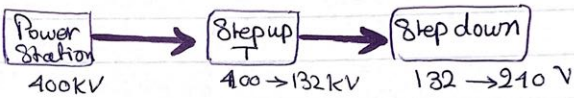
CHAPTER 2

Transformers

▷ Transformer : a device that changes electrical power from a certain frequency and voltage level to same frequency but other voltage level. (AC → AC)

→ Better transmission efficiency for long distance



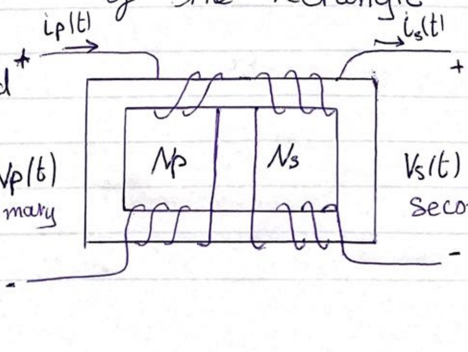
▷ Types :-

1. Step up transformers (Unit)
2. Step down transformers (Substation)
3. Distribution transformers
4. Special Purpose transformers PT , CT (Power, voltage; current)
5. Isolation and impedance matching transformers
   used for protection $N_1 = N_2$

▷ Forms of Transformers

1. Core form : • Simple rectangular (High reliability)
   • Windings wrapped around two sides of the rectangle

Advantage :- If an error occurred → the damage occurred in one transformer we just replace it
But in Shell form it's all damaged
→ Exists at company



compact form

can be used for 3phase connection

→ used for long distance

Exists at load

2. Shell form:-
- Three legged
- Windings wrapped around the center leg : on on Top of the other
- low voltage winding inner

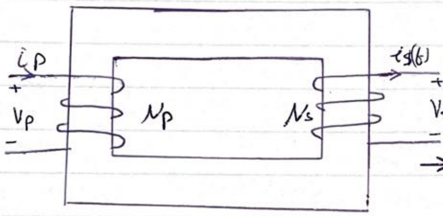
why? to simplify problem of insulating the high voltage windings from the core and to reduce leakage flux

▷ Ideal Transformer → No losses device

$$\frac{V_p(t)}{V_s(t)} = \frac{N_p}{N_s} = a$$

N: number of turns

a: turns ratio



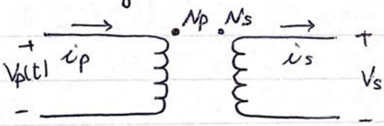
magnitude is changed

$$\frac{i_p(t)}{i_s(t)} = \frac{1}{a}$$

→ Phase angle is not affected

→ frequency not affected

Schematic Symbol of a transformer



Faraday's law

$$V_p = N_p \frac{d\phi}{dt}$$

$$V_s = N_s \frac{d\phi}{dt}$$

▷ Dot convection

for Voltage :- V_1, V_2 are both + or − at the dot → plus sign otherwise use negative

for Current :- I_1, I_2 are both out or into the dot → minus sign otherwise use positive

dot دخول التيار على

coil قبل دخوله على

▷ Power in transformer (Ideal) [Watt]

used by R ← $P_{in} = P_{out}$ (electrical → to any energy)

$I \quad V$ →

$V_p I_p \cos\theta_p = V_s I_s \cos\theta_s$ — equal $= \theta$

P, real Power

used by L, C ← $Q_{in} = Q_{out}$ [VAR] (magnetic field → magnetic field)

$V_p I_p \sin\theta_p = V I \sin\theta_s$

$L \rightarrow j\omega L$

$C \rightarrow \frac{1}{j\omega C}$

Q, reactive Power

Revision

Note :-

S, Complex Power (apparent)

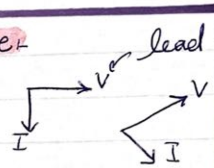
$S = \vec{V}_{rms} \vec{I}^{*}_{rms}$

$= V_{rms} \angle\theta_v \; I_{rms} \angle -\theta_i$

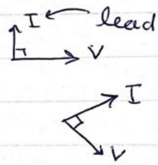
$= V_{rms} I_{rms} \angle \theta_v - \theta_i$

