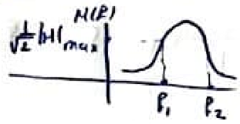
• Definitions for B.W:

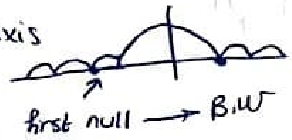
1 Absolute B.W ⟶ 1 Baseband ⟶ $W(B)$, 2 Band pass ⟶ $f_2 - f_1$

2 3dB B.W (Half-power point) ⟶ $\frac{1}{\sqrt{2}} |M(f)|_{max}$



3db = $f_2 - f_1$

3 NULL-to-NULL B.W ⟶ first null with x axis



→ Baseband

4 95% power/energy B.W:

• 3 types of modulation : 1) • Normal AM, • DSB-SC, • SSB } AM 2) Angle Modulation 3) Frequency modulation.

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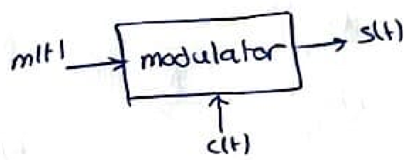
1 Normal AM / Amp. mod.

AM: process in which the carrier amp. is varied linearly with the message.

→ in t-domain

• Normal AM : $s(t) = [1 + K_a m(t)]\, c(t)$



K_a → modulator sensitivity. ($\frac{1}{\text{volt}}$)

1 envelope detector same as message m(t).
2 freq. of s(t) same as c(t).

• envelope of s(t) ⟶ $|A(t)| = |A_c + K_a A_c m(t)|$ → linear relation.

⇒ envelope of s(t) is the same as the message: when;
(1) $f_c \gg f_m$ (f_c at least 10 times greater than f_m)
(2) $|K_a A_m| < 1$ → or it will cause distortion (overmodulation)

define $\mu = K_a A_m$ (mod. index) → for single tone signal modulation

$A_{max} = A_c(1+\mu)$, $A_{min} = A_c(1-\mu)$

or $\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$

• if 1 & 2 are satisfied, we can demod. m(t) from s(t). → o.w → distortion.
⇒ we can use envelope detector.

• $s(t) = \left(1 + \frac{\mu}{A_m} m(t)\right) c(t) = A_c\left(1 + \mu \cos(2\pi f_m t)\right) \cos(2\pi f_c t)$

→ in f-domain

$S(f) = \frac{A_c K_a}{2}\{M(f-f_c) + M(f+f_c)\} + \frac{A_c}{2}\{\delta(f-f_c) + \delta(f+f_c)\}$

⇒ B.W = 2W → double the freq. of m(t).

$$S(f) = \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] + \frac{A_c\mu}{4}\left[\delta(f-f_c-f_m) + \delta(f+f_m+f_c)\right]$$

$$+ \frac{A_c\mu}{4}\left[\delta(f-f_c+f_m) + \delta(f+f_c-f_m)\right]$$

⇒ in terms of μ

- Power efficiency : $\eta = \dfrac{\sum P_{in} \text{ in the sidebands}}{P_{in} \text{ in the sidebands} + P_{in} \text{ in the carrier}}$ (4)

⇒ for normal AM : $P_c = \dfrac{A_c^2}{2}$

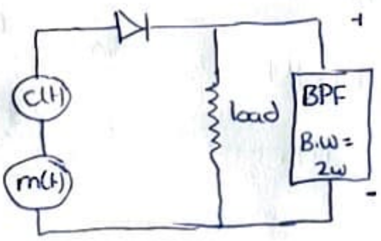
$P_{USB} = \left(\dfrac{A_c\mu}{4}\right)^2 \times 2 = \dfrac{(A_c\mu)^2}{8} = P_{LSB}$

$$\eta = \frac{\frac{(A_c\mu)^2}{4}}{\frac{(A_c\mu)^2}{4} + \frac{A_c^2}{2}} = \frac{\mu^2}{\mu^2+2}$$

- if mod. index $(\mu) = 1 \rightarrow \eta_{max} = \frac{1}{3}$ ($\frac{2}{3}$ → power loss)
- if $\mu = 0 \rightarrow$ no modulation.

- Normal AM is not power efficient & B.W efficient.

