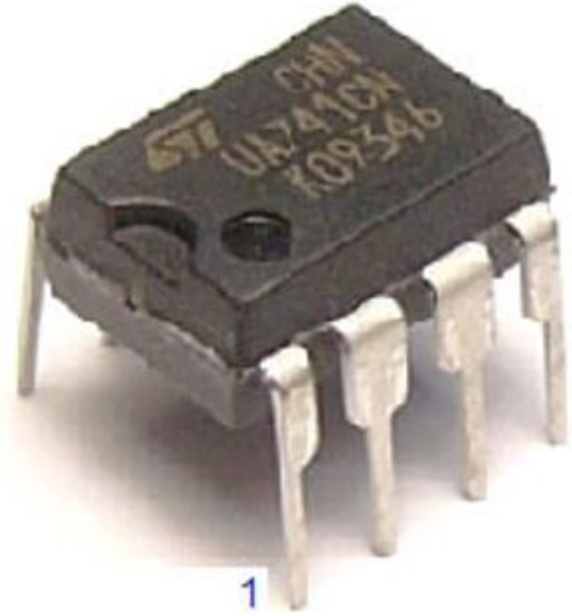
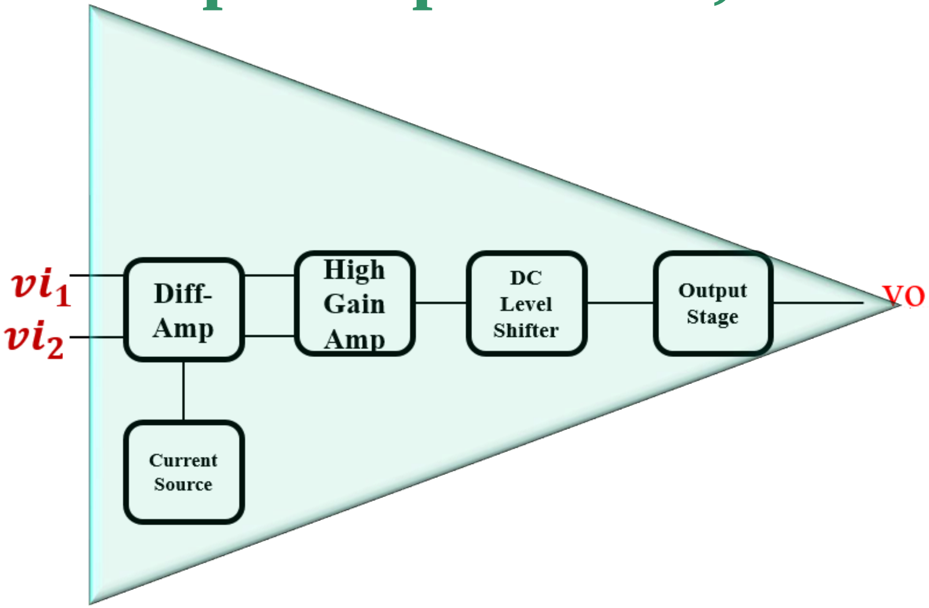
Operational Amplifiers



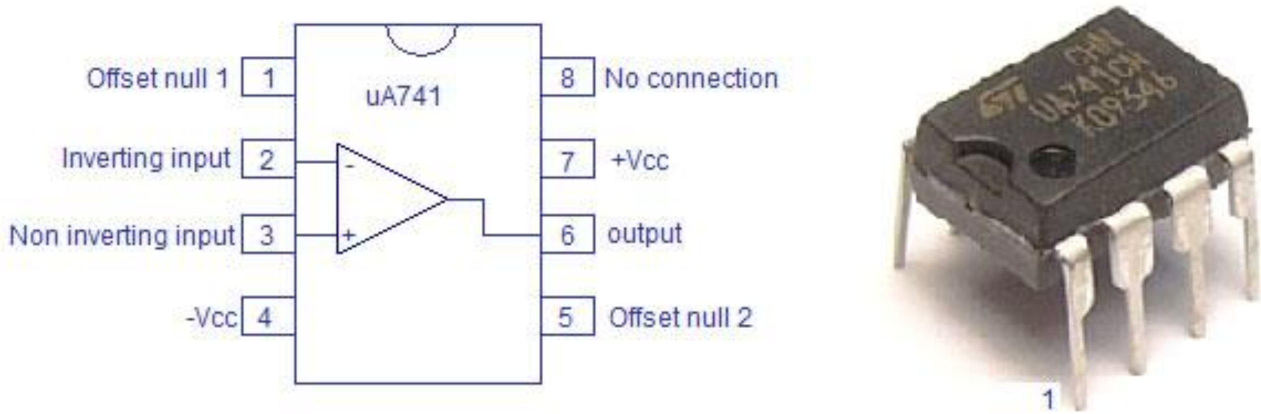
The Operational Amplifier

Designed to do mathematical operations such as addition , subtraction ….

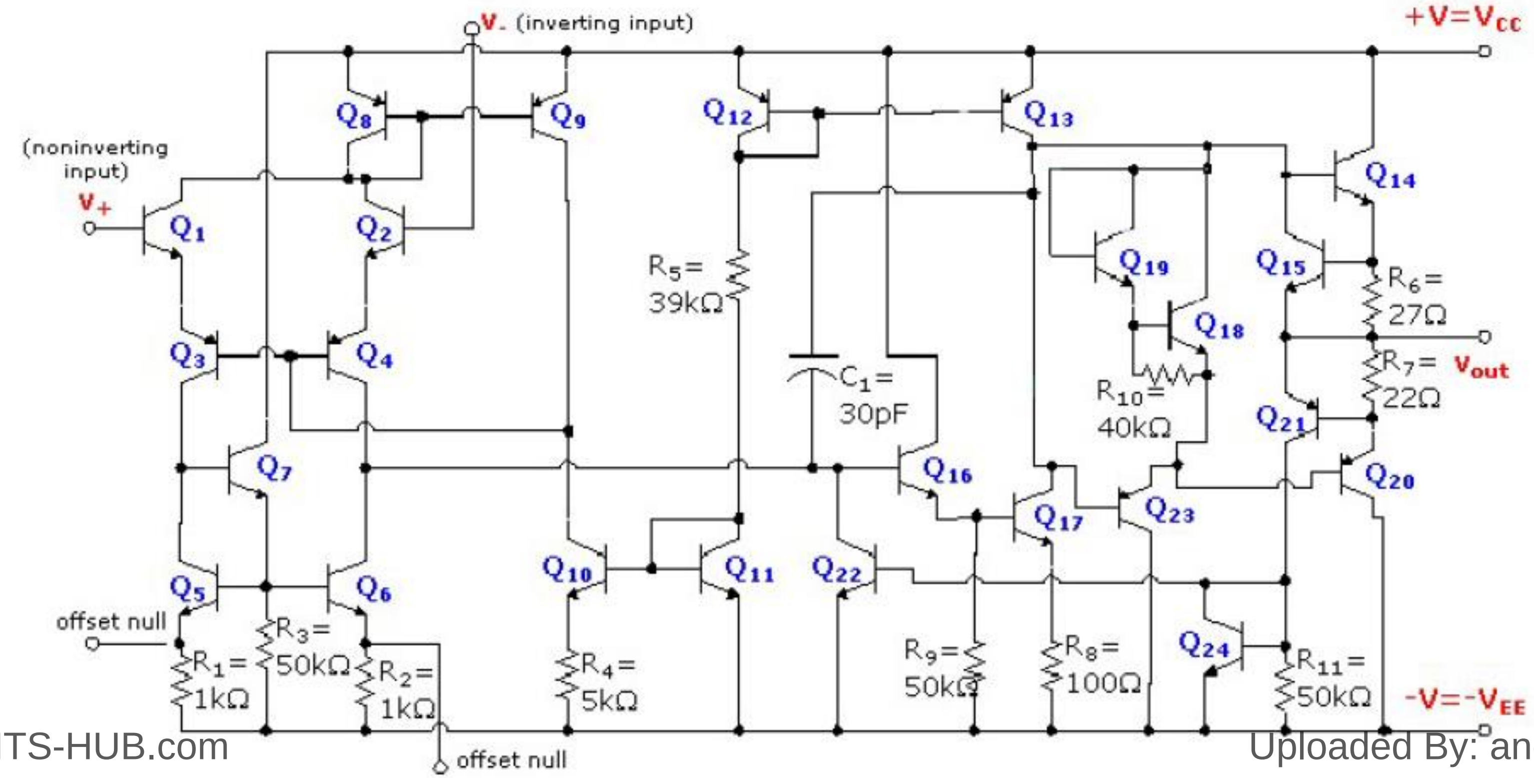
Very high voltage gain ; 200,000
Very High input impedance ; 10M ohm
Very small output impedance ; 75ohm



Operational Amplifiers



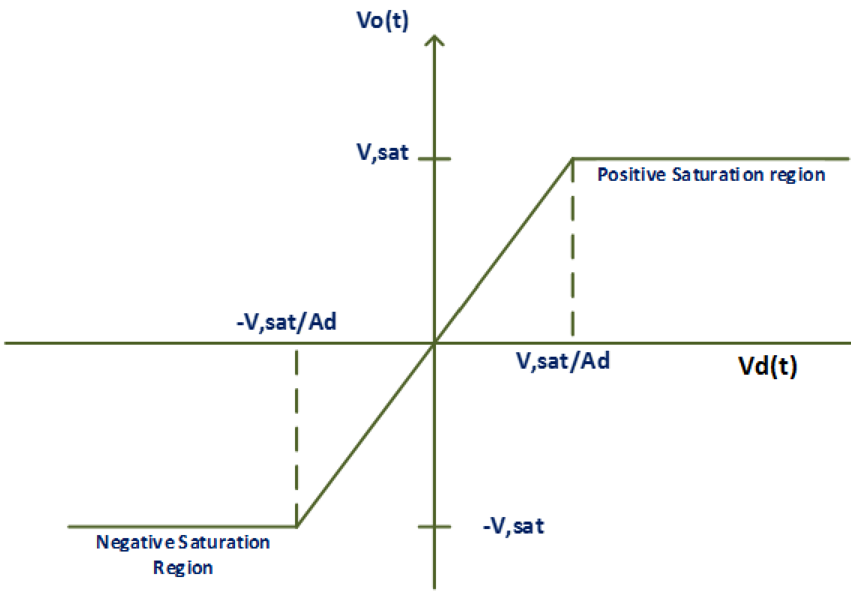
uA741 opamp Pinout and External appearance



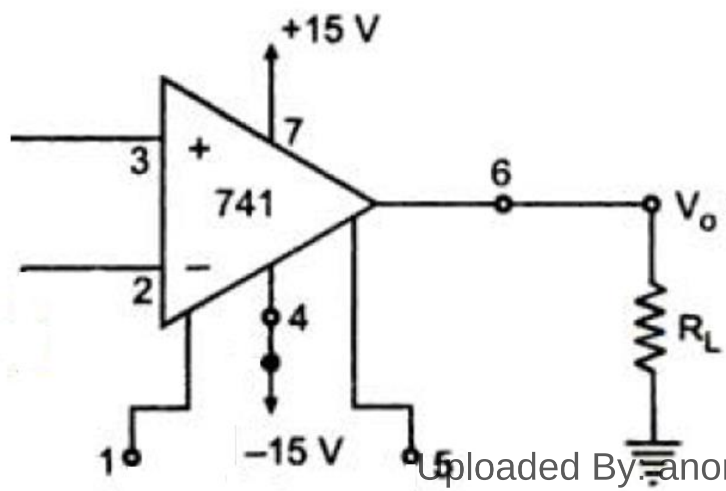
Operational Amplifier

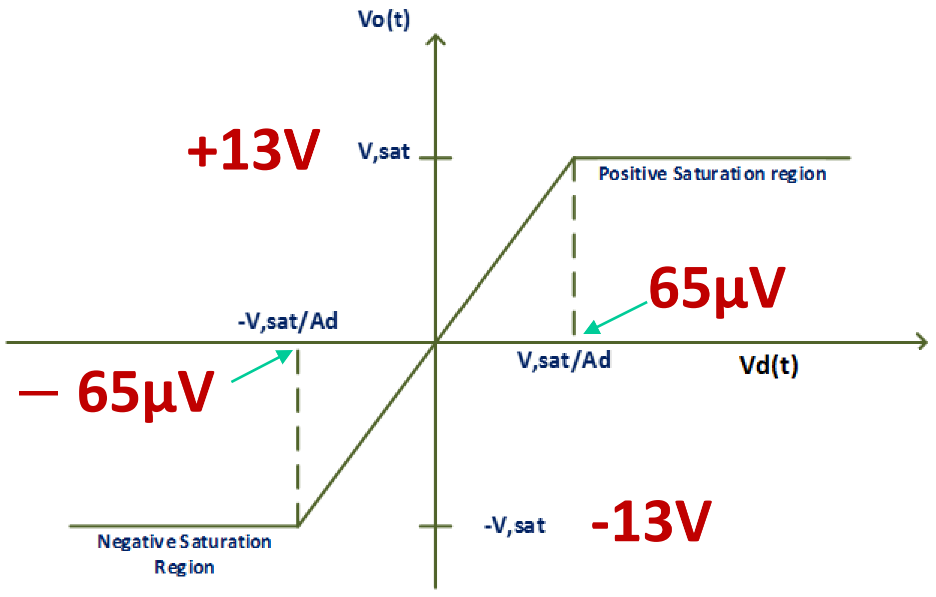
Transfer characteristic Curve:

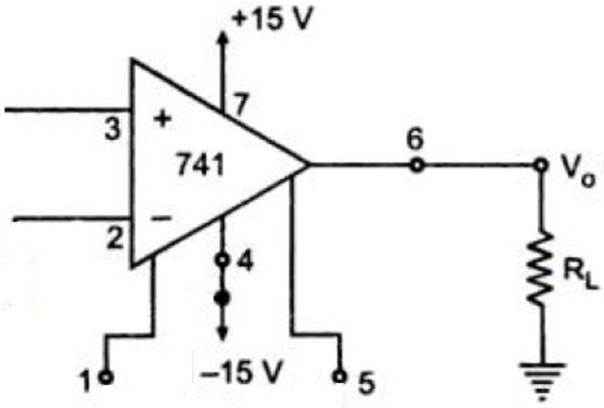
$$V_O = \begin{cases} +V_{sat} & vd > \dfrac{V_{sat}}{A_d} \\ A_d\, vd & \dfrac{V_{sat}}{A_d} > vd > -\dfrac{V_{sat}}{A_d} \\ -V_{sat} & vd < -\dfrac{V_{sat}}{A_d} \end{cases}$$



vd = V(+) - V(-)







vd = V(+) - V(-)

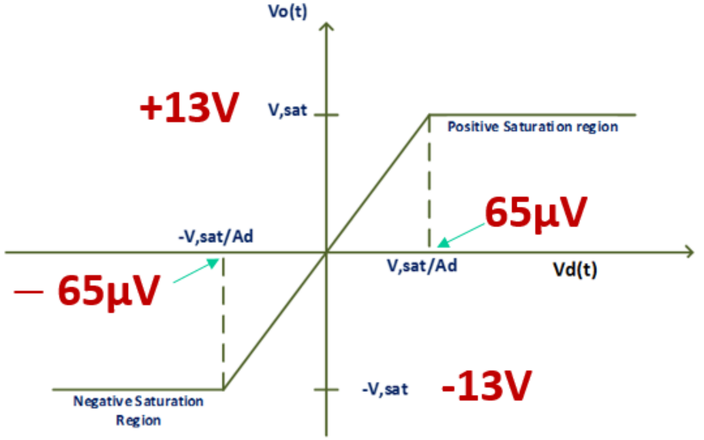
Let $\pm V_{cc} = \pm$ 15V ; $Ad = 200,000$

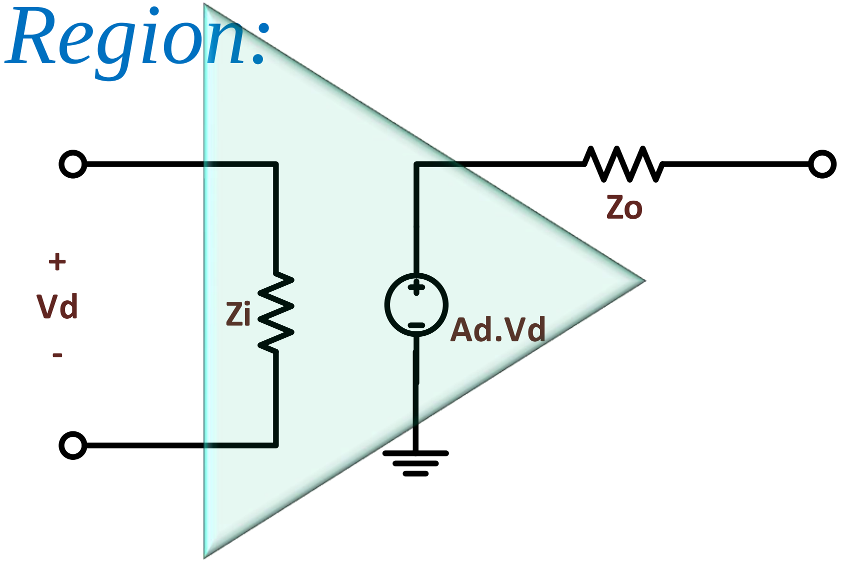
$\therefore \pm V_{sat} = \pm 13V$

$\therefore$ If Vd $> 65\mu V$; $V_O = +13V$

$\therefore$ If Vd $< -65\mu V$; $V_O = -13V$

Op-amp Model in the Linear Region:

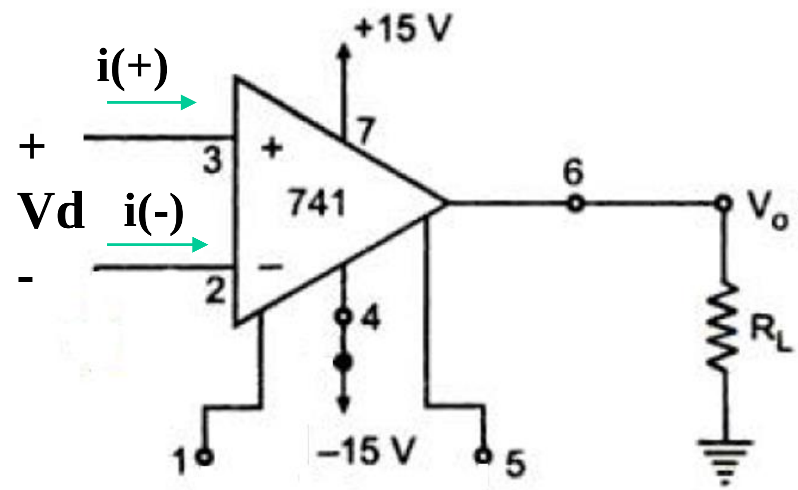




If the Op-Amp is IDEAL and in the Linear Region:

Zi = ∞Ω , Zo = 0Ω , Ad = ∞

Two Assumptions



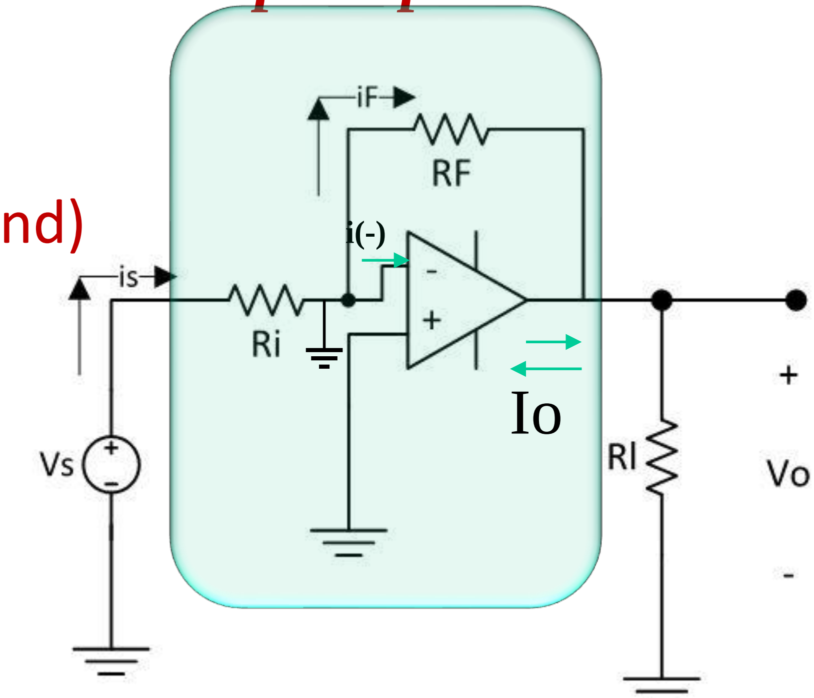
1)Vd ≅ 0 ⇒ V(+) = V(-)

