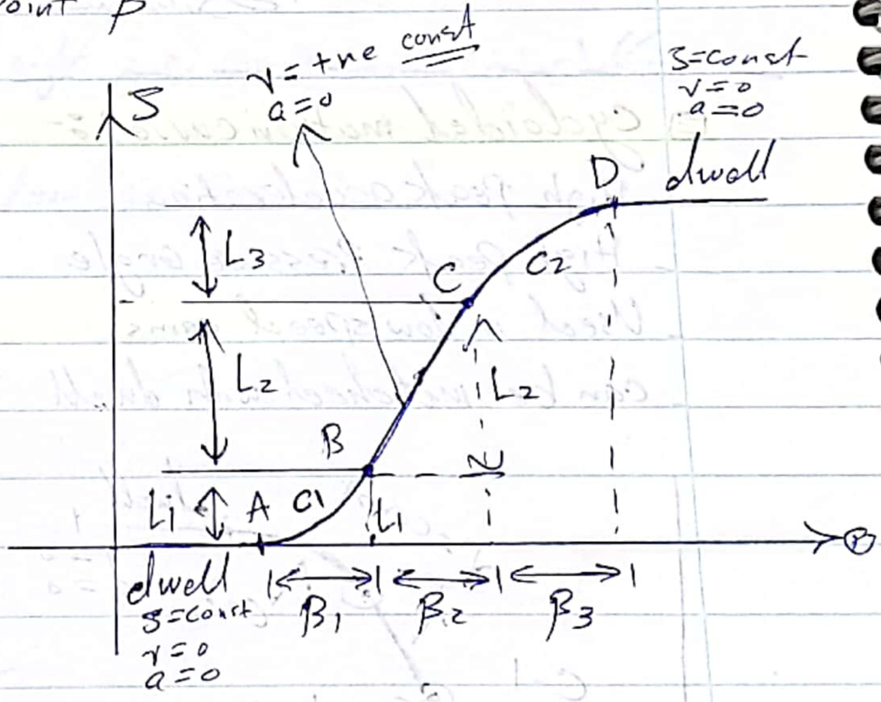
24/5/2021

Ex. 3.23 A follower dewells, rises with acceleration, rises with constant velocity, rises with deceleration, and then dwells again as shown in fig. 3.46 "The motion requirements are as follows:-

Point A	Point B	Point C	Point D
$S=0$	$S=L_1$	$S=L_1+L_2$	$S=L_1+L_2+L_3$
$V=0$	$V=V_1$	$V=V_1$	$V=0$
$A=0$	$A\neq0$	$A=0$	$A=0$

⇒ Recommend the curves to be used for the displacement graph and the relation Between $\beta_1 + \beta_2 + \beta_3$ to match, velocity acceleration at point B

↳ at the ends of the intervals.



at A:

V_A = zero

a_A = zero

☰ A → B

H-1 or C-1 ??

at H-1

At A ⇒ V = zero

$\boxed{a = zero}$ ⇒ But in H-1 ⇒ acceleration $\neq 0$

لذلك H-1 بعمل discontinuity ب مستوى acceleration

(Harmonic not be matched) . لذلك لا يستخدم with dwell.

☐ $\underline{C-1}$ $A \longrightarrow B$ ☐ $P-1$ as $H-1$

At $a \Rightarrow A = 0$ & $V = 0$ ✓

at $B \Rightarrow V = +ve$, $A = zero$ ✓

≪ لذلك نستخدم $C-1$:

☐ $C \rightarrow D$

at C: V = +ve A = zero

at D:- V = zero A = zero ⇒ C-2 ✓

---

* $A \rightarrow B$ (C-1)

$$S = L_1\left(\frac{\theta-\beta_0}{\beta_1} - \frac{1}{\pi}\sin\pi\left(\frac{\theta-\beta_0}{\beta_1}\right)\right) \Rightarrow \text{shift by } \beta_0$$

$$V = \frac{L_1}{\beta_1}\left(1 - \cos\pi\left(\frac{\theta-\beta_0}{\beta_1}\right)\right.$$

$$A = \frac{\pi L_1}{\beta_1^2}\left(\sin\pi\left(\frac{\theta-\beta_0}{\beta_1}\right)\right)$$

---

* $B \rightarrow C$: constant velocity

$$S = S_i + V_c(\theta - \theta_0)$$ ⇒ كم ايجادها في البداية

$$V_c = \frac{\Delta S}{\Delta \theta} = \frac{L_2}{\beta_2} ,$$

$$\hookrightarrow S = L_1 + \frac{L_2}{\beta_2}\left(\theta - (\beta_0 + \beta_1)\right)$$

a = zero from ($B \rightarrow C$).

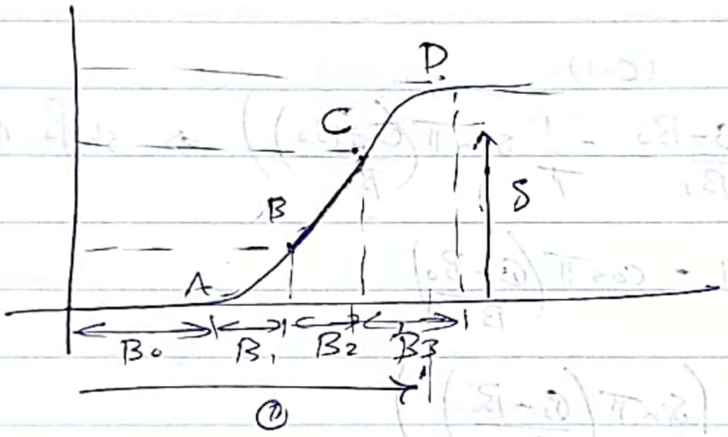
* C ⟶ D (C-2) with shift

$$\beta_0+\beta_1+\beta_2 \le \Theta \le \beta_0+\beta_1+\beta_2+\beta_3$$

$$S=(L_1+L_2)+L_3\left(\frac{\Theta-(\beta_0+\beta_1+\beta_2)}{\beta_3}+\frac{1}{\pi}\sin\pi\frac{\Theta-(\beta_0+\beta_1+\beta_2)}{\beta_3}\right)$$

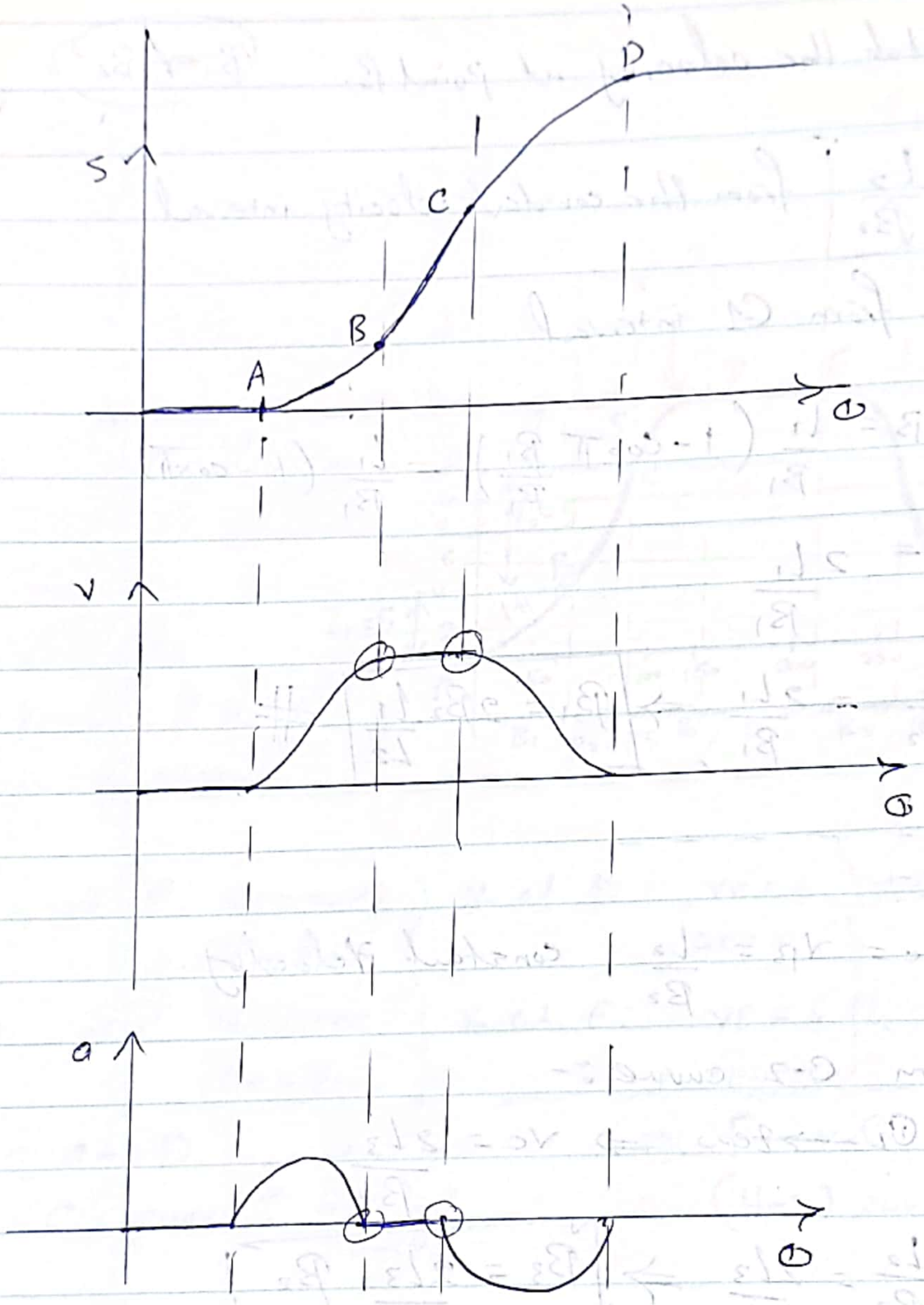
$$A=\frac{-\pi L}{\beta_3^2}\left(\sin\pi\frac{\Theta}{\beta}\right)\longrightarrow \Theta-(\beta_0+\beta_1+\beta_2)$$

↳ Shift in x-axis



اي ان Θ هي زاوية تقع بين D & C
عندما تظهر قيمتها ⇐ يتم ايجاد الازاحة عندها
والسرعة والتسارع كما السابق بعد ايجاد
العلاقات مع Shift .

$$\beta_0+\beta_1+\beta_2 \le \Theta \le \beta_0+\beta_1+\beta_2+\beta_3$$


S
D
C
B
A
θ
v
θ
a
θ

to match the velocity at point B. (β_1 + β_2)

$\frac{\Delta s}{\Delta \phi} = \boxed{V_B = \frac{L_2}{\beta_2}}$ from the constant velocity interval

at B from C1 interval

$$V_B = \frac{L_1}{\beta_1}\left(1 - \cos\pi\frac{\beta_1}{\beta_1}\right) = \frac{L_1}{\beta_1}(1 - \cos\pi)$$

$$= \frac{2L_1}{\beta_1}$$

$$\frac{L_2}{\beta_2} = \frac{2L_1}{\beta_1} \Rightarrow \boxed{\beta_1 = 2\beta_2 \cdot \frac{L_1}{L_2}} \; \#$$

---

at C :-

$V_c = V_B = \frac{L_2}{\beta_2}$ constant velocity.

from C-2 curve :-

$\theta_L \to$ zero $\Rightarrow V_c = \frac{2L_3}{\beta_3}$

$$\frac{L_2}{\beta_2} = \frac{2L_3}{\beta_3} \Rightarrow \boxed{\beta_3 = \frac{2L_3}{L_2}\beta_2}$$

اي اذا اعطى قيمة β_2 فيمكن ايجاد قيمة β_1
واذا اعطى قيمة β_2 فيمكن ايجاد قيمة β_3

Exp:-

A follower:-

① Rises with acceleration

② Rise with const-velocity

③ Rises with deceleration

④ Dewells

⑤ Returns with deceleration.

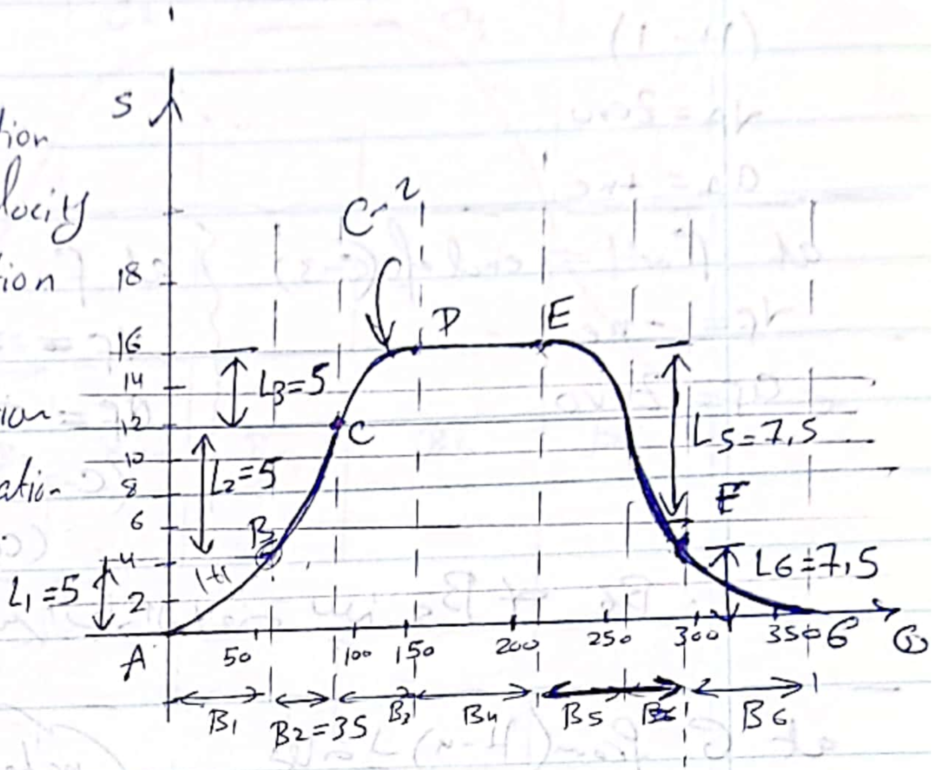
⑥ Returns with acceleration

As shown

given $\beta_2 = 35°$

① Recommend curves

② find all intervels.



* at B : $V_B = +ve$

$a_B = zero$

* at C : $V_C = +ve$

$a_C = zero$

* at D : $V_C = 0$

$a_C = 0$

* at E :- $V_E = 0$

$a_E = 0$

* at F : $V_F = -ve$

$a_F = ??$

☰ C → D

C-2 curve } ⇒ لأنه بعدها
يأتي الـ H
dwell

☰ F → G

(H-4) curve.

☰ E → F :-

C-3 curve ⟹

☰ A - B

H-1 OR C-1

⇓

Better

E $A \to B$

(H-1)

$v_A = zero$

$a_A = +ve$

---

at F :- ⇒ end of (C-3)

$v_F = -ve$

$a_F = zero$

at F :- start of (H-4)

$V_F = -ve$ ✓

$a_F = zero$ ✓

نفس قيم نهاية (C-3)

(Continuity)

⇐ اي بنقدر ايجاد العلاقة بين B_5 & B_6 .

---

at G from (H-4) نهاية لـ

$v_G = zero$

$A_G = +ve$

note that

$A_G = A_A$

لانه تغير كل 360° درجة وكذلك السرعة .

مثلاً لو استخدمنا بدل H4 ← C4

من F → G

at F :- (C-4)

$V_F = -ve$ ⇒ يتناسب مع (C-3)!

$A_F = zero$

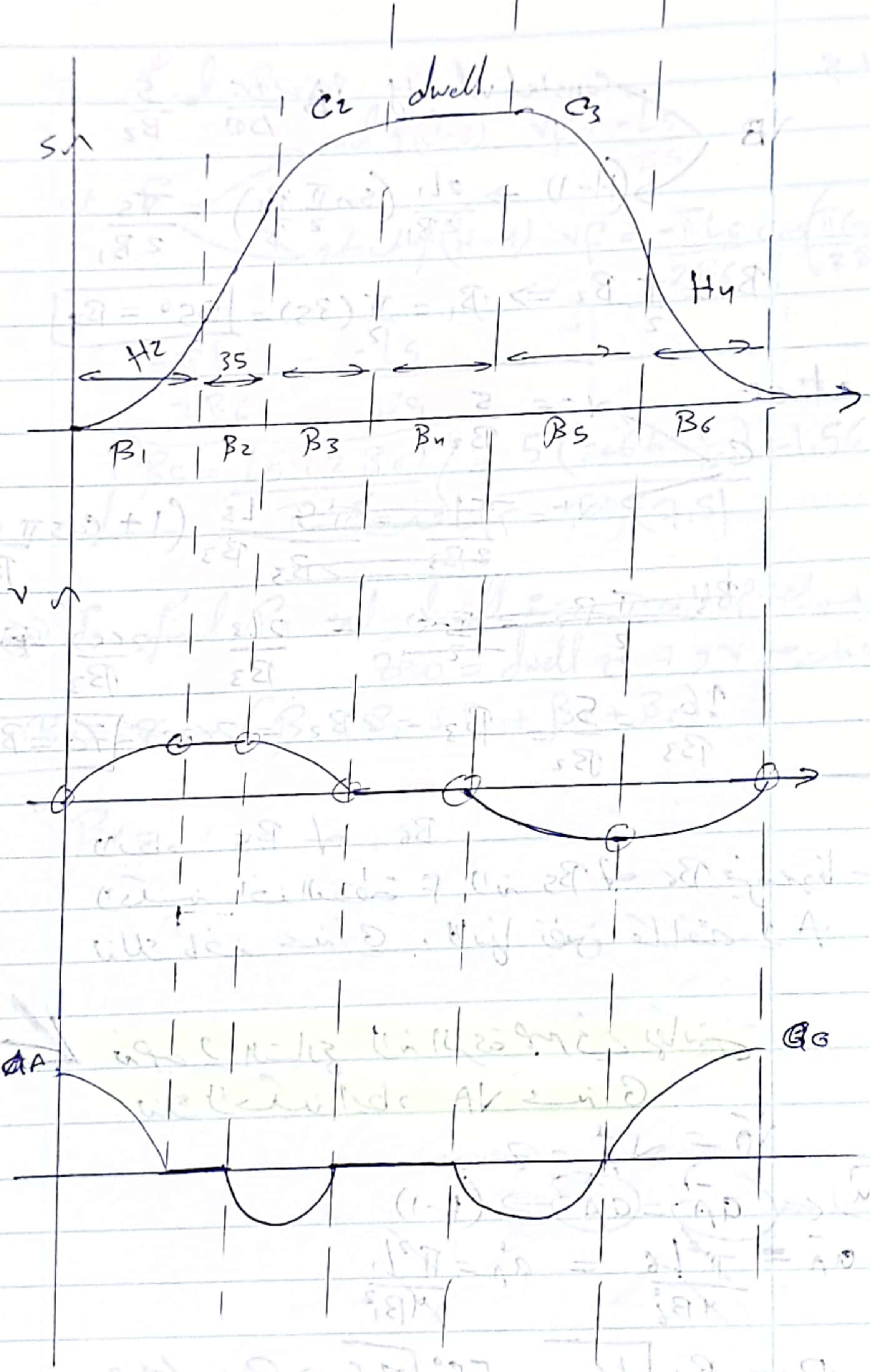
ولكن اذا استخدمت (C-4) من G ← F

يجب استخدام (C-1) من (B ← A)

لانه (C-4) ⇐ نهايته $a=0$ & $v=0$

و (C-1) ⇐ بدايته $a=0$ & $v=0$

وهكذا يتحقق الاتصال #


S
C2
dwell
C3
H2
35
H4
B1
B2
B3
B4
B5
B6
V
aA
aG

29/5/2021

at B:-

$V_B \to$ Constant velocity $V_B = \frac{\Delta S}{\Delta \theta} = \frac{5}{\beta_2}$

$V_B \to (H-1) \to \frac{2L_1}{2\beta_1}\left(\sin\frac{\pi}{2}\frac{\beta_1}{\beta_1}\right) = \frac{\pi S}{2\beta_1}$

$$\beta_1 = \frac{\pi}{2}\beta_2 \Rightarrow \beta_1 = \frac{\pi}{2}(35) = \boxed{55^\circ = \beta_1}$$

---

at C:-

$C \to V^- = \frac{5}{\beta_2}$

$C \to V^+ = ~~\frac{\pi L_3}{2\beta_3} = \frac{\pi S}{2\beta_3}~~ \frac{L_3}{\beta_3}\left(1 + \cos\pi\frac{\theta}{\beta_3}\right)$

$~~\beta_3 = \frac{\pi}{2}\beta_2 = \frac{\pi}{2}6~~ = \frac{2l_3}{\beta_3} = \frac{2(5)}{\beta_3}$

$$\frac{10}{\beta_3} = \frac{5}{\beta_2} = \beta_3 = 2\beta_2 = 2(35) = \boxed{70 = \beta_3}$$

لايجاد B5 & B6

لا يمكن اخذ النقطة F لانه B5 & B6 غير معروفة —

لذلك ناخذ عند G . لانها نفس مكافئه في A.