- Generation of normal AM :

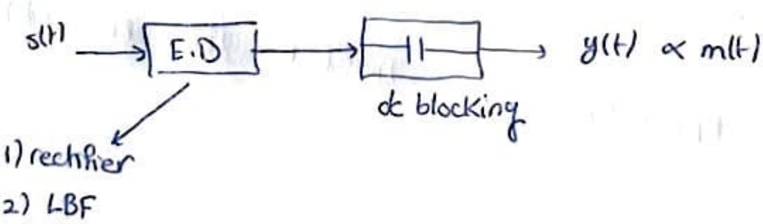


$s(t) = \frac{A_c}{2}\left[1 + \frac{4}{\pi A_c} m(t)\right] \cos(2\pi f_c t)$

BPF centered at f_c

$K_a = \frac{4}{\pi A_c}$

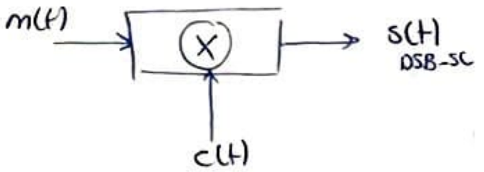
- Demodulation :



- $\frac{1}{f_c} \ll \tau = RC \ll \frac{1}{W}$

② Double sideband supressed carrier (DSB-SC)

- by product modulator:



$s_{DSB-SC}(t) = \underline{m(t)\,c(t) = A_c\, m(t) \cos(2\pi f_c t)}$

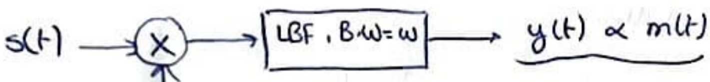
⇒ we can't use envelope detector: → this will cause distortion → we will use <u>coherent demod.</u>

- no impulses at f_c (carrier) • power η = 100%. • B.W for s(t) = $2w$

- DSB-SC = (LSB + USB) only.

$$\Rightarrow S_{DSB-SC}(f) = \frac{1}{2} M(f-f_c) + \frac{1}{2} M(f+f_c)$$

- DSB-SC demodulation (coherent demod / synchronized)



(local oscillator) → $\bar{c}(t) = \bar{A}_c \cos(2\pi \bar{f}_c t + \phi)$

- missynchronization → 1) $\bar{f}_c \neq f_c$ 2) $\phi \neq 0$ 3) $\bar{A}_c \neq A_c$

1) case 1 : perfect coherent demod. → when $A_c \neq \bar{A}_c$

$\Rightarrow y(t) = \frac{A_c \bar{A}_c}{2} m(t)$ → recovered without distortion.

2) case 2 : non-coherent demod → ① constant phase shift → $\phi \neq 0$

$\Rightarrow y(t) = \frac{A_c \bar{A}_c}{2} \cos\phi\, m(t)$ → unless $\phi \neq \frac{\pi}{2}$ → it $y(t) \propto m(t)$ <u>with some attenuation.</u>

- if $\phi = \frac{\pi}{2}$ → <u>message disappears.</u>

② $f_c \neq \bar{f}_c$

$y(t) = \frac{A_c \bar{A}_c}{2} m(t) \cos(2\pi \Delta f t)$ → distortion. → $y(t) \neq m(t)$

- Generation → using ring modulator.

[3] Single sideband (SSB):

$$s(t) = A_c\, m(t) \cos(2\pi f_c t) \pm A_c\, \hat{m}(t) \sin(2\pi f_c t)$$

$\hat{m}(t)$ ↳ hilbert transform (phase shifter)

$+ \rightarrow$ LSB , $- \rightarrow$ USB

- SSB Generation : ① through freq. discrimination. → 1) DSB - SC
  2) BPF

$$S_{SSB}(f) = S_{DSB-SC}(f) * H(f)$$

$m(t) \rightarrow B.W = w$

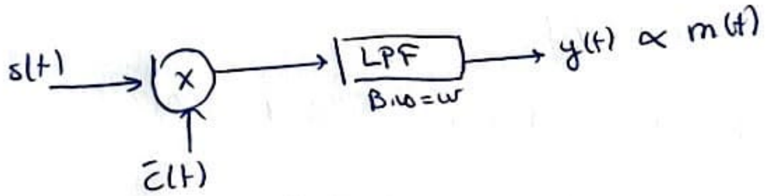
$s(t)_{DSB-SC} \rightarrow BW = 2w$

$H(f) \rightarrow B.W = w ,\ g = 1$

$S(t)_{SSB} \rightarrow BW = W$

[2] phase discrimination

- SSB demodulation → coherent demodulator.