Note:-

$+jQ$ L :



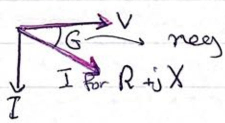
$-jQ$ C :

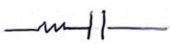


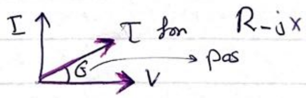
$S = P + jQ = V_{rms} I_{rms} \underbrace{\cos\theta}_{PF} + V_{rms} I_{rms} \sin\theta \, j$

Type of loads

RL 



RC 

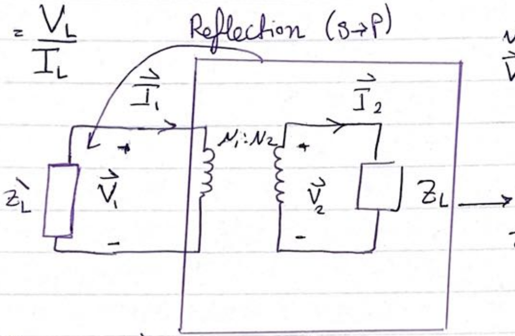


▷ Impedance Transformation

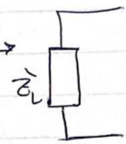
Impedance: ratio of the phasor voltage across it to the phasor current flowing through it

Ω

$Z_L = \dfrac{V_L}{I_L}$



Note:- $\vec{V} = V \angle \theta$ ← phase angle
↑ magnitude



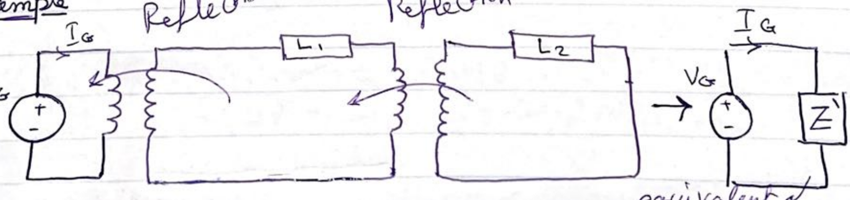
$$Z_L' = \frac{\vec{V}_1}{\vec{I}_1} = \frac{a\vec{V}_2}{\frac{\vec{I}_2}{a}} = a^2 \frac{\vec{V}_2}{\vec{I}_2} = a^2 Z_L$$

- Reflection (P→S)

$$Z_L' = \frac{1}{a^2} Z_L$$

→ Note we can use this to reduce complexity of questions

Example



equivalent
$I_{load}, V_{load} \neq I_G, V_G$

▷ losses

for a real transformer there are losses to be considered

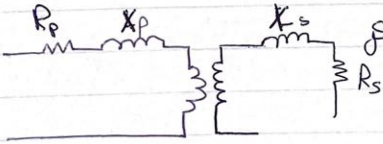
Winding ← 1. Copper losses : Resistive heating losses in windings (R_p and R_s)
R_p and R_s

Core R_c

2. Eddy current losses :- " " " in the core (R_c)
↳ $\propto V^2$ voltage applied to transformer

3. Hysteresis : associated with the rearrangement of the magnetic domains in the core during each half cycle.
Complex, non linear function of the voltage applied to the transformer

Winding ← 4. leakage flux : fluxes and which escape the core and pass through only one of the transformer windings are leakage fluxes. They then produce self inductance in the primary and secondary coils
X_p and X_s



بحكي تمثيلها على شكل

R_p, R_s → copper losses
X_p, X_s → leakage flux

$$e_{Lp}(t) = N_p \frac{d\phi_{Lp}}{dt} = N_p^2 P \frac{di}{dt} \quad / \quad e_{Ls}(t) = N_s \frac{d\phi_{Ls}}{dt} = N_s^2 P \frac{di}{dt}$$

leakage flux is proportional to current flow in primary and secondary windings →

leakage flux

so $\Phi_{Lp} = (\mathcal{P}N_p)\, i_p$ $\qquad \Phi_{Ls} = (\mathcal{P}N_s)\, i_s$

where: $\mathcal{P} = 1/\mathcal{R}$ permeance of flux path

Re arranging equations:-

self inductance in primary coil $= L_p$

$$e_{Lp}(t) = N_p \frac{d}{dt}(\mathcal{P}N_p)\, i_p = \boxed{N_p^2\mathcal{P}}\, \frac{di_p}{dt}$$