2) i(+) = i(-) = 0

Op-amp Linear Applications:

1. Inverting Amplifier

Op-Amp is ideal



a) Since $V(+) = 0$; $\therefore$ $V(-) = 0$

(Virtual ground)

And $i_s = \frac{v_s}{R_i}$

b) Since $i(-) = 0$; $\therefore$ $i_F = i_S$

c) $V_O = -R_F i_F$

$V_O = -R_F i_S$

$$V_O = -\frac{R_F}{R_i} V_S$$

$$\therefore A_v = \frac{V_O}{V_S} = -\frac{R_F}{R_i}$$

$$+V_{sat} > V_O > -V_{sat}$$

$$Io < Io,max$$

The voltage gain does not depend on RL

Op-Amp Linear Applications:

1. Inverting Amplifier

Design an inverting amplifier to provide A_v = -200

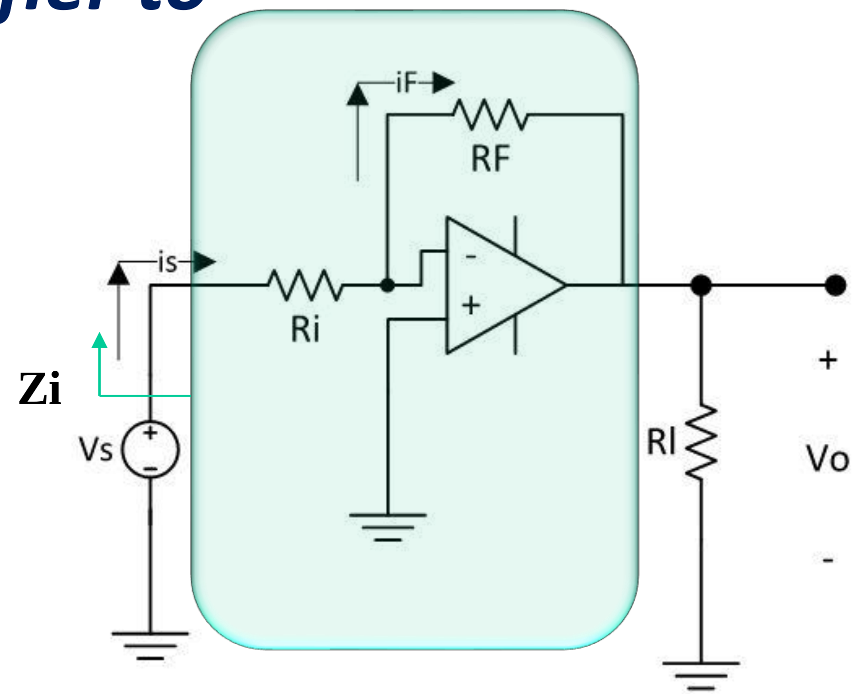
Solution :

$$A_v = -\frac{R_F}{R_i} = -200$$

$$\therefore \frac{R_F}{R_i} = 200$$

Let R_i = 20K

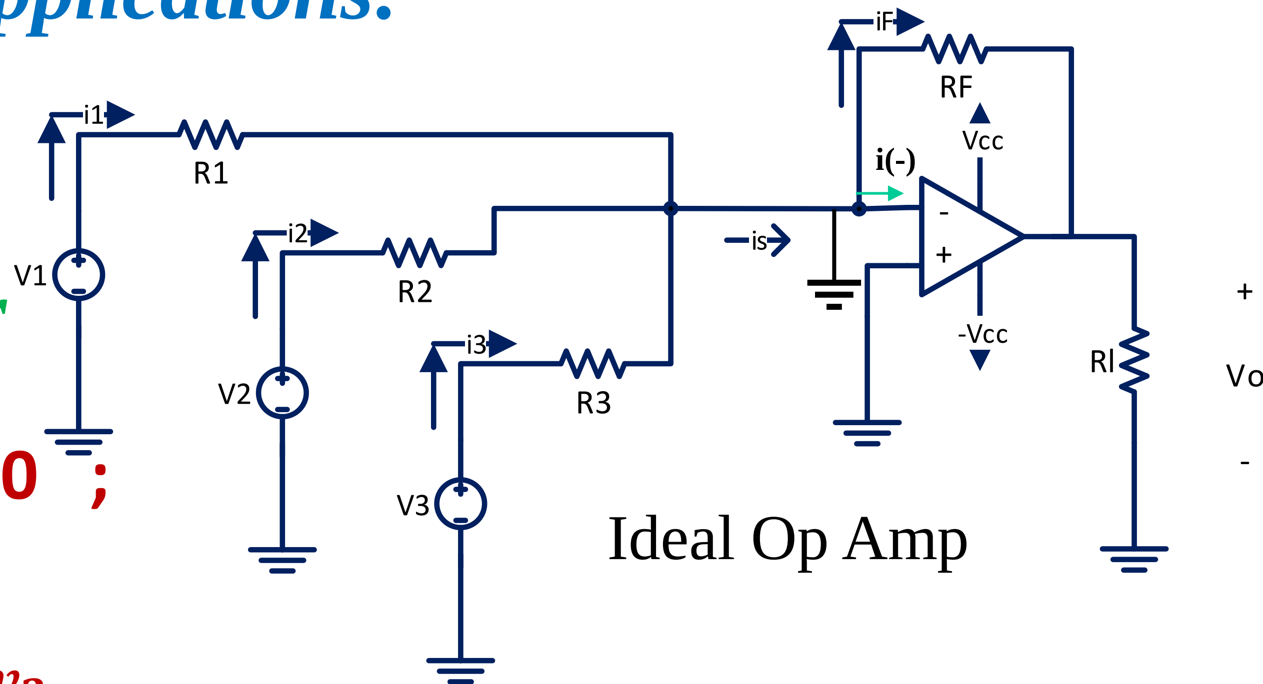
$$\therefore R_F = 4000K$$



$$Zi = R_i$$

Op-amp Linear Applications:

2. Inverting Adder



1- Since V(+) = 0 ;

∴ V(-) = 0

$$i_1 = \frac{v_1}{R_1} \quad ; \quad i_2 = \frac{v_2}{R_2} \quad ;$$

$$i_3 = \frac{v_3}{R_3}$$

$$i_s = i_1 + i_2 + i_3$$

2- Since i(-) =0 ; ∴ $i_F = i_S$

3- $V_O = -R_F i_F$

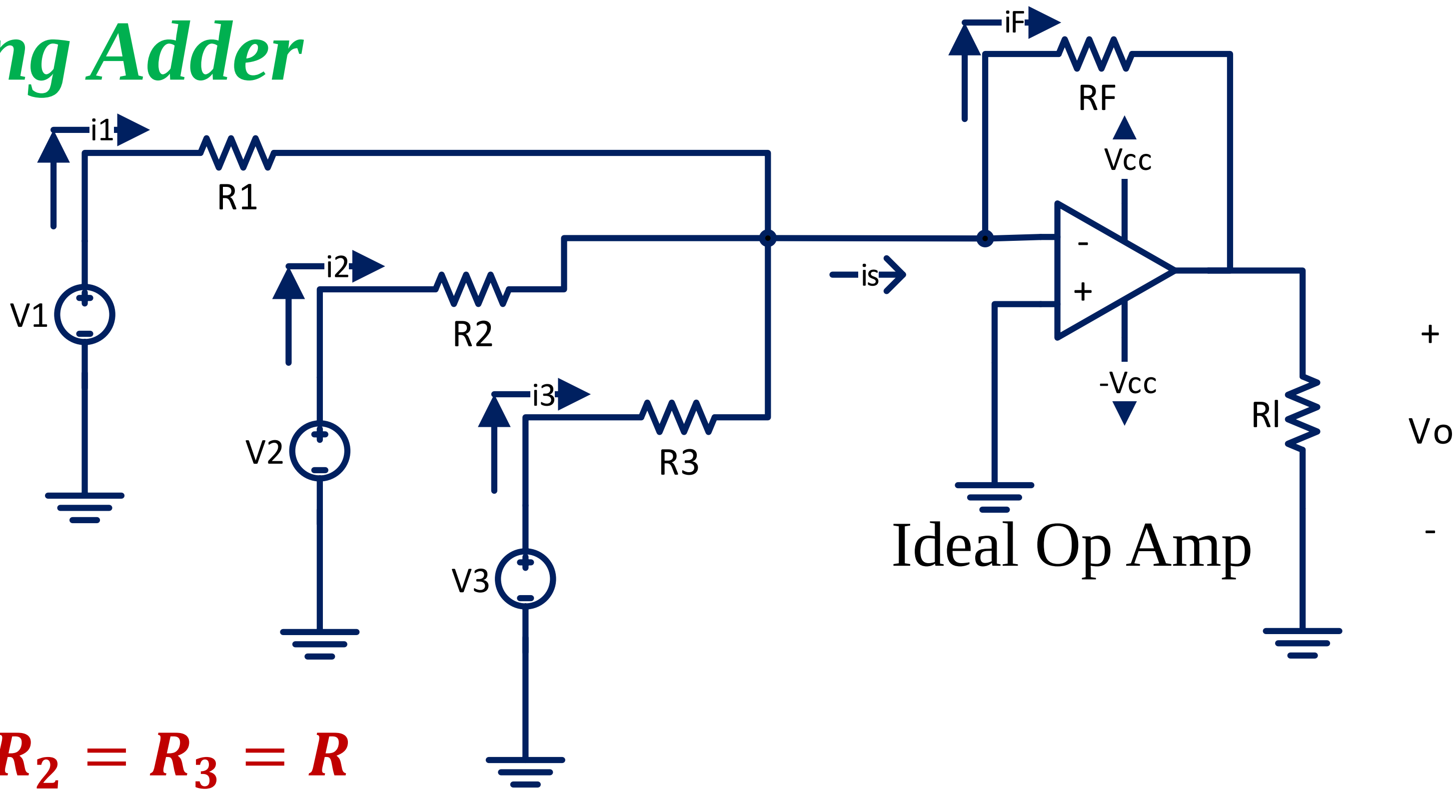
$$V_O = -\left(\frac{R_F}{R_1}V_1 + \frac{R_F}{R_2}V_2 + \frac{R_F}{R_3}V_3\right)$$

$$If \quad R_1 = R_2 = R_3 = R_F = R$$

$$V_O = -(V_1 + V_2 + V_3)$$

Op-amp Linear Applications:

2. Inverting Adder



$If \quad R_1 = R_2 = R_3 = R$

And $R_F = \frac{R}{n} = \frac{R}{3}$

Averager

$\therefore \quad V_O = -\left(\frac{V_1 + V_2 + V_3}{3}\right)$

3. Non Inverting Amplifier

1- Since V(+) = V_s

∴ V(-) = V_s

$$\therefore i_i = \frac{V_s}{R_i}$$

2- Since i(-) = 0 ;

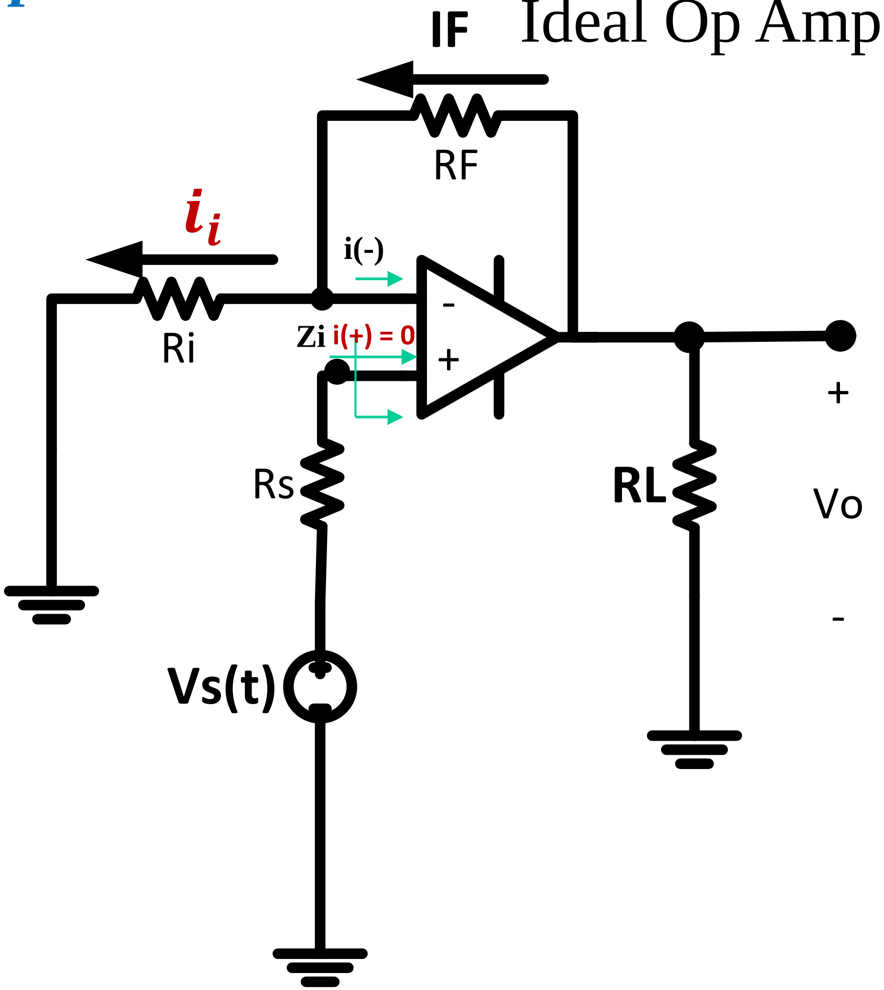
$$\therefore \quad i_F = i_i$$

$$3 - V_O = R_F i_F + R_i i_i$$

$$\Rightarrow \quad V_O = (1 + \frac{R_F}{R_i})\, V_s$$

$$V_O = (1 + \frac{R_F}{R_i})V(+)$$

Ideal Op Amp

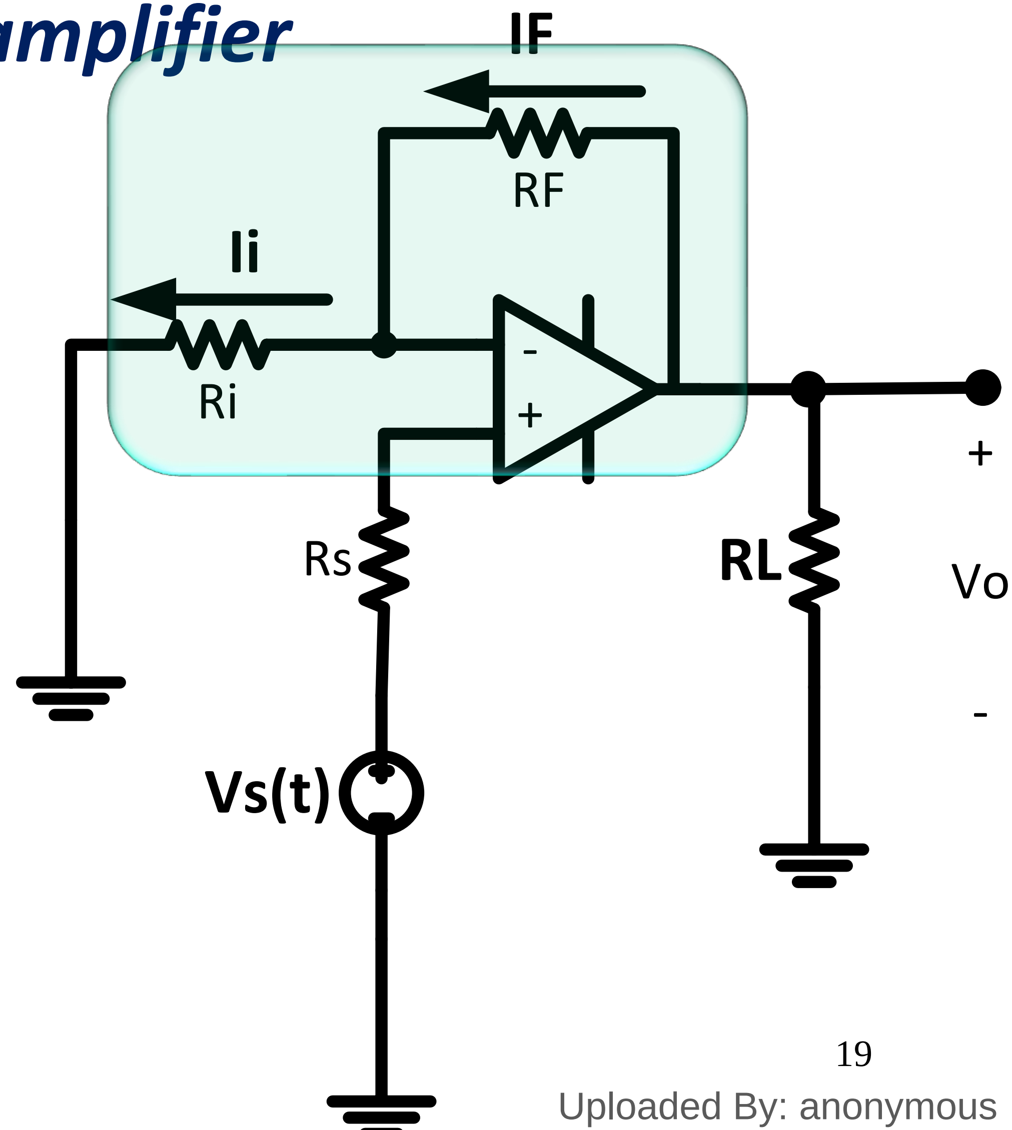


Zi = ∞

Op-amp Linear Applications:

Non Inverting Amplifier

Design a non inverting amplifier to provide A_v= 100



Solution :

$$A_v = 1 + \frac{R_F}{R_i} = 100$$

$$\therefore \frac{R_F}{R_i} = 99$$

Let R_i =10K

$\therefore R_F$ = 990K

Op-amp Linear Applications:

4. Buffer , Unity Gain

1- Since V(+) = V_s ; ∴ V(-) = V_s

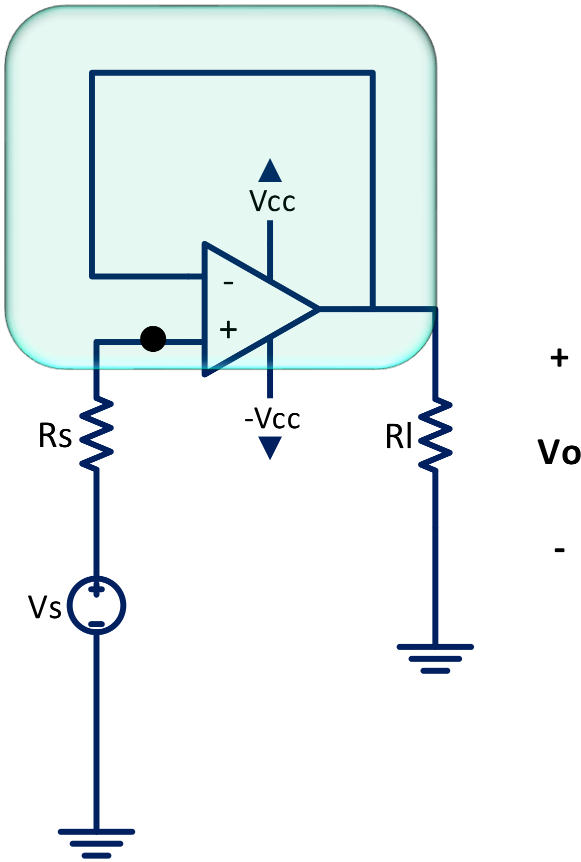
2- $V_O =$ V(-) = V_s

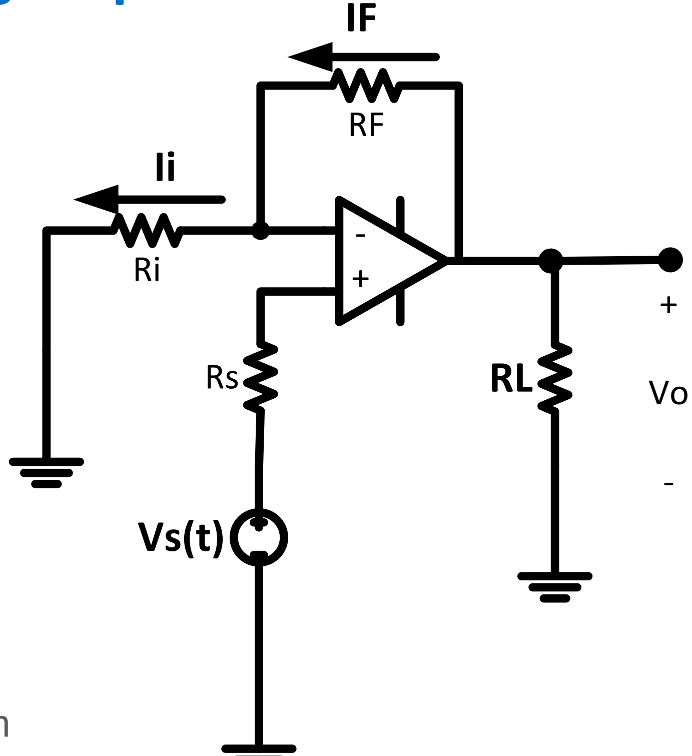
Buffer is a special Case of

non inverting amplifier for which:

$R_i = \infty$

$R_F = 0$

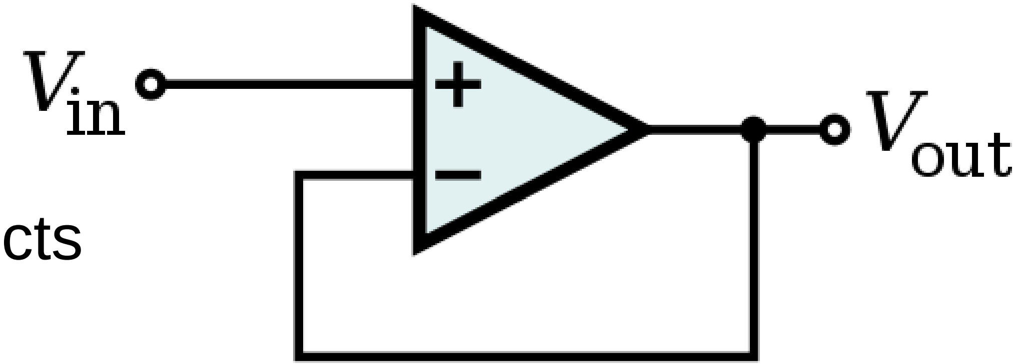


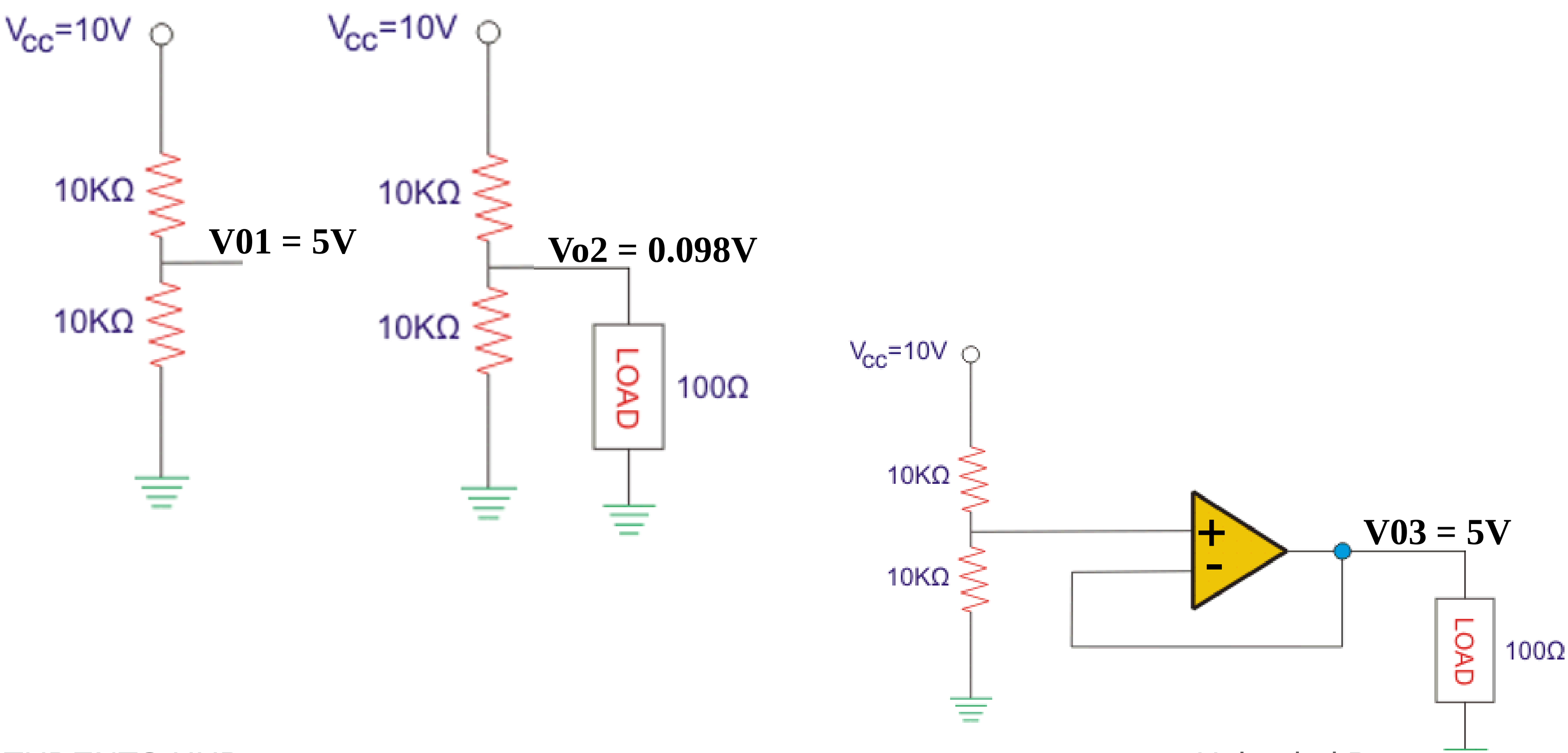


Op-Amp Buffer

Vout = Vin

Isolates loading effects

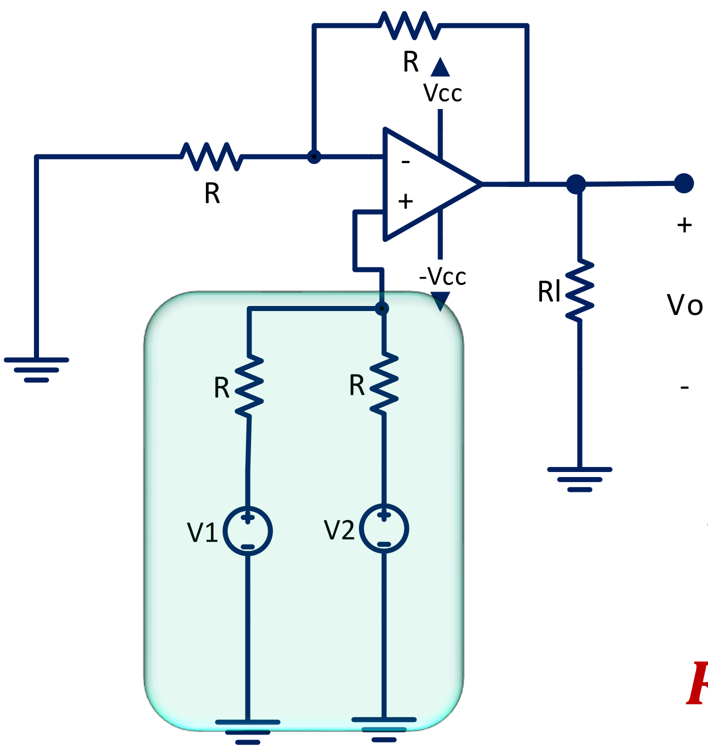


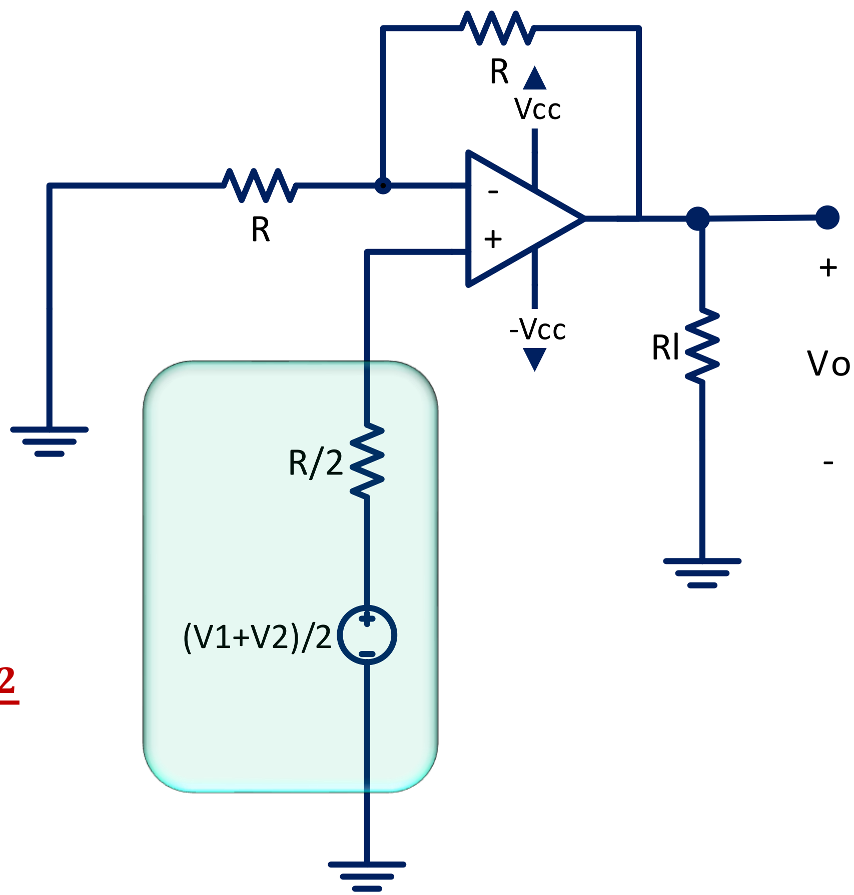


Op-amp Linear Applications:

5. Non-inverting Adder

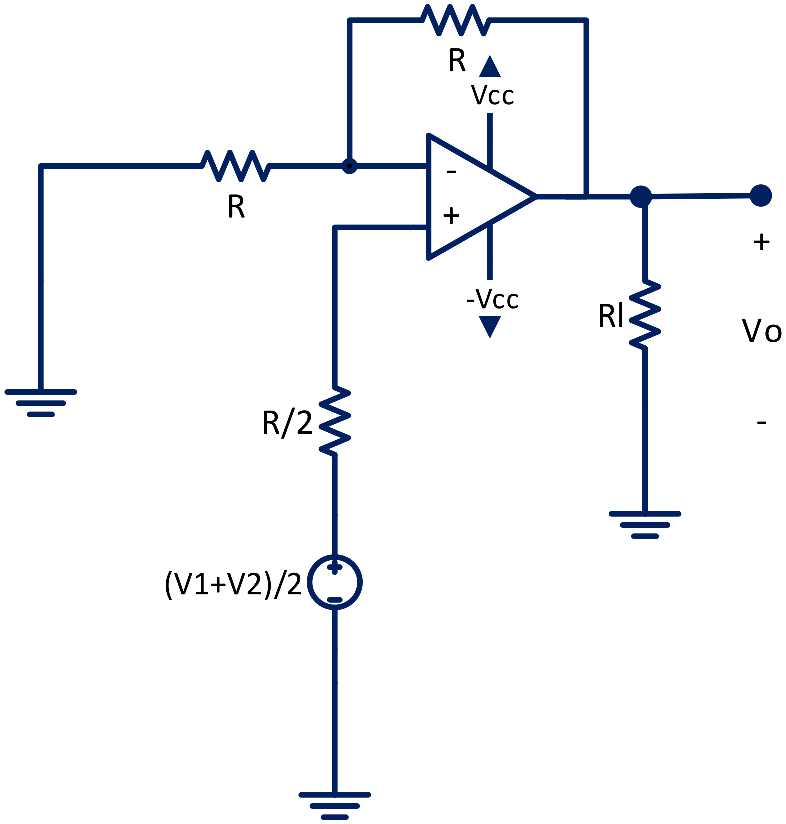
Ideal Op Amp





$$V_{TH} = \frac{V_1 + V_2}{2}$$

$$R_{TH} = \frac{R}{2}$$



$$V_O = \left(1+\frac{R}{R}\right)\left(\frac{V_1+V_2}{2}\right) = V_1 + V_2$$

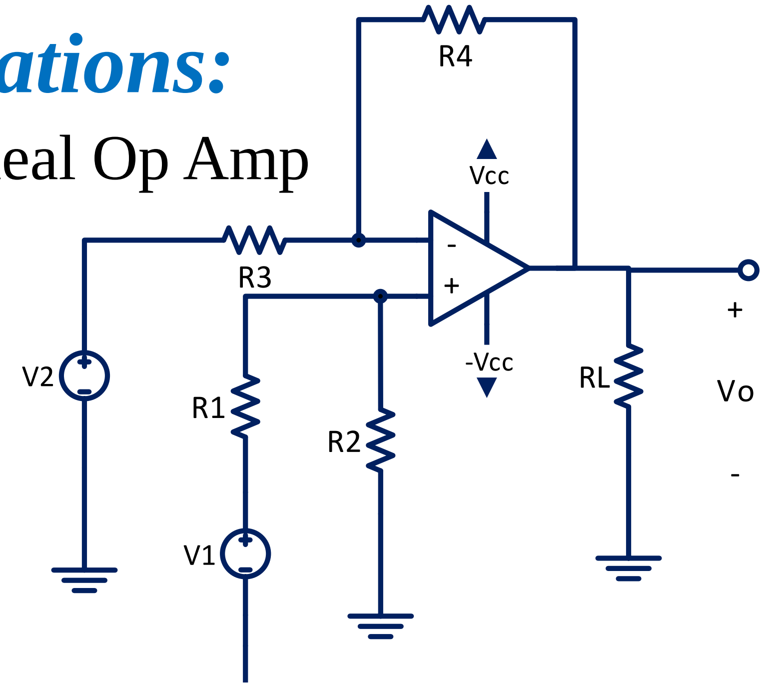
If we have n signal :

let $R_F = (n\text{-}1)R$

$$V_O = V_1 + V_2 + \ldots\ldots\ldots\ldots\ldots + V_N$$

Op-amp Linear Applications:

Ideal Op Amp



6. Voltage Subtraction

Using superposition

a) Let $V_1 = 0$

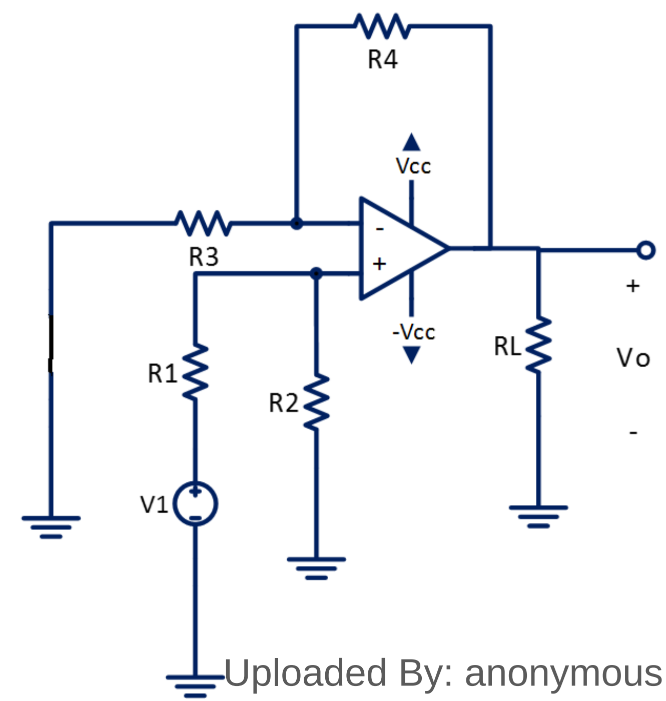
$$\therefore V_{o1} = -\frac{R_4}{R_3} V_2$$ **(Inverting amplifier)**

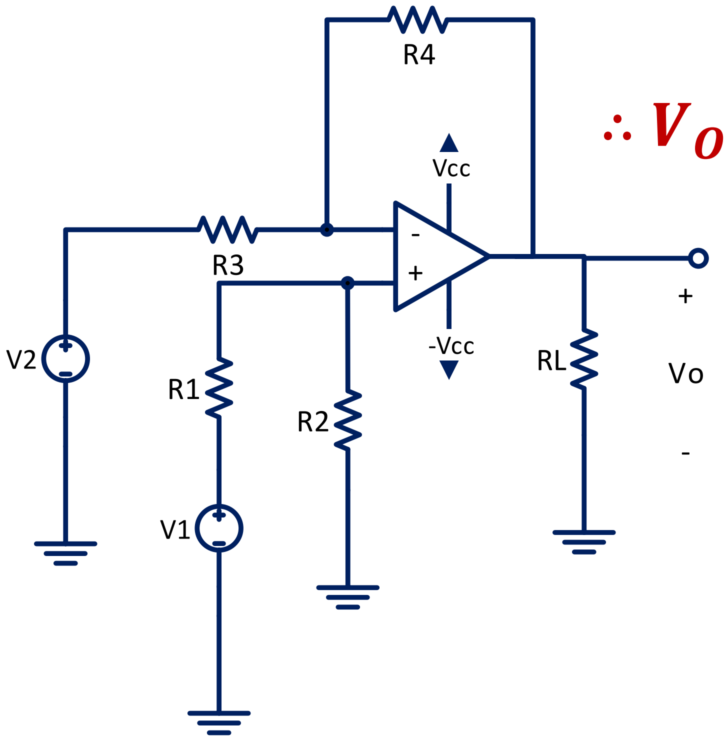
b) Let $V_2 = 0$

$$\therefore V_{o2} = (1+\frac{R_4}{R_3})(\frac{R_2}{R_1+R_2}) V_1$$

(non inverting amplifier)

$$\therefore V_O = (1+\frac{R_4}{R_3})(\frac{R_2}{R_1+R_2}) V_1 - \frac{R_4}{R_3} V_2$$





$$\therefore V_O = \left(1 + \frac{R_4}{R_3}\right)\left(\frac{R_2}{R_1+R_2}\right) V_1 - \frac{R_4}{R_3} V_2$$

$$V_O = aV_1 - bV_2$$

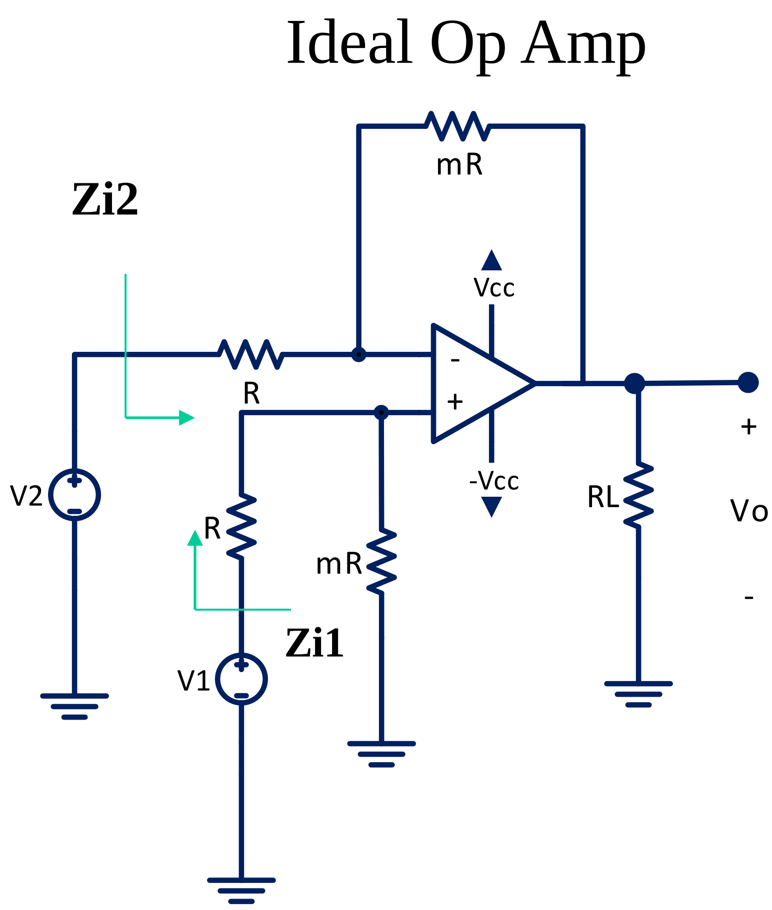
If $R_1 = R_3 = R$
and $R_2 = R_4 = \text{m}R$

$$V_O = \text{m}(V_1 - V_2)$$

Basic Difference Amplifier

Op-amp Linear Applications:

Basic Difference Amplifier



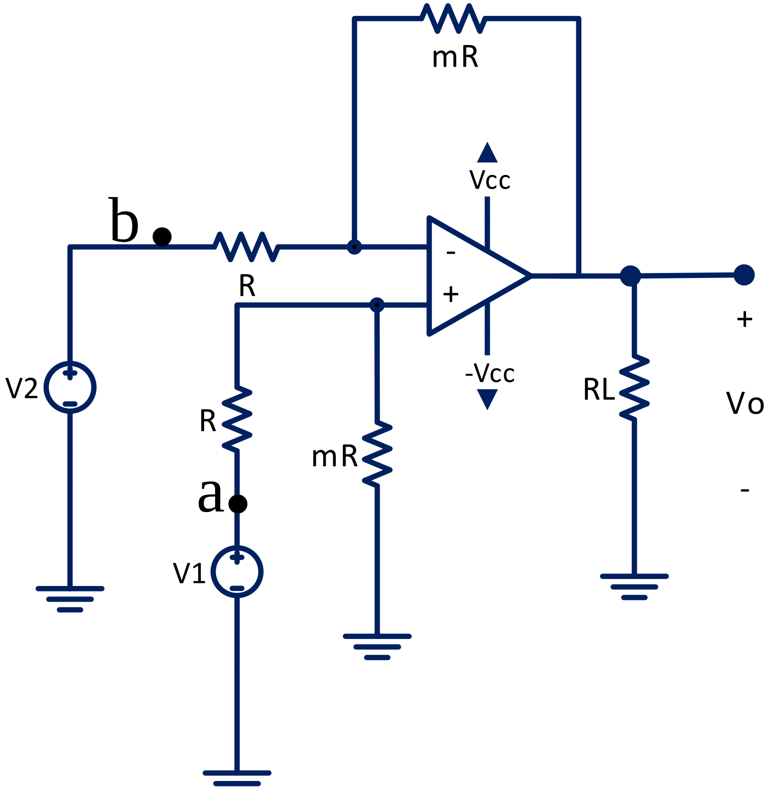
$$V_O = m(V_1 - V_2)$$

$$Z_{i1} = R+mR$$

$$Z_{i2} = R$$

*It has low input impedance

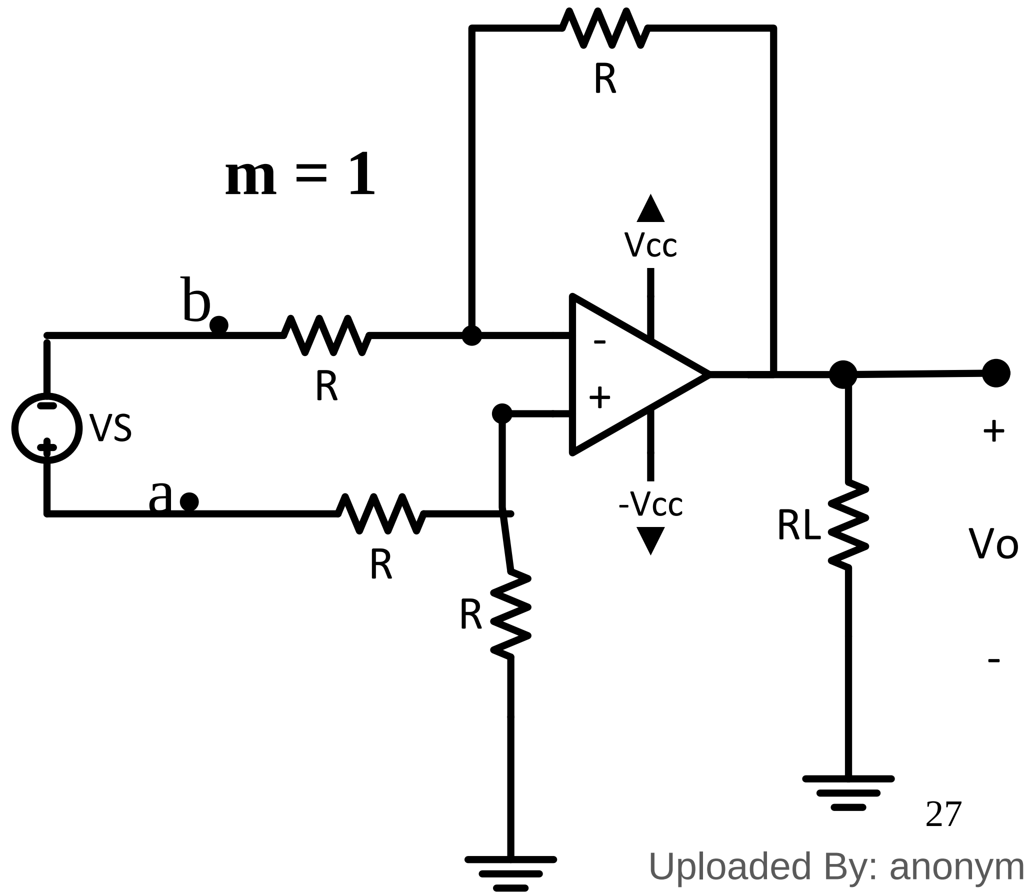
*To Change the gain, we must change two resistors.



$$V_o = m(V_1 - V_2)$$

$$V_o = m\ V_{ab}$$

m = 1



$$V_{ab} = V_1 - V_2$$

$$V_{ab} = V_s$$

$$\therefore \mathbf{Vo = Vs}$$

Instrumentation Amplifier:

$I = \frac{V_1 - V_2}{aR}$

V_{o1} = (R+ aR + R)I

$V_{o1} = (1+\frac{2}{a})\,(V_1 - V_2)$

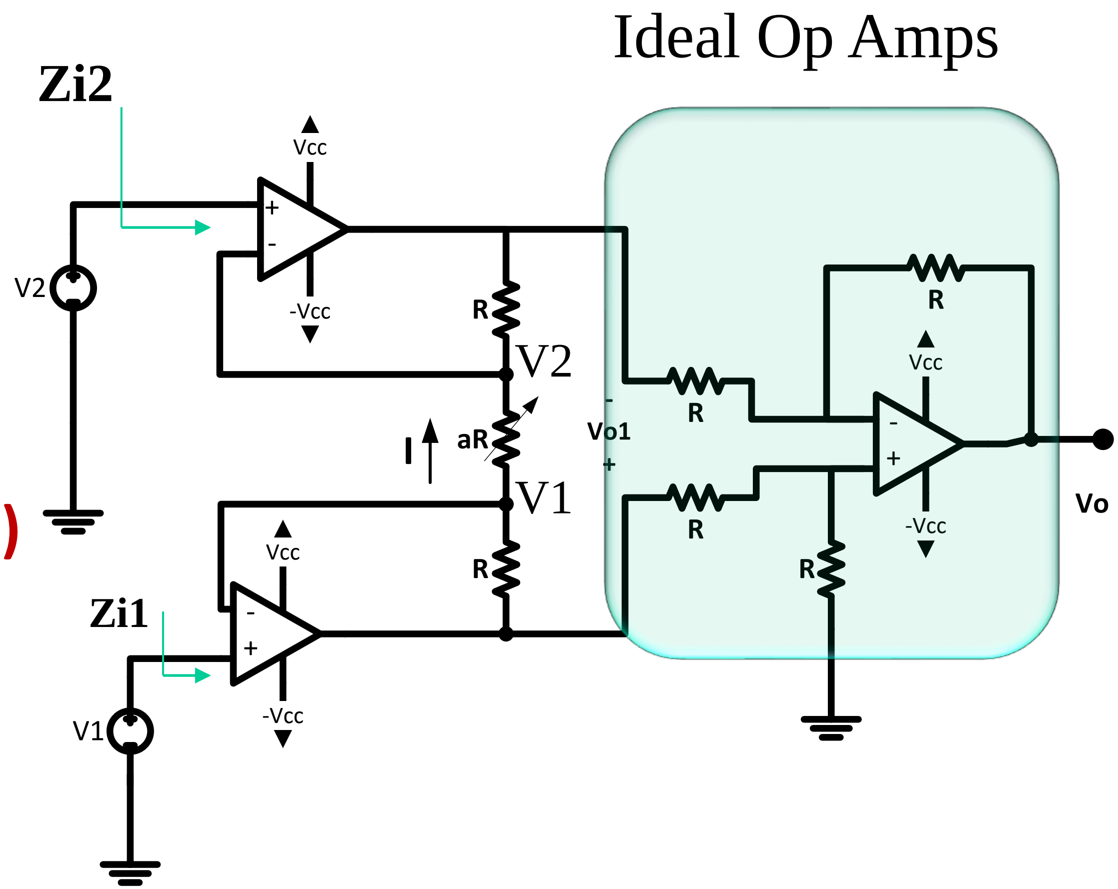
$V_o = V_{o1} = (1+\frac{2}{a})\,(V_1 - V_2)$