نذهب في التسارع لانه الحركة صفرية نهائياً

لذلك لا يمكن ايجاد VA عند G

$$V_A^- = V_A^+ = \text{zero}$$

$$(H-4) \Leftarrow a_A^- = a_A^+ \Rightarrow (H-1)$$

$$a_A^- = \frac{\cancel{\pi^2} L_6}{\cancel{4}\beta_6^2} = a_A^+ = \frac{\cancel{\pi^2} L_1}{\cancel{4}\beta_1^2}$$

$$\beta_6 = \beta_1\sqrt{\frac{L_6}{L_1}} = 55^\circ\sqrt{\frac{7.5}{5}} \Rightarrow \beta_6 = 67.3$$

To find $\underline{\beta_5}$:-

at F → end of (C-3) $V_f = \frac{-L_5}{\beta_5}$

at F → start of (H-4) $V_f = \frac{-\pi L_6}{2\beta_6}\cos\left(\frac{\pi(0)}{2\beta_6}\right)$

$$\frac{-\pi L_6}{2\beta_6} = \frac{-L_5}{\beta_5}$$

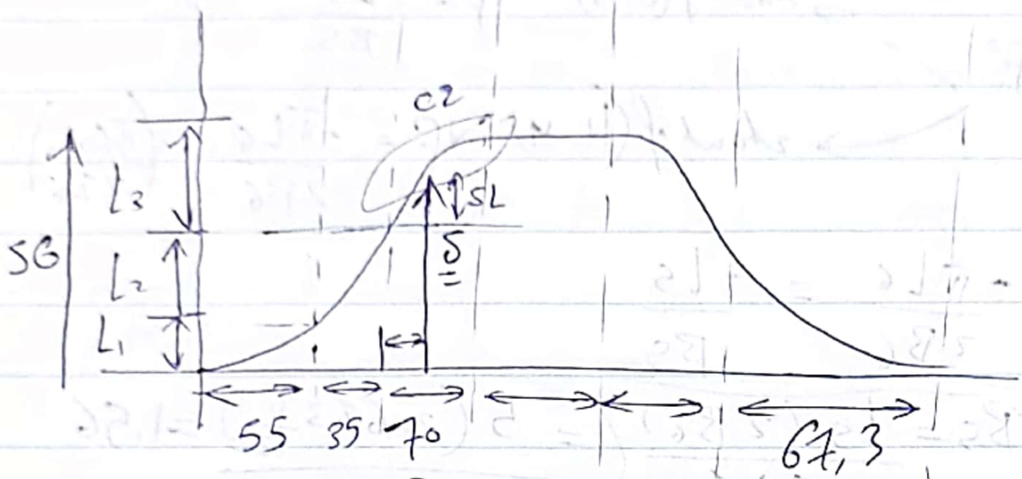
$$\boxed{\beta_5 = \frac{\cancel{L_5}(2\beta_6)}{\pi \cancel{L_6}}} = \frac{5\left(2*67.3\frac{\pi}{180}\right)}{\pi \cancel{180}(7.5)} = 1.56$$

▤ ≫ To find β_4 at dwell :- لايمكن ايجادها لانه العلاقات مع v و a مع $\underline{zero}$ = dwell

لذلك → $\beta_4 = 360 - (\beta_1 + \beta_2 + \beta_3 + \beta_5 + \beta_6)$

$\beta_4 =$ ✓

بعد ايجاد القيم :-



Determine δ @ $\beta = 120$??

$$\beta_1 + \beta_2 \leq 120 \leq \beta_1 + \beta_2 + \beta_3$$

$$\theta_L = 120 - 90 \Rightarrow \boxed{\theta_L = 30}$$

$$\delta_L = l_3 \left(\frac{\theta_L}{\beta_3} + \frac{1}{\pi} \sin \pi \frac{\theta}{\beta_3} \right)$$

$$= 5 \left(\frac{30}{70} + \frac{1}{\pi} \sin 180 \frac{30}{70} \right)$$

خارج
sin
بنعوض
180 قطعا

$$\delta_L = 3,69 \rightarrow \text{unit}$$

$$\delta G = (l_1 + l_2) + \delta L$$

$$= 10 + 3.69$$

$$= \boxed{13.69}$$

$$v\Big|_{120} = \frac{L_3}{\beta_3}\left(1 + \cos 180 \cdot \frac{30}{70}\right)$$

$$= \frac{5}{70\left(\frac{\pi}{180}\right)}\left(1 + \cos 180 \ \frac{30}{70}\right)$$

تم التحويل لانه β_3 بالدرجات نحولها

$$= \boxed{5 \text{ unit/radins}} \ \#$$

\# Acceleration لـ وكذلك الشي

بداية curve

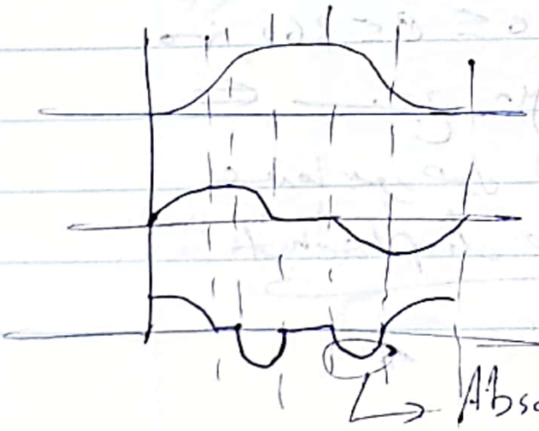
$$v_c = v_{max} = \frac{l_2}{\beta_2} \ (\text{from } (C-2))$$

$$= \frac{5}{\left(35 \ast \frac{\pi}{180}\right)} = \boxed{8.185 = v_c} \ \#$$

اذا طلب Absolute Maximum Velocity ← عندما ينزل

→ (C-3)

$$v_f = \frac{-L_6}{\beta_6} \ ✓$$



← اي نعود الى الرسم

اي مقارنة مع

نوع H or C بي

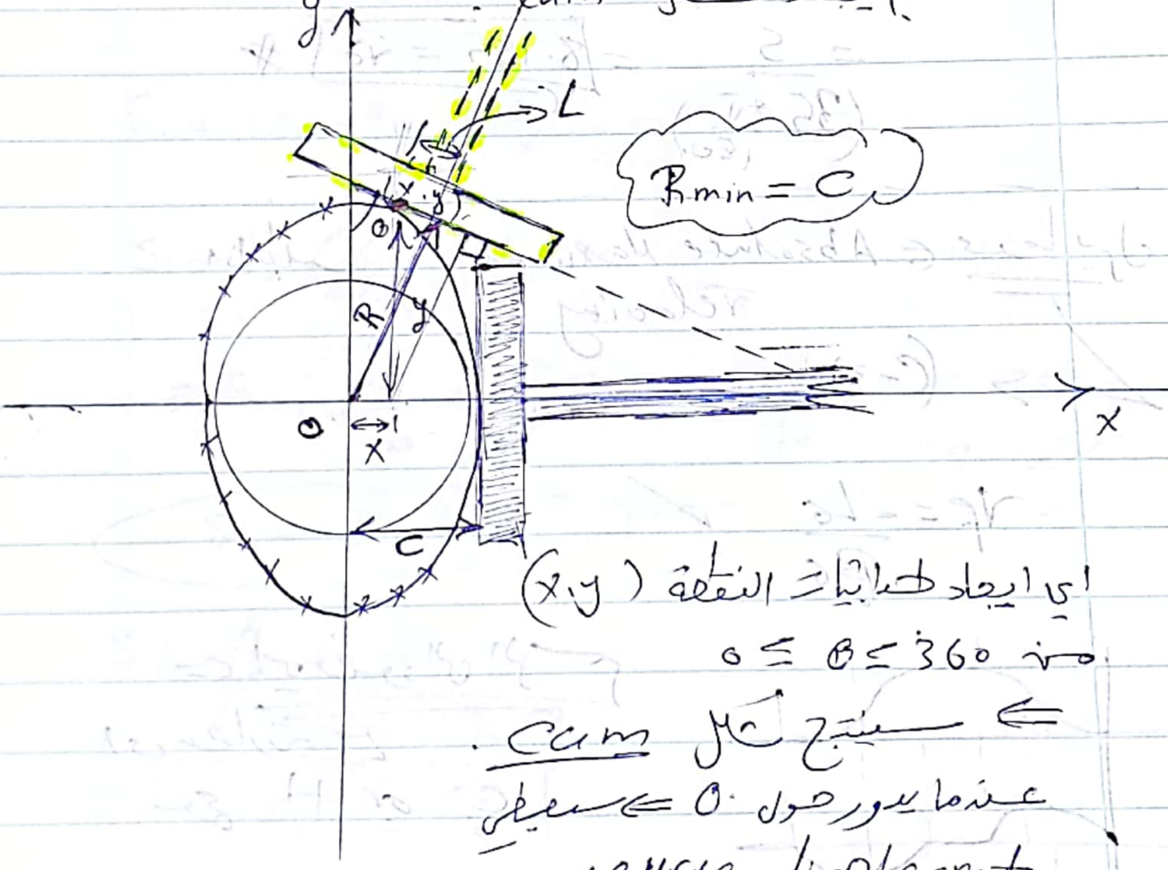
→ Absolute maximum acceleration

عندما ينزل

▤ Design: flat-faced follower.

* find cam countour. such that. When this cam rotates about it's center O ⇒ the follower is moving according the prescribed motion defined by the displacement curve.

* اي الفكرة بدلاً من تدوير cam و follower
بمقدار 360 درجة ⇐ لايجاد الاحداثيات
⇐ فانه تدوير follower حول cam أجل.
وايجاد احداثيات ال contact point على امتداد 360
سيتم ايجاد شكل cam

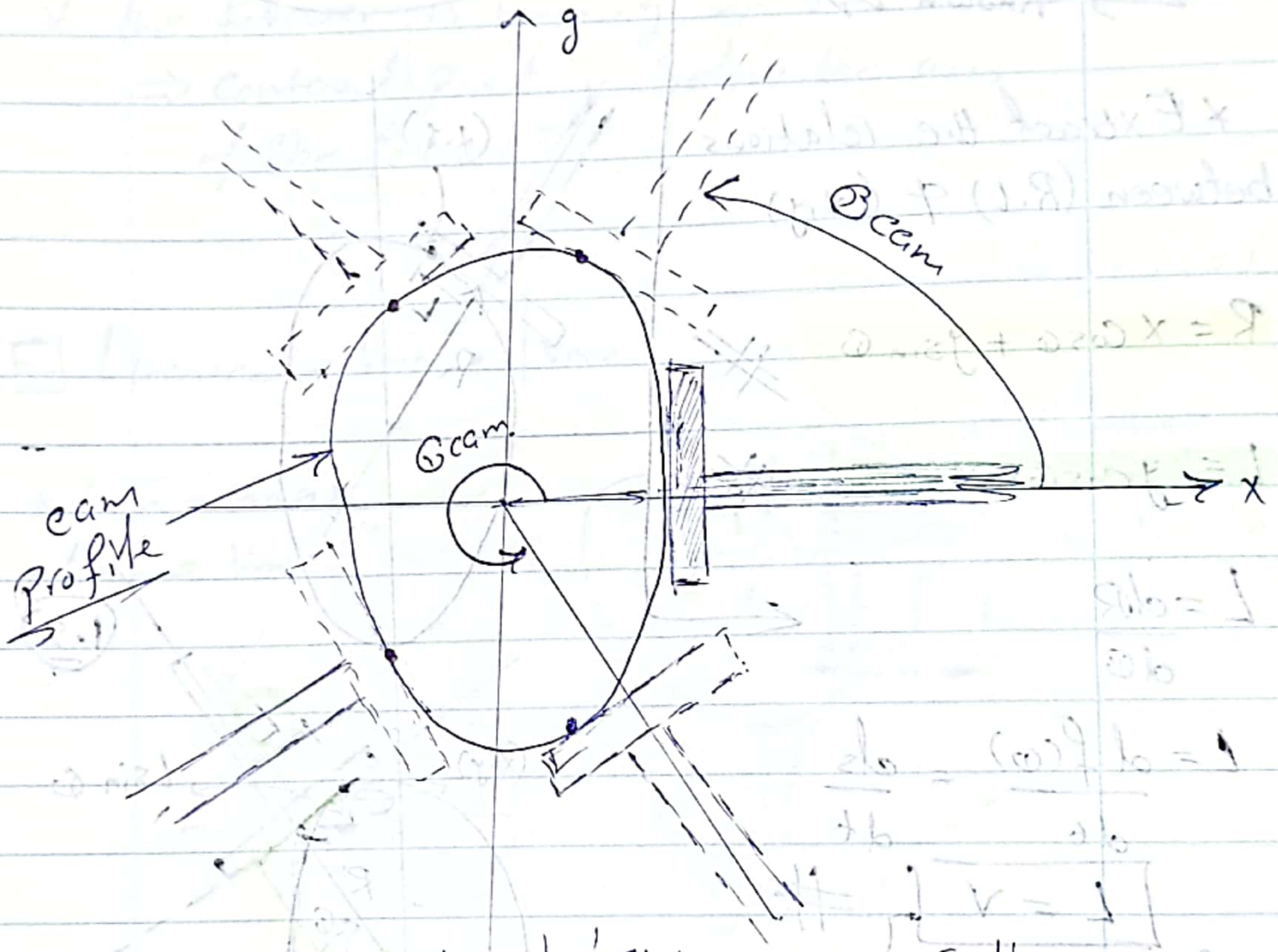


اي ايجاد احداثيات النقطة (x,y)
من $0 \leq \theta \leq 360$
⇐ سينتج شكل cam.
عندما يدور حول O ⇐ سيعطي
* curve displacmt

* R: distance from the center of rotation of the cam to the face of the follower

* C: min. distance from the center of rotation of the cam to the face of the follower

* L: perpendicular distance from the axis of the follower and the contact point



* اي حسب زاوية. cam اذا استطعنا ايجاد
احداثيات = النقطة (x,y) عند موقع ⇐ نستطيع ايجاد المسار
اي شكل cam [cam profile]

* $\boxed{R_{min} = C}$

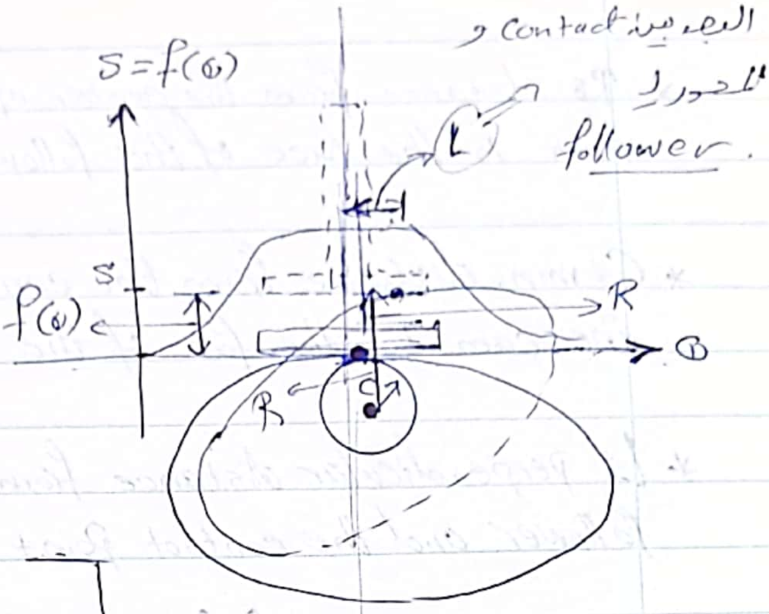
$R = C + f(\theta)$

$\boxed{R = C + S}$

$f(\theta) = S \Rightarrow$ Designed

$\Rightarrow \boxed{R = C + f(\theta)}$

Known ✓✓



* Extract the relations between (R.L) & (x,y)

$R = x\cos\theta + y\sin\theta$ #

$L = y\cos\theta - x\sin\theta$ #

$L = \frac{dR}{d\theta}$

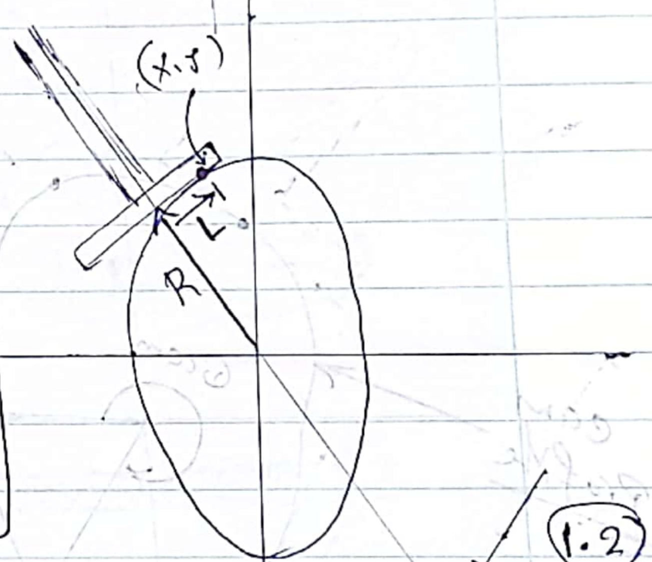
$L = \frac{d\,f(\theta)}{dt} = \frac{ds}{dt}$

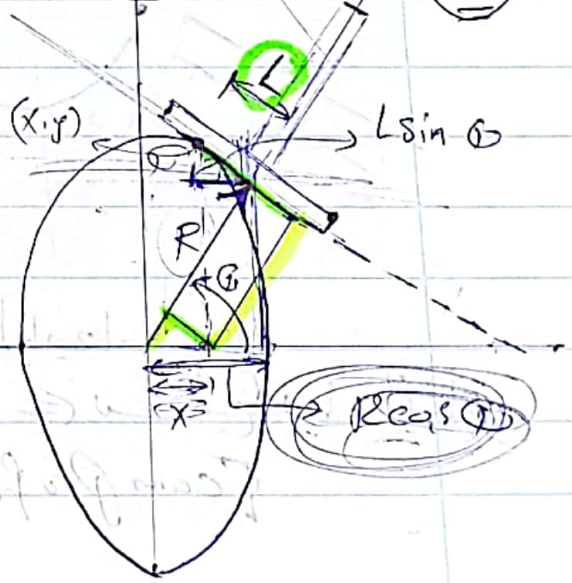
$\boxed{L = V}$ #

اذا ال follower

يصعد لفوق فهو موجب V +

و هنا بصير ال L ⇐ Positive





$L = v$

└> follower ① rizing ⇒ $v = +ve$ ⇒ $L = +ve$

⇒ contact point is above the x-axis of the follower.

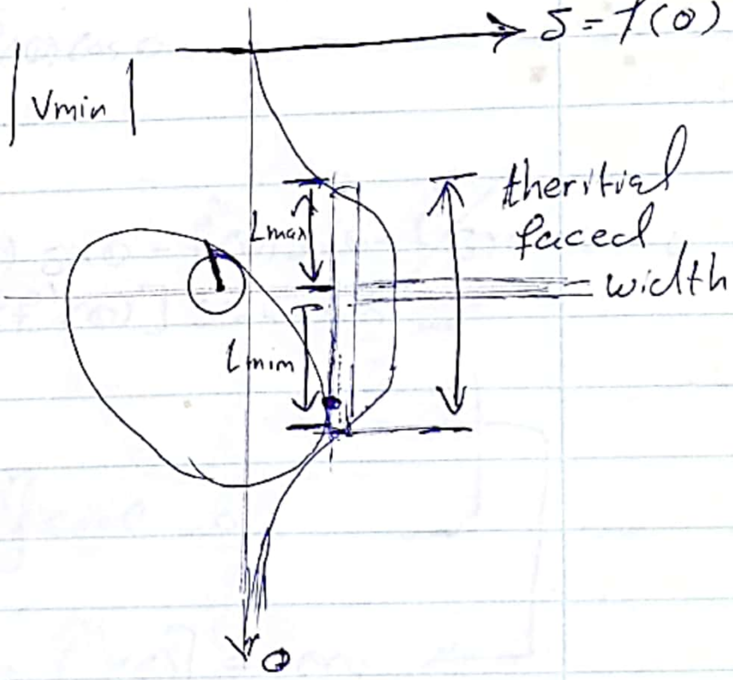
* $v = 0$ ⇒ $L = 0$ ⇒ contact point at the axis of the follower.

* the follower is lowering ⇒ $v = -ve$ ⇒ $L = -ve$

⇒ contact point is below the axis of the follower

$L_{theritical} = V_{max} + |V_{min}|$

* $L_{max} = V_{max}$
* $L_{min} = V_{min}$



مسألة الرسمة 1.2 :-

$x = R\cos\theta - l\sin\theta$

$y = R\sin\theta + L\cos\theta$

$\Rightarrow x = \checkmark$

$y = \checkmark$

* لذلك كل النقاط أصبحت محروقة

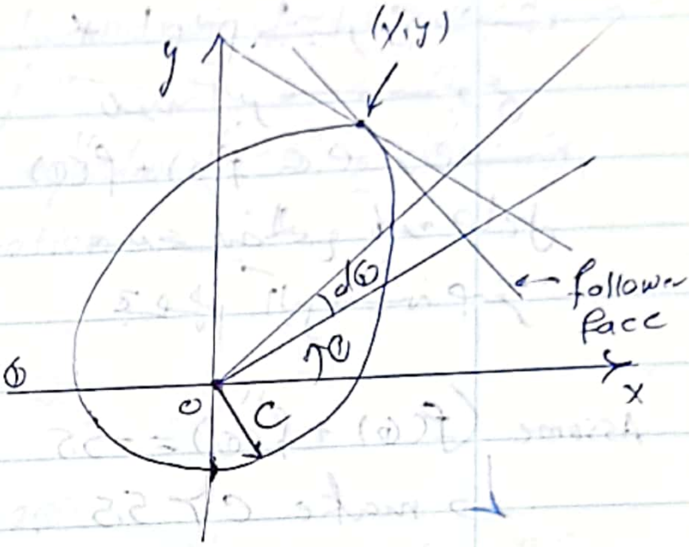
31/5/2021

cusp :-

التدبب الناتج عن C
اذا كانت صغيرة فتكون
الـ cam مدببة

Flat-faced follower
minimum C to avoid cusp

x θ is changing, however
the coordinates of the
contact point (x, y) are
not !!



$x = R\cos\theta - l\sin\theta$

$x = (C + f(\theta))\cos\theta - f'(\theta)\sin\theta$

$dx/d\theta = 0$

$y = R\sin\theta + l\cos\theta$

$= (C + f(\theta))\sin\theta + f'(\theta)\cos\theta$

$dy/d\theta = 0$

$\rightarrow \cancel{f'(\theta)\cos\theta} - C + f(\theta)\sin\theta - f''(\theta)\sin\theta - \cancel{f'(\theta)\cos\theta} = 0$

$-[C + f(\theta) + f''(\theta)]\sin\theta = 0$

$\rightarrow$ - -

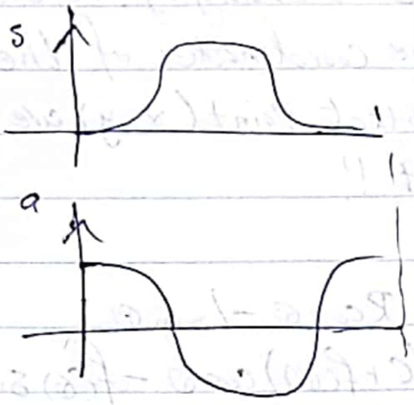
$[C + f(\theta) + f'']\cos\theta = 0$

$[C + f(\theta) + f''(\theta)] = \text{zero}$

$\rightarrow$ cusp occurs

▤ To avoid cusps:-

$$c + f(\theta) + f''(\theta) > 0$$



$+ \; C > 0$

* اي يتم اختيار C بحيث
تكون البرنامج موجب
حتى نعوض $\in f(\theta) + f''(\theta)$
عند عملية الجمع تكون حاصل
جمع البرنامج موجب

Assume $(f(\theta) + f''(\theta)) = -5.5$

$\hookrightarrow$ make $c > 5.5$

Exp:-

$$c + f(\theta) + f''(\theta) > 0$$

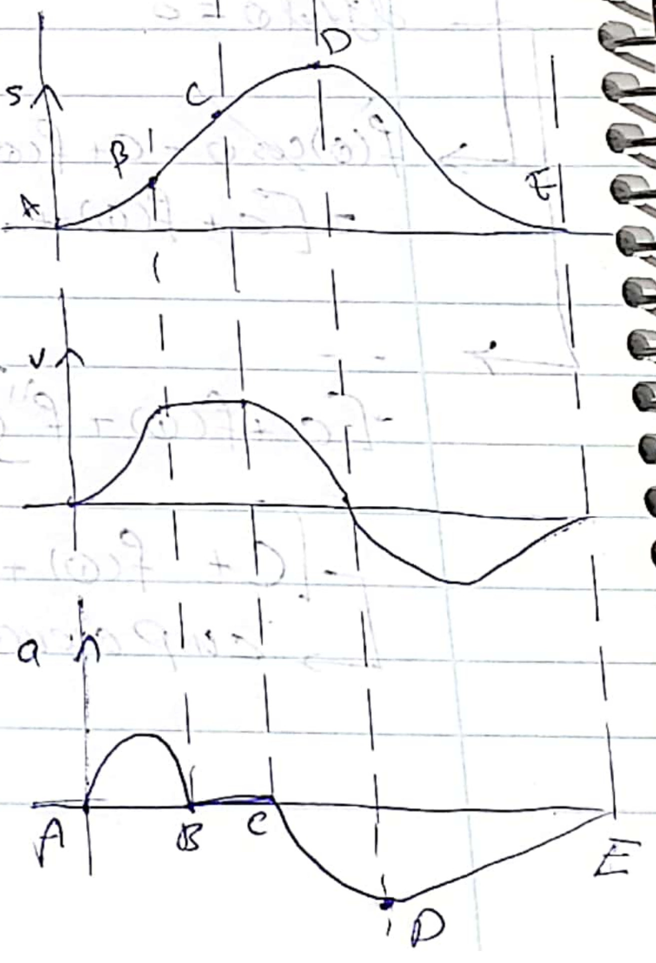
$(f(\theta) + f''(\theta))_{min}$

$\underset{\text{+ve}}{f(\theta)}$ $\quad$ $\underset{\text{-ve}}{f''(\theta)}$ $\hookrightarrow (f''(\theta))_{min}$

the minimum value of the $f(\theta) + f''(\theta)$ is close to D point $f''(\theta)$ at D

$\Rightarrow (f(\theta) + f''(\theta))$ at D

$c >$ the value



Example 3.3. As an example of how the minimum radius C and the length of the follower face are determined, consider a flat-faced radial follower which moves out and back 50.8 mm with simple harmonic motion for half a revolution of the cam. two motion cycles of the follower occur for one revolution of the cam.

only one displacement equation (H-5) is necessry to specify the follower motion:

$$S = \frac{L}{2}\left(1 - \cos \frac{\pi \theta}{\beta}\right)$$

$L = 50mm$ & $\beta = \frac{\pi}{2}$

$$S = f(\theta) = 25.4\ (1 - \cos 2\theta)$$

$$f'(\theta) = 50.8 \sin 2\theta \quad \& \quad f''(\theta) = 101.6 \cos 2\theta$$

To find the minimum radius, the sum $C + f(\theta) + f''(\theta)$ must be greater than zero

$$\boxed{C + 25.4 + 76.2 \cos 2\theta > 0}$$

$(25.4 + 76.2 \cos 2\theta)_{min}$

$\cos 2\theta = -1 \Rightarrow 2\theta = \pi \Rightarrow \boxed{\theta = \pi/2}$

will be minumim at $\theta = \pi/2$

$$25.4 + 76.2 \cos\left(2\left(\frac{\pi}{2}\right)\right)$$

$$25.4 - 76.2 = \boxed{-50.8\ mm}$$

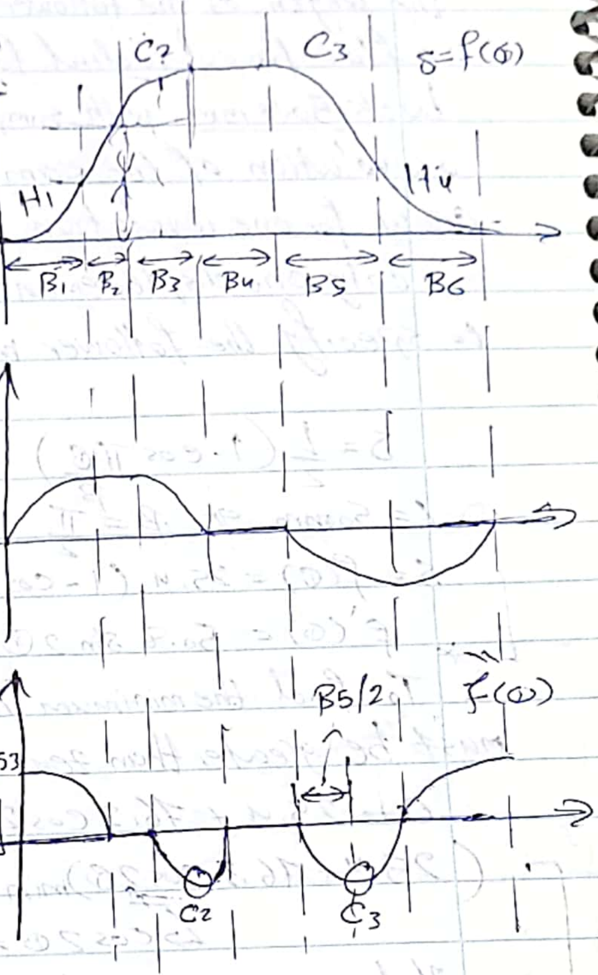
$C > 50.8\ mm \Rightarrow$ cusp will be prevented.

* أي الا حتى يوجد مش تكون اقل شي وعندها

تكون # $(f(\theta) + f''(\theta))_{min}$ بلاقي من الاشتقاقه $= 0$ $\frac{d(f(\theta) + f''(\theta))}{d\theta}$

* لايجاد القيم القصوى

هذا آخر السؤال الذي تم حله في البداية :-



$\left(f(\theta) + f''(\theta) \right) \Big|_{\frac{\beta_5}{2}}$

$s \Big|_{\beta_5/2} = 7.5 + 7.5 \left[1 - \frac{1}{2} + \frac{1}{\pi} \sin \frac{\pi}{2} \right]$

$s = 13.64 \, mm$

$a_{min} = f''(\theta) \Big|_{\beta_5/2} = \frac{-\pi L_5}{\beta_5^2} = \frac{-\pi(7.5)}{\left(85.17 * \frac{\pi}{180}\right)^2}$

$= -10.53 \, mm/rad^2$

$s + f''$

$\left(f(\theta) + f''(\theta) \right)_{min} = 13.64 - 10.53$

C3 - interval $= \boxed{3.11 mm}$

+ ve

اقل قيمة ⇐ موجبة

لذلك اي قيمة لـ c تكون مناسبة

alowys ⇒ $c + f(\theta) + f''(\theta) > 0$

regerdlles to the value of c.

⇐ افحص interval of (C-2)

(c2) interval:-

$$\left(f(0) + f''(0)\right)\Big| \Rightarrow$$

~~$s\Big|_{\frac{\beta_3}{2}} = 5+5+L_3\left(\frac{\theta}{\beta_3} + \frac{1}{\pi} + \sin\frac{\theta}{\beta_3}\right)$~~ $\theta \Rightarrow \frac{\beta_3}{2}$

~~$\frac{\beta_3}{2} = 5+5+5\left(1 + \frac{1}{\pi} + \cancel{s}\ 1\ \sin\pi\frac{\beta_3}{2\beta_3}\right)$~~

~~$= 10 + 5\left(2 + \frac{1}{\pi}\right)$~~

~~$= 21.6$~~

$$s\Big|_{\frac{\beta_3}{2}} = L\left(\frac{\theta}{\beta} + \frac{1}{\pi}\sin\pi\frac{\theta}{\beta}\right) \quad \theta_L = \frac{\beta}{2}$$

$$s\Big|_{\frac{\beta_3}{2}} = 5\left(1 + \frac{1}{\pi}\sin\frac{\pi}{2}\right) = 6.59 \text{ local}$$

$$s\Big|_{\frac{\beta_3}{2}} = 5+5+6.59 = \boxed{16.5} \ \#$$

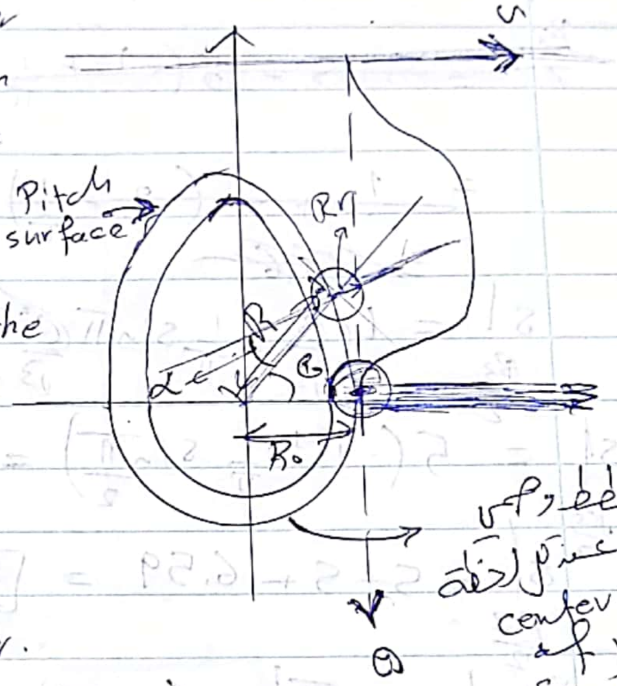
$$a = f'(0)\Big|_{\frac{\beta_2}{2}} = \frac{-\pi L}{\left(\beta_3 \frac{\pi}{180}\right)}\left(\sin\frac{\pi}{2}\right) = \checkmark$$

$$f(1) + f''(1) \cancel{\neq} C \ \checkmark$$

Design: cam Profile, radial roller follower.

>> find cam contour, such that when this cam rotates about it's center θ ⇒ the follower is moving according the prescribed motion defined by the displacement curve.

* R: distance from the center of the rotation of the cam to the ~~face of the follower~~ center of the roller.



مخطط رسم بيبين مسار الحركة لـ center of roller أين يقع؟!

* R_o: min. distance from the center of rotation of the cam to the center of the roller

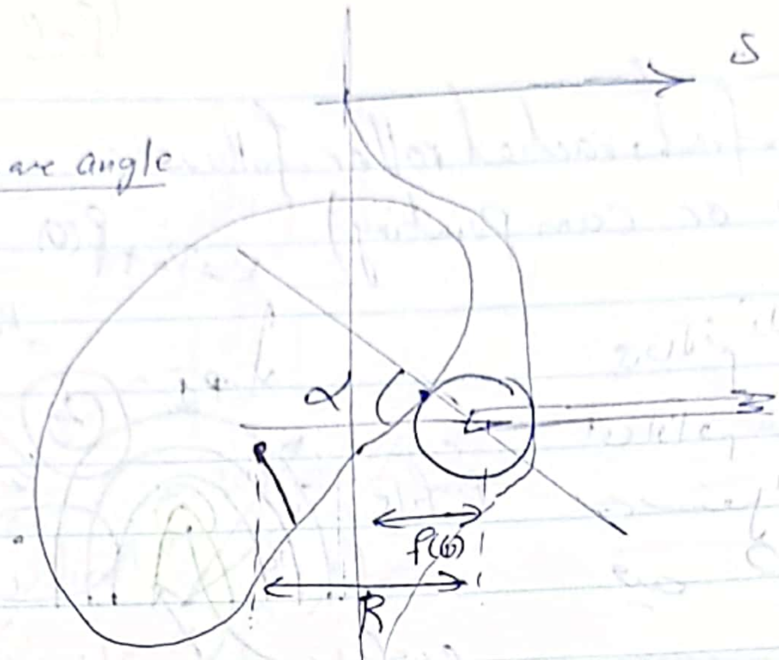
* R_r: radius of the roller.
* α: pressure angle.

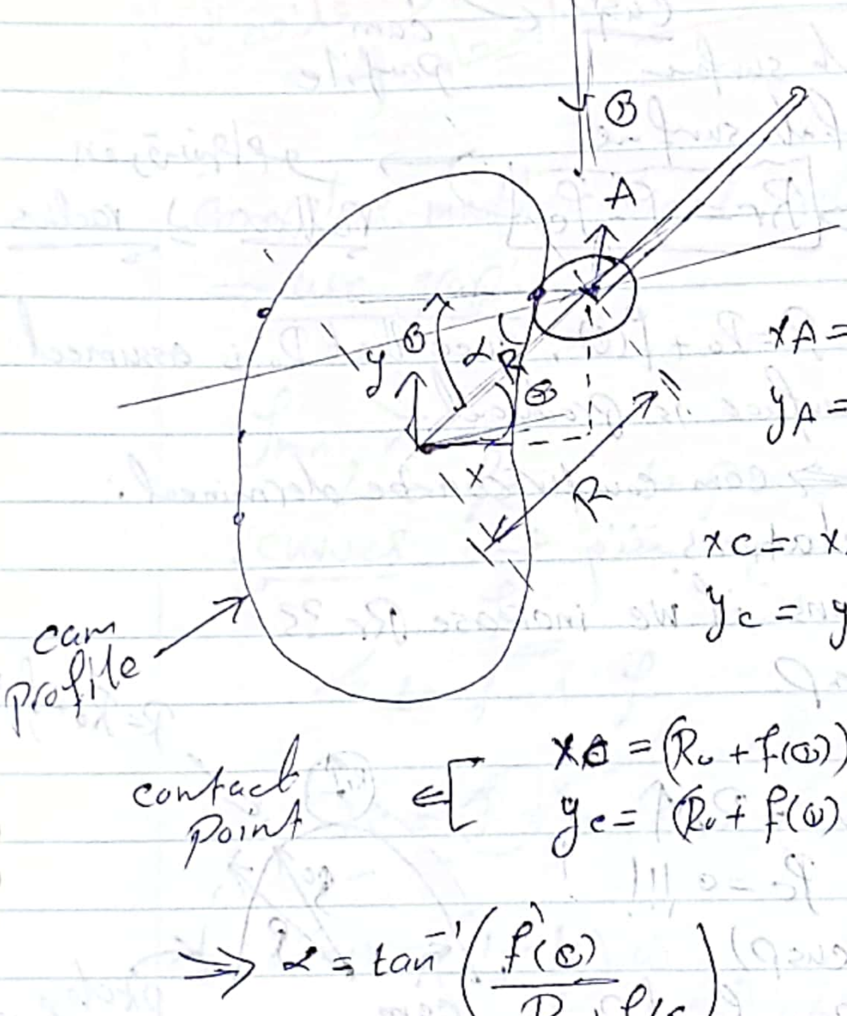
$$R = R_o + f(\theta) \Rightarrow$$

* العلاقات: كما في السابق.

* To find contact point

*α: pressure angle





$x_A = R\cos\theta$

$y_A = R\sin\theta$

$x_C = x_A - R_r\cos(\theta - \alpha)$

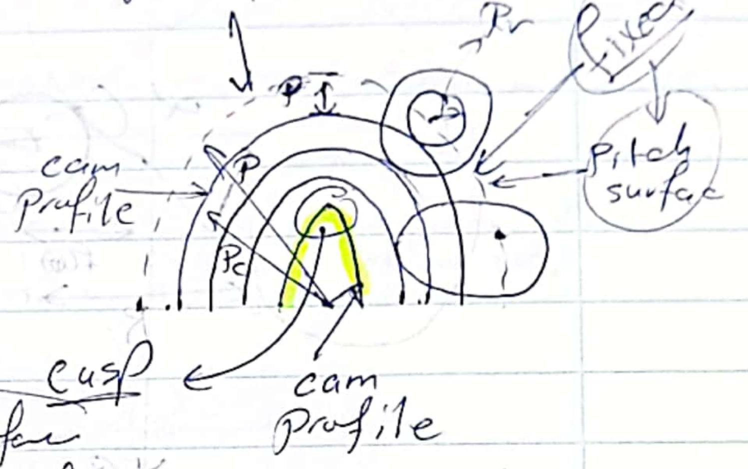
$y_c = y_A - R_r\sin(\theta - \alpha)$

contact point $\in \left[\begin{array}{l} x_c = (R_o + f(\theta))\cos\theta - R_r\cos(\theta - \alpha) \\ y_c = (R_o + f(\theta))\sin\theta - R_r\sin(\theta - \alpha) \end{array} \right.$

$$\Rightarrow \alpha = \tan^{-1}\left(\frac{f'(\theta)}{R_o + f(\theta)}\right)$$

$\rho_c =$ (ρ-pitch)

cam Profial, radial roller follower
(cusp or cam Pointing)



Rr ما لازم تكبير
اي اذا تم عملية التكبير
⇐ ستصل الى حد يكون
cusp فيه

ρ: Radius of pitch surface
ρc: radius of profial surface

$\boxed{\rho = \rho_c + R_r} \Rightarrow \boxed{R_r = \rho - \rho_c}$ → الفرق بينهم هو roller ل radius

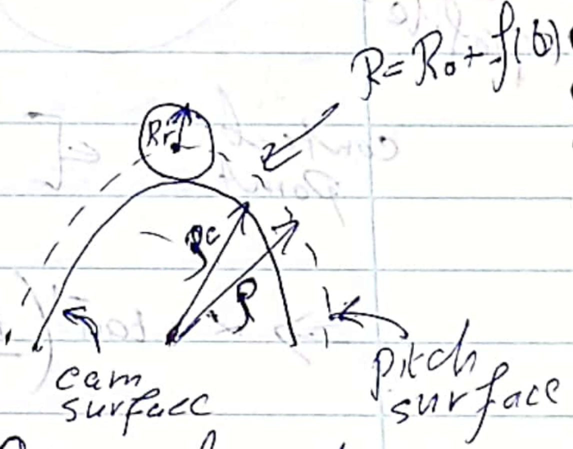
ρ is defined by $R = R_o + f(\theta)$, once that R_o is assumed then the pitch surface is produced

* ⇒ Assuming Rr ⇒ cam countur can be dermined:
recall (x,y)-relations

\# What happens if we increase Rr ??
↳ cusp.

$\rho(\text{fixed}) = \rho_c \downarrow + R_r \uparrow$

If $R_r \uparrow$ to $\rho \Rightarrow \rho_c = 0$!!!
cam pointing (cusp)



* to avoid cusp → find ρmin
and insure that $R_r < \rho_{min}$, $\rho_c > 0$ always!

$$\rho = \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

← ولكنه صعبة

Polar coordinate ⇐ لذلك هناك علاقة التركيب لقطبي لايجاد ρ.

complex mathematical Relations

⇒ use graphs

$\rho_{min} > R_r$

$\hookrightarrow \rho_c \Rightarrow$ to prevent cusp

. curves اي سيتم استخدام هذه لايجاد $\underline{\underline{\rho_{min}}}$

$\hookrightarrow$ to find ρ_c

$$X_c = \boxed{R_x} - R_r \cos(\theta - \alpha) \quad \boxed{R_x} \to (R_o + f(x_o)) \cos\theta$$

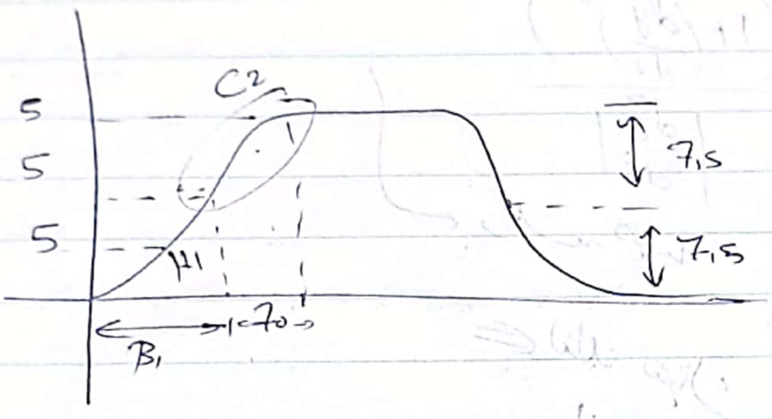
$=$

$$\omega_{max} = \left(1 + \frac{k}{2}\right)\omega$$

$$\omega_{min} = \left(1 - \frac{\boxed{k}}{2}\right)\omega \qquad \boxed{k} \to \text{[illegible]}$$

5/6/2021

من السؤال السابق.



$L_1 = 5cm$

$R_o = 10cm$

$\frac{L}{R_o} = \frac{5}{10} = \boxed{0.5}$. + $\boxed{\beta_1 = 55^\circ}$

ثم نعينها على المنحنى لـ Harmonic

ثم نسقطها على محور السينات وايجاد التقاطع

مع curve لـ $\frac{L}{R_o}$

ثم ايجاد الانعكاس على محور الصادات . $\left(\frac{\rho_{min}}{R_o}\right)$

$\frac{\rho_{min}}{R_o} = 0.56 \Rightarrow \rho_{min} = 0.56 \times 10$

$\boxed{\rho_{min} = 5.6} > R_r$

to avoid cusp

for C_2 :-

$R_o = 10$ cm

$\frac{L}{R_o} = \frac{5}{10} = \boxed{0.5}$ ← ايجاد حالي curve.

ثم ايجاد $\underline{\underline{7o}}$ من محور السينات.

⇐ ومن ثم ايجاد حالي الصادات

$\frac{\rho_{min}}{R_o} = 0.64 \Rightarrow \rho_{min} = 0.64 * 10 = 6.4$

$R_r < 6.4$ cm

radius of roller less than 6.4 to aviod cusp.

C_2 :- $R_r < 6.4$ cm

H_1 :- $H_1 < \boxed{5.6 \text{ cm}}$ ⇒ critical to aviod cusp

← اي أخذنا اقل وحدة قيام.

▢ Cam profile, radial roller follower
(Maximum pressure angle (α_{max}) !!!)

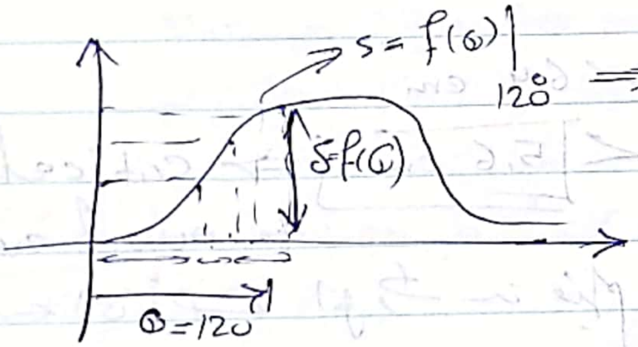
$$\alpha = \tan^{-1}\left[\frac{f'(\theta)}{R_0 + f(\theta)}\right]$$

⇒ complex mathematical relations.

* Use graphs (Nomogram).

Exp:- Determine $\alpha\Big|_{\theta_{cam}=120} = \tan^{-1}\left(\frac{f'(\theta)}{R_0+f(\theta)}\right)$

& $R_0 =$



⇒ find $s=f(\theta)$ & $f'(\theta)$ @ 120° :-

↳ then determin the α !!

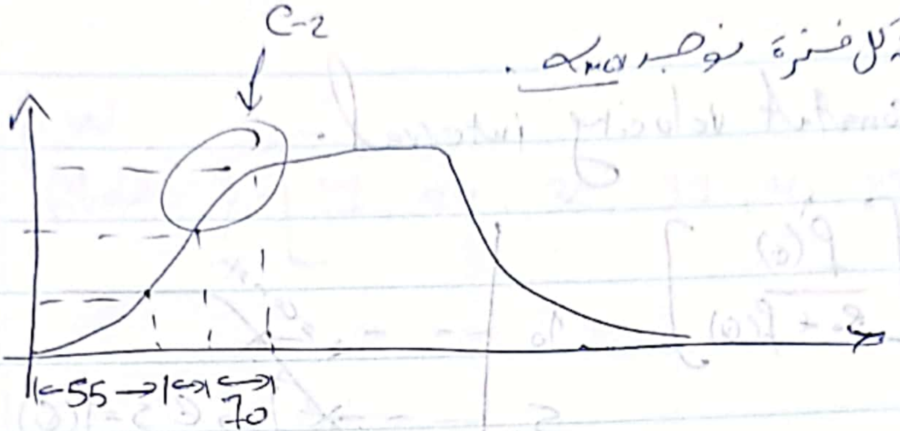
$f'(120)=5$, $f(\theta)=13.69$

$$\alpha = \tan^{-1}\left(\frac{5}{10+13.69}\right) \Rightarrow \boxed{\alpha = 11.9°} \#$$

$\alpha_{max} \leq 30°$
$\alpha_{max} \leq 32°$] ⇒ to find α_{max} !!
use (Nomogram)

↳

اي عند كل فترة نوجد $\underline{\alpha_{max}}$.



ⓐ $\beta = 70 \Rightarrow$ interval $\underline{\underline{3}}$:-

$\frac{L}{R_0} = \frac{5}{10} = \boxed{0.5}$ ⓑ $\boxed{C-2} \rightarrow$ يتم تعيينه في القطر من الاعلى

ويتم تعيينه $\underline{\beta = 70}$ في نصف القطر من الاعلى والتوصيل منه النقطة ⇐ ونقطة التقاطع مع النصف الآخر تعين α_{max} .

in C_2 interval $\boxed{\alpha_{max} = 34^\circ}$ too High !!

لتجنب ذلك ⇐

$R_0 \uparrow \Rightarrow \frac{L}{R_0} \downarrow$

اي يتم تغيير في قيمة R_0 ⇐ حتى يتم تقليل $\underline{\text{Pressurangle}}$.

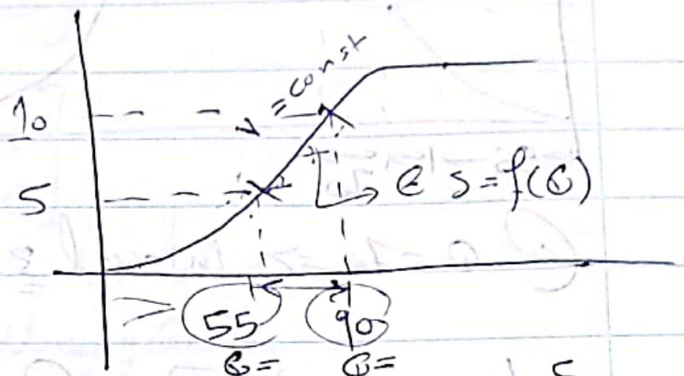
* اذا طلب قيمة R_0 بحيث قيمة α_{max} لا تزيد عن $\underline{25^\circ}$ ∀ $\underline{\underline{\beta = 70}}$

⇐ يتم ايجاد قيمة R_0 بالتوصيل بينها .

وإذا كان في Harmonic تكون الحسابات مختلفة نظراً للقطر في الدائرة .

* at constant velocity interval.

$$\alpha = \tan^{-1}\left[\frac{f'(\theta)}{R_0 + f(\theta)}\right]$$



$$v = \frac{5}{55 * \frac{\pi}{180}}$$

f(55) = 5cm
f(90) = 10cm

$$\alpha(55) = \tan^{-1}\left[\frac{✓}{10+5}\right]$$

$$\alpha(90) = \tan^{-1}\left[\frac{✓}{10+10}\right]$$

ال slope
اي كل ما كبر ال slope
α ↑

slope ↑ ⇒ α↑

$\alpha(55) > \alpha(90)$

slope @ 55 > slope @ 90.

* اي αmax تكون عند بداية interval ال
ال constant velocity ،

* اي مش constant velocity ال لا نذهب الى ال Nomogram
⇐ بل نذهب الى بداية ال interval و نوجد αmax
▤ اي لايجاد القوة بشكل مباشر .

H.W
Chapter 3:- [17, 22, 36, 37, 42, 43, 47]

☰. Gear Trains.

تحويل

$$\frac{9000}{60} rpm = 150 \frac{rev}{s}$$

* simple gear train

$\omega_1 r_1 = \omega_2 r_2$

$\omega_2 r_2 = \omega_3 r_3$

مثلاً سرعة دوران عجل السيارة :-

220 Km/h ??

= 60 m/s

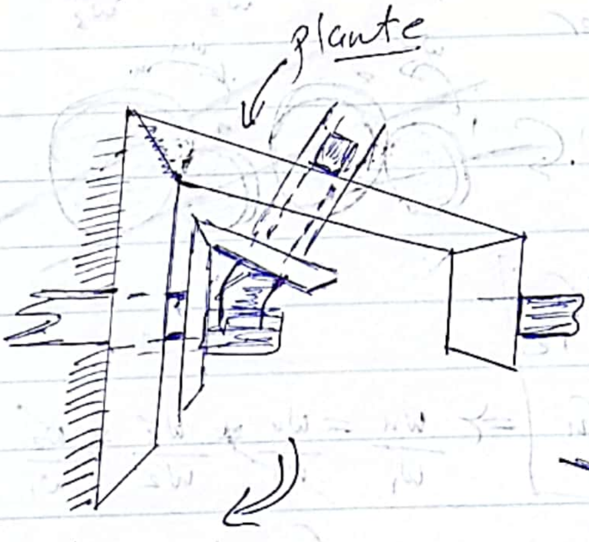
$$\omega = \frac{v}{r} = \frac{60}{0.4} = 152 \, rad/s \, * \frac{rev}{2\pi \, rad}$$

$$= \boxed{24 \, rev/s}$$

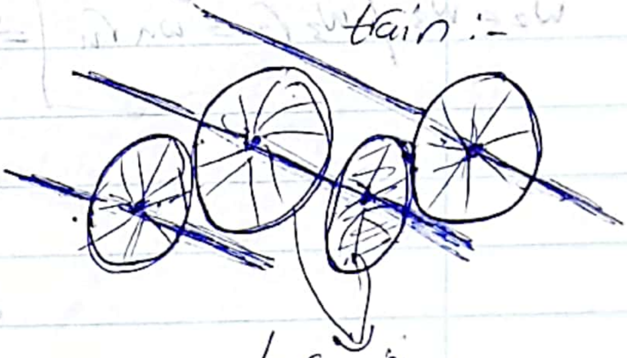
$\omega_1 : \omega_2$
10 : 1 $\Rightarrow \frac{r_2}{r_1} = 10$ in the simple gear

وهذا غير مقبول في reduction في السيارات لذلك

هناك ما يسمى compound gear train :-



☰ planetary gear

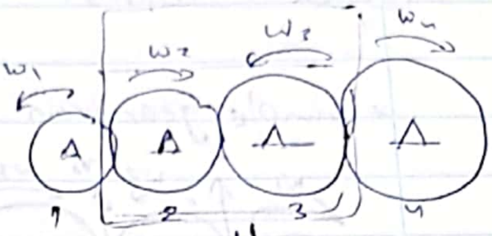


نفس angular

لازم ع نفس shaft

Simple gear Train.

⇒ One gear per shaft



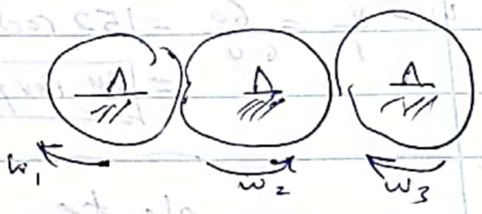
$$w_1 r_1 = \bar{w}_2 r_2$$
$$w_2 r_2 = \bar{w}_3 r_3$$
$$w_3 r_3 = \bar{w}_4 r_4$$
$$\Rightarrow w_1 r_1 = w_4 r_4$$

↓ idle gears

اي انها لا تأثر ع الحركة ووضعها اذا كان هنالك مسافة مثلاً بين الاول والرابع / او لعكس اتجاه الحركة

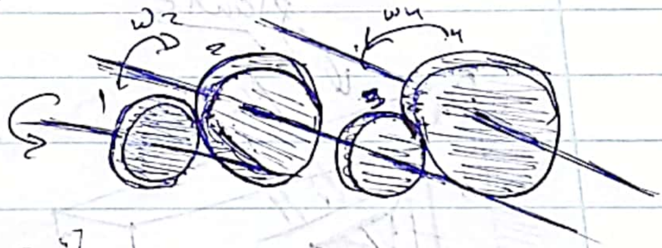
لجعله يدور بنفس الاتجاه

Compound gear train.



⇒ More than one gear per shaft.

* see videos.



$$w_1 r_1 = \bar{w}_2 r_2 \Rightarrow \frac{w_2}{w_1} = \frac{r_1}{r_2}$$

$$w_2 = w_3 \;/\; w_3 r_3 = \bar{w}_4 r_4 \Rightarrow \frac{w_4}{w_1} = \frac{w_4}{\cancel{w_3}} * \frac{w_3}{w_2} * \frac{w_2}{w_1}$$

$$= \frac{r_3}{r_4} \cdot 1 \cdot \frac{r_1}{r_2} = \boxed{\frac{r_3 r_1}{r_4 r_2}}$$

اي هنا اصبح هنالك فائدة وهي reduction.

⇐ ولكن نعتبر radius لكل gears غير متغير
لانه هناك بديل عن ذلك
وهي عدد الاسنان لكل gear.

$$\Rightarrow \frac{\omega_4}{\omega_1} = \frac{\omega_4}{\omega_3}\,\frac{\omega_3}{\omega_2}\,\frac{\omega_2}{\omega_1}$$

$$= \frac{r_3}{r_4}\cdot\frac{r_1}{r_2} = \boxed{\frac{N_3}{N_4}\cdot\frac{N_1}{N_2}} \;!!$$

لانه كلهم يجب انه يكونوا بنفس الخطوة او
النعومة اي standard لكل gear.

*Diametral pitch P: the number of teeth of a gear per inch of it's pitch diameter.

$$P = \frac{N}{D}$$

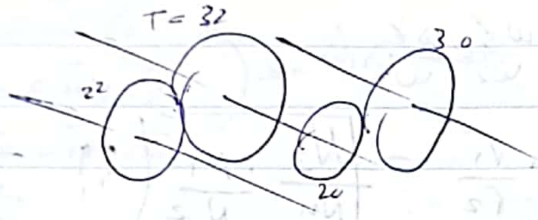
*Module m: Pitch diameter divided by number of teeth. It is the inverse of diametral pitch.

$$m = \frac{D}{N}$$

⟹ To be able to mesh two gears they must have the same Diametral pitch or Module.

الله

$$\frac{r_1}{r_2} = \frac{D_1}{D_2} = \frac{N_1}{N_2} \Leftarrow \boxed{\frac{N_1}{D_1} = \frac{N_2}{D_2}} \quad \left\{ \frac{r_2}{r_1} = \frac{N_2}{N_1} \right\}$$



$$\frac{\omega_4}{\omega_1} = \frac{N_1}{N_2} \cdot \frac{N_3}{N_4} = \frac{22 \times 20}{32 \times 30} = \frac{44}{96} = \frac{22}{48} = \boxed{0.45} = \frac{\omega_4}{\omega_1}$$

اي اذا input يدور دورة واحدة

فانه output بيدور 0.45 دورة .

▤ velocity Ratio :-

$$\text{velocity Ratio} = \frac{\omega_{out}}{\omega_{in}} = \frac{\omega_4}{\omega_1} = VR$$

$$VR = \frac{\omega_{out}}{\omega_{in}} = \frac{\text{Product of number of teeth of Driver gears}}{\text{Product of number of teeth of Driven gears}}$$

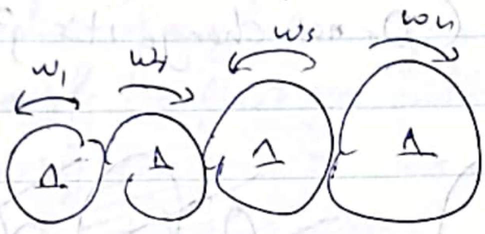
⇐ اي العلاقة عكسية

$$\frac{\omega_4}{\omega_1} = \frac{\Pi \text{ Driver}}{\Pi \text{ Driven}} = \frac{N_1 \times N_3}{N_2 \times N_4}$$

اي في compound gear ∈ 1 يقود 2 و 3 تقود 4 (Driver)

و 2 يتبع 1 و 4 يتبع 3 (Driven)

simple gear وكذلك بالنسبة ل



اي دائماً عند

mesh هناك

negative sign

$$\frac{w_4}{w_1} = \frac{N_1 \times N_2 \times N_3}{N_2 \times N_3 \times N_4} = \boxed{\frac{-N_1}{N_4}}$$

$$e = \frac{W_{in}}{W_{out}} = \frac{\prod \text{Driven}}{\prod \text{Driver}}$$

⇒ العلاقة العامة

حطينا الحلقة

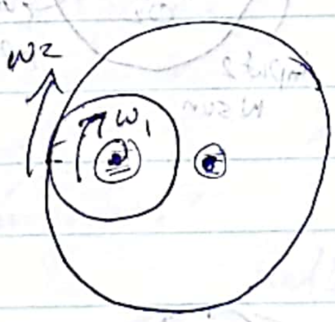
دكسه في gear عالسكة

Internal gears

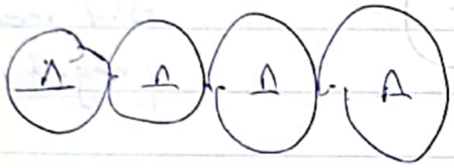
⇐ يتم استخدام نفس العلاقة السابقة, ولكن لا يوجد

negative sign

لانها بنفس الاتجاه.



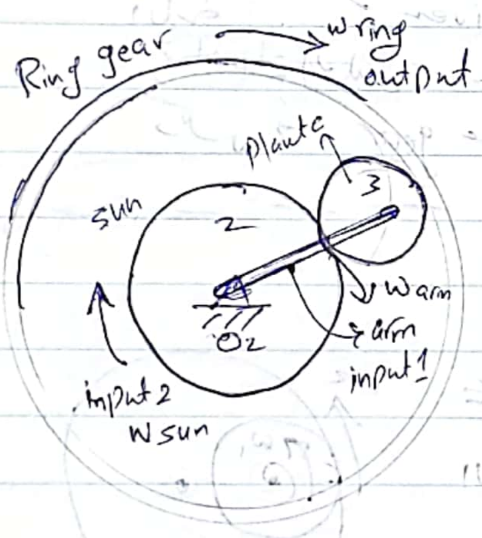
* Remarks:- in all previous example all axes of rotation are fixed ! (Do not change it's position)

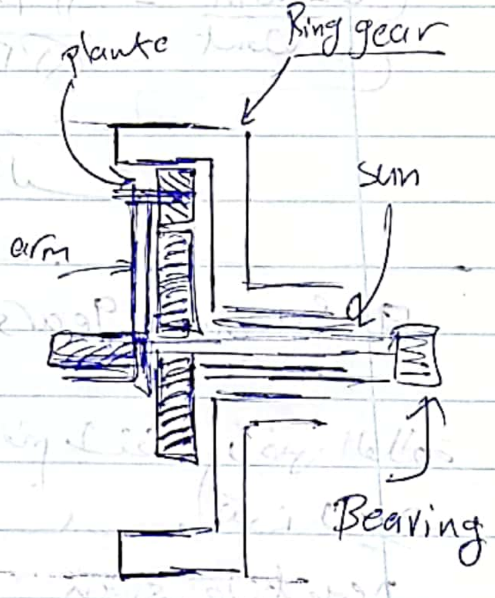


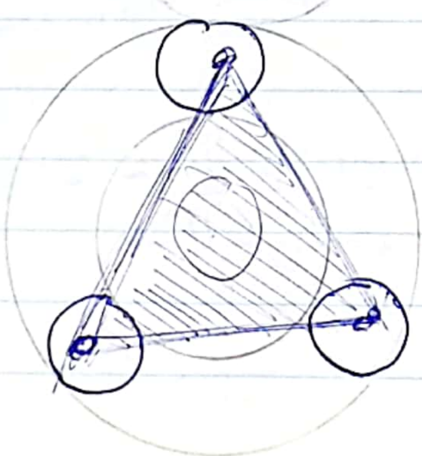
⇒ To increase velocity ratio:-

→ * Moving axes of Rotation

→ * Planetary Gears.







* هنا صار ع 3 Plante

⇒ بيعطي ثلاث أضعاف

Torque وتكسني

عملية التحليل وجد

تكفي

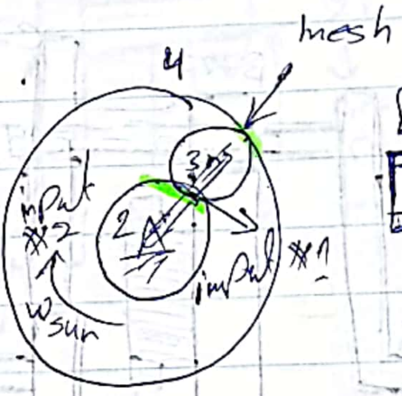
10/5/2021

Planetary gears:-

Procedure of anaylish:-

① Identify the plant gear and the arm (gear 3)

② Identify the gears that are directly in mesh with the planet (gears 2 & 4)



* Relation to the arm all gears are rotating about a fixed axis.

$$\Rightarrow (VR)_{arm} = \frac{\omega_{out/arm}}{\omega_{in/arm}} = \frac{\Pi \text{ of driver}}{\Pi \text{ of driven}} = \frac{\omega_{out} - \omega_{arm}}{\omega_{in} - \omega_{arm}} = \frac{\Pi \text{ Driver}}{\Pi \text{ Driven}}$$

$$V_{B/A} = V_B - V_A$$

$$\vec{\omega}_{3/2} = \vec{\omega}_3 - \vec{\omega}_2$$

Exp:-

$\omega_{arm} = 2 rad/s$ ccw

gear 2 is fixed ⇒ Detmine $\underline{\underline{\omega_4}}$??

$$= \frac{\omega_{out} - \omega_{arm}}{\omega_{in} - \omega_{arm}} = \frac{\Pi \text{ Driver}}{\Pi \text{ Driven}} = \frac{\omega_4 - \overset{2rad/s}{\omega_{arm}}}{\underset{\text{fixed = zero}}{\omega_2} - \underset{2rad/s}{\omega_{arm}}} = \frac{-N_2 * N_3}{N_3 * N_4}$$

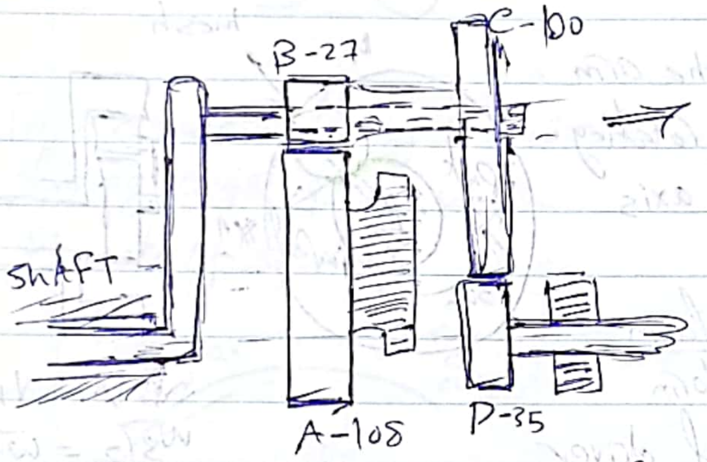
$\omega_4 =$ ______

⇐ اي لحل هذه الاسئلة :-

* يجب معرفة اي gear ⇐ d axis معلوما

اي arm اي عنصر ثابت :

* اي اهم شي يجب ان نستطيع تحديد arm :



⟶ planetary gears (plantes)

B ↙ ↘ C

in mesh with :-

B	C
A	D
input	output

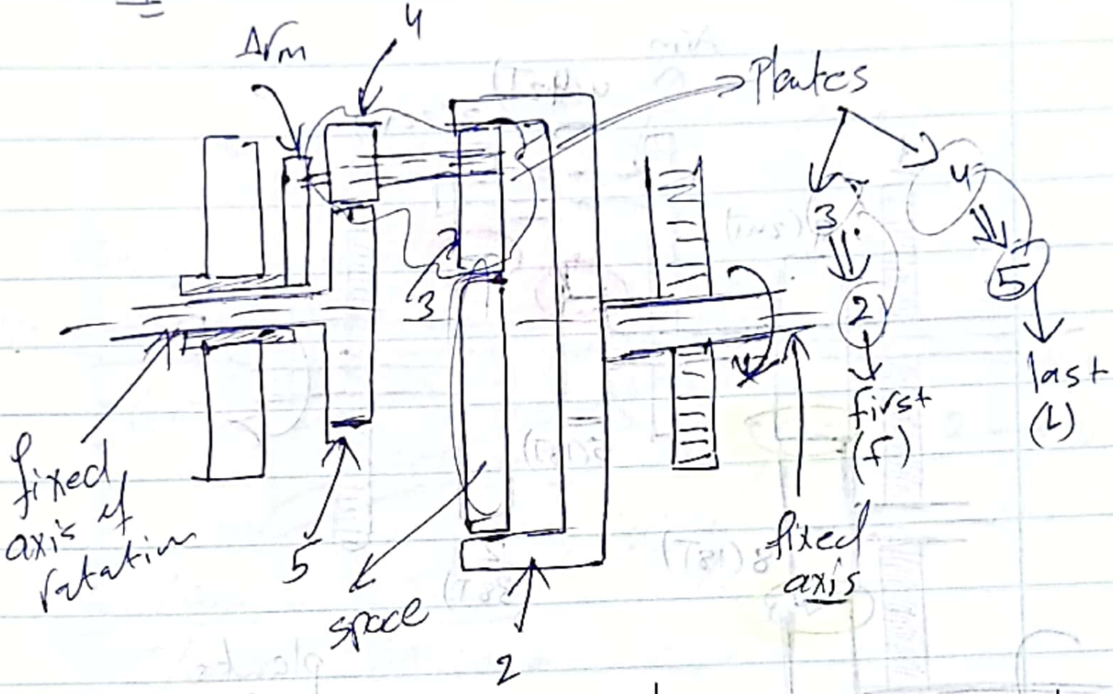
$$\frac{w_D/arm}{w_A/arm} = \frac{w_{out}/arm}{w_{in}/arm} = \frac{\prod Driver}{\prod Driven} = -\frac{N_A}{N_B} * \frac{N_C}{N_D} = \frac{w_D - w_{arm}}{w_A - w_{arm}}$$

external ان
gear.

zero
fixed

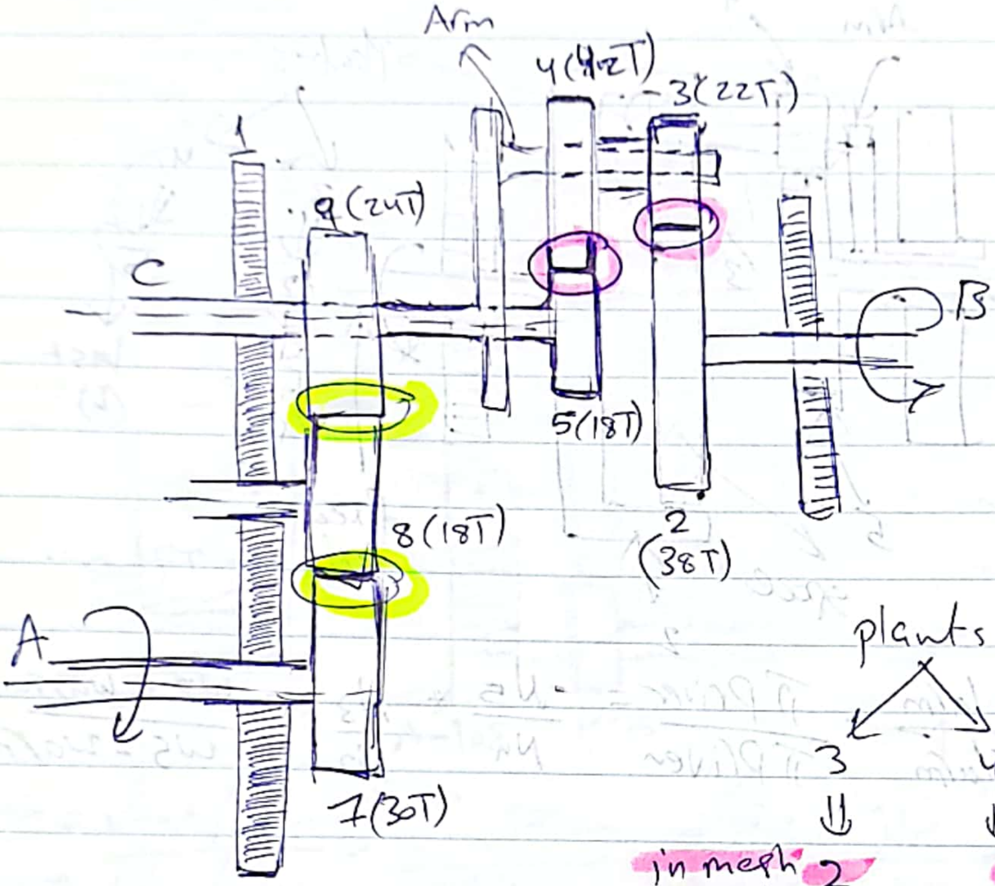
given

Exp :



$$\frac{w_L/arm}{w_F/arm} = \frac{\Pi\, Driver}{\Pi\, Driven} = \frac{-N_5}{N_4} * \frac{N_3}{N_2} = \frac{w_2 - w_{arm}}{w_5 - w_{arm}}$$

Question ※7:-



plants

3 4

⇓ ⇓

in mesh with 2 5

⇓ ⇓

first last

$\omega_{arm} = \omega_A$

$$\frac{\omega_9}{\omega_7} = \frac{\Pi \text{Driver}}{\Pi \text{Driven}} = -\frac{N_7}{\cancel{N_8}} \times -\frac{\cancel{N_8}}{N_9} = \frac{N_7}{N_9}$$

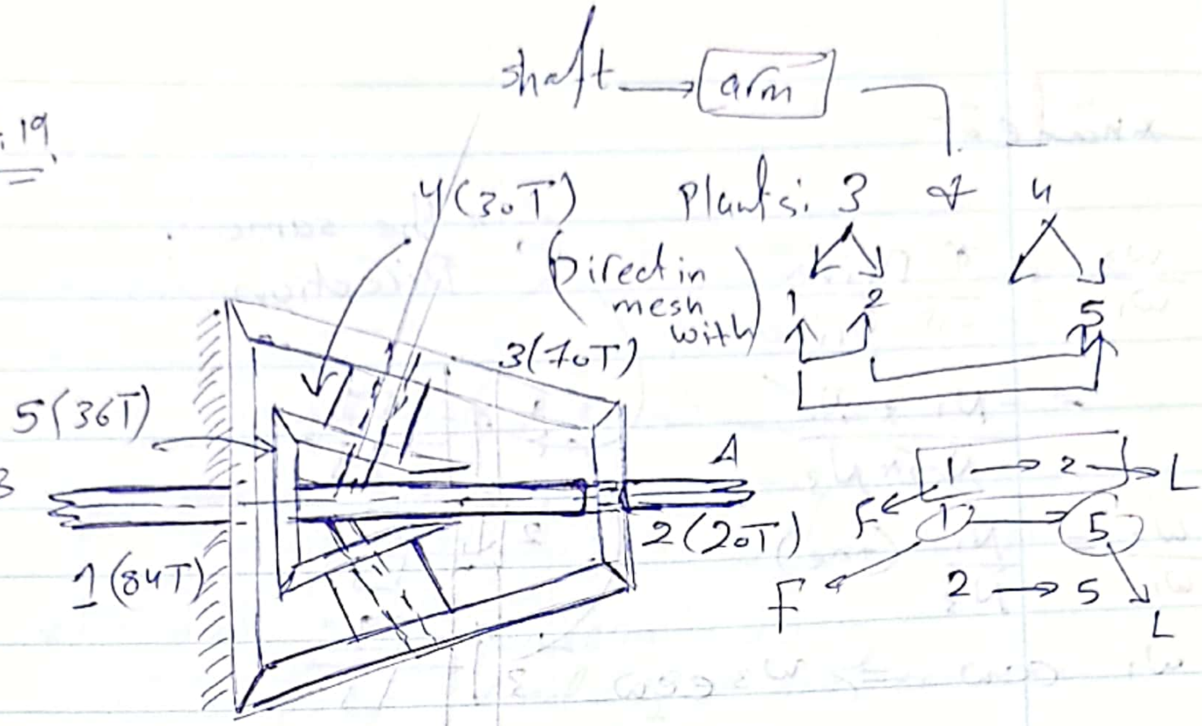
$\boxed{\omega_7 = \omega_A}$

given ω_A & ω_B ⇒ find ω_c?.?

$$\Rightarrow \frac{\overset{\omega_c}{\omega_5} - \omega_{arm}}{\underset{\omega_B}{\omega_2} - \omega_{arm}} = \frac{\omega_{L/arm}}{\omega_{F/arm}} = \frac{\Pi \text{Driver}}{\Pi \text{Driven}} = \frac{-N_2 \times -N_4}{N_3 \times N_5}$$

17/5/2021

7.19



$$\frac{w_L/\text{arm}}{w_F/\text{arm}} = \frac{\Pi\,\text{Driver}}{\Pi\,\text{Driven}}$$

$$\frac{w_2 - w_{arm}}{w_1 - w_{arm}} = \frac{w_2/\text{arm}}{w_1/\text{arm}} = \frac{w_L/\text{arm}}{w_F/\text{arm}} = \frac{\Pi\,\text{Driver}}{\Pi\,\text{Driven}} = \frac{-N_1}{N_3} * \frac{N_3}{N}$$

fixed = zero

$$= \frac{-N_1}{N_2}$$

Remember

w_A & w_6

$$\frac{w_A - w_6}{0 - w_{arm}} = \frac{w_L - w_{arm}}{w_1 - w_{arm}} = \frac{-N_1}{N_2}$$

By (1 → 5)

$$\frac{w_B - w_6}{0 - w_6} \quad \frac{w_5 - w_{arm}}{w_1 - w_{arm}} = \frac{w_L/\text{arm}}{w_F/\text{arm}} = \frac{\Pi\,\text{Driver}}{\Pi\,\text{Driven}} = \frac{-N_1}{N_3} * \frac{N_4}{N_5}$$

$$= \frac{N_1 N_4}{N_3 N_5}$$

w_B & w_6

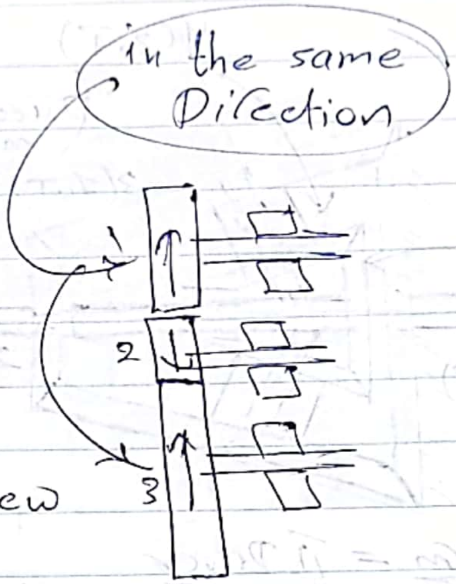
*note :-

$$\frac{w_3}{w_1} = \frac{\Pi \text{ Driver}}{\Pi \text{ Driven}}$$

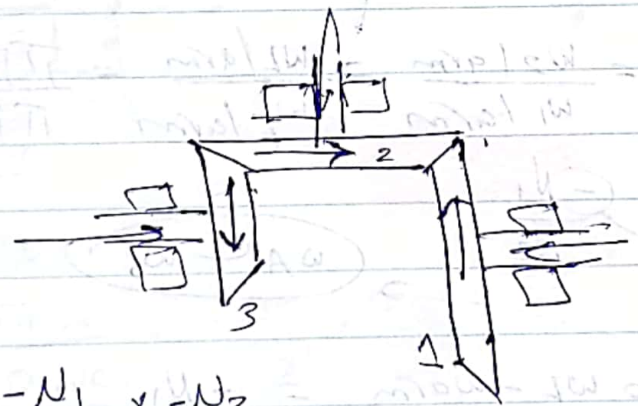
$$= \frac{-N_1 * \cancel{N_2}}{\cancel{N_2} * N_3}$$

$$\frac{w_3}{w_1} = \frac{N_1}{N_3} \quad (+ve)$$

w_1 ccw => w_3 ccw



: bevel gear وتسمى حالة ≪



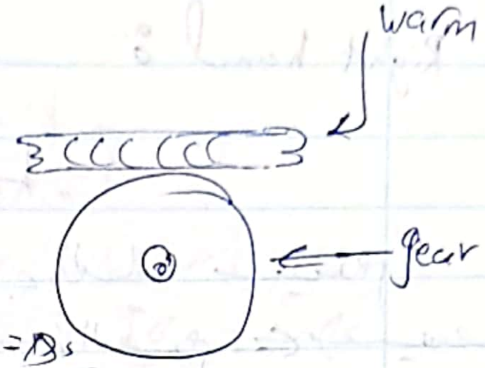
$$\frac{w_3}{w_1} = \frac{-N_1}{N_2} \times \frac{-N_2}{N_3}$$

ط are w_1 & w_3 in the same direction ??

$$\frac{w_3}{w_1} = \left(\frac{N_1}{N_3}\right)$$

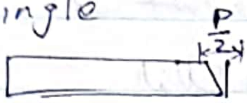
* لذلك ليست بنفس الاتجاه رغم أن الحالة كانت سواء العلاقة لهذا وفقا الاشارة السابقة

Worm gears:-



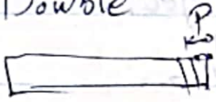
⇒ in mechanical Drawing:-

* single — one teach



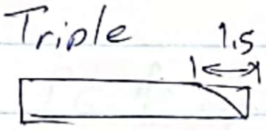
⇒ $lead = P = \Delta s$

* Double — one term — one turn w



⇒ $lead = 2P = \Delta s$ / one turn w

* Triple

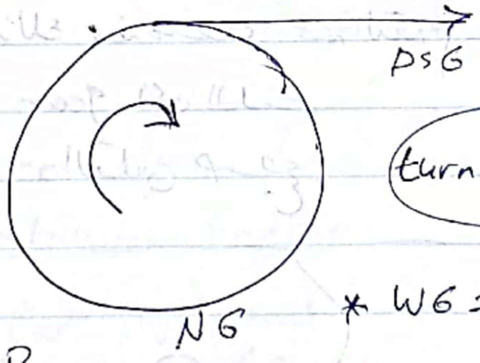


⇒ one term

$lead = 3P = \Delta 3$ / one turn w

---

1 turn ⇒ ΔSG = circumference $= NG * P$



$$\Delta SG = \frac{NG * P}{turn\ G}$$

$$\boxed{turn\ G = \frac{NG * P}{\Delta SG}}$$

$$* \ WG = \frac{NGP}{VG}$$

$$1\ turn\ w = \frac{Nw * P}{\Delta Sw}$$

$$* \ Ww = \frac{Nw * P}{Vw}$$

$NG = 40$

$Nw = 2$

$$\frac{WG}{Ww} = \frac{2}{40} = \boxed{\frac{1}{20}}$$

$Vw = VG$

$$\frac{Nw * P}{Ww} = \frac{NG * P}{WG} \Rightarrow \boxed{\frac{WG}{Ww} = \frac{Nw}{NG}}$$

معناه اذا لف warm

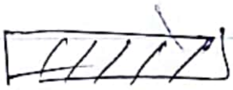
20 لفة، فانه

gear بلف لفة وحدة

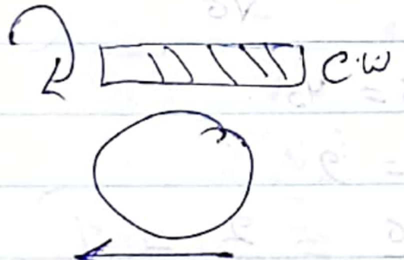
Right hand :-

* عند تدويره مع عقارب الساعة سيتقدم
لذلك عند منعه عن التقدم ⇐ سيسحب
gear الى الخلف
والعكس صحيح.

left hand :-



* عند تدويره عكس عقارب الساعة سيتقدم
لذلك عند منعه عن التقدم ⇐ سيسحب
gear الى الخلف
والعكس صحيح.

*


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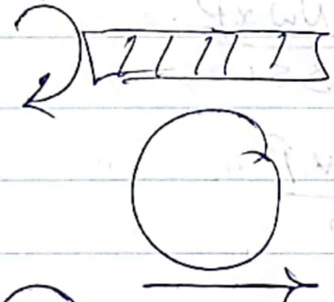

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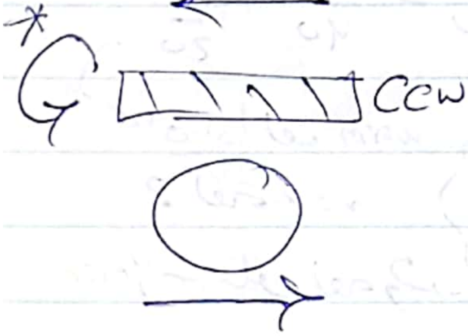

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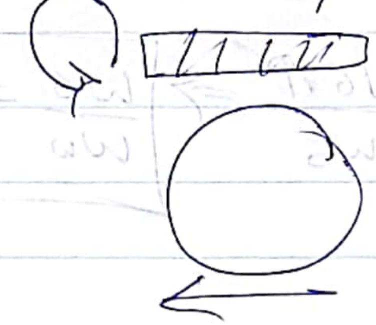


☰ Flywheels ...

In Dynamics $\boxed{T = \frac{1}{2} I_G \omega^2}$

الطاقة المخزنة في اي جسم يدور

وما نريده هو زيادة ... Kintal Energy لذلك نعمل

على رفع I_G (Moment of Inertia)

⇐ ونعني بها تركيز المادة حول محورها ...

كلما كان تركيز المادة ابعد عن محورها

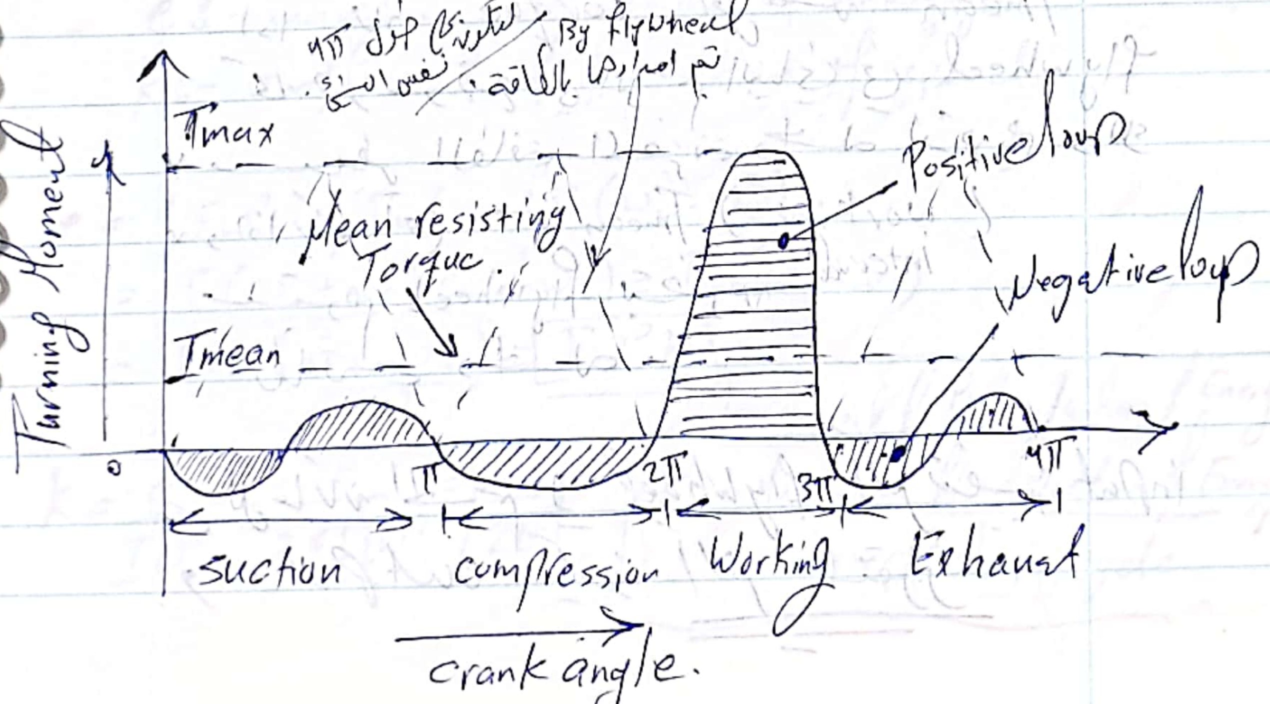
⇐ تكون moment of Inertia أعلى.

$I_G \uparrow \Rightarrow \int r^2 dm$

وهذا ما نريده ... لانه هدفنا عمل خزانة طاقة.

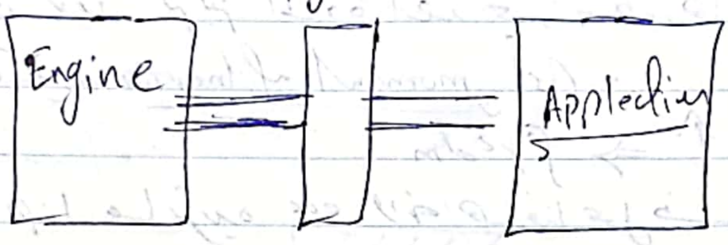
* وكذلك الأمر بالنسبة لِ capacitor في الدوائر الالكترونية ...

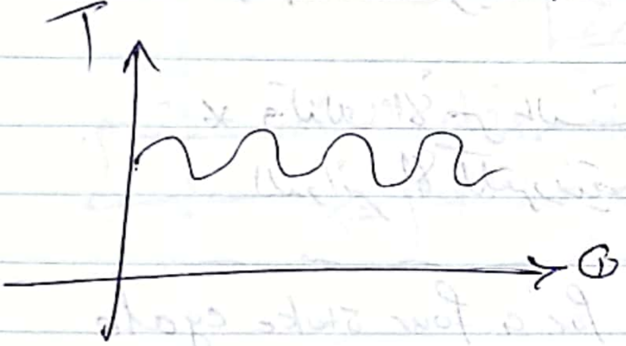
☰ Turning moment digram for a four stoke cycle internal combusition Engine



* اي في محرك السيارة كما الرسم السابق
يكون 0.25 الدورة سوجب طاقة 0.75 // سالب
طاقة ⇐ لذلك لا نريد ان نشعر بقوة
ضربات محرك السيارة اثناء working

* وكذلك التركيب :- flywheel





اي بناء آلى Tmean والتي سيتم التعرف لها.
اذا كان torque اقل من Tmean
كما هو موضح في الرسم البياني في flywheel
* سيعطي الطاقة المخزنة داخله لتعويض ذلك
* و اذا كان اكبر من Tmean (working interval)
سيقوم flywheel بتخزين
الطاقة داخله .

كل ما كان الحجم لـ flyWheel اكبر تغير "input"
و output يكون اكثر استقرار.

Aim:-

الاختلاف

- ⇒ * To contral the variation in speed during each cycle of an engine.
- ⇒ * It is attached to the Crankshaft and acts as a reservoir of Energy because of it's inertia.
- ⇒ * When the supplied energy is more than the required, it stores energy.
- ⇒ * When the required energy is more than supplied, it releases energy.

Definitions:-

I :- moment of inertia of the flyWheel.
ω_1 :- maximum speed
ω_2 :- minimum speed.
ω :- mean speed $\longrightarrow \frac{\omega_1+\omega_2}{2}$
E :- Kinetic Energy of the flyWheel.
e :- maximum fluctuation of energy.
k :- coefficient of fluctution of speed.

$$e = \frac{1}{2}I\omega_1^2 - \frac{1}{2}I\omega_2^2 = \frac{1}{2}I(\omega_1^2 - \omega_2^2)$$

$$= I\frac{(\omega_1+\omega_2)}{2}(\omega_1-\omega_2) = I\omega(\omega_1-\omega_2)$$

$$= I\omega^2\left(\frac{\omega_1-\omega_2}{\omega}\right) = \boxed{I\omega^2 k}$$

$$k = \frac{e}{I\omega^2} = \frac{e}{2*\frac{1}{2}I\omega^2} = \boxed{\frac{e}{2E}}$$

coffeicent of fluactulian of

* K_e = Maximum fluatuation of / work done/cycle

$$\boxed{E = \frac{1}{2} I_G \omega^2}$$

e :- التغير في الطاقة

e :- energy supplied by the flywheel

$$e = \frac{1}{2} I_G (\omega_1^2 - \omega_2^2)$$

$\longrightarrow I \uparrow \Rightarrow \Delta\omega \downarrow$

اي كل ما كبرت I_G ⇐ كل ما قل التغير في angular velocity.

* If $e = 100 J$

$$\frac{1}{2} I (\omega_1^2 - \omega_2^2) = 100$$

$$(\omega_1^2 - \omega_2^2) = \frac{2 * 100}{I} \Rightarrow I \uparrow \Leftrightarrow (\omega_1^2 - \omega_2^2) \downarrow$$

$$k = \frac{\omega_1 - \omega_2}{\omega} * 100$$

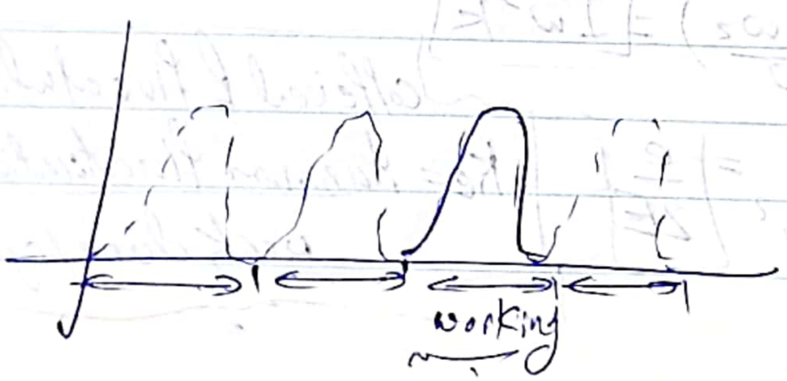
العلاقة بين e و k :-

$$* \quad \boxed{k = \frac{e}{I\omega^2} = \frac{e}{2 * \frac{1}{2} I \omega^2} = \frac{e}{2E}}$$

- - - - - - - - - - - - -

اي الهدف من هذه المادة :-

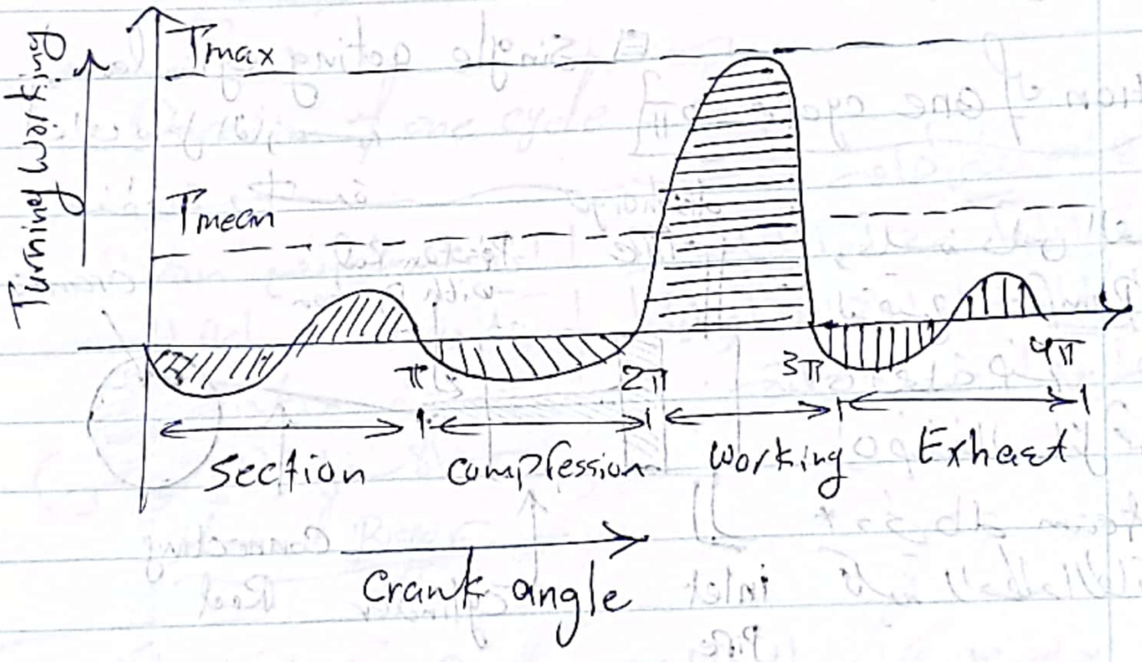
ايجاد حجم flywheel المناسب بحيث تردد (angular velocity) لل (output shaft) لا تزيد عن نسبة معينة.



(٢) ☰ Internal Combustion engine. (four stroke).

* We know that in a four stroke cycle internal combustion engine there is one working stroke after the crank has turned through two revolutions 720° (or 4π radians).

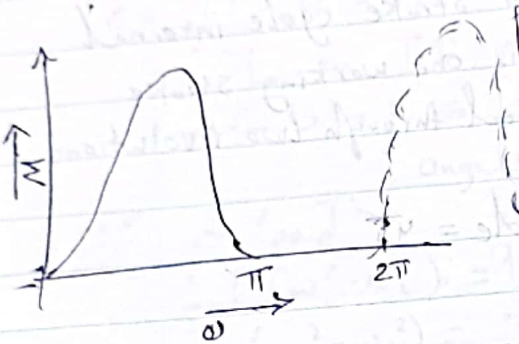
⇒ Duration of on cycle = 4π

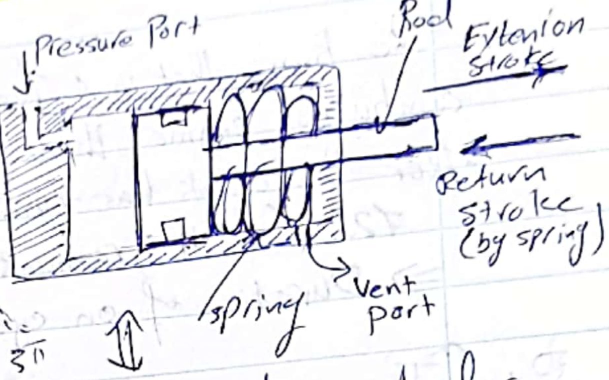


أي في حالة working يكون هناك Torque وكمان في
باقي الفترات لا يوجد عزم ⇐ لذلك لتم تشغيلها
تقوم flywheel بتزويد engine بالطاقة اللازمة
لذلك.

② steam Engine → single acting cylinder
→ Double acting cylinder

Steam engine : Single acting cylinder.



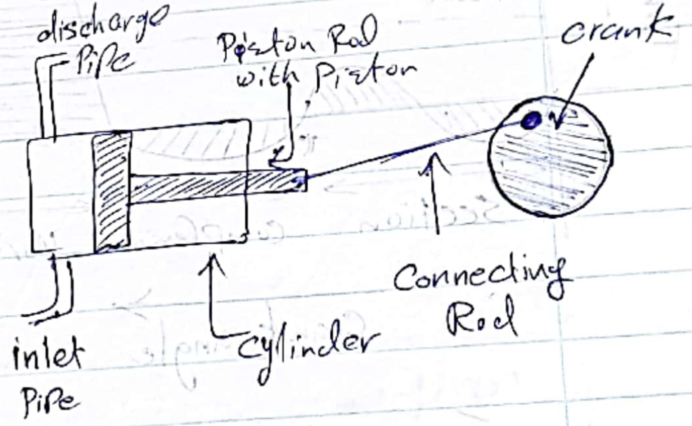


Single acting cylinder ⇐ أي يعطي الطاقة
منطقة واحدة.

Duration of one cycle = 2π

⇐ وعليه نكس العملية
لتصبح Pump
أي لعملية ضخ.
اضغط السائل ثم تسريره

* دي طلع steam
تكبيرة لاعطاء الطاقة.

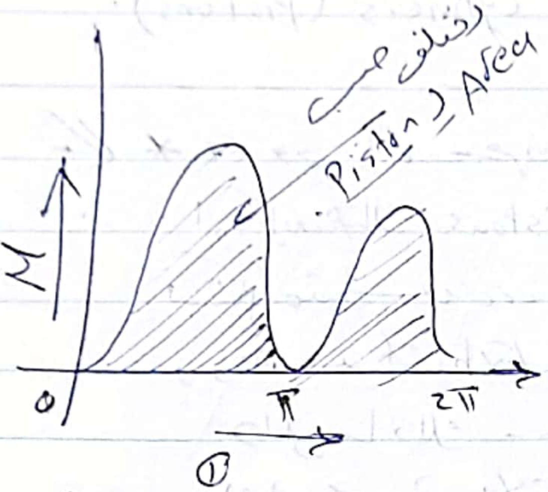


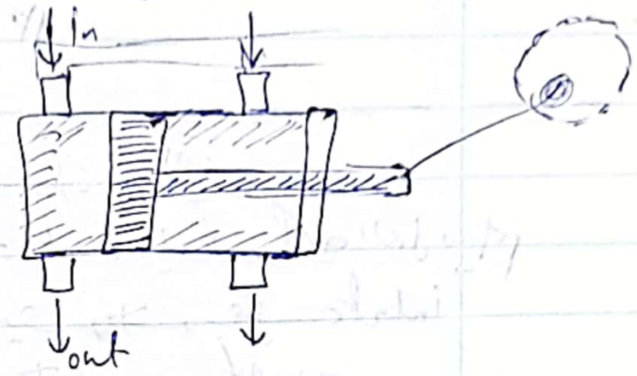
⇐ يعطي طاقة عندما يرتفع
ويعود عن طريقة spring.

أي انه يعطي الطاقة في
اتجاه السلبية فقط أي
عكس الاتجاه اذا كان معكوس
ام لا.

⇐ * كل 2π ⇐ cycle تكرر نفسها.

Steam Engine:- Double acting cylinder.





Double acting cylinder

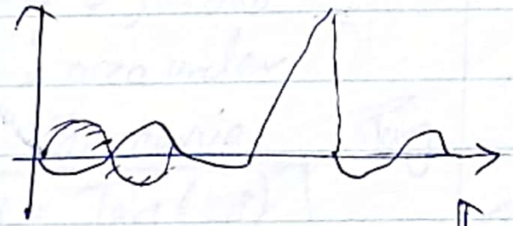
اي انه يعطي طاقة ذهاباً واياباً .. اما البابغ يعطي طاقة اما ذهاباً او اياباً عن طريق اتجاه العملية.

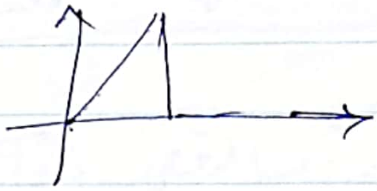
* Duration of one cycle = 2π

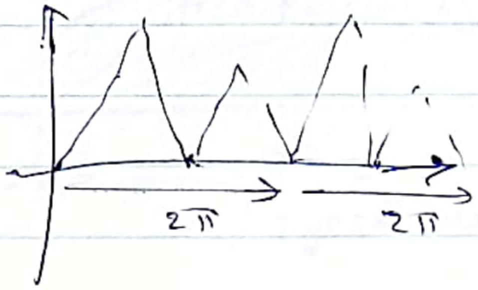
اي عندما يرتفع Piston تكون الطاقة اعلى لانه مساحة Piston اكبر من الاسفل (لا وجود عمود).

Momat Bigger ---

* لتسهيل عملية التعامل مع Digram علينا اعتبارها كمثلثات





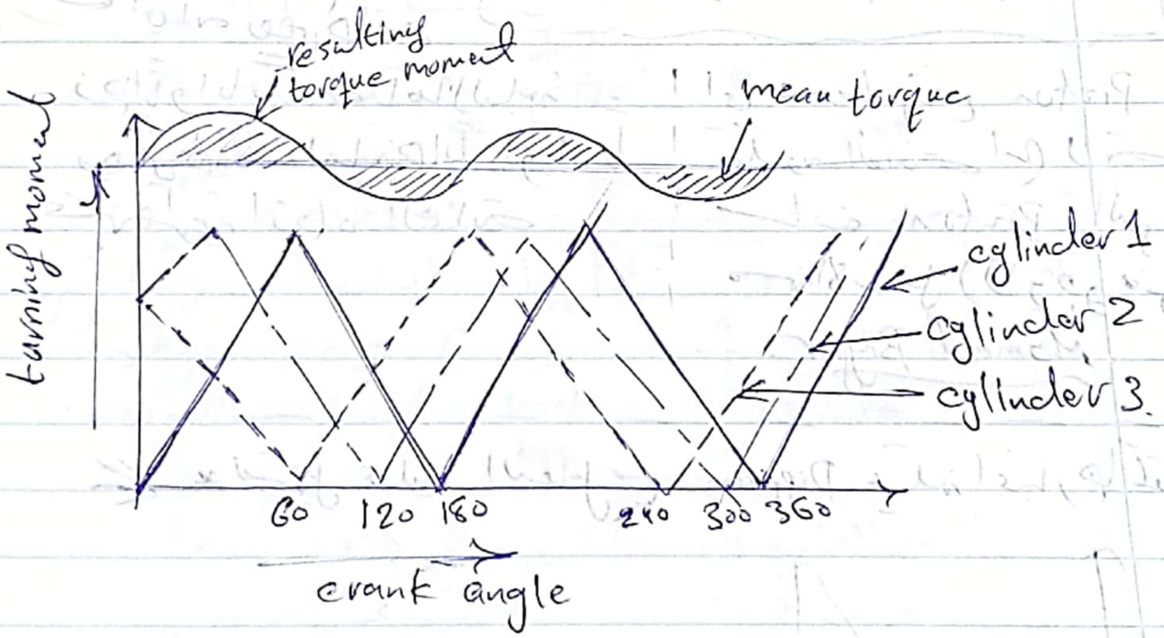


$$* \quad \text{crank angle} = \frac{\text{duration of one cycle}}{\#\ \text{of cyliners (Pistons)}}.$$

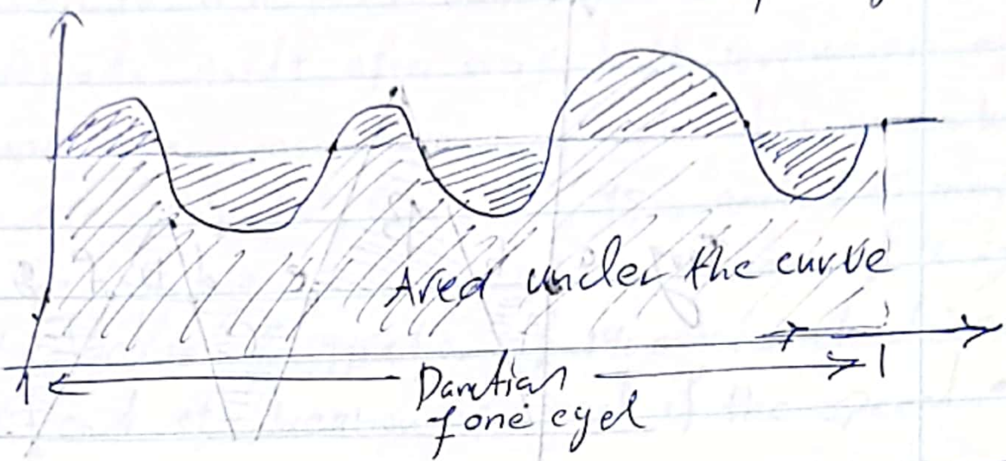
⇐ * ⇐ في حالة محرك السيارة مثلاً :

اذا كان هناك 4-Pistons ⇐ تكون آلية العمل ستتم بشكل متناسق ... بحيث واحد يعمل intake وواحد Exhast ، وواحد working - - - وإلى ما ذلك - - بحيث تكون هناك تناسق في العمل وستتم الانتقال من ذلك - - -

⇓



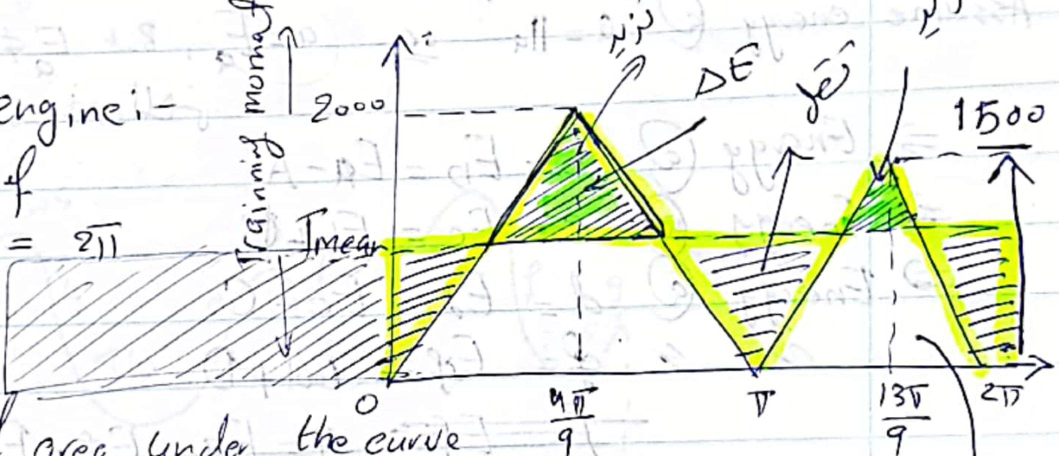
⟹ Averag Torque = $\frac{\text{area under the curve}}{\text{Duration f on cycle}}$



Exp:-

* steam engine:-

Duration of one cycle = 2π

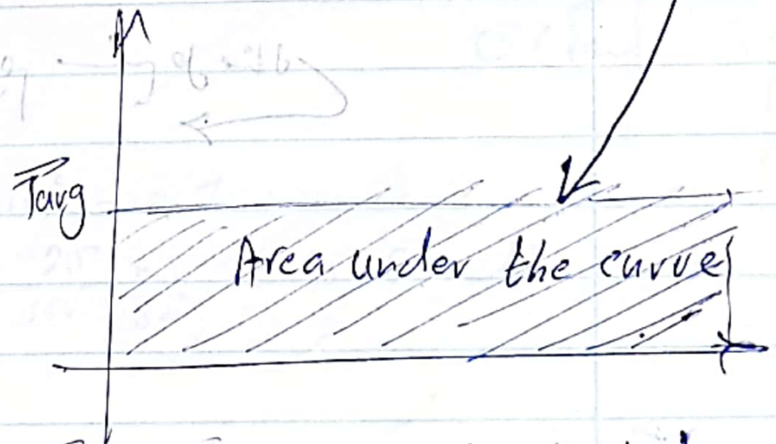


$T_{avg} = \frac{\text{Total area under the curve}}{\text{Duration of one cycle}}$ ← crank angle →

Energy = $\int M d\theta$

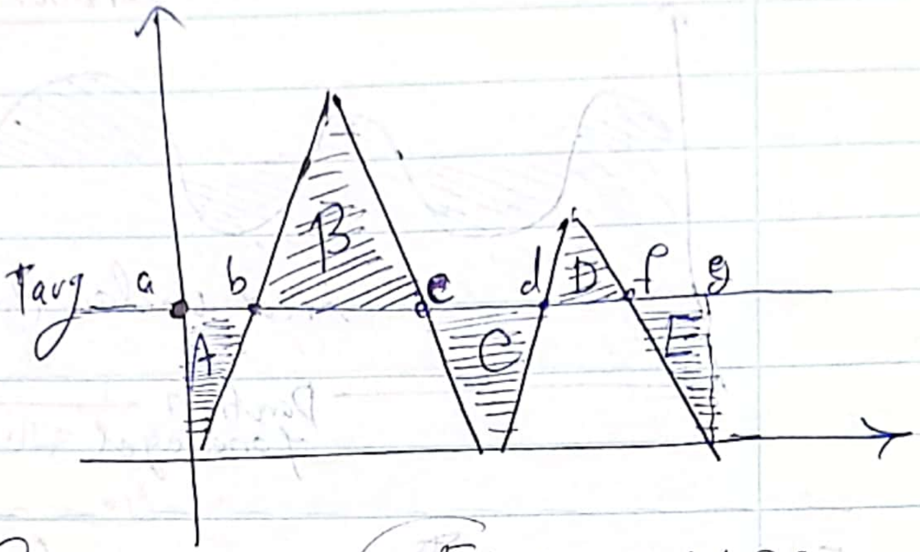
= area under the curve

$\boxed{\text{Energy} = T_{avg}(2\pi)}$



* أي انه عملية النقص والزيادة تقوم بتعويض بعضها لتعطي Tavg .

⇐ * لحل هذه المشكلة



Assume energy @ $a = H_a$ or $a = E_a$, But $E_a \neq \frac{1}{2} I \omega^2$ } اي بس نفس المعنى

⇒ Energy @ $b = E_b = E_a - A$

⇒ Energy @ $c = E_c = E_b + B$

⇒ Energy @ $d = E_d = E_c - C$

" " $f = E_f = E_d + D$

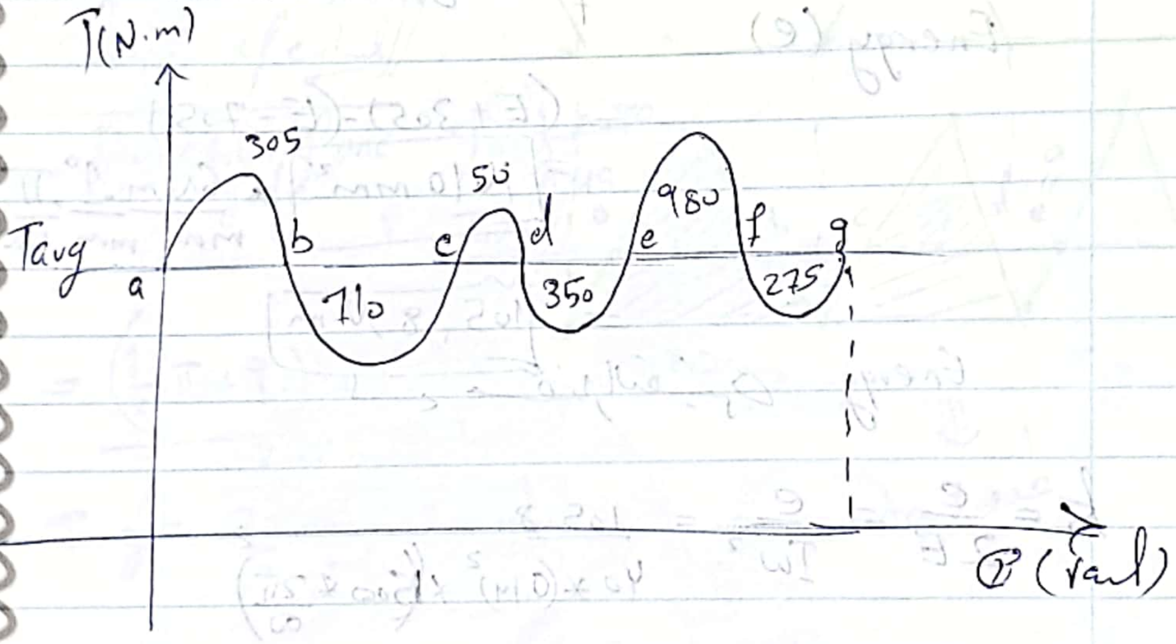
~~f E_f =~~ $g = E_g = E_f - E$

~~g~~ $E_g = E_a$

لمعرفة اي يلزم سنشرح حل مثال

Exp. The turning-moment digram for a petral engine is drawn to a vertical scale of 1mm to 6 N.m. and a repeats itself after every half revolution of engine. The areas above and below the mean torque line are 305, 710, 50, 350, 980, and 275 mm^2

→ The rotating parts amount to a mass of 40 kg at a radius of gyration of 140 mm. calculate the cofficient of flucation of speed if the speed of the engine is 1500 rpm.



$I = mk^2 = 40 * (0.14)^2 = 0.784$

$w = 1500 \text{ rpm} * \frac{2\pi}{rev} \frac{min}{80} = 106.5 \text{ rad/s}$

* Assume energy @ $a = E_a = E$

* energy @ $\boxed{b = E + 305}$ $\Rightarrow E_{max}$

" " $c = (E+305) - 710 = E - 405\ mm^2$

" " $d = (E-405) + 50 = E - 355\ mm^2$

" " $e = (E-355) - 350 = \boxed{E - 705\ mm^2}$ E_{min}

" " $f = (E-705) + 980 = E + 275\ mm^2$

" " $g = E + 275 - 275 = \boxed{E}$ #

* maximum fluctuation Energy (e) $= E_{max} - E_{min}$

$= (E+305) - (E-705)$

$= \boxed{1010\ mm^2} * \frac{6N.m}{mm} \cdot \frac{1^\circ}{mm} \cdot \frac{\pi}{180}$

$= \boxed{105.8\ N.m}$

مقدار التغير في Energy

$$k = \frac{e}{2E} = \frac{e}{I\omega^2} = \frac{105.8}{40 * (0.14)^2 * \left(500 * \frac{2\pi}{60}\right)^2}$$

$k = 5.45 \times 10^{-3} = \boxed{0.545\ \%}$

نسبة التغير في angular speed لل output shaft لن تزيد عن $\boxed{0.5\%}$.

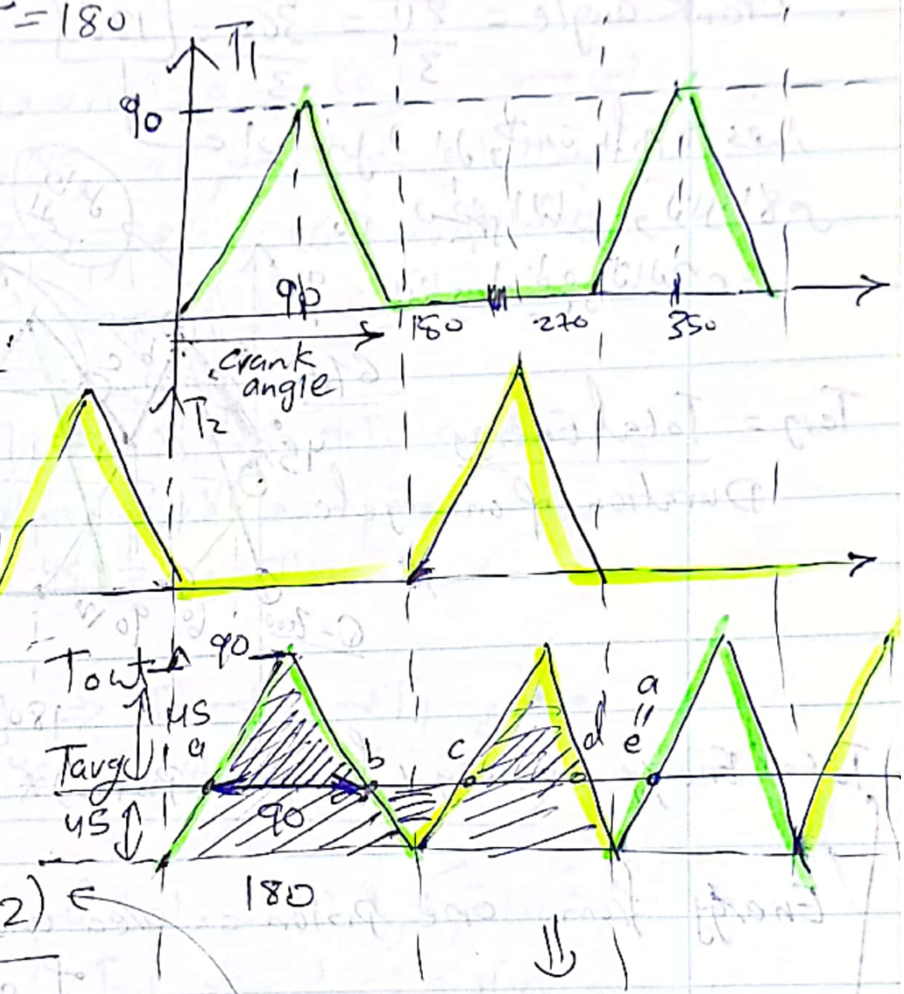
* اي اذا طلب في الامتحان ان نسبة التغير لا تزيد عن قيمة معينة هنا تكون قيمة K . لايجاد حجم ال flywheel المناسب.

$$* \text{ Crank angle} = \frac{\text{Period of one cycle}}{\# \text{ of cylinders}}$$ { (Two cylinder) Single acting

$$= \frac{2\pi}{2} = \pi = 180$$

$T_{out} = T_1 + T_2$

$$T_{avg} = \frac{(\text{area under the curve of one cylinder}) * (\text{number of cylinders})}{\text{Duration of one cycle}}$$



$$= \frac{\left(\frac{1}{2}\pi * 90\right) * (2)}{2\pi}$$ ←

$$T_{avg} = \frac{90}{2} = \boxed{45 \text{ N.m}}$$

2 = Pistons عدد

$$\frac{180}{90} = \frac{x}{45} \Rightarrow 90$$

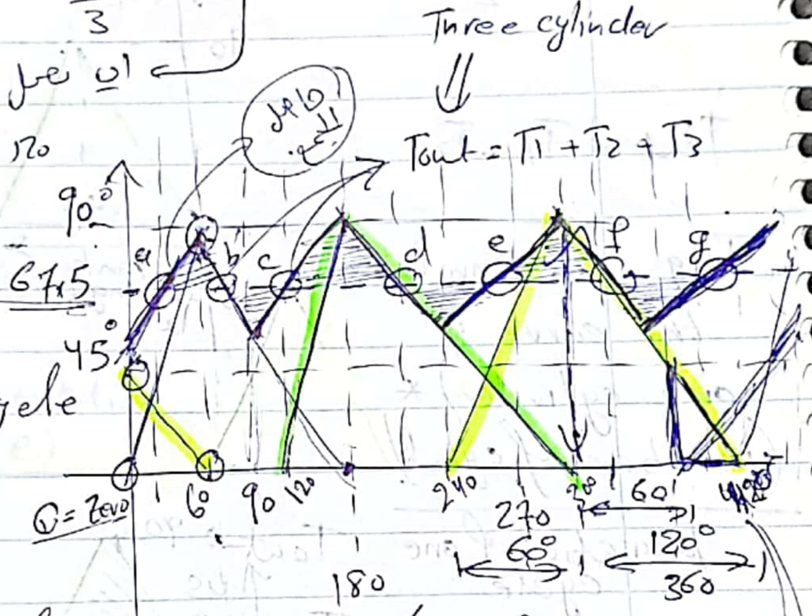
Example 16.8.

Three cylinder (single acting) $T_{max} = 90$ N.m

. Crank angle $= \frac{2\pi}{3} = \frac{360}{3} = \boxed{120^\circ}$

اي نعمل ل اول وحدة shift بمقدار 120 لنظام الثاني وكذلك الامر للثاني لنظام الثالث.

$T_{avg} = \frac{\text{Total Energy}}{\text{Duration of one cycle}}$



Total Energy = (Energy from one Piston) * (# of Piston)

Energy from one piston = area under the curve of one Piston

$= \frac{1}{2}\pi * 90 = \boxed{45\pi \text{ N.}}$

$T.E = 3 * 45\pi = \boxed{135\pi \text{ N}}$

$T_{avg} = \frac{135\pi}{2\pi} = \boxed{67.5 \text{ N}}$ #

$\frac{y}{90} = \frac{60}{120}$

$\boxed{y = 45^\circ}$

#

$420 - 360 = ⑥⓪$ #

$T_{out} = T_1 + T_2 + T_3$

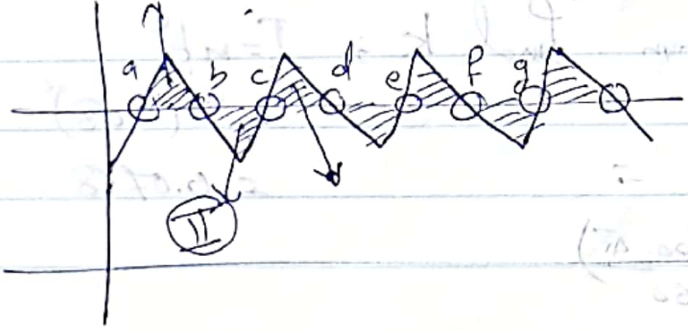
أين من $[0 \rightarrow 60]$ الأزرق والأصفر ⇐ لهما قيم
عند البداية ⇐ الأصفر لأنه 45 , الأزرق zero
وعند النهاية (60) ⇐ الأصفر = zero + الأزرق = 60 .
⇐ لذلك يكون حاصل الجمع linear.

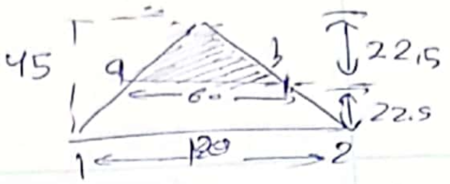
$[60 \rightarrow 120] \Rightarrow T_3 + T_2 + T_1$ مجموع
⇐ مجموعهم فقط الأزرق لأنه الأخضر والأصفر = zero
وقيمته قيمة الأزرق = 45°.

$[120 \rightarrow 180] \Rightarrow T_3 + T_2 + T_1$ مجموع
⇐ الأخضر صفر ، الأزرق = 45 ، الأصفر = zero

• هكذا يتم تكرار العملية وإيجاد قيم $T_3 + T_2 + T_1$
من كل فترة لإيجاد T_{out}.

⇐ لذلك average torque ، وتعامله
مع Actual torque :
(I) تم تطبيق الخطوات على السابقة





$\frac{120}{45} = \frac{ab}{22.5} \Rightarrow$ $\boxed{ab = 60°}$

assume energy at a = E

" " " b = E_a + area

$= E_a + \frac{1}{2}(60 . \frac{\pi}{180})22.5$

$= E_a + \frac{45\pi}{12}$

وكذلك الأمر بالنسبة
لباقي المثلثات لايجاد
قيم القاعدة

" " " c = $E_b - \text{area}_{II} = E_a$

" " " d = $E_a + \frac{45\pi}{12}$

" " " e = E_a

" " " f = $E_a + \frac{45\pi}{12}$

" " " g = E_a = 135πN

↳ Energy @ a

ليس نفسها Energy for flyWheel

$E_{max} = E_a + \frac{45\pi}{12}$; $E_{min} = E_a$

$e = E_{max} - E_{min}$

$= \frac{45\pi}{12} = \boxed{11.78 \text{ J}}$

① Power developed

$P = T_{avg} . w = 67.5 * 600 \frac{2\pi}{60} =$ ✓

② m = 12 , k_o = 80mm find k ; $I = m k_o^2$

$= 12(0.08)^2$

$= 0.0768$

$k = \frac{e}{I w^2} = \frac{11.78}{(0.0768)(600 . \frac{2\pi}{60})} =$

③ coefficient of fluctuation of energy.

$$K_E = \frac{\text{Max fluctation of Energy}}{\text{workdone/cycle}} = \frac{11.78}{424}$$

Area

↳ 3 * Area of one piston

number of piston

$$= 0.0278 = \boxed{2.78\ \%}$$

④ Maximum anguler accelration of flywheel:

α = maximum anguler acceleration of the flywheel.

$$T_{max} - T_{min} = I\alpha = mk^2 . \alpha$$

$$90 - 67.5 = 12 * (0.08)^2 * \alpha = 0.077$$

$$\alpha = \frac{90 - 67.5}{0.077} \Rightarrow \boxed{\alpha = 292\ rad/s^2}$$

H.W

- single acting cylinder
  - → 1 piston
  - → 2 pistons
- Double acting cylinder
  - → 1 piston
  - → 2 pistons

Machine Balancing

* Balancing of Rotary :-

① - static Balancing (force Balancing ; all masses in the same plane)

② - Dynamic Balancing (force + Moment Balancing)

* Balancing of mechanism .(four Bar

normally $w = \text{constant}$

$\alpha = \text{zero}$

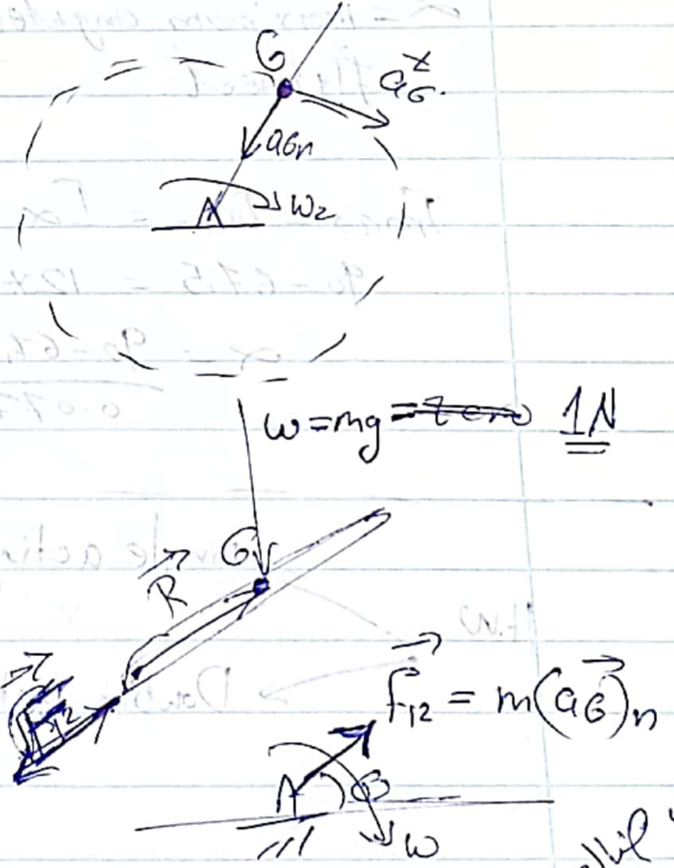
$\vec{a_G} = \vec{(a_G)_n} + \cancel{(a_G)_t}$

$* \, a_{G_n} = w^2 R$

* weight is negleatable

$\sum \vec{f} = m \vec{a_G}$

$$\boxed{\vec{F} = m \vec{(a_G)_n}}$$



if 12200 rpm

$(a_G)_n = w^2 R = \boxed{1880 N}$ * 1oogm

$= \boxed{158 N}$

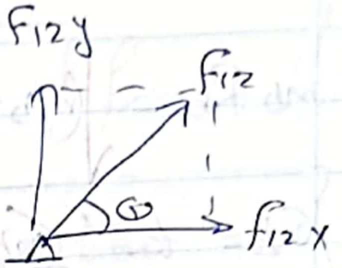
لذلك weigh بنتجاهله

في mechanism ... لا يؤثر على الحل .

* أي قانون action & Reaction

$f_{12}x = f_{12} \cos\theta$

$f_{12}y = f_{12} \sin\theta$



⇒ the problem rises when the center of mass is not the center of rotation (has an offset from the center of rotation)

$$f_{12} = m\,a\,G_n = m R w^2$$

m balancing ← $\boxed{m_b} R_b w^2 = m R w^2$

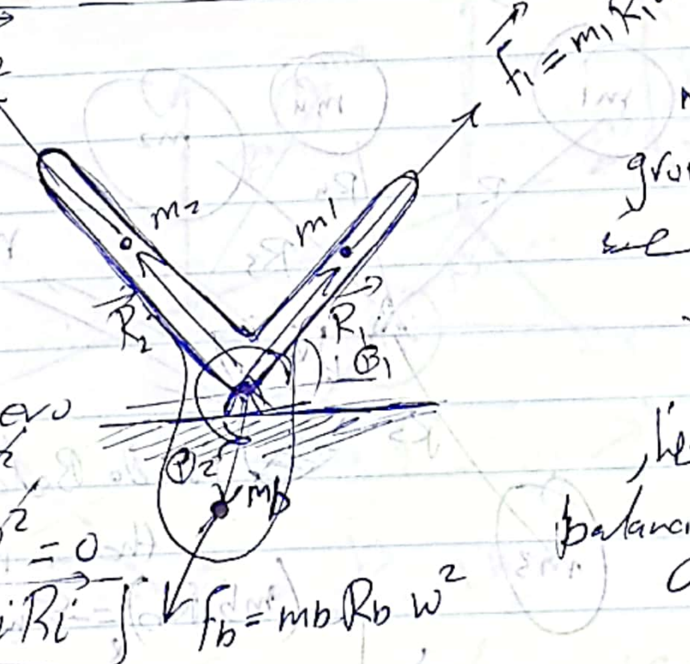
$$-m_b \vec{R_b} w^2 + m \vec{R} w^2 = zero$$

$$-m_b \vec{R_b} + m \vec{R} = zero$$

* R_b تكون عكس R 81 عليها

$m_2 R_2 w^2 = \vec{f_2}$

$\vec{f_1} = m_1 R_1 w^2$



$f_b = m_b R_b w^2$

اي اشي Mass يتحرك عن ground عشان قوة مساوية بالحجم R.

$\Sigma \vec{F} = zero$

$f_1 + f_2 + f_b = zero$

$m_1 \vec{R_1} \cancel{w^2} + m_2 \vec{R_2} \cancel{w^2} + m_b R_b \cancel{w^2} = 0$

$$\boxed{m_b R_b = -\Sigma m_i R_i}$$

المطلوب هو اختار كتلة لعمل balancing

$$\boxed{\begin{aligned}(m_b R_b)x &= -\Sigma m_i R_i x \\ (m_b R_b)y &= -\Sigma m_i R_i y\end{aligned}}$$

* $R_i x = R_i \cos\theta_i$
* $R_i y = R_i \sin\theta_i$

$$\Rightarrow m_b R_b = \sqrt{(m_b R_b)_y^2 + (m_b R_b)_x^2}$$

$$\Rightarrow \boxed{\theta_b = \tan^{-1}\frac{(m_b R_b)_y}{(m_b R_b)_x}} \quad \#$$

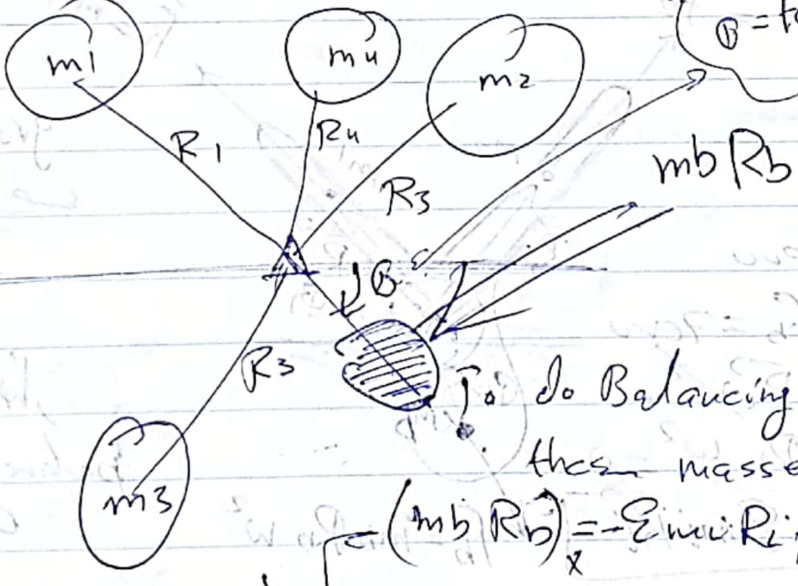
EXP:-

1	m	R	theta	L	mRcosθ Fx	mRsinθ Fy	Fy²L Mx	Fx²L My
1	1.2	1,135	113.4	0.854	-0.541	1.250	1.067	0,402
2	1.8	0.822	48.8	1.701	0.975	1.113	1.894	-1,558
3	2.4	1.04	251.4	2,396	-0.796	-2.366	-5,668	1.908
B	1	0.904	75,270	3.097	0.230	0.874	2.707	-0.712
A	1	0.883	81.347		0.133	0.872		

$\Rightarrow$ mb

Rb

$$\theta = \tan^{-1}\frac{(m_b R_b)_y}{(m_b R_b)_x}$$



To do Balancing for these masses:-

$$\rightarrow \begin{cases} (m_b R_b)_x = -\Sigma m_i R_{ix} \\ (m_b R_b)_y = -\Sigma m_i R_{iy} \end{cases}$$

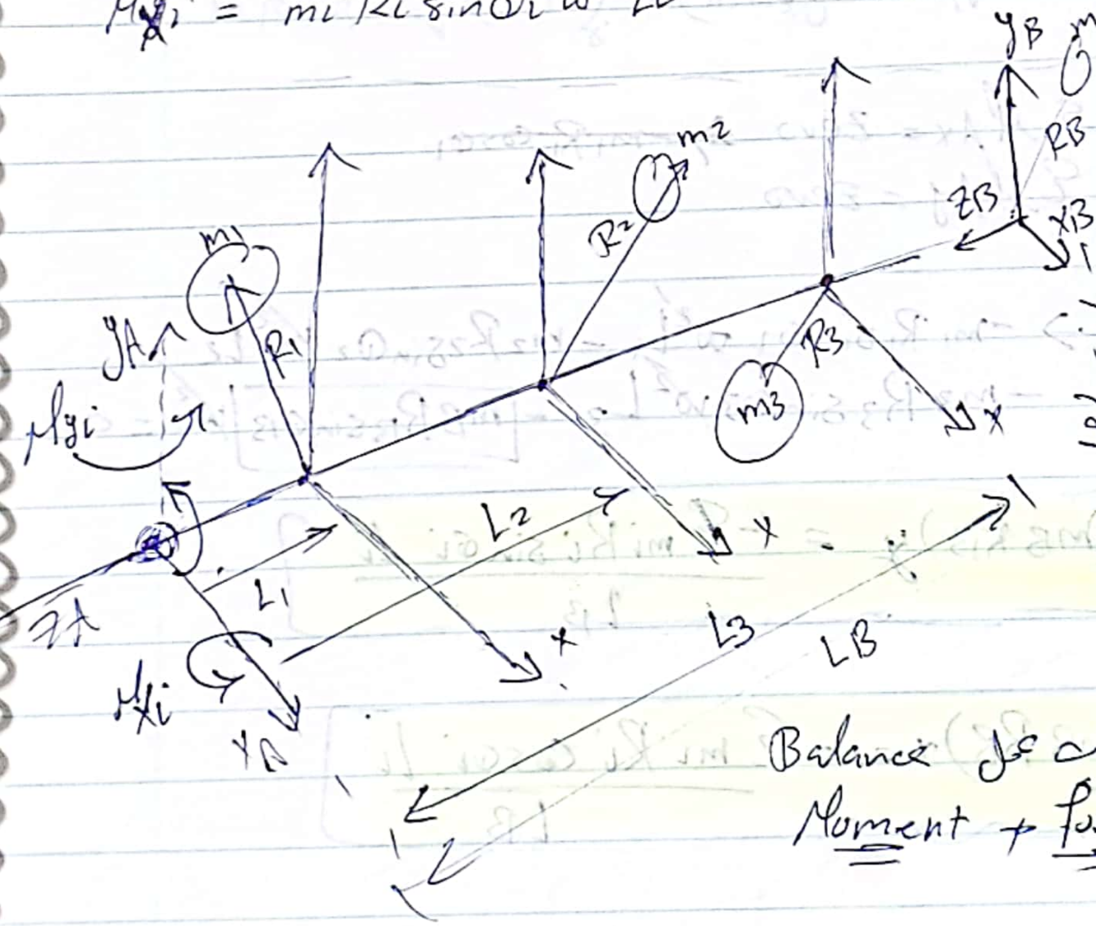
▣ Dynamic Balancing [Balance of force + Moment]

⇒ masses rotate in defferent planes.

$$f_{x2} = m_2 R_2 \cos\theta_2 \, \omega^2$$
$$f_{y2} = m_2 R_2 \sin\theta_2 \, \omega^2$$

$$\left[\begin{array}{l} M_{y2} = -f_{x2} L_2 = -m_2 R_2 \cos\theta_2 \, \omega^2 L_2 \\ M_{x2} = -f_{y2} L_2 = m_2 R_2 \sin\theta_2 \, \omega^2 L_2 \end{array} \right.$$

$$M_{yi} = \cancel{F_x} - m_i R_i \cos\theta_i \, \omega^2 L_i$$
$$M_{xi} = m_i R_i \sin\theta_i \, \omega^2 L_i$$



* إذا $\sum F = 0$ ـــلا يعني عمل
Balance (لل) لازم Moment +

لذلك يجب عمل Balance
Moment + force ل

* اي الاتنين mass واحد بعمل **Balance**

لذلك يجب اضافة ⇐ two masses ⇐ in two plane

لعمل Balancing.

⇒ two masses are added in two different planes.

⇒* mass in plane B to Balance the moment about the axis of plane A.

⇒* mass in plane A to Balance the forces.

$$L_A = zero$$
$$M_{xA} = zero$$
$$M_{yA} = zero$$

$$\Sigma M_{Ax} = zero \Rightarrow \cancel{-m_1 R_1 \cos\theta_1}$$
$$\Sigma M_{Ay} = zero$$

$$\rightarrow -m_1 R_1 \sin\theta_1 \omega^2 L_1 - m_2 R_2 \sin\theta_2 \omega^2 L_2 - m_3 R_3 \sin\theta_3 \omega^2 L_3 - \boxed{m_B R_B \sin\theta_B} \omega^2 = 0$$

$$* \boxed{(m_B R_B)_y = \frac{-\Sigma m_i R_i \sin\theta_i l_i}{L_B}}$$

$$* \boxed{(m_B R_B)_x = -\frac{\Sigma m_i R_i \cos\theta_i l_i}{L_B}}$$

$$\left[\begin{array}{l} * \ (m_A R_A)_x = -\sum m_i R_i \cos\theta_i \\ * \ (m_A R_A)_y = -\sum m_i R_i \sin\theta_i \end{array}\right.$$

MBRB cosθB included

Exp. ⇒ ك نفس الرسمة السابقة.
ونفس n الجدول.

$$(m_B R_B)_x = -\frac{\sum m_i R_i \cos\theta_i \, L_i}{L_B}$$

$$(m_B R_B)_y = 0.874$$

$$m_B R_B = \sqrt{(m_B R_B)_x^2 + (m_B R_B)_y^2}$$