$e_{Lp}(t) = L_p \dfrac{di_p}{dt}$ and similarly $e_{Ls}(t) = L_s \dfrac{di_s}{dt}$

Core excitation effects :- → Primary circuit only

experimental

Magnetization current : $i_m \propto$ Voltage applied to core and lagging X_m by 90° across primary voltage source (in the unsaturated region)

+ hysteresis current

Eddy current (core loss current) : $i_{h+e} \propto$ voltage applied

~ ~ in phase with R_c across ~ ~

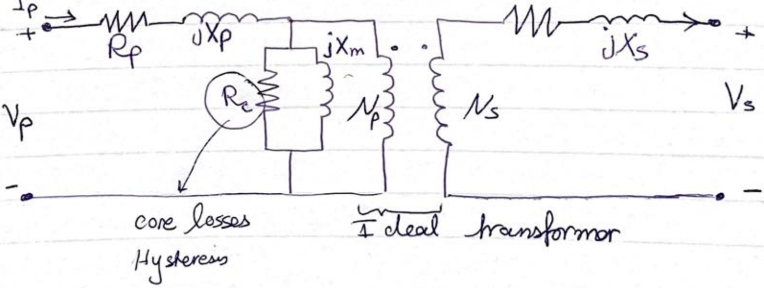
~

$I_{h+e} \rightarrow V_p$

$I_m \downarrow$

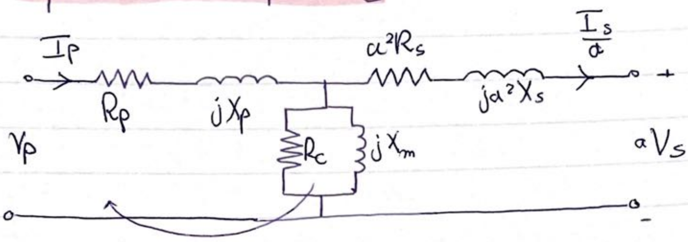
X_m, R_c are applied voltage modeled as loads

▷ Equivalent Circuit

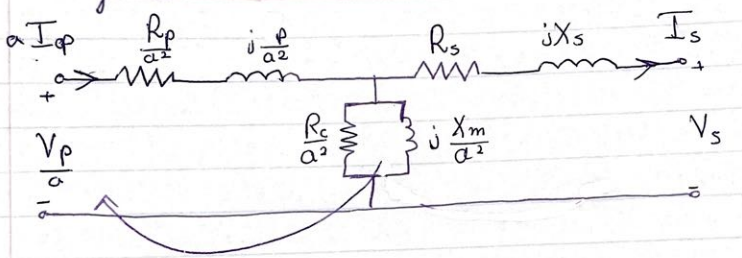


Using impedence Reflection :-

Referred to primary

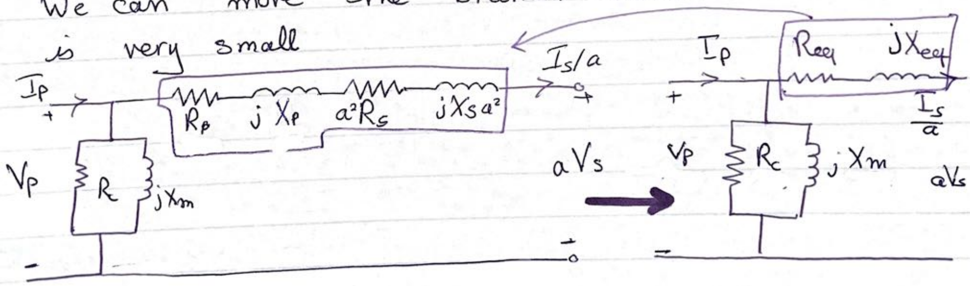


Referred to Secondary

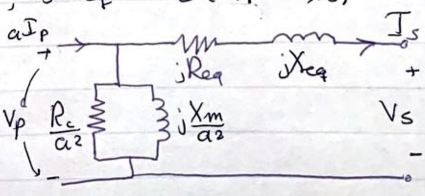


Note:-

We can more the branch since i across it is very small



$R_{eq} = R_p + a^2R_s$, $jX_{eq} = j(X_p + a^2X_s)$

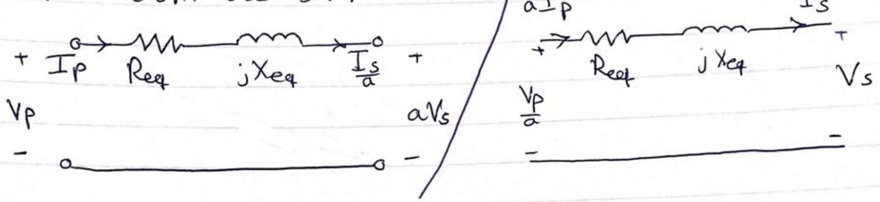


for Secondary :-

$R_{eq} = \frac{R_p}{a^2} + R_s$

$X_{eq} = \frac{X_p}{a^2} + X_s$

- For more simplicity consider I_m small, impedence is very high so open circuit (We don't use it)



▷ To find losses (values of transformer model components)

$R_c =$

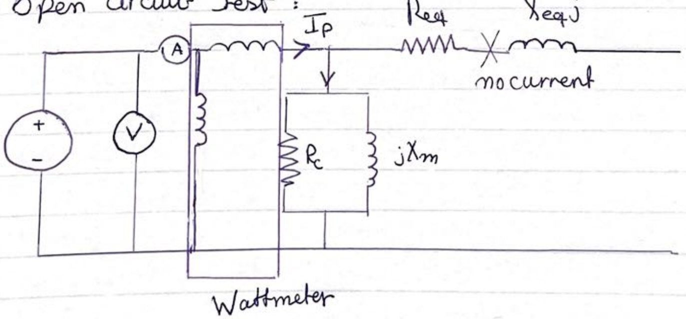
$X_m j =$

$R_{eq} =$

$X_{eq} j =$

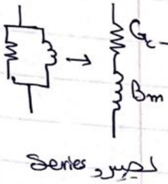
→ We use open circuit test and short circuit test

- Open circuit test :



So I_p goes to R_c, jX_m only and we can find them How? page 9

- We notice that R_c and jX_m are parallel



→ Admittance of the branch $Y = Z^{-1}$

$$Y = \frac{1}{R_c} + \frac{1}{jX_m} = G_c - jB_m = Y \angle -\theta$$

angle How to find it?

How does open circuit works?

W → measures real Power
A → " Current
V → " Voltage

from these we find θ

$$P = V_{oc} I_{oc} \cos\theta$$
$$P = V_{oc} I_{oc} \, PF$$

→ Magnitude of $Y = \frac{I_{oc}}{V_{oc}}$ (refered to primary)

→ Angle $\theta = \cos^{-1} \frac{P_{o.c}}{V_{oc} I_{o.c}}$

- So Y is found, thus G_c, B_m can be calculated

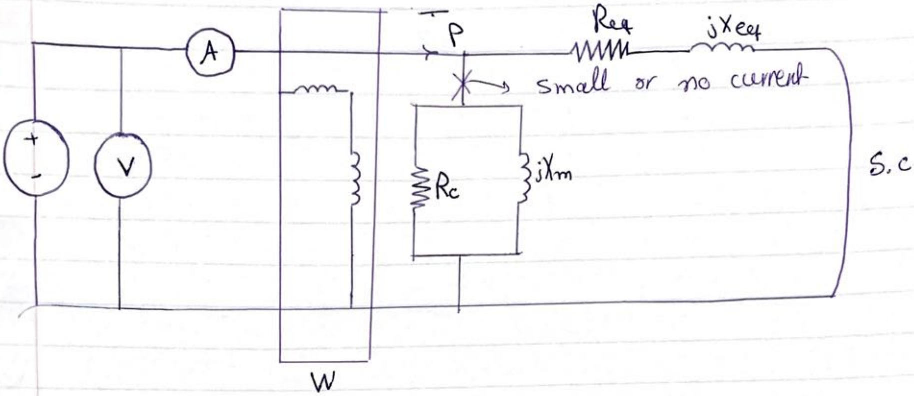
Question: Why is θ negative? Because the transforemer is an inductor and so angle is :-