$Z_{i1} = \infty$

$Z_{i2} = \infty$

To change the gain → change a

$a \ll 1$



Measuring small resistance change

$$E_1 = \frac{R_1}{R1+R1} \text{ E} = \frac{E}{2}$$

$$E_2 = \frac{R}{R+R+\Delta R} \text{ E} = \frac{R}{2R+\Delta R} \text{ E}$$

$$E_1 - E_2 = \frac{E}{2}\left(\frac{\Delta R}{2R+\Delta R}\right)$$

ΔR is very small

$$E_1 - E_2 = \frac{E}{2}\frac{\Delta R}{2R}$$

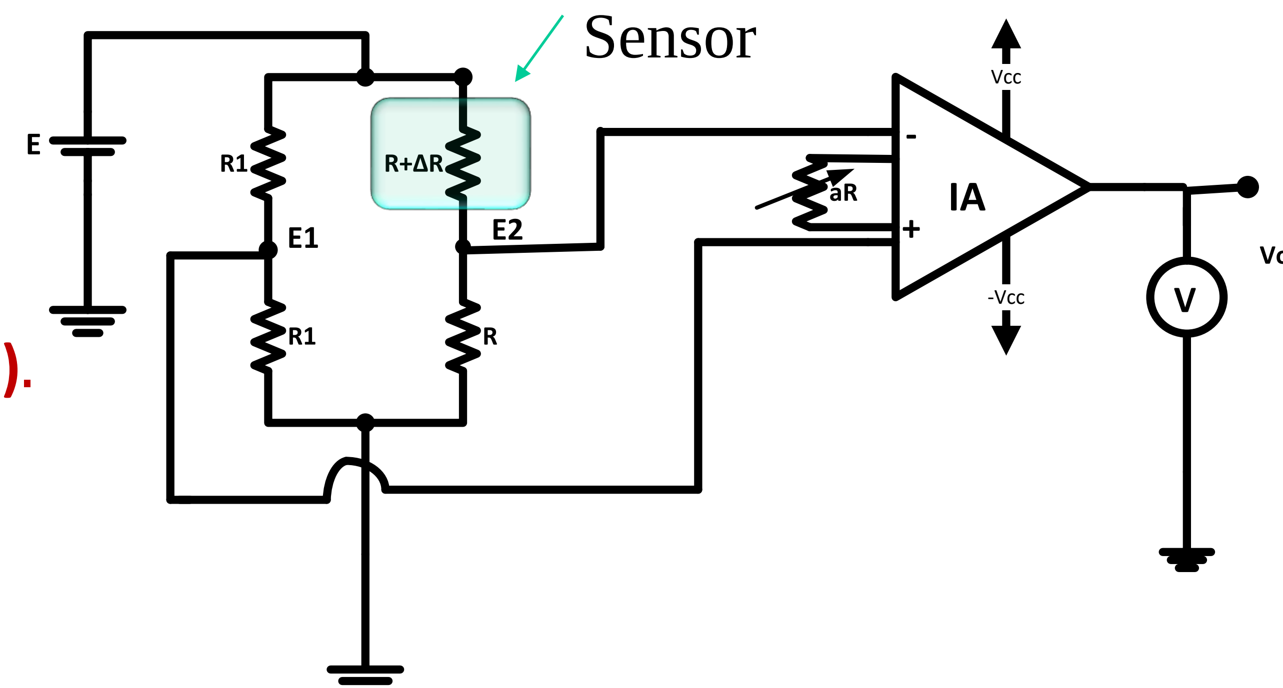
$$V_o = (1+\frac{2}{a})\,(E_1 - E_2).$$

Let $(1+\frac{2}{a}) = 400$

$$V_o = (400)\left(\frac{\Delta R}{4R}\right)\text{E}$$

$$V_o = 100 \text{ E}\left(\frac{\Delta R}{R}\right)$$

$$V_o = (1+\frac{2}{a})\,(E_1 - E_2).$$



Measuring small resistance change

$V_o = 100 \ E\left(\frac{\Delta R}{R}\right)$

If R = 10K

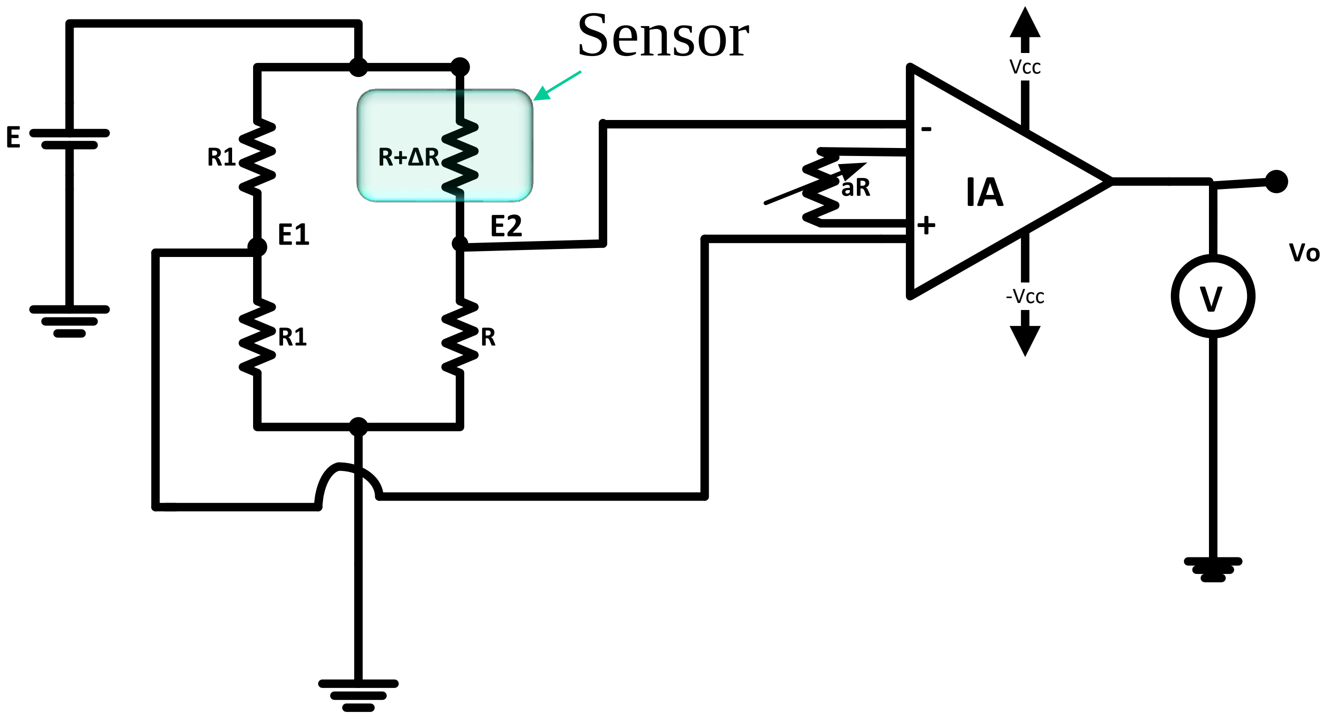
ΔR = 0.1K

E = 1V

$\frac{\Delta R}{R}$ = 1% change

$V_o = 1V$

∴ 1V per 1% change in resistance



If Vo = 2V ⟹ ΔR = 0.2K

If Vo = 0.1V ⟹ ΔR = 0.01K

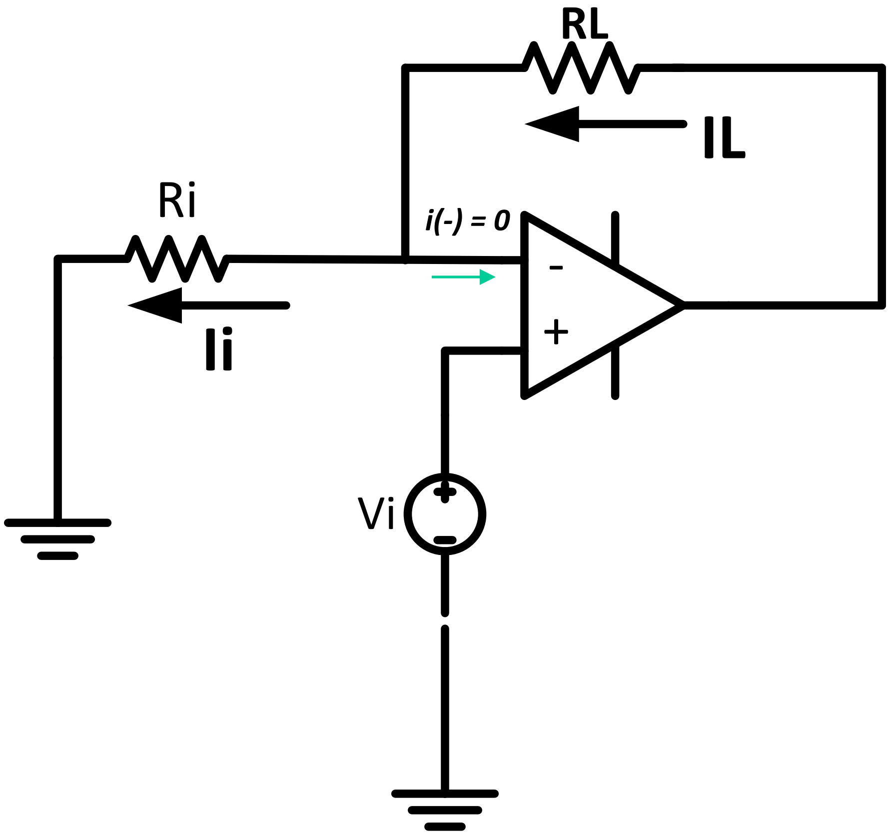
If Vo = 0.01V ⟹ ΔR = 0.001K

If Vo = 1mV ⟹ ΔR = 0.0001K

Voltage to current Converter

a) Floating load

Ideal Op Amp



Since V(+) = Vi

∴ V(-) = Vi

$$I_i = \frac{V_i}{R_i}$$

$$I_L = I_i = \frac{V_i}{R_i}$$

High-Resistance DC Voltmeter

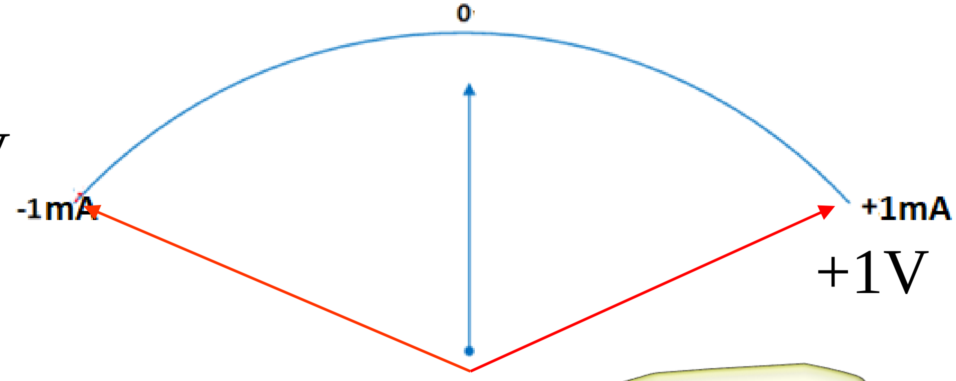
$$I_m = I_i = \frac{V_i}{R_i}$$

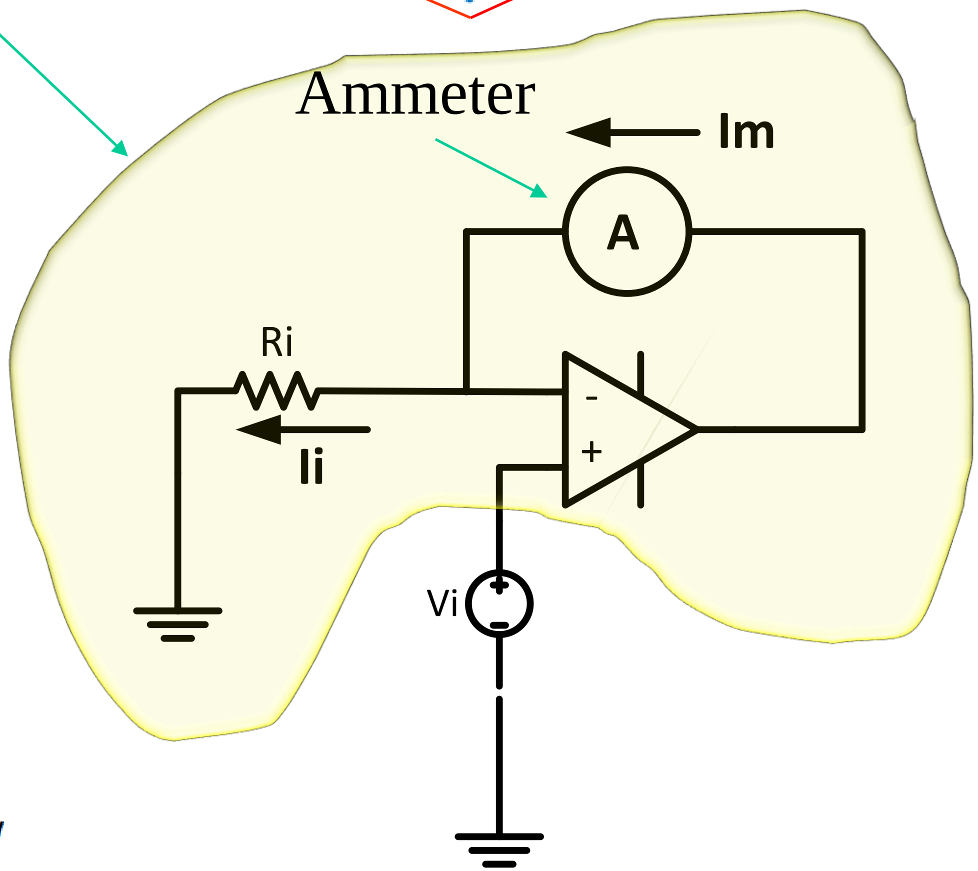
If V_i =+1V → $I_m =$ +1mA

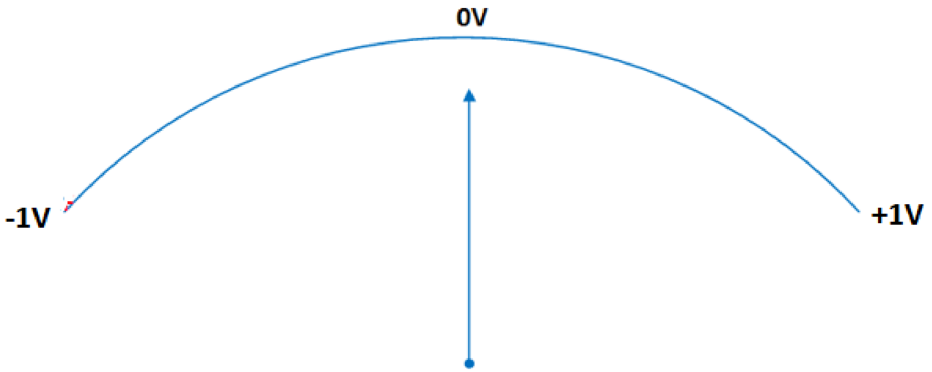
$R_i =$ 1k

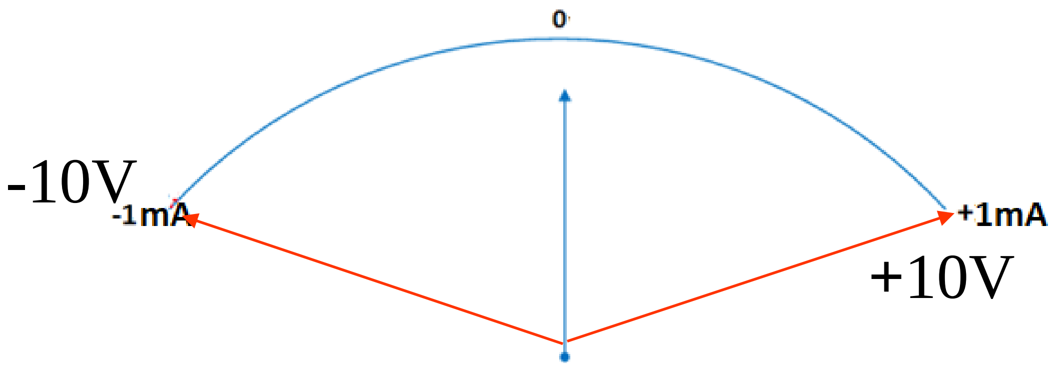
If V_i =-1 V → $I_m =$ -1mA

$R_i =$ 1k







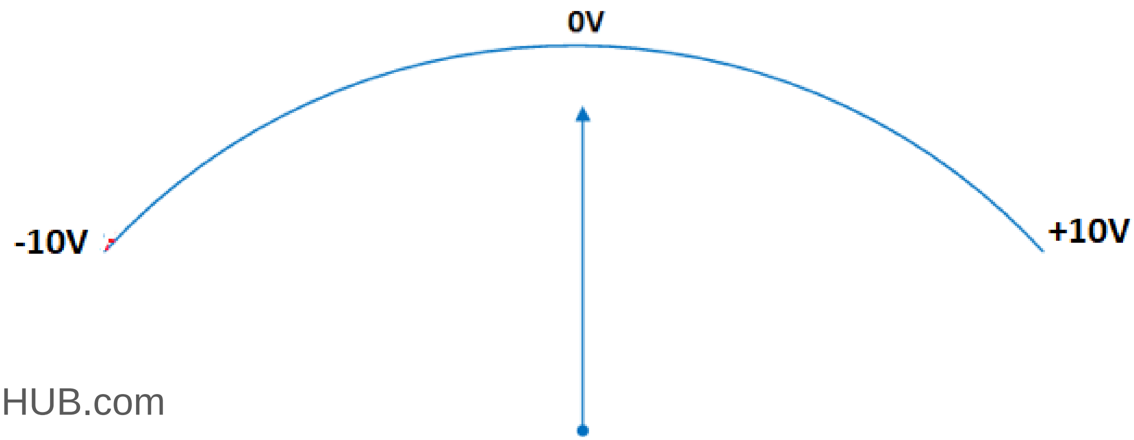


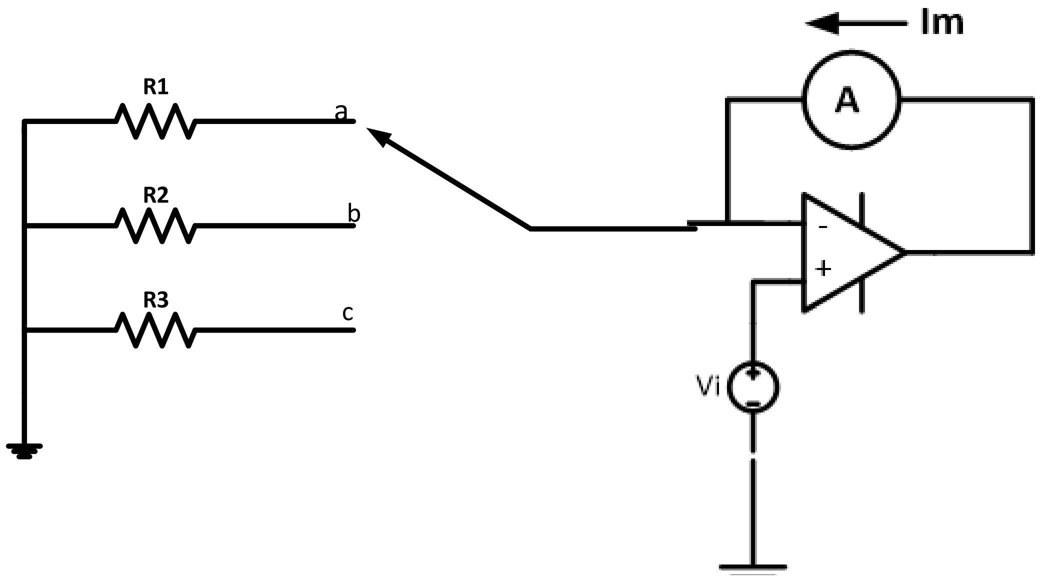
If V_i =+10 V → I_m = +1mA

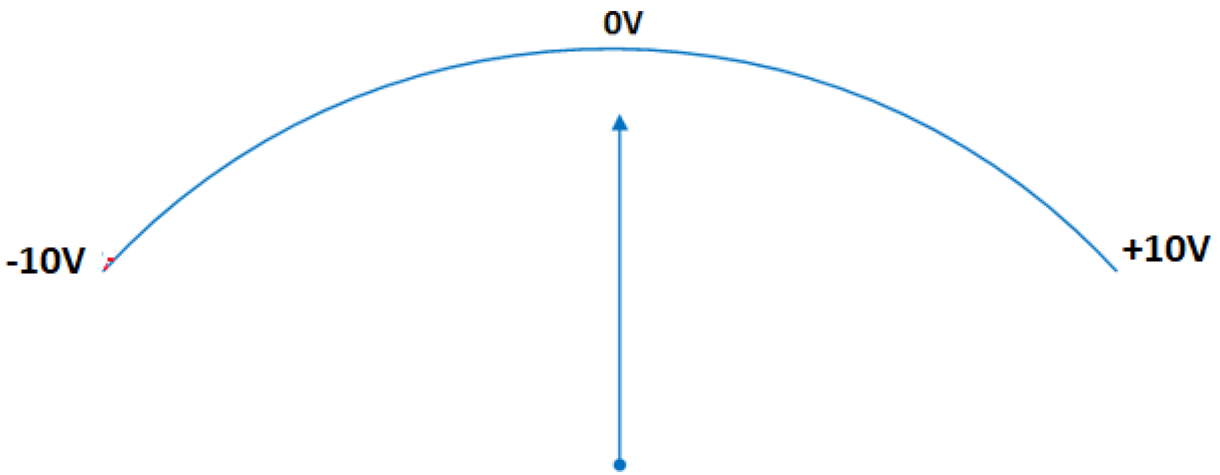
R_i = 10k

If V_i =-10 V → I_m = -1mA

R_i = 10k




Im
A
R1
R2
R3
a
b
c
-
+
Vi


0V
-10V
+10V

Voltage to current Converter

Ideal Op Amp

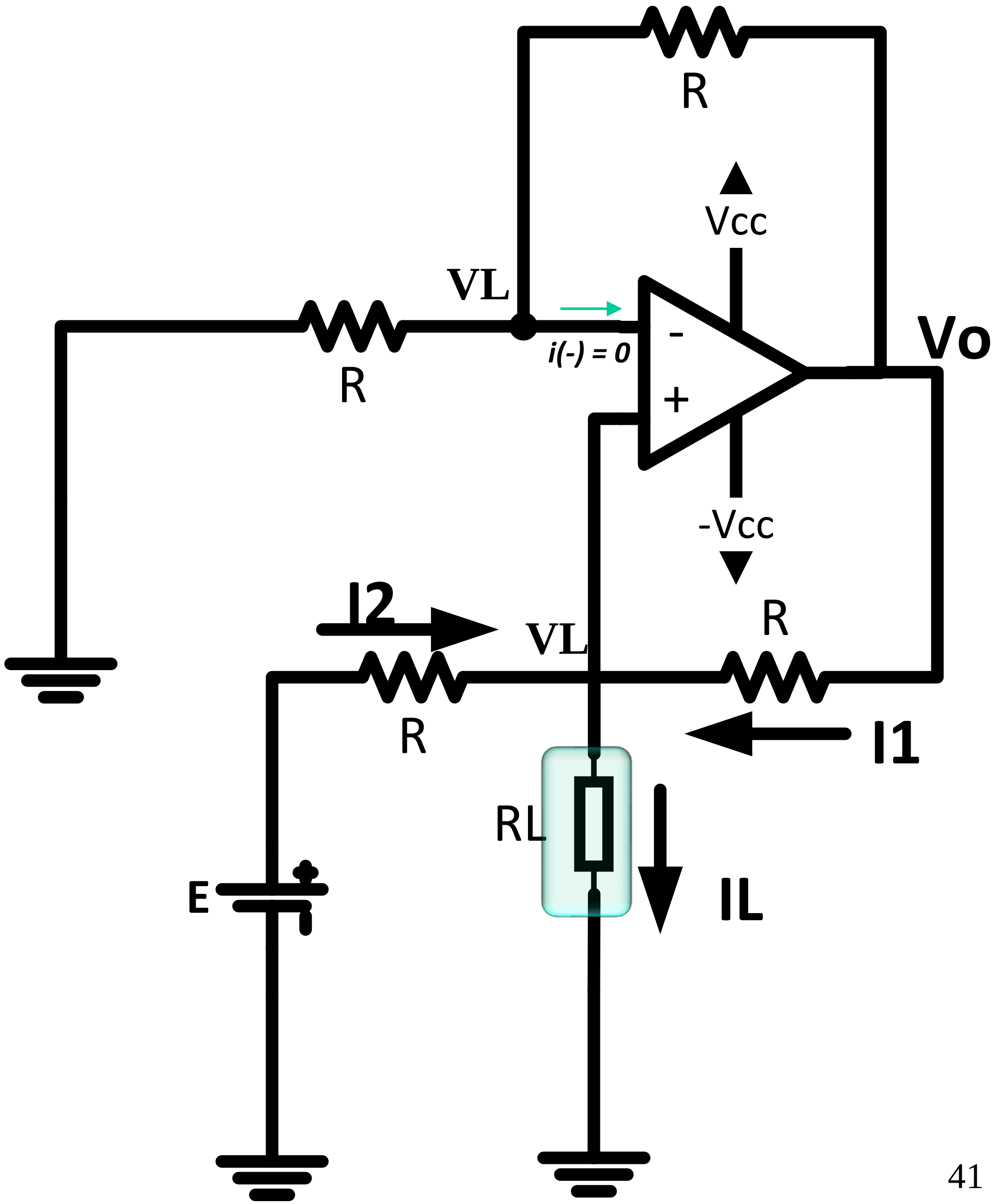
b) Grounded load

$$V_L = V(-) = \frac{1}{2}\ V_o$$

$$I_L = I_1 + I_2$$

$$I_L = \frac{V_o - V_L}{R} + \frac{E - V_L}{R}$$

$$\therefore I_L = \frac{E}{R}$$



Current to Voltage Converter

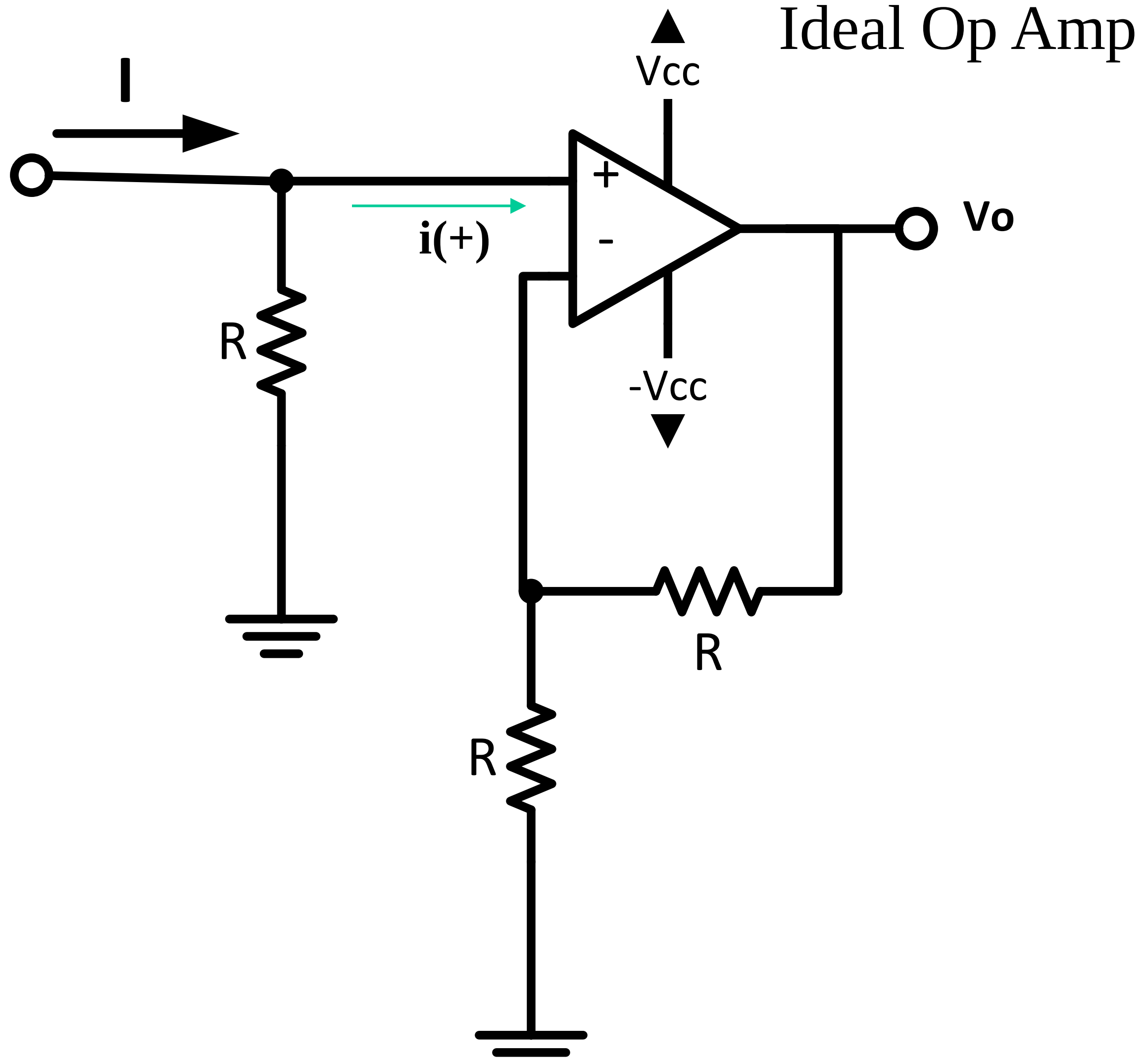
$V(+) = R\,I$

$V_o = (1 + \frac{R}{R})\ V(+)$

$V_o = (1 + \frac{R}{R})\ R\,I$

$V_o = K\,I$

K = 2R



Integrator

- So far, the input and feedback components have been resistors. If the feedback component used is a capacitor,, the resulting connection is called an *integrator.*

Since V(+) = 0 ; ∴ V(-) = 0

(Virtual ground)