① $A_c \neq \tilde{A}_c$

$$y(t) = \frac{A_c \tilde{A}_c}{2} m(t) \rightarrow y(t) \propto m(t)$$

② $\phi \neq 0$

$$y(t) = \frac{A_c \tilde{A}_c}{2} \left(m(t)\left(\cos(2w_c t + \phi) + \cos\phi\right)\right) - \frac{A_c \tilde{A}_c}{2} \hat{m}(t)\left[\sin(2w_c t + \phi) + \sin\phi\right]$$

$\neq m(t) \rightarrow$ distortion.

② $\tilde{f}_c \neq f_c$

$$y(t) = \frac{A_c \tilde{A}_c}{2} m(t) \cos(2\pi \Delta f t) + \frac{A_c \tilde{A}_c}{2} \hat{m}(t) \sin(2\pi \Delta f t) \neq m(t) \rightarrow \text{distortion.}$$

2 Angle modulation : → 1) Freq. mod. → $S_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int^{t} m(\alpha)\, d\alpha\right]$

→ 2) phase mod. → $S_{PM}(t) = A_c \cos(2\pi f_c t + k_p m(t)]$

⇒ In angle mod → the angle of the carrier is varried according to the message, while the amp. is maintained constant.

$$s(t) = A_c \cos(\theta_i(t))$$

↑ constant; $\theta_i(t)$ ← instant. phase.

→ it needs more B.W

⇒ either the phase or the time derivative of the phase is varried linearly with the message.

$$s(t) = A_c \cos(2\pi f_c t + \theta(t))$$

f_c ↓ carrier freq.; $\theta(t)$ ↳ phase

- instant. freq. $f_i(t) = \frac{1}{2\pi} \frac{d\,\theta_i(t)}{dt}$

$$\frac{1}{2\pi} \cdot \frac{d\,\theta_i(t)}{dt} = \frac{d}{dt}\left\{2\pi f_c t + \theta(t)\right\} \cdot \frac{1}{2\pi} = f_c + \frac{1}{2\pi}\frac{d}{dt}\theta(t) = f_i(t)$$

---

① PM : type of angle mod. → $\theta_i(t)$ is varried linearly with the message.

$$S_{PM}(t) = A_c \cos(2\pi f_c t + K_p m(t))$$

⇒ $\theta_i(t) = 2\pi f_c t + K_p m(t)$

K_p ↳ phase sens. (rad/volt)

⇒ peak - peak phase deviation ⇒ $\Delta\theta_{max} = \theta_i(t) - 2\pi f_c t = |K_p m(t)|_{max}$.

⇒ $f_i(t) = f_c + \frac{1}{2\pi} K_p\left(\frac{d}{dt} m(t)\right)$

⇒ peak freq. deviation $= \frac{K_p}{2\pi}\left(\frac{d}{dt} m(t)\right)_{max}$.

② Freq. mod. (FM) → type of angle mod. → $f_i(t)$ is varried linearly with the message.

$$s_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi K_f \int_0^t m(\alpha)\, d\alpha\right)$$

inst. freq. ⇒ $f_i(t) = f_c + K_f\, m(t)$ (K_f → freq. sens. (Hz/volt))

⇒ peak freq. deviation ⇒ $\Delta f_{max} = |K_f\, m(t)|_{max}$.

⇒ $\theta_i(t) = 2\pi f_c t + 2\pi K_f \int_0^t m(\alpha)\, d\alpha$

⇒ peak phase deviation ⇒ $\Delta\theta_{max} = 2\pi K_f \left|\int_0^t m(\alpha)\, d\alpha\right|_{max}$

- P_{avg} for either PM / FM $= \frac{A_c^2}{2}$

  - $A\cos(\theta + \frac{\pi}{2}) \Rightarrow -A\sin(\theta)$
  - $A\cos(\theta - \frac{\pi}{2}) \Rightarrow A\sin(\theta)$

- for single tone freq. mod. → $m(t) = A_m \cos(2\pi f_m t)$

⇒ $s_{FM}(t) = A_c \cos\left(\omega_c t + \frac{K_f A_m}{f_m} \sin(2\pi f_m t)\right)$

⇒ β = modulation index $= \frac{K_f A_m}{f_m} = \frac{\Delta f}{f_m}$

$$s_{FM}(t) = A_c \cos(\omega_c t + \beta \sin(2\pi f_m t))$$

→ at the best case: $B.W_{FM} \geq B.W_{AM}$ → $B.W_{FM} = B.W_{AM}$

⇒ $B.W = 2\Delta f$ → false start.

- Bessel function: $C_n = \sum J_n(\beta)$, $|J_n(\beta)| = |J_{-n}(\beta)|$

$$s_{FM}(t) = A_c \sum J_n(\beta) \cos(2\pi(f_c + n f_m)t)$$

$$S_{FM}(f) = \frac{A_c}{2} \sum J_n(\beta) \left(\delta(f - f_c - n f_m) + \delta(f + f_c + n f_m)\right)$$

- any term has the form: $A_c \sum J_n(\beta) \cos(2\pi(f_c + n f_m)t)$, $P_{avg} = \frac{A_c^2}{2}$
- power for each term: $\frac{(A_c \sum J_n(\beta))^2}{2}$

B.W → 99% of the power.

· **Carson's Rule:** 98% → B.W for FM / PM

$$B.W = 2(\beta + 1) f_m$$

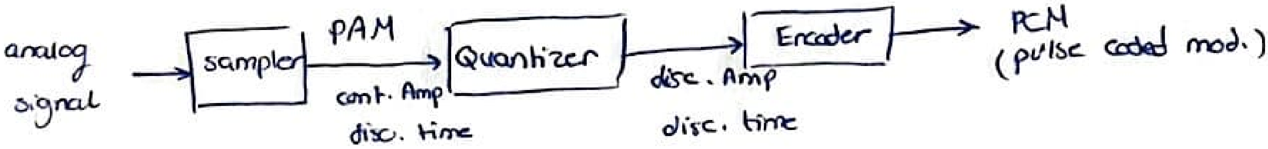
$$= 2(\Delta f + f_m)$$

⇒ when $\beta \ll 1$ (narrowband) ⇒ $B.W = 2 f_m$

⇒ when $\beta > 1$ (wideband) ⇒ $B.W = 2(\beta + 1) f_m$

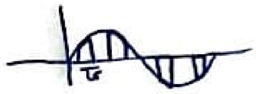
⇒ if we doubled the amp. of m(t) → doubles the B.W

• **Pulse mod.** : Transition from Analog to digital signal.



① Sampler → ideal sampler

→ natural sampler



→ flat-top sampler (sample & hold)

Ts : sampling period.

① Ideal sampling: it must be a bandlimited signal.

⇒ $x(t) \rightarrow \otimes \rightarrow x_s(t) = \sum_{K=-\infty}^{\infty} x(KT_s)\,\delta(t-KT_s)$

$g(t) = \sum \delta(t-KT_s)$

$G(f) = \frac{1}{T_s} \sum_{k} \delta(f-Kf_s)$

$X_s(f) = f_s \sum_{k} X(f-Kf_s)$

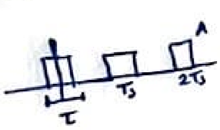
. to recover $X(f)$ from $X_s(f)$ → $f_s \geq 2w$ (nyquiste rate.)

. if the signal $x(t)$ was not bandlimited → we use antialiasing LPF with $B.W = w$ before entering the sampler.

② Natural sampling:

$x(t) \rightarrow \otimes \rightarrow x_s(t) = x(t)\,g_p(t)$

$g_p(t) \rightarrow$



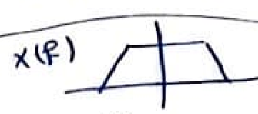
duty cycle $= \frac{\tau}{T_s}$

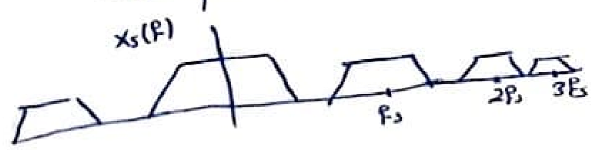
$g_p(t) = C_0 + \sum_{n=1}^{\infty} 2|C_n| \cos(2\pi n f_s t)$