$$0 < \cancel{\theta_v} - \theta_i < 90$$
$$0$$
$$0 < -\theta_i < 90$$

→ so θ_i need to be negative

$$Y = \frac{I \angle -\theta_i}{V \angle 0} = \frac{I}{V} \angle -\theta_i$$

- Short circuit test



$$Z = R_{eq} + X_{eq}j = \frac{V_{s.c}}{I_{s.c}} \angle \theta$$

measured from V, A

measured from W

$$\theta = \cos^{-1} \frac{P_{s.c}}{I_{s.c} V_{s.c}}$$

Question: why is θ positive : Because $I_{s.c} = |I_{s.c}| \angle -\theta$ as explained previously meaning that $Z = \frac{V_{s.c} \angle 0}{I_{s.c} \angle -\theta}$ and so $Z = \frac{V_{s.c}}{I_{s.c}} \angle \theta$

Note:

In open circuit test: it is preformed on the low voltage side because devices gives limited values of voltage (to reduce voltage to be measured) measured

(Same for short circuit meaning that we reduce the current value to be measured by doing it on high voltage side)

▷ Per unit System

Voltage, current, power and impedance are measured in decimal fraction of same base value

$$\longrightarrow \quad \text{Quantity}_{\text{perunit}} = \frac{\text{Actual value}}{\text{Base value of Quantity}}$$

قيمة موحدة لكل ال System

Advantages

1) simplify calculations

2) Equivalent circuit can be simplified (like we did in impedance reflection to get rid of winding)

Usually base value of Power and Voltage

only magnitude so all of them are of unit VA

$$P_{app} = P_{base\,1\phi} = Q_{base\,1\phi} = S_{base\,1\phi} \rightarrow \text{Given}$$

one phase:

$$I_{base} = \frac{S_{base\,1\phi}}{V_{base\,\underline{\underline{LN}}}}$$ ← line to neutral (single phase)

$$Z_{base} = R_{base} = X_{base} = \frac{V_{base\,LN}}{I_{base}}$$

$$Y_{base} = G_{base} = B_{base} = \frac{1}{Z_{base}}$$

For three phase:-

$$S_{base\,1\phi} = \frac{S_{base\,3\phi}}{3}$$

$$V_{base\,LN} = \frac{V_{base\,LL}}{\sqrt{3}}$$ ← line to line

$$I_{base} = \frac{S_{base\,3\phi}}{\sqrt{3}\,V_{base\,LL}}, \quad Z_{base} = \frac{V_{base\,LL}^2}{S_{base\,3\phi}} = \frac{V_{base\,LN}}{I_{base}}$$

$$R_{base} = X_{base} = Z_{base} = \frac{1}{Y_{base}}$$

▷ Voltage regulation

Full load voltage regulation: It is used to compare the output voltage at no load with the output Voltage at full load

$$VR = \frac{V_{S,nl} - V_{S,fL}}{V_{S,fL}} \times 100\%$$

For Ideal Transformer

$VR = 0\%$

$V_{nL} = V_{fL} = \frac{V_P}{a}$

• It is used to compare transformers

At the no load, Primary side or Secondary :- $V_S = \frac{V_P}{a} = V_{nL}$

$V_{S,nL}$: Voltage at no load (output)

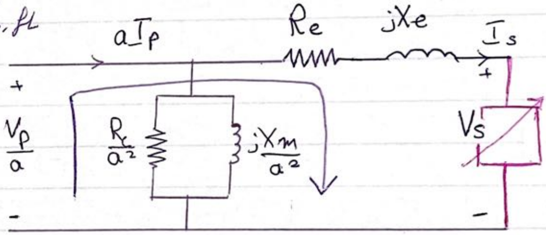
$V_{S,fL}$: Voltage at full load (")

احسن اشي يعني دائماً أقل نسب voltage

• To calculate $V_{S,nL}$, $V_{S,fL}$

$$-\frac{V_P}{a} + I_S(R_e + X_e j) + V_S = 0$$

No load : $\boxed{\frac{V_P}{a} = V_S}$

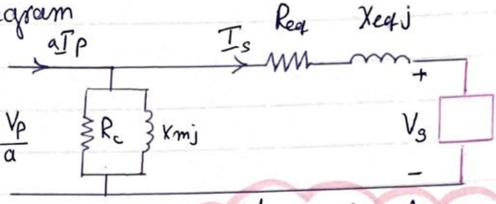


▶ With load (As Drawn with this)

$$V_S = V_{P/a} - \underbrace{I_S(Z_e)}_{\text{voltage drop}}$$

load ↑ → current ↑

▷ Transformer phasor diagram



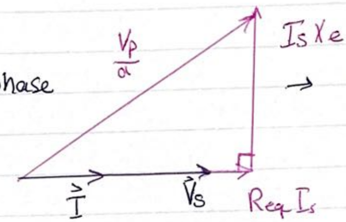
Types of loads:-

1. Resistive load:- (R)

Taking $\vec{V}_s$ as reference

$$-\frac{\vec{V}_p}{a} + \vec{I}_s(R_{eq} + X_{eq}j) + \vec{V}_s = 0$$

R load means: V_s, I_s are in phase



$\rightarrow V_{p}/a > V_s$

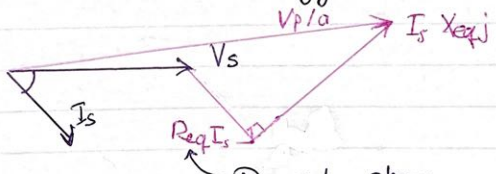
VR > 0 (smaller than VR lag)

2. Inductive load:- (R + Xj) (lagging PF) → VR > 0

$V_p/a > V_s$

$$-\frac{V_p}{a} + \vec{I}_s(R_{eq} + X_{eq}j) + \vec{V}_s = 0$$

L load means:- I_s is lagging from V_s

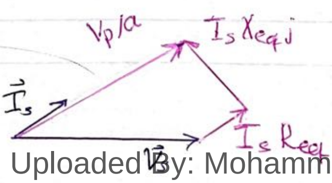


Does not change direction

3. Capacitive load:- (R − Xj)

C load: I_s is leading (leading PF) → VR < 0

$V_s > V_p/a$



→ When we have high capacitive load, V_s is large and so since current moves from high voltage to lower voltage it will reverse it's direction.

- This is undesirable and needs to be prevented

▷ Transformer efficiency

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{P_{out}}{P_{loss} + P_{out}} \times 100\%$$

For transformers generators and motors

P_{loss} → Copper losses, Core losses, Eddy current losses

for a transformer:- $\boxed{\eta = \frac{V_s I_s \cos\theta}{P_{cu} + P_{core} + V_s I_s \cos\theta} \times 100\%}$

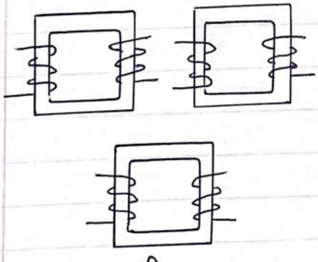
▷ Three-phase transformer calculations, wye Y or delta Δ

Important Example

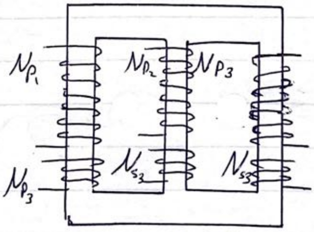
To connect a three phase transfer we use:

Core form

Shell form



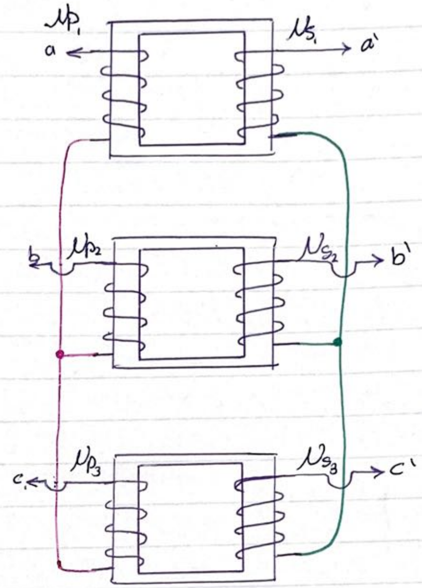
Core form



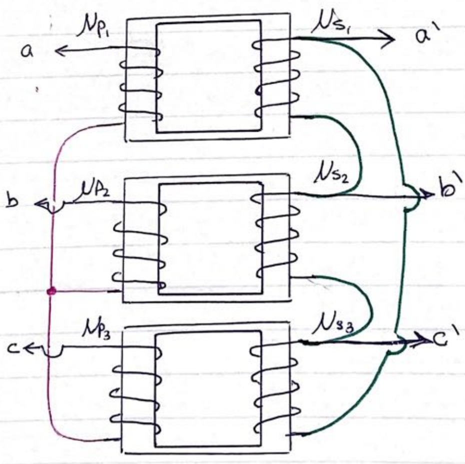
Shell form

Connections:-

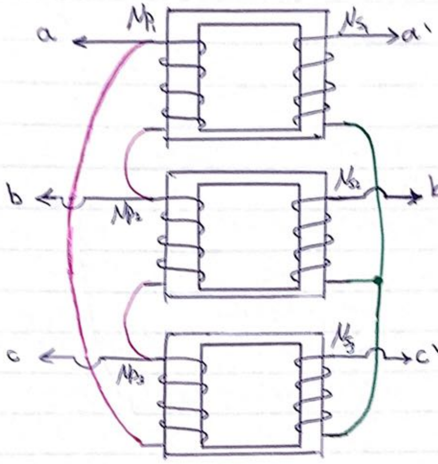
Y-Y



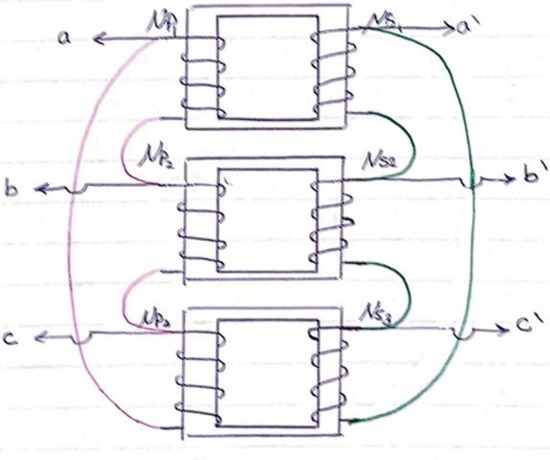
Y- Δ



$\Delta - Y$:-



$\Delta - \Delta$:-



Impedance, voltage regulation, efficiency and similar calculation for 3-phase are done on a per-phase basis using same techniques

For Δ $\quad V_{\phi P} = V_L \quad$ (phase primary) (line) $\quad I_{\phi P} = \frac{I_L}{\sqrt{3}} \quad S_{\phi P} = \frac{S}{3}$

For Y $\quad V_{\phi P} = \frac{V_L}{\sqrt{3}} \quad$ (phase second) $\quad I_{\phi P} = I_L \quad S_{\phi P} = \frac{S}{3}$

○ Turns ratio $= \frac{V_{LP}}{V_{LS}}$ (primary) (secondary)

for Y–Y
$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}\,V_{\phi P}}{\sqrt{3}\,V_{\phi S}} = a$$

for Δ–Δ
$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{V_{\phi S}} = a$$

for Y–Δ or Δ–Y
$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}\,V_{\phi P}}{V_{\phi S}} = \sqrt{3}a$$
$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{V_{\phi S}} = \frac{1}{\sqrt{3}}a$$