

Chapter 5

$$Q_1: (a) \int \sin(5x) dx = -\frac{\cos 5x}{5} + C$$

$$(b) \int \tan^2 x dx = \int (\sec^2 x - 1) dx$$

$$= \tan x - x + C$$

$$(c) \int (1 + \cot^2 \theta) d\theta = \int \csc^2 \theta d\theta$$

$$= -\cot \theta + C$$

$$(d) \int \frac{\csc \theta}{\csc \theta - \sin \theta} d\theta =$$

$$\int \frac{\frac{1}{\sin \theta}}{\frac{1}{\sin \theta} - \sin \theta} d\theta = \int \frac{\frac{1}{\sin \theta}}{\frac{1 - \sin^2 \theta}{\sin \theta}} d\theta$$

$$= \int \frac{1}{1 - \sin^2 \theta} d\theta = \int \frac{1}{\cos^2 \theta} d\theta$$

$$\int \sec^2 \theta d\theta = \tan \theta + C$$

Q2 (a) $y = \int_1^x \frac{dt}{t}$

$$y' = \frac{1}{x}$$

(b) $y = \int_0^{\sqrt{x}} \cos t \, dt \Rightarrow$

$$y' = \frac{1 \cos x}{2\sqrt{x}}$$

(c) $y = \int_{\tan x}^0 \frac{dt}{1+t^2}$

$$y = - \int_0^{\tan x} \frac{dt}{1+t^2}$$

$$y' = - \frac{\sec^2 x}{1+t^2} = \frac{\sec^2 x}{-1-t^2}$$

Q3 Find the linearization of $g(x) = 3 + \int_1^{x^2} \sec(t-1) \, dt$ at $x=1$

$$g(x) = 3 + \int_1^{x^2} \sec(t-1) \, dt \Rightarrow g(1) = 3 + \int_1^{(-1)^2} \sec(t-1) \, dt \Rightarrow 3$$

$$g'(x) = 2x \cdot \sec(x^2-1) \Rightarrow g'(1) = 2 \sec 0 \Rightarrow 2$$

$$L(x) = g(a) + g'(a)(x-a)$$

$$= 3 + 2(x-1)$$

$$= 3 + 2x - 2$$

$$= 1 + 2x$$

$$Q4 \quad (a) \quad \int_1^{\sqrt{2}} \left(\frac{s^2 + \sqrt{s}}{s^2} \right) ds = \int_1^{\sqrt{2}} (1 + s^{-\frac{3}{2}}) ds$$

$$s + \frac{s^{-\frac{1}{2}}}{-\frac{1}{2}} \Big|_1^{\sqrt{2}} =$$

$$\left(\sqrt{2} - \frac{2}{\sqrt{2}} \right) - \left(1 - 2 \right)$$

$$(0) - (-1)$$

$$= 1$$

$$(b) \int_0^{\frac{\pi}{6}} (\sec x + \tan x)^2 dx$$

$$\int_0^{\frac{\pi}{6}} (\sec^2 x + \sec x \tan x + \tan^2 x) dx$$

$$\int_0^{\frac{\pi}{6}} (\sec^2 x + \sec x \tan x + \sec^2 x - 1) dx$$

$$\int_0^{\frac{\pi}{6}} (2 \sec^2 x + \sec x \tan x - 1) dx$$

$$2 \tan x + \sec x - x \Big|_0^{\frac{\pi}{6}}$$

$$\left(2 \tan \frac{\pi}{6} + \sec \frac{\pi}{6} - \frac{\pi}{6} \right) - (2 \tan 0 + \sec 0 - 0)$$

$$= \left(\frac{2}{\sqrt{3}} + \frac{2}{\sqrt{3}} - \frac{\pi}{6} \right) - (0 + 1 - 0)$$

$$= \frac{4}{\sqrt{3}} - \frac{\pi}{6} - 1$$

$$c) \int_0^{\pi} (\cos x + |\cos x|) dx$$

$$\int_0^{\pi} \cos x dx + \int_0^{\pi} |\cos x| dx$$

$$\int_0^{\pi} \cos x dx + \int_0^{\frac{\pi}{2}} \cos x dx + \int_{\frac{\pi}{2}}^{\pi} -\cos x dx$$

$$\left(\sin x \Big|_0^{\pi} \right) + \left(\sin x \Big|_0^{\frac{\pi}{2}} \right) + \left(-\sin x \Big|_{\frac{\pi}{2}}^{\pi} \right)$$

$$(0-0) + (1-0) + (-0-(-1))$$

$$1+1=2$$

$$\text{Q5 (a)} \int \frac{dx}{\sqrt{x}(1+\sqrt{x})^2} \quad \begin{aligned} \sqrt{x} &= y \\ dy &= \frac{1}{2\sqrt{x}} dx \\ dx &= 2y dy \end{aligned}$$

$$\begin{aligned} &= \int \frac{2y \cdot 2\sqrt{x}}{y(1+y)^2} dy = \int \frac{2y dy}{y(1+y)^2} \\ &= \int 2(1+y)^{-2} dy = \frac{2(1+y)^{-1}}{-1} + C \\ &= \frac{2(1+\sqrt{x})^{-1}}{-1} + C \end{aligned}$$

$$\text{(b)} \int \frac{\sec z \tan z}{\sqrt{\sec z}} dz \quad \begin{aligned} y &= \sec z \\ dy &= \sec z \tan z dz \end{aligned}$$

$$\int \frac{\sec z \tan z}{\sqrt{y}} \cdot \frac{dy}{\sec z \tan z} = \int dy (y)^{\frac{1}{2}}$$

$$\frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = 2\sqrt{y} + C = 2\sqrt{\sec z} + C$$

$$\begin{aligned} \text{(c)} \int \sqrt{\left(\frac{x-1}{x^5}\right)} dx &= \int \sqrt{\frac{x}{x^5} - \frac{1}{x^5}} dx \\ &= \int \sqrt{\frac{1}{x^4} - \frac{1}{x^5}} dx = \int \sqrt{\frac{1}{x^4} \left(1 - \frac{1}{x}\right)} dx \end{aligned}$$

$$= \int \frac{1}{x^2} \sqrt{1 - \frac{1}{x}} dx \quad \begin{aligned} y &= \frac{1}{x} \\ dy &= -\frac{1}{x^2} dx \end{aligned}$$

$$= \int \frac{1}{x^2} \sqrt{1-y} \cdot (-x^2) dy = \int -\sqrt{1-y} dy$$

$$-\frac{2}{3} (1-y)^{\frac{3}{2}} + C = -\frac{2}{3} \left(1 - \frac{1}{x}\right)^{\frac{3}{2}} + C$$

$$(d) \int x^3 \sqrt{x^2+1} dx \quad y = x^2+1$$

$$dy = 2x dx$$

$$\int x^3 \sqrt{y} \cdot \frac{dy}{2x}$$

$$\frac{1}{2} \int x^2 \sqrt{y} dy$$

$$\frac{1}{2} \int (y-1) \sqrt{y} dy = \frac{1}{2} \int (y^{3/2} - y^{1/2}) dy$$

$$= \frac{1}{2} \times \frac{2}{5/2} y^{5/2} - \frac{2 \times 3}{2} y^{3/2} + C$$

$$\frac{1}{5} y^{5/2} - 3 y^{3/2} + C$$

$$\frac{1}{5} (x^2+1)^{5/2} - 3 (x^2+1)^{3/2} + C$$

Q6 (a) $y = x^2 - 2x$, $y = x$ $x = x^2 - 2x$

$$\int_0^3 (x - (x^2 - 2x)) dx \quad \left| \begin{array}{l} \Rightarrow 0 = x^2 - 3x \\ 0 = x(x-3) \\ x = 0, 3 \end{array} \right.$$

$$\int_0^3 (x - x^2 + 2x) dx = \int_0^3 (3x - x^2) dx$$

$$\left. \frac{3x^2}{2} - \frac{x^3}{3} \right|_0^3 = \left(\frac{3 \times 9}{2} - \frac{27}{3} \right) - (0 - 0)$$

$$\frac{27}{2} - \frac{27}{3} = \frac{27}{6}$$

(b)

$$y = x^2$$

$$y = -x^2$$

UPLOADED BY AHMAD JUNDI

$$x^2 = -x^2 + 4x \Rightarrow 2x^2 - 4x = 0$$

$$x^2 - 2x = 0$$

$$x(x-2) = 0$$

$$x = 0, x = 2$$

$$\int_0^2 (-x^2 + 4x - x^2) dx$$

$$= \int_0^2 (-2x^2 + 4x) dx = -\frac{2x^3}{3} + \frac{4x^2}{2}$$

$$= -\frac{2 \times 8}{3} + 2 \times 4$$

$$= -\frac{16}{3} + \frac{8 \times 3}{3} = \frac{8}{3}$$

$$(c) x = y^2, x = 3 - 2y^2$$

$$y^2 = 3 - 2y^2$$

$$0 = 3 - 3y^2$$

$$y = \pm 1$$

$$\int_{-1}^1 (3 - 2y^2 - y^2) dy = \int_{-1}^1 (3 - 3y^2) dy$$

$$3y - y^3 \Big|_{-1}^1$$

$$= (3 - 1) - (-3 + 1)$$