Since i(-) = 0

$$i_{i(t)} = \frac{V_{i(t)}}{R}$$

$$i_{i(t)} = i_{f(t)} = \frac{V_{i(t)}}{R}$$

$$V_{o(t)} = -Vc(t)$$

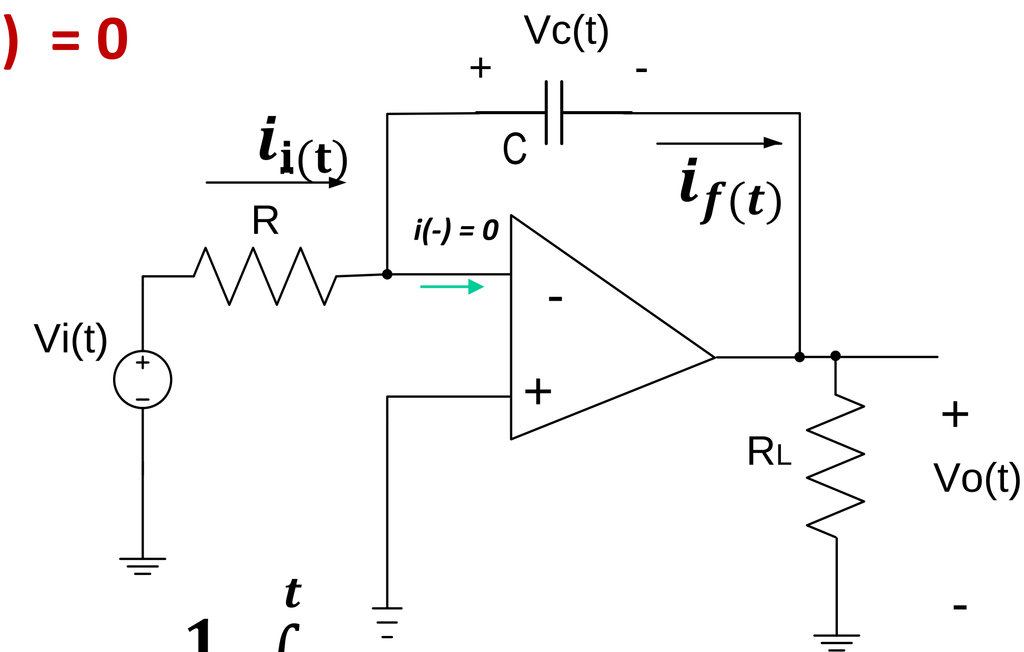
$$Vc(t) = \frac{1}{C}\int_0^t i_f(t)dt + V_C\,(0^-)$$

$$if\ V_C(0^-) = 0$$

$$V_{o(t)} = -\frac{1}{C}\int_0^t \frac{V_i(t)}{R_i}dt$$

$$V_{o(t)} = -\frac{1}{RC}\int_0^t V_i(t)dt$$

Ideal Op Amp



Differentiator

Since V(+) = 0 ; ∴ V(-) = 0

$$\mathbf{i}_{\mathbf{i}(\mathbf{t})} = i_{C(t)} = C\frac{dV_i(t)}{dt}$$

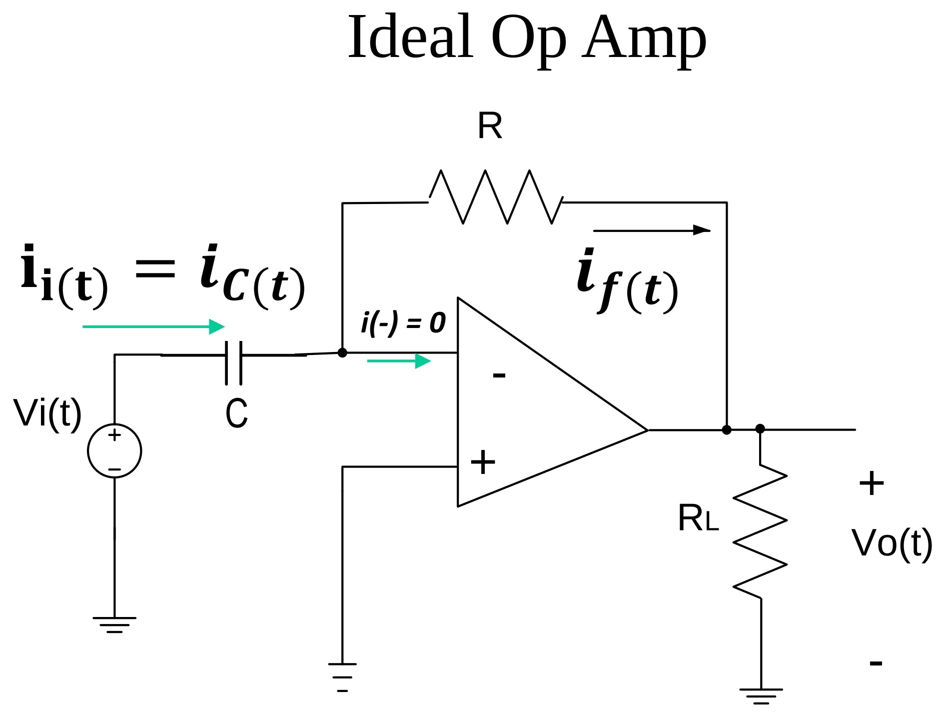
Since i(-) = 0

$$i_{C(t)} = i_{f(t)}$$

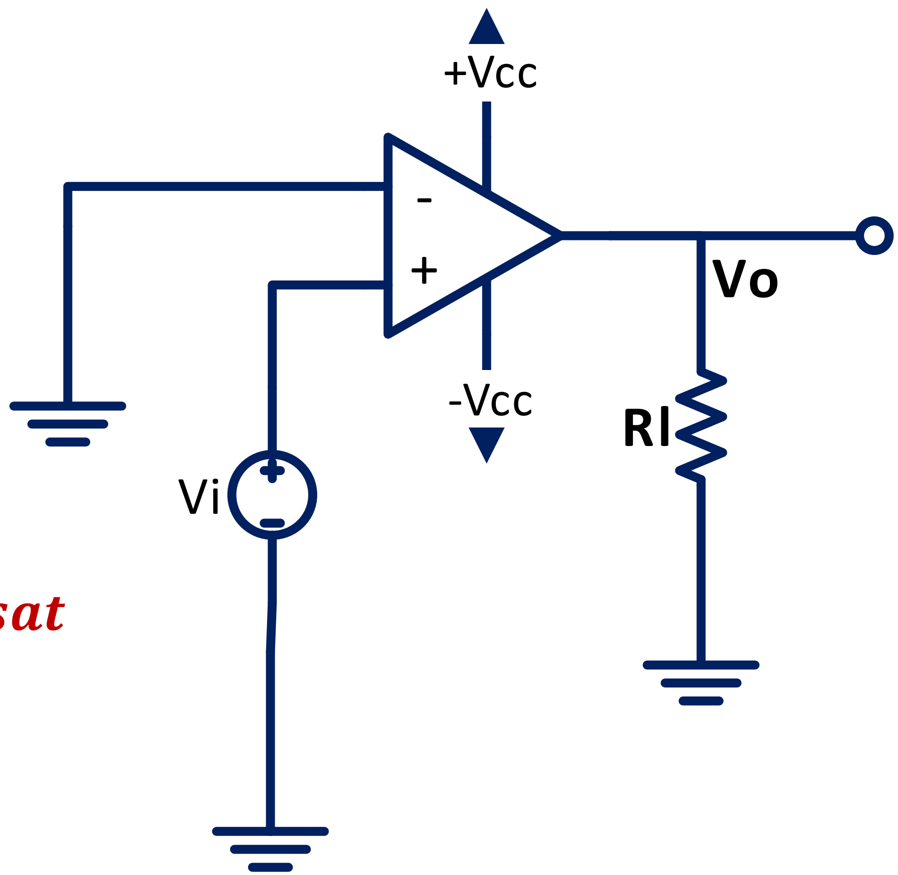
$$V_{o(t)} = -i_f(t)R$$

$$Vo(t) = -\left(C\frac{dV_i(t)}{dt}\right)(R)$$

$$V_{o(t)} = -RC\frac{dV_i(t)}{dt}$$



Comparator : Zero –Level detector



Exact analysis:

When $vd > 65\mu V$; V_O=+ V_{sat}

When $vd < -65\mu V$; V_O= - V_{sat}

Approximate analysis

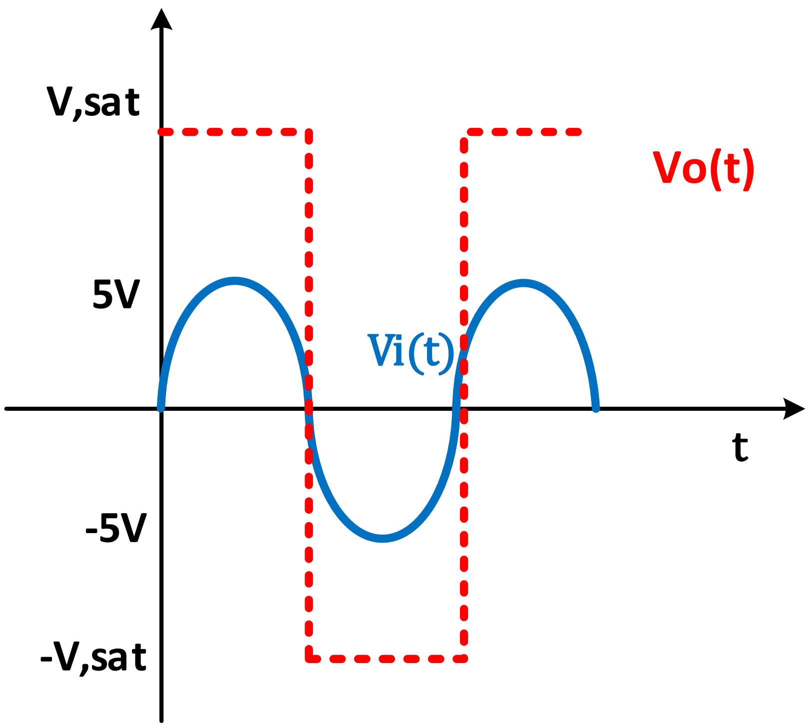
When $vd > 0V$; V_O = + V_{sat}

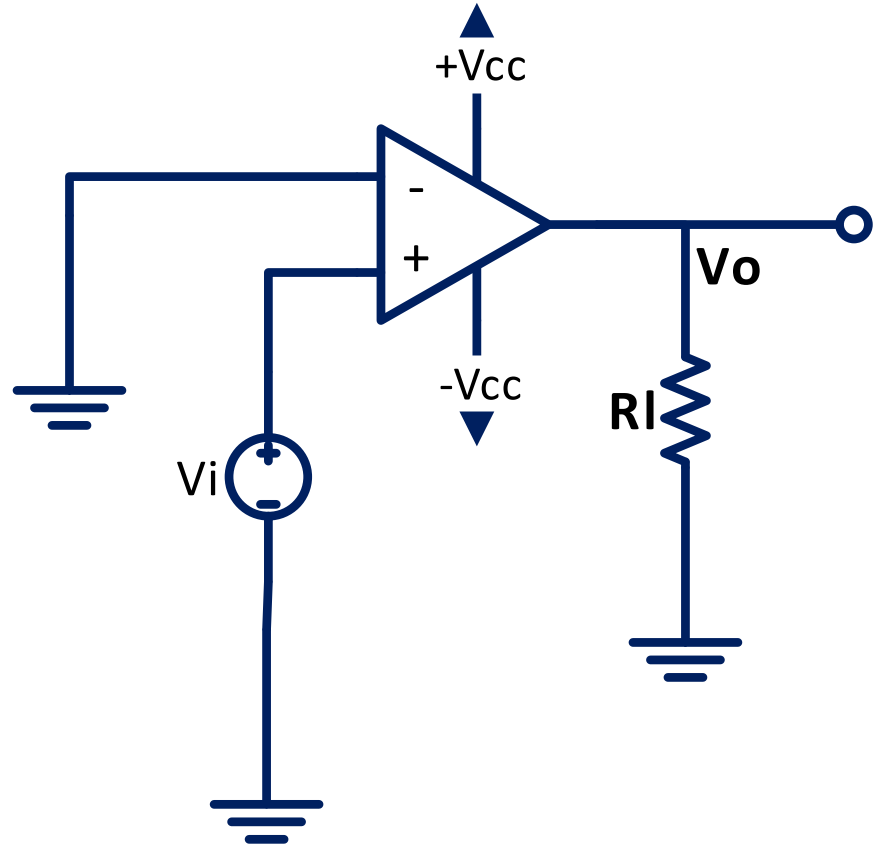
When $vd < 0V$; V_O = - V_{sat}

Comparator : Zero –Level detector

V_i (t) = $5 sin\, \omega t$ v

$\pm\ V_{sat} = \pm 13$ v





When $vd > 0V$; $V_O = + V_{sat}$

When $vd < 0V$; $V_O = - V_{sat}$

$vd = V_i$

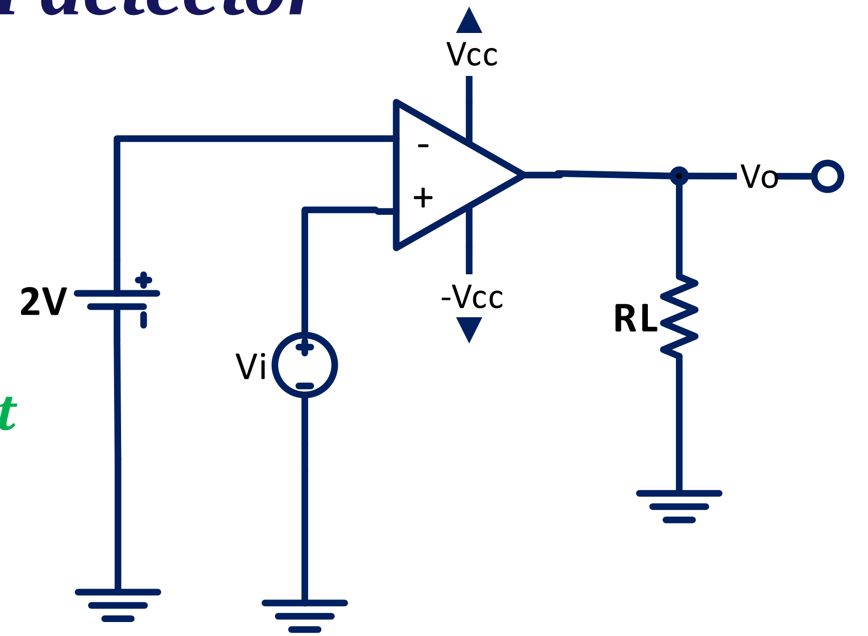
∴ When $V_i > 0V$; $V_O = + V_{sat}$

∴ When $V_i < 0V$; $V_O = - V_{sat}$

Non Zero –Level detector

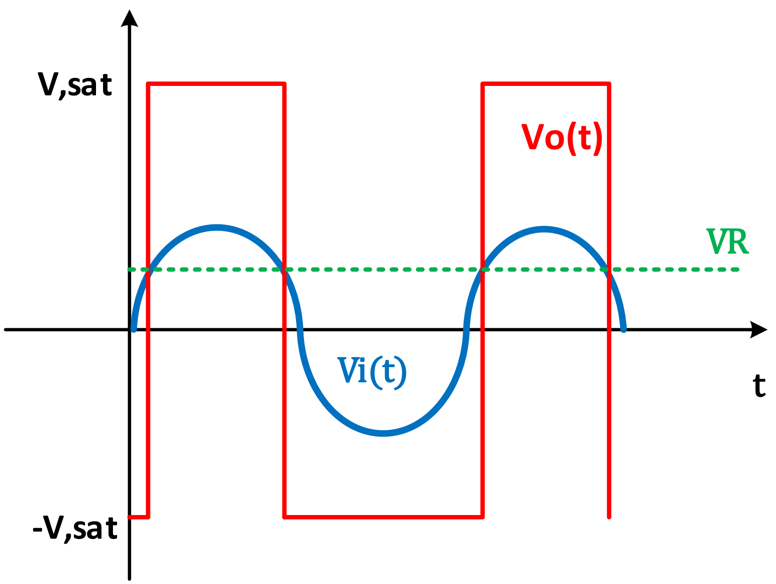
V_i (t) = 5$sin\,\omega t$ v

$\pm\ V_{sat} = \pm 13$ v



When $V_i > 2V$; $V_O = +V_{sat}$

When $V_i < 2V$; $V_O = -V_{sat}$



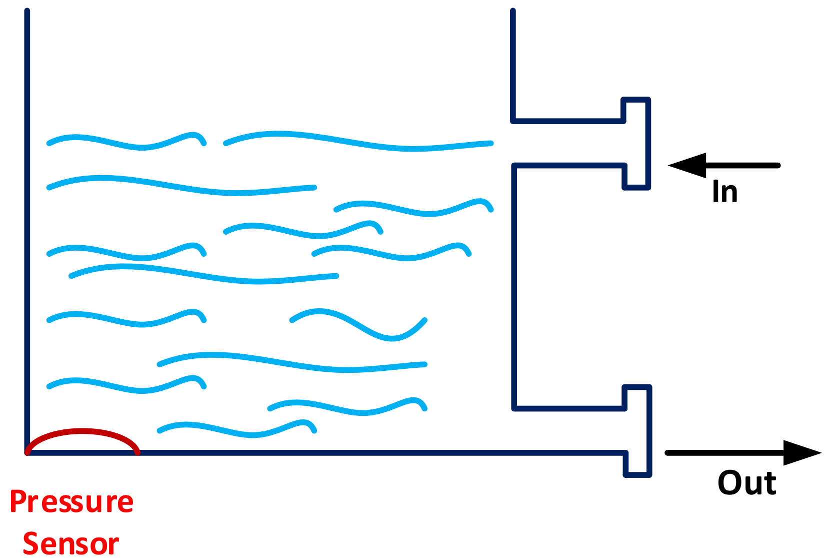
When $vd > 0V$; $V_O = +V_{sat}$

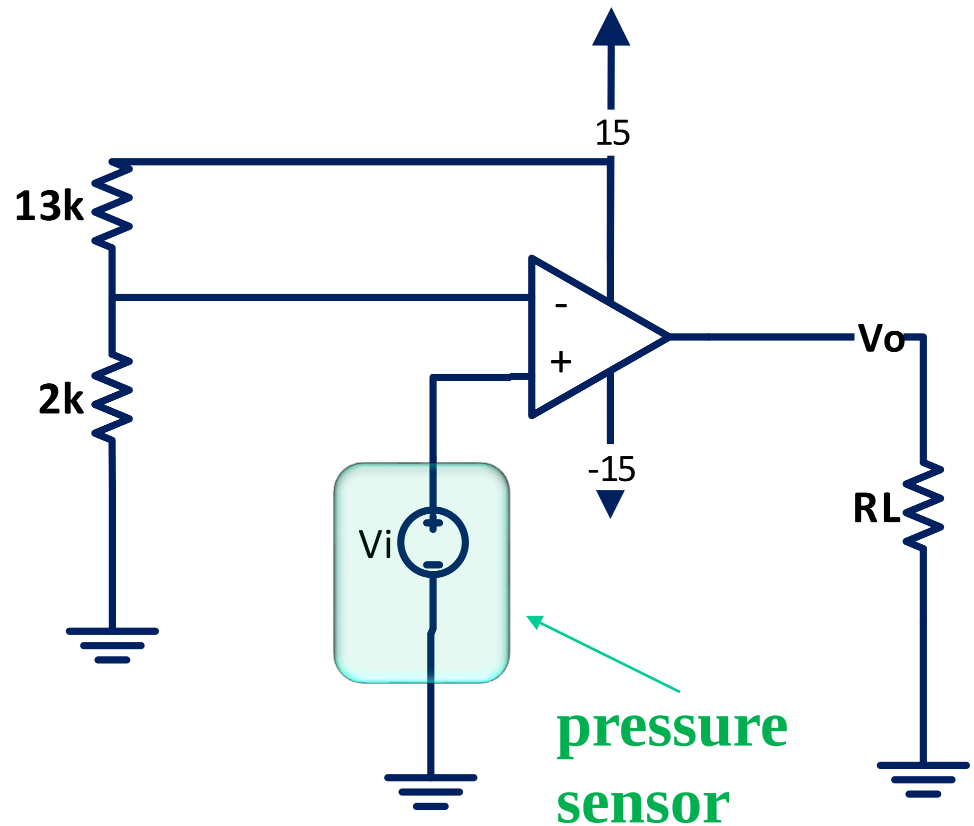
When $vd < 0V$; $V_O = -V_{sat}$

$vd = V_i - 2$

Practical Non Zero –Level detector

Application





The pressure sensor generates a voltage proportional to the water level in the tank

$V(-) = 2V$

When water level reaches the maximum allowable lever

➡ $Vi = 2V$

When V_i >2V ; V_O=+ V_{sat}

When V_i <2V ; V_O= - V_{sat}

Voltage–Level detector with LEDs:

When $V_i > 2V$; $V_O = +V_{sat}$

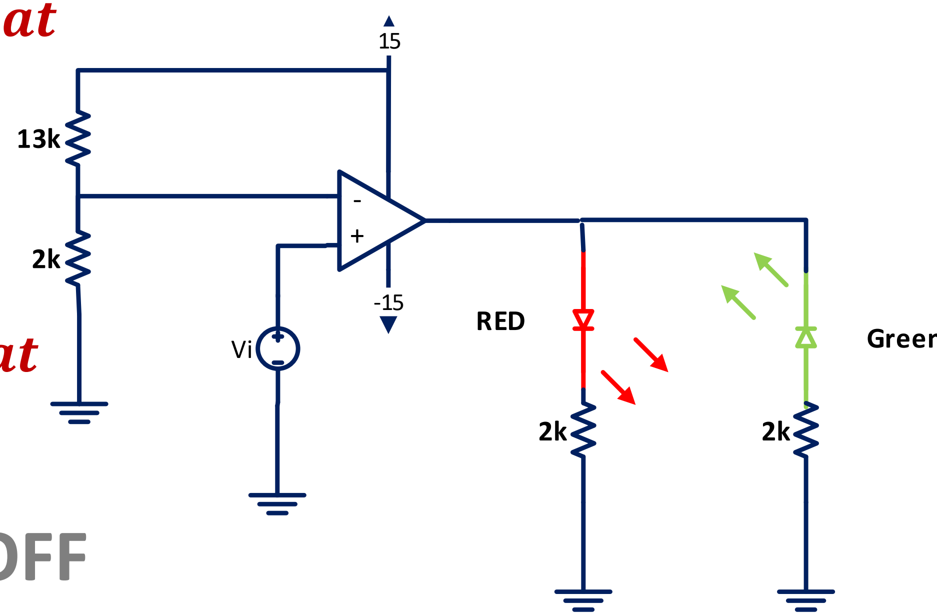
∴ Red LED is ON

∴ green LED is OFF

When $V_i < 2V$; $V_O = -V_{sat}$

∴ green LED is ON

∴ Red LED is OFF



When $V_i = 2V$; $V_O = 0$

∴ green LED and the Red LED are OFF

Over Temperature sensing Circuit

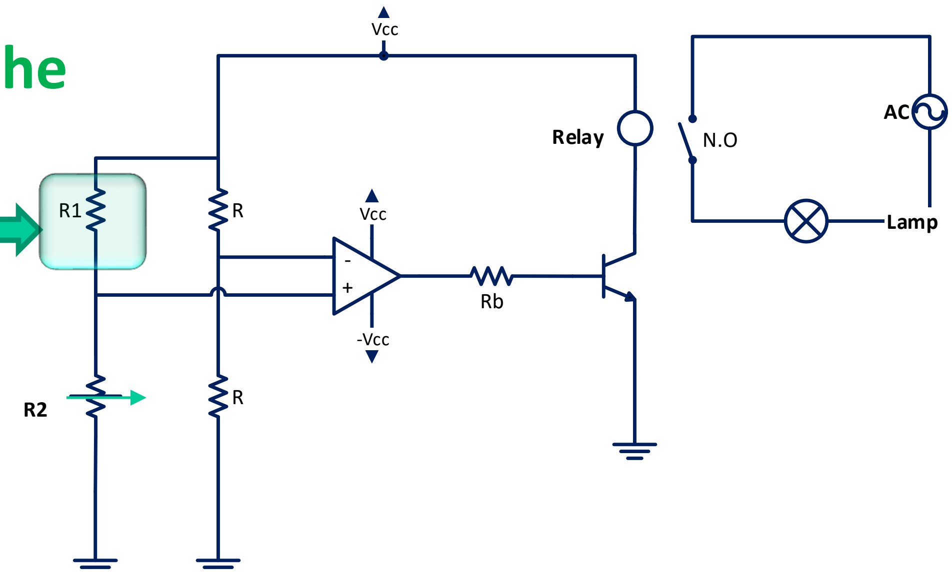
R1 ↑ as T ↓

R1 = Resistance of the thermistor.

R2 is set equal to the resistance of the thermistor at the critical temp.

R = 100k

This is the thermistor



1)At Normal temperature ($T < T_C$)

$R_1 > R_2$

$V(+) = \frac{R_2}{R_1 + R_2} V_{CC} < \frac{1}{2} V_{CC}$

$V(-) = \frac{1}{2} V_{CC}$

∴ V(-) > V(+) , ∴ $V_{op} = -V_{sat}$

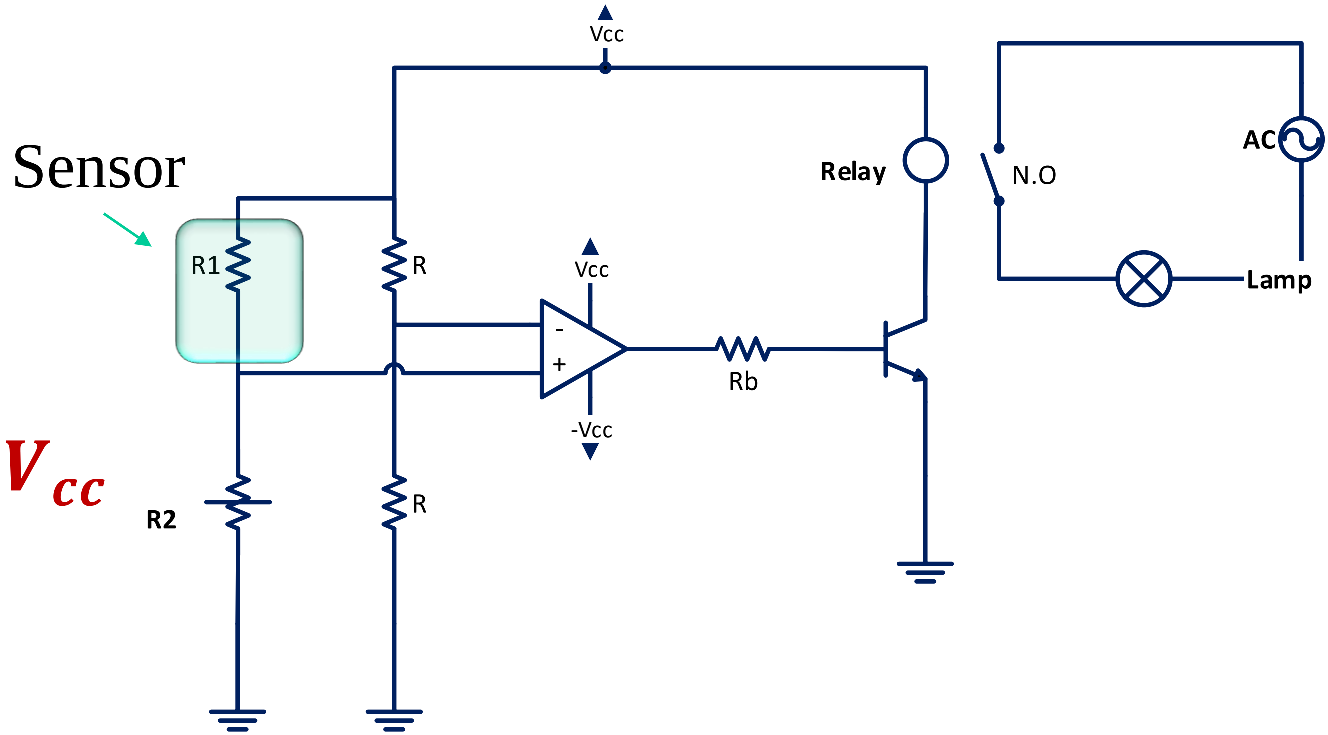
∴ transistor is cut off ; $I_C = 0$

∴ Relay is denergized

∴ switch is open

∴ Lamp is OFF

Over Temperature sensing Circuit



2) When $T = T_C$

$R_1 = R_2$

$V(+) = V(-) = \frac{1}{2} V_{cc}$

$\therefore V_{op} = 0$

∴ transistor is in cut off ;

$I_C = 0$

∴ Relay is denergized

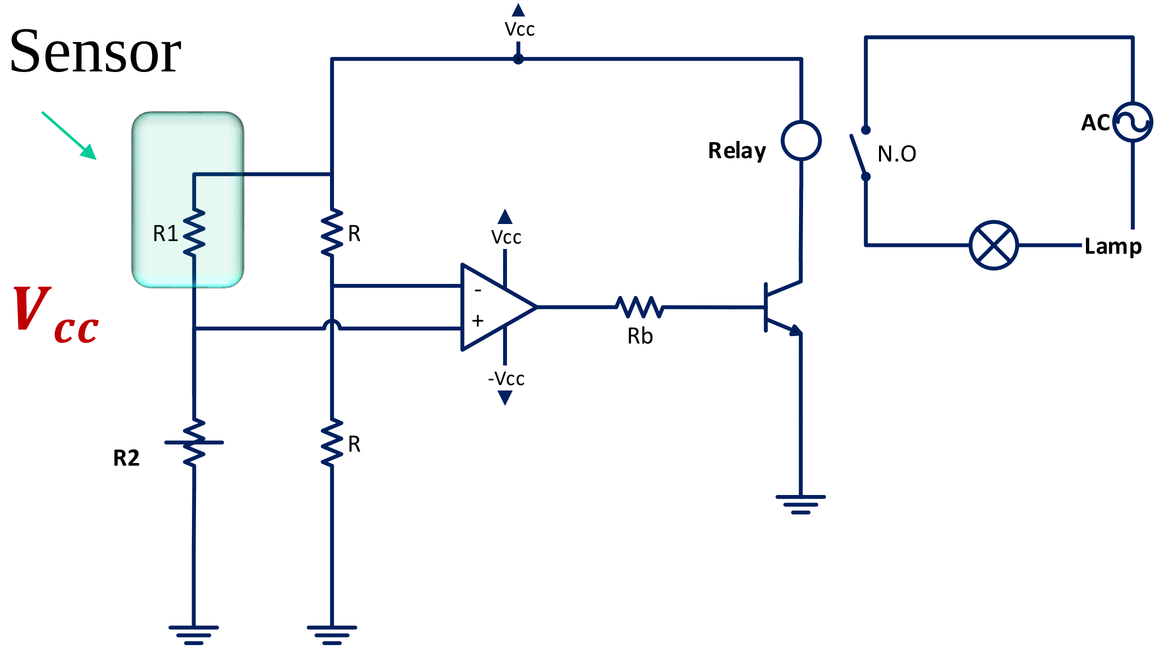
∴ switch is open

∴ Lamp is OFF

Over Temperature sensing Circuit

3- When T > T_C

$R_1 < R_2$



$$V(+) = \frac{R_2}{R_1 + R_2} V_{cc} > \frac{1}{2} V_{cc}$$

$$V(-) = \frac{1}{2} V_{cc}$$

∴ V(+) > V(-)

∴ transistor is conducting

∴ V_{op} = + Vsat

∴ Relay is energized

∴ Lamp is on

Schmitt Trigger Comparator

1. Assume $V_O = +V_{sat}$

V(-) = Vi

$$V(+) = \frac{R_2}{R_1+R_2}(+V_{sat}) = V_{UT}$$

$$vd > 0$$

$$\frac{R_2}{R_1 + R_2}(+V_{sat}) - V_i > 0$$

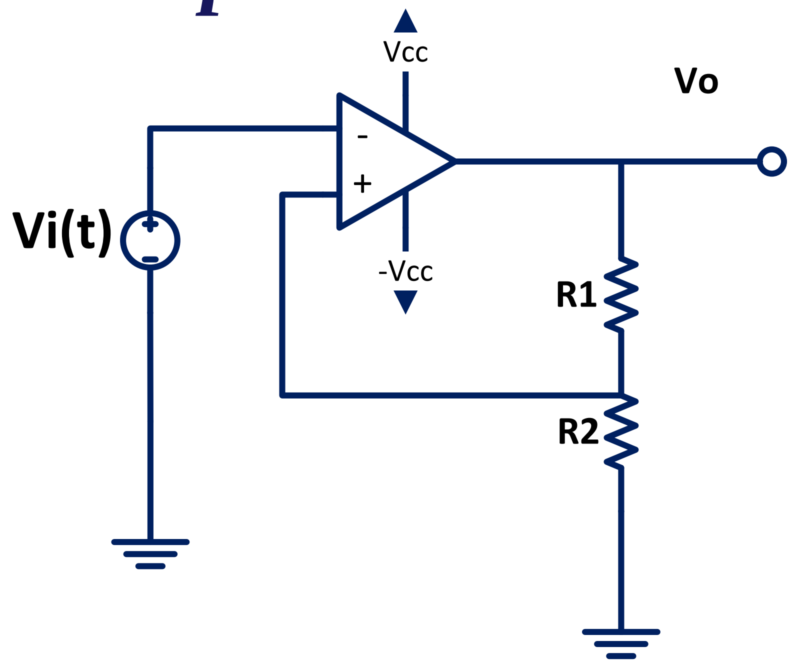
$$\therefore V_i < \frac{R_2}{R_1 + R_2}(+V_{sat})$$

$\therefore$ **as long as** $V_i < \frac{R_2}{R_1 + R_2}(+V_{sat})$

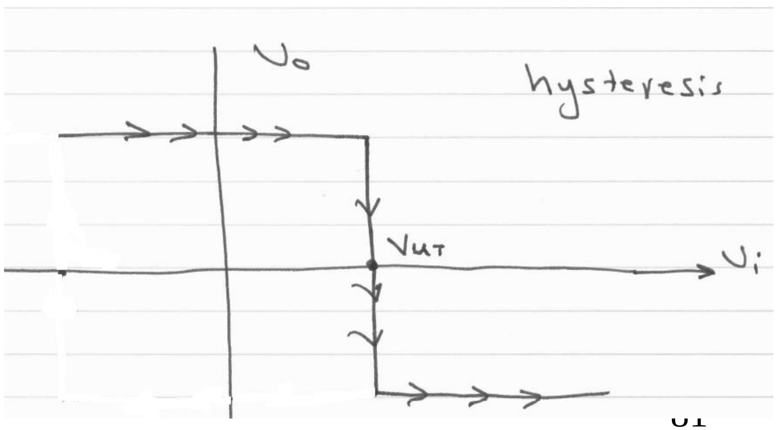
$$V_O = +V_{sat}$$

But when $V_i > \frac{R_2}{R_1 + R_2}(+V_{sat})$

V_O **switch to** $(-V_{sat})$



Inverting Schmitt trigger comparator



Schmitt Trigger Comparator

2. Assume $V_O = -V_{sat}$

$V(-) = V_i$

$V(+) = \frac{R_2}{R_1+R_2}(-V_{sat}) = V_{LT}$

$v_d < 0$

$\frac{R_2}{R_1+R_2}(-V_{sat}) - V_i < 0$

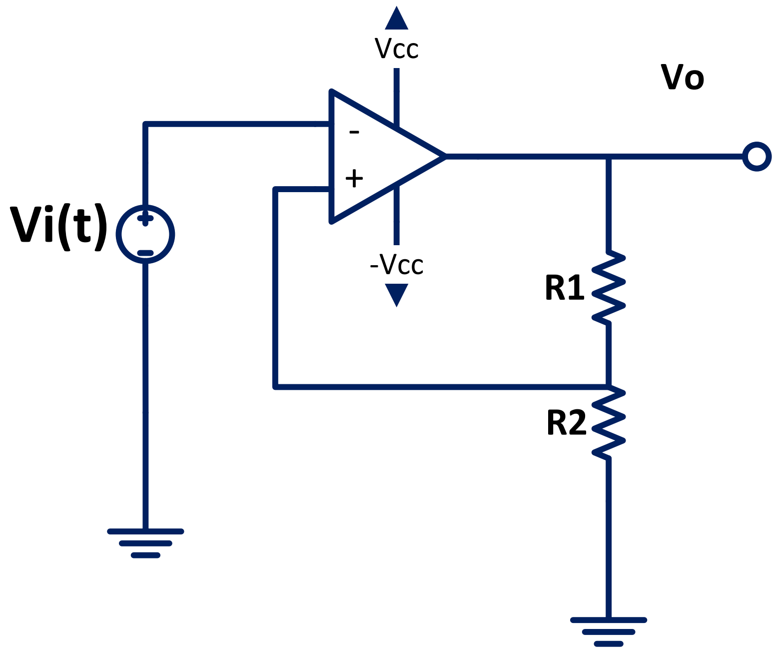
$\therefore V_i > \frac{R_2}{R_1+R_2}(-V_{sat})$

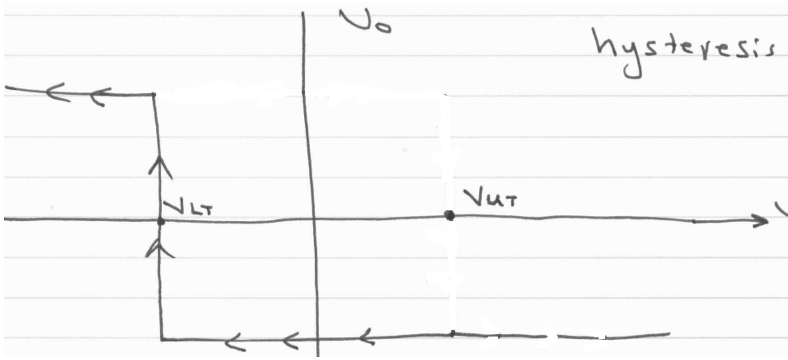
$\therefore$ as long as $V_i > \frac{R_2}{R_1+R_2}(-V_{sat})$

$V_O = -V_{sat}$

But when $V_i < \frac{R_2}{R_1+R_2}(-V_{sat})$

V_O switch to $(+V_{sat})$





Schmitt Trigger Comparator

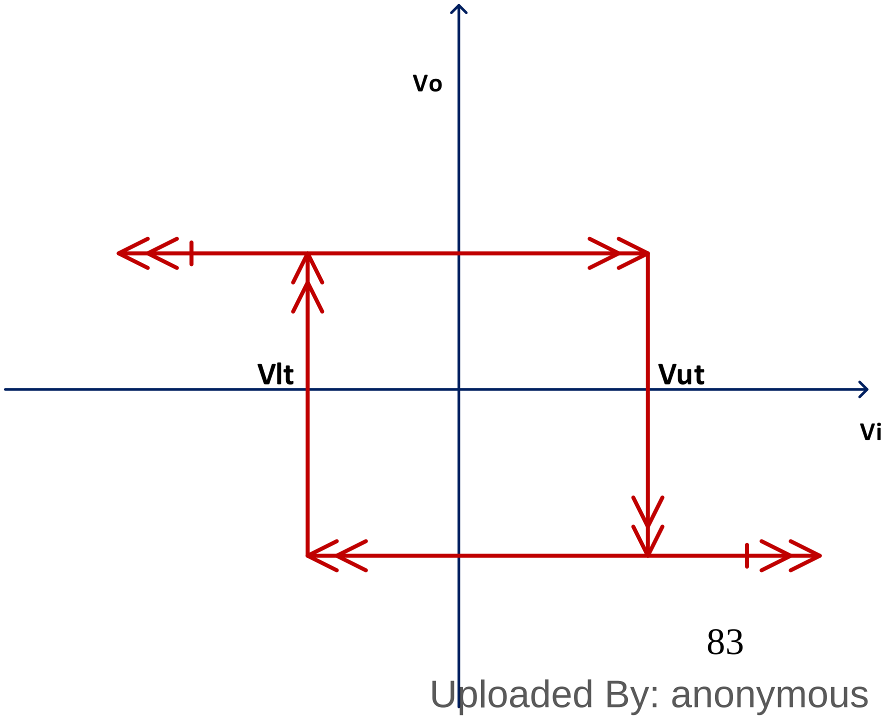
$$\text{Hysteresis voltage} \equiv V_H = V_{UT} - V_{LT}$$

$V_{UT} \equiv$ *Upper Threshold voltage*

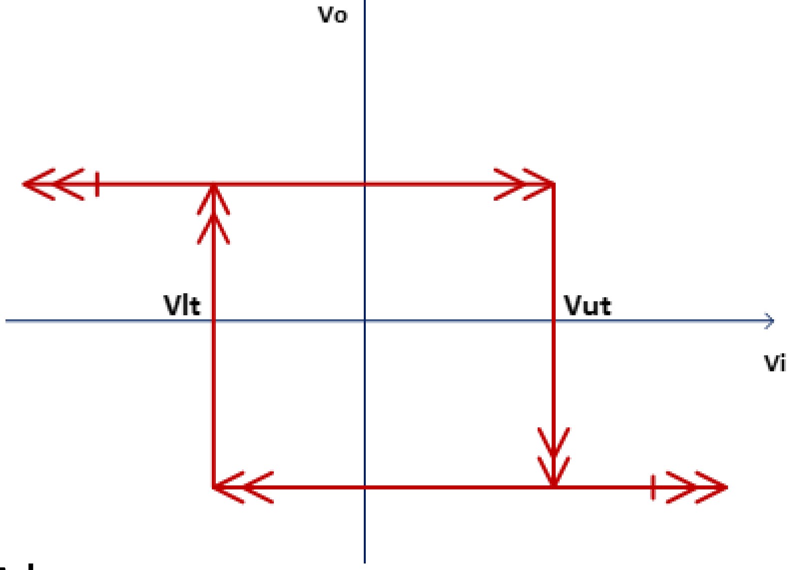
$$V_{UT} = \frac{R_2}{R_1+R_2}(+V_{sat})$$

$V_{LT} \equiv$ *Lower Threshold voltage*

$$V_{LT} = \frac{R_2}{R_1+R_2}(-V_{sat})$$



Schmitt Trigger Comparator



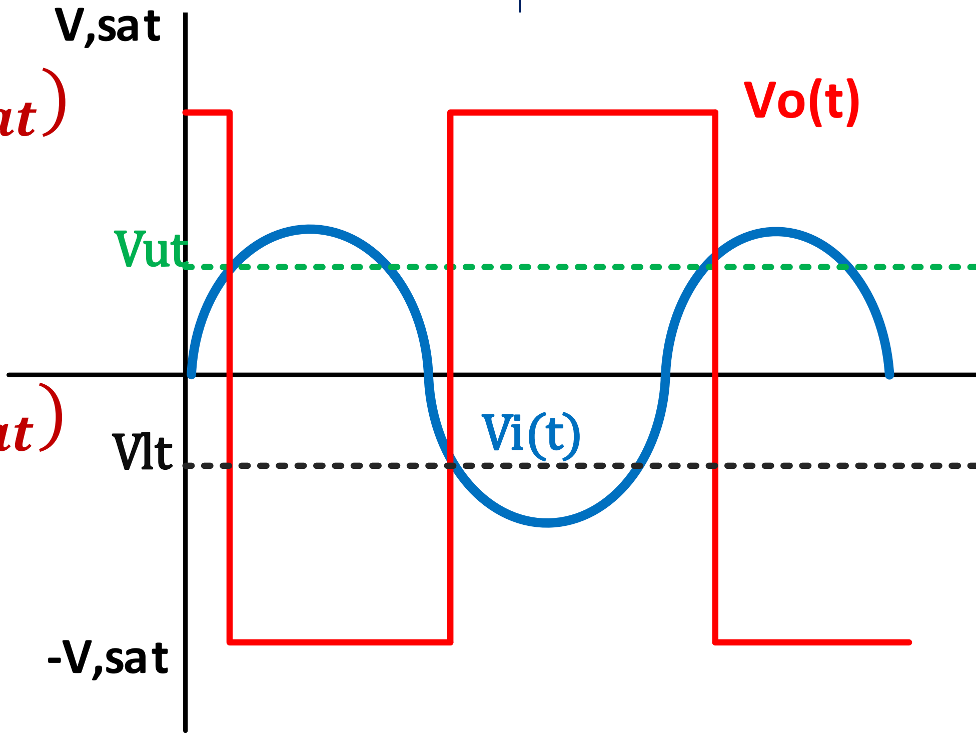
Signal Wave form

when $V_i < \frac{R_2}{R_1 + R_2}(-V_{sat})$

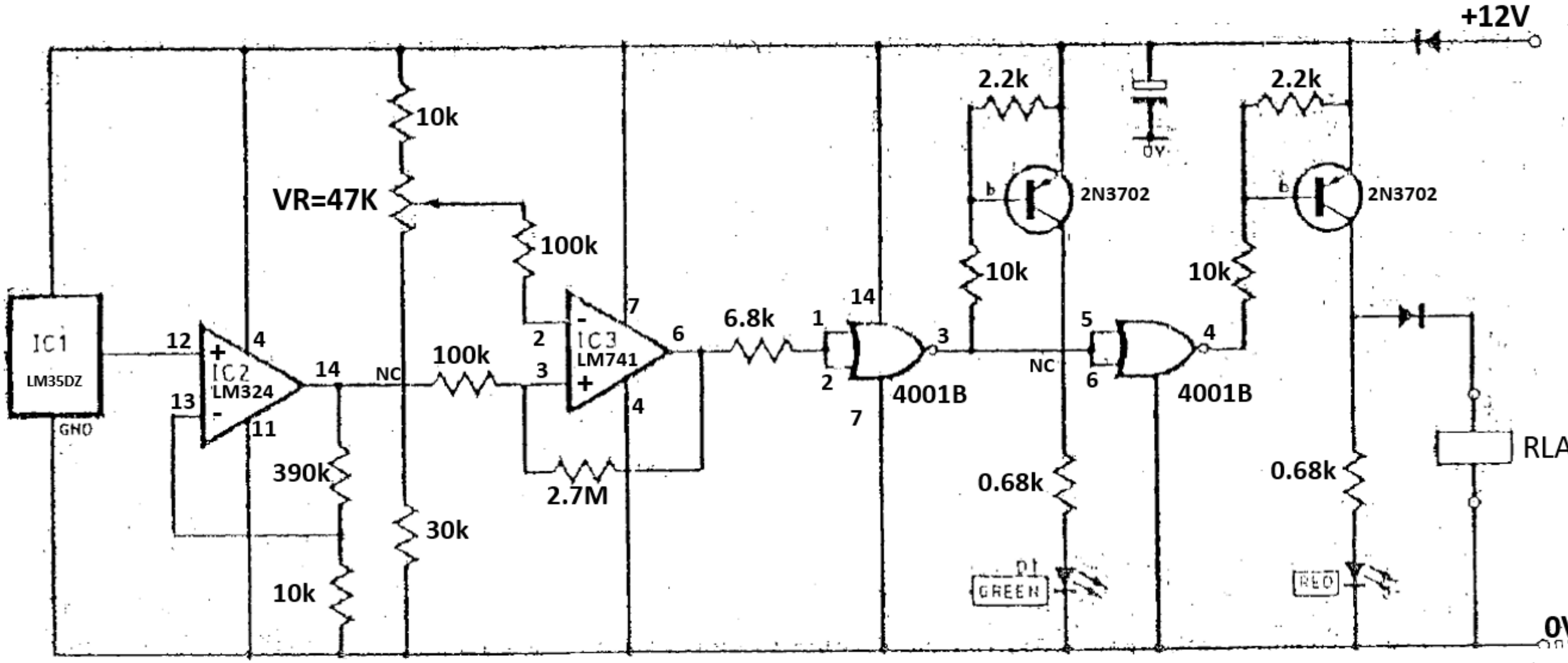
V_O switch to $(+V_{sat})$

when $V_i > \frac{R_2}{R_1 + R_2}(+V_{sat})$

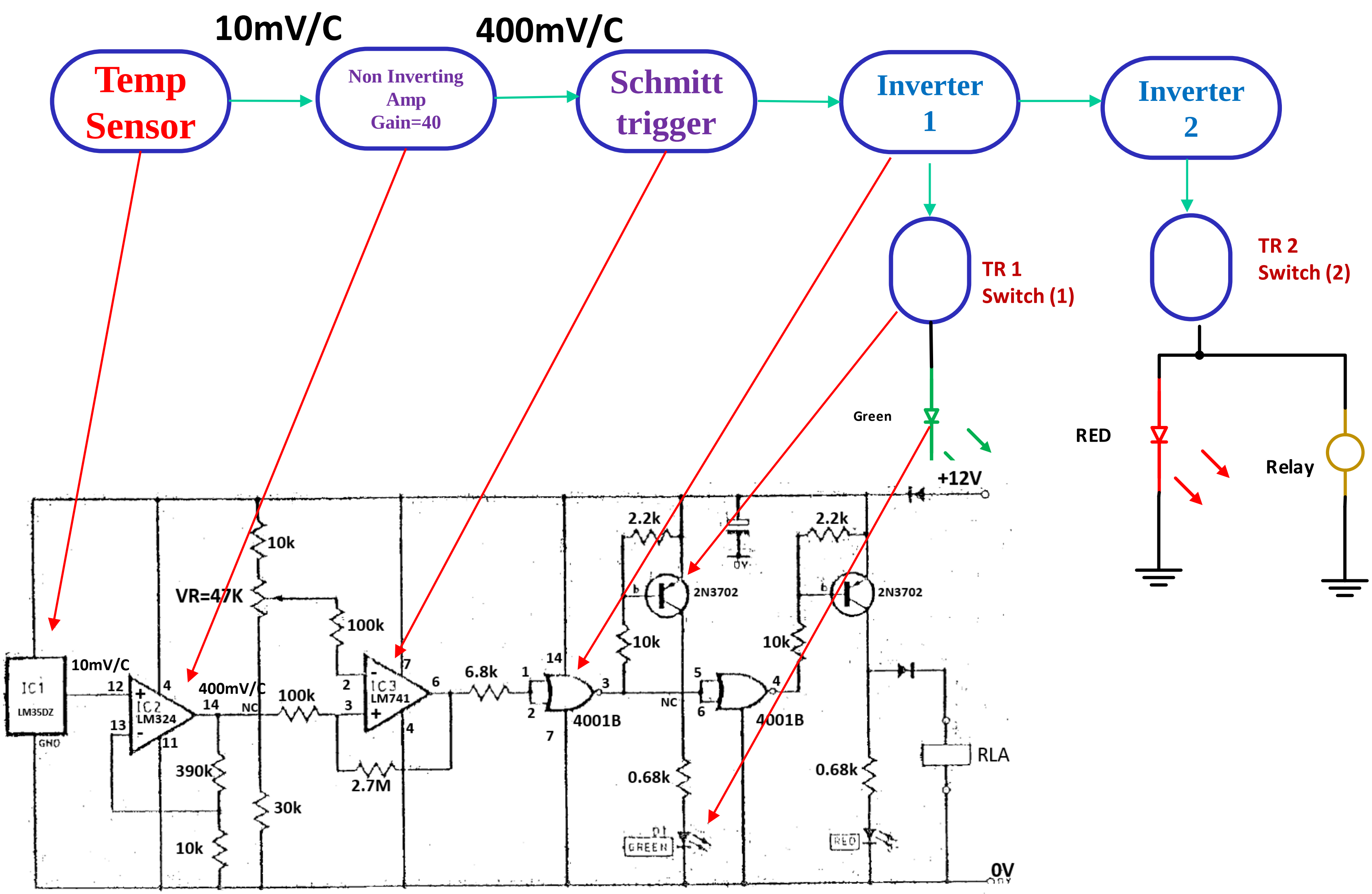
V_O switch to $(-V_{sat})$



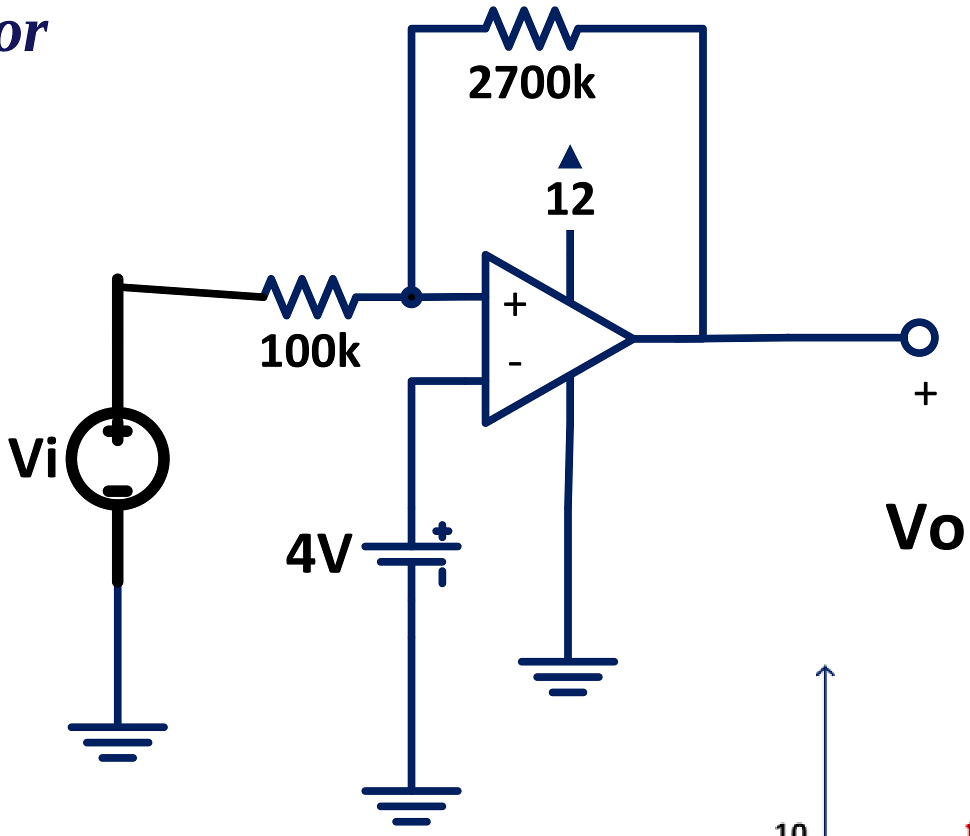
Room Thermostat



Room Thermostat



Room Thermostat



$V_i = 400\text{mV/C}$

1. Let $V_O = +V_{sat} = +10$

$vd > 0$

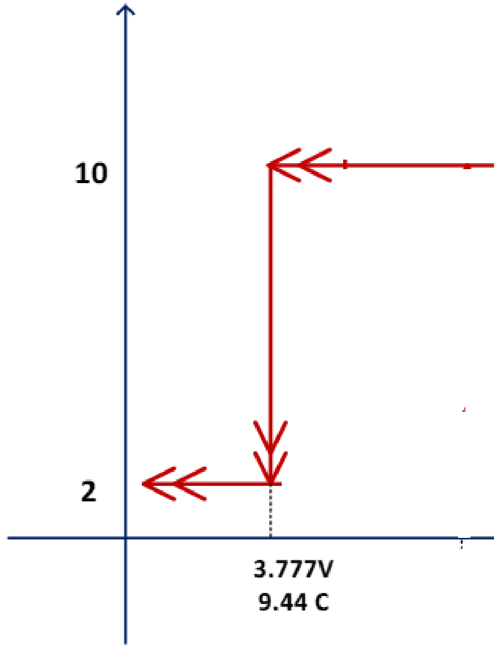
$V(-) = 4v$

$$V(+) = \frac{100K}{100K+2700K}(+V_{sat}) + \frac{2700K}{2700K+100K} V_i$$

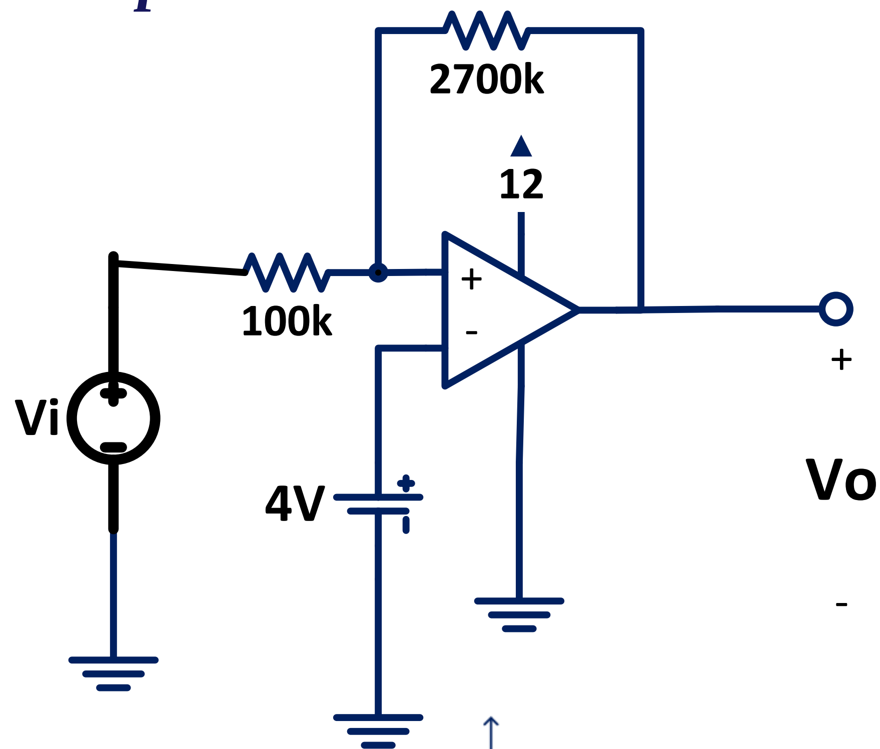
For $vd > 0$; $V_i > 3.777V$

∴ As long as $V_i > 3.777V$; $V_O = +V_{sat}$

But when $V_i < 3.777V$; V_O switch to $(-V_{sat})$



Schmitt Trigger Comparator



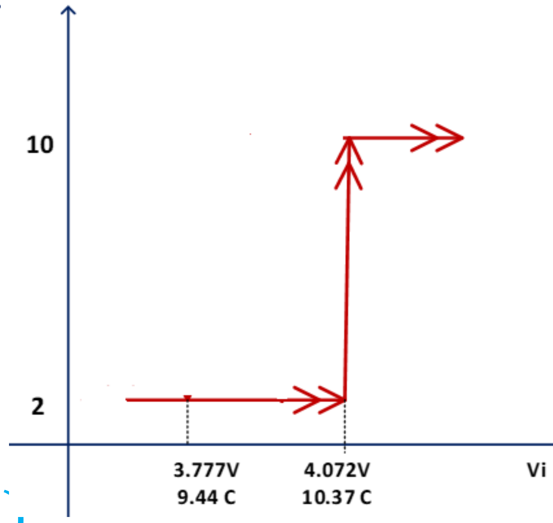
2. Let $V_O = -V_{sat} = +2$

$vd < 0$

$V(-) = 4V$

$$V(+) = \frac{100K}{100K+2700K}(-V_{sat}) + \frac{2700K}{2700K+100K} V_i$$

For $vd<0$; $V_i < 4.072V$

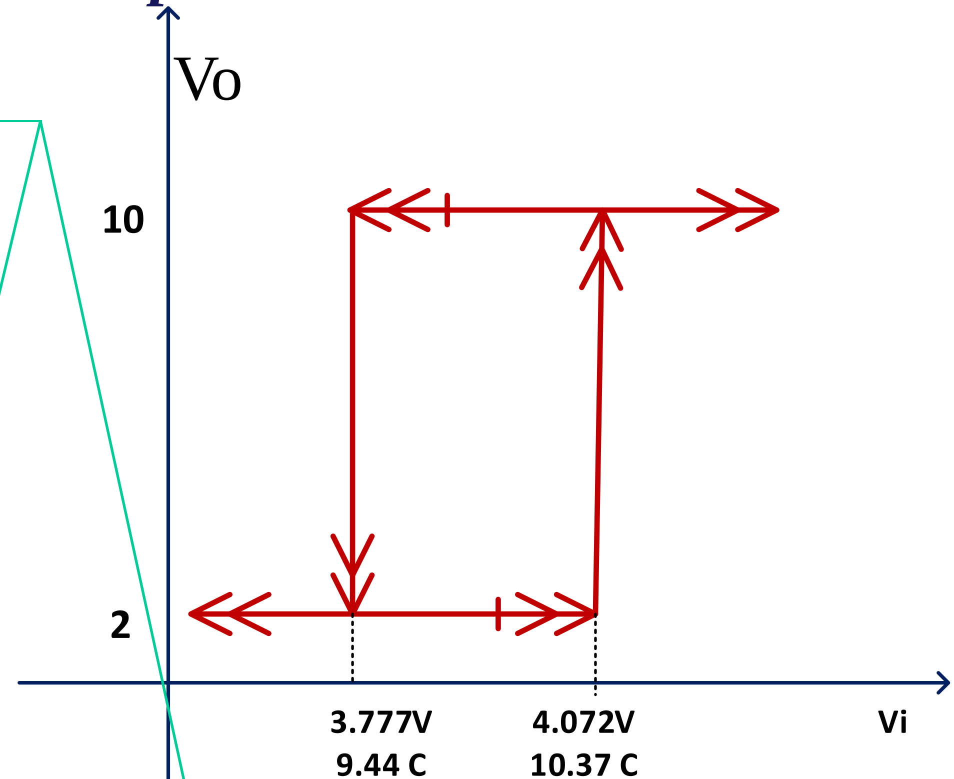


∴ As long as $V_i < 4.072V$; $V_O = -V_{sat}$

But when $V_i > 4.072V$; V_O switch to $(+V_{sat})$

Schmitt Trigger Comparator

1)When T > 10.37 C , