Chapter 11 : Inferences about population variance

11.1 : Inferences about population variance .

σ^2 : population variance $\qquad$ s^2 : sample variance .

σ : population st. dev. $\qquad$ s : sample st. dev .

- $\sigma^2 = \dfrac{\sum_{i=1}^{N} (x_i - \mu)^2}{N}$

- $s^2 = \dfrac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$.

RMK :

Data Normal : $\dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \overset{\text{has}}{\sim} \underset{\text{Normal}}{N} (\overset{\text{Mean}}{0}, \overset{\text{variance}}{1})$.

CLT : $\dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$ approx. $N(0,1)$ for large n .

sampling Distribution of $\bar{x}$.

★ sampling Distribution of $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$

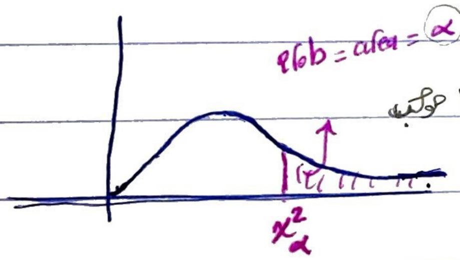
In words, $\frac{(n-1)S^2}{\sigma^2}$ has chi-squared distribution with $n-1$ degrees of freedom

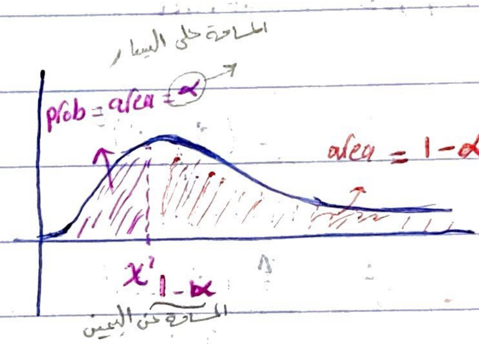
- $df = n-1$
- $S^2 = \frac{\Sigma(x_i-\bar{x})^2}{n-1}$

مراجعة الحسابات بالآلة الحاسبة
↳ $\bar{x}, S, S^2$

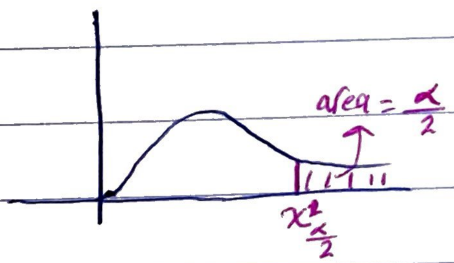
$df = n-1$

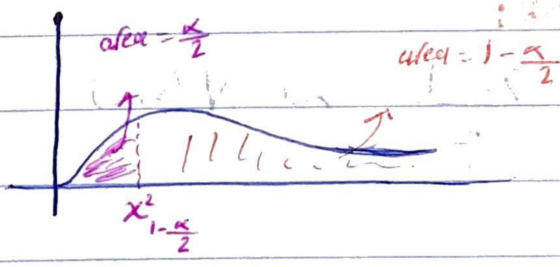
- chi-squared distribution : المساحة على اليمين





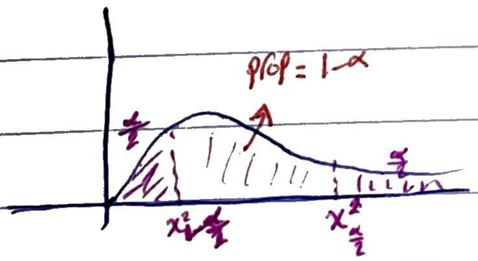
يستخدم جدول ... table 3 - chi-squared d...





- $\chi^2 = \frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$

chi-squared dis., $df = n-1$



$(1-\alpha)$ CI for σ^2 :

$P(\chi^2_{1-\frac{\alpha}{2}} \leq \chi^2 \leq \chi^2_{\frac{\alpha}{2}}) = 1-\alpha$

$$\Rightarrow P\left(\chi^2_{1-\frac{\alpha}{2}} \leq \frac{(n-1)S^2}{\sigma^2} \leq \chi^2_{\frac{\alpha}{2}} \right) = 1-\alpha .$$

بشتغل عليها ضرب وقسمة وطرح

$n>1 \ , \ S^2 > 0 .$

$$\rightarrow P\left(\frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}}} \leq \sigma^2 \leq \frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2}}} \right) = 1-\alpha .$$

وصلنا بنسوي $(1-\alpha)CI$

$$\rightarrow (1-\alpha)\,CI \text{ for } \sigma^2 = \left[\frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}}} \ , \ \frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2}}} \right].$$

★ summary:

$$(1-\alpha)\,CI \text{ for } \sigma^2 = \left[\frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}}} \ , \ \frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2}}} \right], \text{ where}$$

$$df = n-1 \ , \quad S^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} .$$

Assuming:

1. The sample is random.
2. The sample come from a Normal population.

★ Testing Hypothesis :

① LTT

$H_0 : \sigma^2 \geq \sigma_0^2$

$H_1 : \sigma^2 < \sigma_0^2$

② UTT

$H_0 : \sigma^2 \leq \sigma_0^2$

$H_1 : \sigma^2 > \sigma_0^2$

③ TTT

$H_0 : \sigma^2 = \sigma_0^2$

$H_1 : \sigma^2 \neq \sigma_0^2$.

- Test statistic :

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} \quad , \quad df = n-1$$

σ_0^2 : hypothesis value .

- Reject H_0 if :

① $\chi^2 < \chi^2_{1-\alpha}$ ② $\chi^2 > \chi^2_{\alpha}$ ③ $\chi^2 > \chi^2_{\frac{\alpha}{2}}$ or $\chi^2 < \chi^2_{1-\frac{\alpha}{2}}$

→ Reject H_0 if p-value $\leq \alpha$.

★ "Assuming same assumption used for C.I."

Exp: 330 ml .

Data set : $n = 20$

$s^2 = 2.5$ → point estimator for σ^2

→ $s = 1.58$ → point estimator for σ

Find :

1. 95% CI for σ^2 : $\quad$ ($1-\alpha$) → $\alpha = 0.05$

$$\left[\frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2}}} , \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2}}} \right] .$$

n ✓ $\quad$ s^2 ✓ $\quad$ χ^2 : بجيبها من الجدول

$df = n-1 = 19$, $1-\alpha = 0.95$ → $\alpha = 0.05$.

want $\frac{\alpha}{2} = 0.025$

هيك بستخدم الجدول → $1-\frac{\alpha}{2} = 0.975$.

Table 3 :

$df = 19$: $\chi^2_{0.025} = 32.825$. $\quad \chi^2_{0.975} = 8.907$.

هيك عوضنا مباشر .

$$\Rightarrow 95\% \text{ CI for } \sigma^2 = \left[\frac{(19)(2.5)}{32.825} , \frac{(19)(2.5)}{8.907} \right] = [1.45 , 5.33] .$$

هون من σ^2 **2.** 95% CI for σ : $\left[\sqrt{1.45} , \sqrt{5.33} \right]$

بنوخذ الجذر

$$= [1.20 , 2.31] .$$

★ ما بطلع معي قيم سالبة ، لأن $\frac{(n-1)s^2}{\sigma^2}$

exp : Taxi servise σ_0^2

بنعتبر أوزن الجذر ونحوها σ

$H_0 : \sigma^2 \leq 4$

$H_1 : \sigma^2 > 4$

upper tail test .

Data set : $n = 24$

$s^2 = 4.9$

$\alpha = 0.05$

Solution :

Test statistic : $\chi^2 = \dfrac{(n-1)\, s^2}{\sigma_0^2} = \dfrac{23\,(4.9)}{4} = \boxed{28.18}$.

upper tail test :

↳ critical value : $\chi^2_{\alpha} = \chi^2_{0.05}$ with $df = 23$.

$\chi^2_{0.05} = 35.172$

We Regect H_0 if $\chi^2 > \chi^2_{0.05}$ → $28.18 < 35.172$.

So we don't Regect H_0 $(\alpha = 0.05)$.

By P-value :

بنروح عند $df = 23$ وبنشوف χ^2 بين شو وشو محصورة

→ $\text{p-value} \in [0.10 , 0.90]$.

$\text{p-value} > \alpha$

$[0.10 , 0.90] > 0.05$

So Don't Regect H_0 $(\alpha = 0.05)$

!!! done