$C_0 = A\frac{\tau}{T_s}$, $C_n = \frac{A}{\pi n} \sin(n\pi \frac{\tau}{T_s})$

$\uparrow n$, $|C_n| \downarrow$

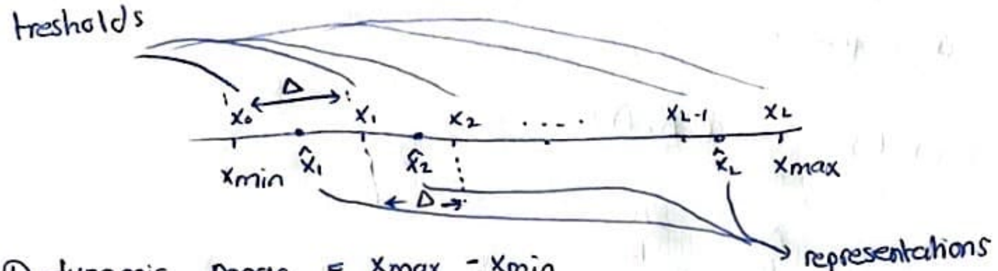
$X_s(f) = C_0 X(f) + \sum_{n=1}^{\infty} |C_n| \left[X(f-nf_s) + X(f+nf_s) \right]$





• Quantization process: converting cont. Amp. → discrete Amp.

$$x(kT_s) \longrightarrow \hat{x}(kT_s)$$



① dynamic range = xmax − xmin

② step size $\Delta = \frac{x_{max} - x_{min}}{L}$, spacing between tresholds / representations.

③ L = # of levels = 2^n , n → number of bits.

• Uniform Quantizer: spacing of L regions are equal to Δ, and spacing between rep. are equal to Δ.

• Quantization error = e = $x - \hat{x}$ = $x - Q(x)$

→ error must be $|e| < \frac{\Delta}{2}$ → $-\frac{\Delta}{2} < e < \frac{\Delta}{2}$

• $T_s = \frac{1}{f_s}$, $f_s \geq 2w$

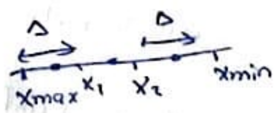
• steps for questions:

① f_s (make sures it is ≥ 2w)

② take sample every T_s & find $x(kT_s)$

③ $\Delta = \frac{x_{max} - x_{min}}{L}$

④ find $Q(kT_s)$ → $Q(x)$ → by using



⑤ find error (e)

• if we decreases # of L → Δ↓ → e↓

• # of bits (n) = $\log_2 L$

• data rates = f_s * # of bits (n) (bit / sec) ⇒ $f_s = 2w$

- Signal - to - Quantization noise ratio (SQNR):

- Distortion $D = \frac{\Delta^2}{12}$ → depends on the design of the Quantizer.

- $SQNR = \frac{P_x}{D}$ → avg. power

Case: if $x(t) = A\cos(2\pi f_c t)$, $(-A, A)$

→ $D = \frac{\Delta^2}{12}$, $\Delta = \frac{2A}{L}$

⇒ $P_x = \frac{A^2}{2}$

⇒ $SQNR = \frac{A^2/2}{\Delta^2/12} = \frac{6A^2}{\Delta^2}$ → $SQNR = \frac{3}{2} L^2$

but $L = 2^n$

⇒ $SQNR = \frac{3}{2} 2^{(2n)}$

- In dB scale → $SQNR_{dB} = 10 \log_{10} SQNR = 6.02n + 1.76$

- Notes: ① $SQNR = \frac{3}{2} \cdot 2^{2n}$ → for every bit added, SQNR increases exp.

② $SQNR_{dB} = 6.02n + 1.76$ → SQNR increases 6.02 for every bit added.

- strong signals → if the message range is equal to the dynamic range.

→ o.w → it is a weak signal.

→ if the signal is $\left(\frac{A}{2}\right)\cos(2\pi f_c t)$

$\frac{A}{2}$ → weak signal → $SQNR_{dB} = 6.02n - 4.77$

$SQNR = \frac{12}{32} L^2 = \frac{12}{32} \cdot 2^{2n}$

→ $SQNR_A \gg SQNR_{\frac{A}{2}}$ → drawbacks of uniform Quantizer.

- non-uniform Quantizer: employs the μ-law. with $\mu = 255$

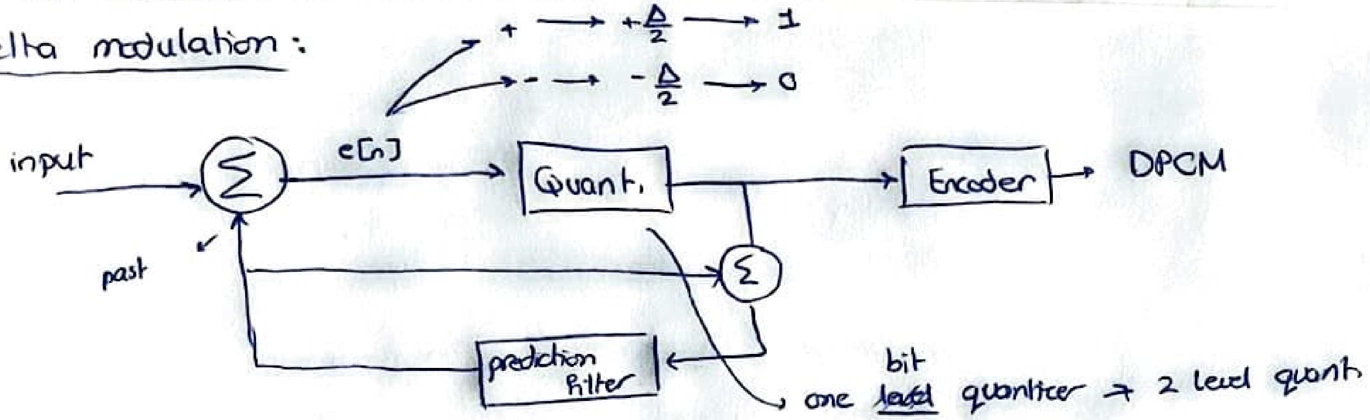
in compressor

$$\frac{y'}{y'_{max}} = \frac{\left(e^{\frac{x'}{x'_{max}}(\ln(1+\mu))}\right) - 1}{\mu \, sgn(x')}$$

in expandor

$$y = y_{max}\left[\frac{\ln\left[1 + \mu\left(\frac{|x|}{x_{max}}\right)\right]}{\ln(1+\mu)}\right] sgn(x)$$

• Delta modulation :



- line encoding : 0 → —— 0V
  1 → ⊓ 5V

0111010 → [Encode] →
↓
PCM
DM
DPCM

⇒ two design considerations :
1) DC component X
2) clocking → self synchronization

① Unipolar non-return to zero :
1 → $s_1(t)$ → A ⊓
0 → $s_2(t)$ → 0 —

→ it has DC value
→ lack of synch. for long 1's / 0's series

② polar non-return to zero :
1 → $s_1(t)$ → A ⊓
0 → $s_2(t)$ → -A ⊔

→ No DC value
→ Poor synch.

③ Polar return to zero :
1 → $s_1(t)$ → A
0 → $s_2(t)$ → -A

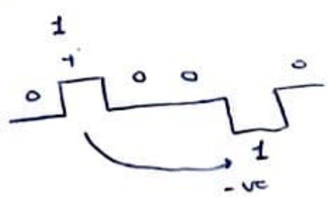
→ No DC
→ synchronization ✓
→ twice the B.W → $B.W \propto \frac{1}{\text{pulse width } (\tau)}$

→ $B.W_{RZ} = 2\ B.W_{NRZ}$

④ Manchester encoding :
1 → $s_1(t)$ →
0 → $s_2(t)$ →

→ if 1 comes after 1 → it needs double the B.W

⑤ Bipolar encoding :
0 → 0
1 → +ve $\xrightarrow{next}$ 1 → -ve



→ No clk X
→ No DC

⑥ 2B1Q : take 2 bit at a time.

00 → -3

01 → -1

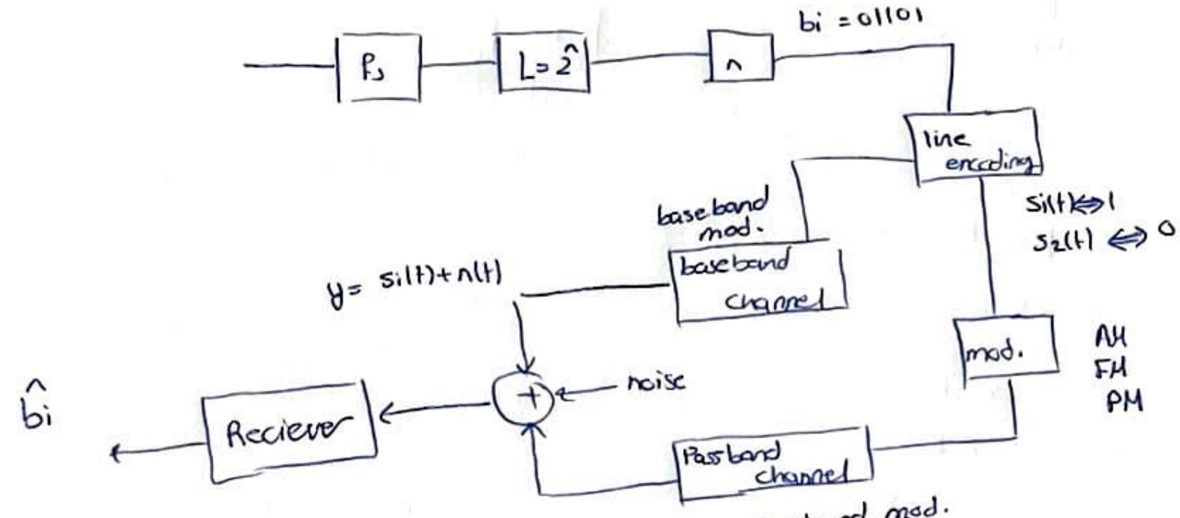
10 → 1

11 → 3

(2 Bipolar to 1 Quaternary)

- **Optimum Reciever:** minimize P_e (Probability of error)

$P_e = P(b_i = 1, \hat{b}_i = 0) + P(b_i = 0, \hat{b}_i = 1)$ → Bit error rate (BER)



$\hat{b}_i \neq b_i$ → error

- the time for each signal is called "symbol duration T"

⇒ data rate $r_b = \frac{1}{T}$

- channel noise is (AWGN), thermal noise. → PSD = $\frac{N_o}{2}$ $(10^{-12} \to 10^{12})$

- Basic elements of the reciever:

① filter → P_e minimized ($h(t)$??) → minimize the effect of the noise.

② sampler → (T ??) → bit duration

③ Treshold comparator → λ, $> \lambda \to ①$, $< \lambda \to ⓪$ → (λ ??)

- $P(b_i = 0) = P(b_i = 1) = \frac{1}{2}$ → $P(s_1(t)) = P(s_2(t)) = \frac{1}{2}$

[1] $h(t) = s_1(T - t) - s_2(T - t)$ $\quad 0 \leq t \leq T$

[2] $KT = t_s$ (sampling time) → $T = \frac{1}{r_b}$

→ $E = V^2 T$

[3] $\lambda^*_{\text{optimum}} = \frac{E_1 - E_2}{2}$

Q →

$P_e^* = Q\left(\sqrt{\frac{\int_0^T (s_1 - s_2)^2 dt}{2N_o}}\right)$ → noise power

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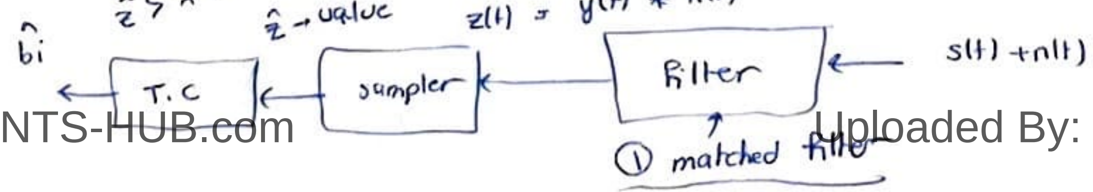
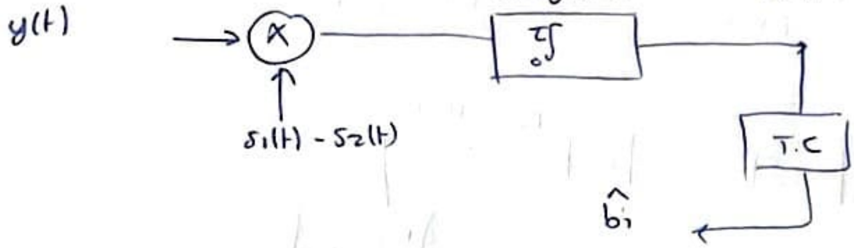
$\hat{z} < \lambda \to 0$

$\hat{z} > \lambda \to 1$



② correlator :

$z(t) = \int_0^t y(t)\,[s_1(t) - s_2(t)]\,dt$



- average energy per bit $E_b = \frac{1}{2}E_1 + \frac{1}{2}E_2$
- $\uparrow$ SNR $\rightarrow$ $P_b^* \downarrow$
- in sinc $\rightarrow$ $R_b \rightarrow$ 90% B.W
  $2R_b \rightarrow$ 95% B.W
  $3R_b \rightarrow$ > 95% B.W

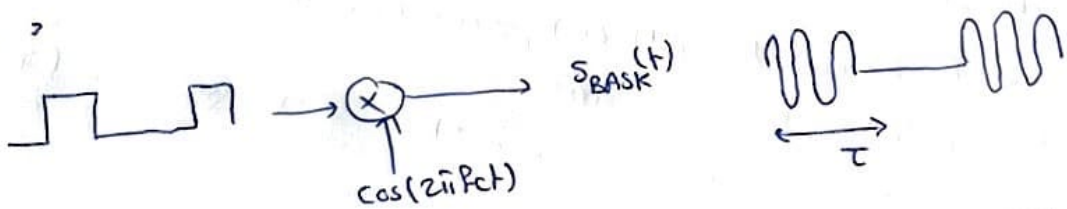
- Binary Amp. shift keying (BASK)

$1 \rightarrow s_1(t) = A\cos(2\pi f_c t) \rightarrow E_1 = \frac{A^2 T}{2}$ $\quad 0 \le t \le T$

$0 \rightarrow s_2(t) = 0$

$E_b = \frac{A^2 T}{4}$, $T = n\frac{1}{f_c}$

- Generation :



$\Rightarrow P_b^* = Q\left(\sqrt{\frac{E_b}{N_o}}\right)$

if we want to send $R_b \rightarrow$ B.W = $2 \cdot R_b$ (90%)

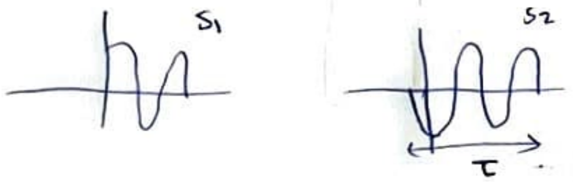
B.W = $4 \cdot R_b$ (95%)

- Binary phase shift Keying (BPSK)

$1 \rightarrow s_1(t) = A\cos(2\pi f_c t) \quad 0 \leq t \leq \tau$

$0 \rightarrow s_2(t) = A\cos(2\pi f_c t + \pi) = -A\cos(2\pi f_c t) \quad 0 \leq t \leq \tau$

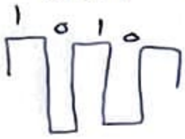
$= -s_1(t)$

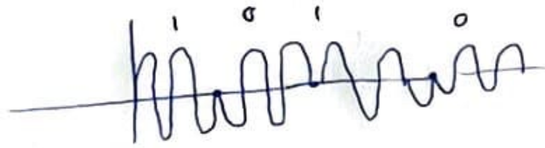


$\tau = \frac{K}{f_c}$

$S_{BPAM}(t) \longrightarrow \otimes \longrightarrow S_{BPSK}$

$\uparrow A\cos(2\pi f_c t)$





↙ polar NRZ

$S_{BPSK}(t) = S_{BPAM}(t) * \cos(2\pi f_c t)$

$E_1 = E_2 = \frac{A^2\tau}{2} = E_b$

$P_b^* = Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$

$90\% \rightarrow 2R_b$ same as BASK

- Binary freq. shift Keying (BFSK)

$b_i \longrightarrow$ [map PNRZ] $\longrightarrow$ [Voltage controlled oscillator] $\longrightarrow$

same as BPSK

$f_1(t) = f_c + \Delta f \rightarrow 1$

$f_2(t) = f_c - \Delta f \rightarrow 0$

$f_1(t) = f_c + (Kf)m(t)$, $m(t) \rightarrow 1,\ -1$; $Kf \rightarrow \Delta f$

B.W 90% $= 2\Delta f + 2R_b$

$1 \leftrightarrow s_1(t) = A\cos(2\pi(f_c + \Delta f)t)$

$0 \leftrightarrow s_2(t) = A\cos(2\pi(f_c - \Delta f)t)$

95% B.W $= 2\Delta + 4R_b$

$P_b^* = Q\left(\sqrt{\frac{E_b}{N_o}}\right)$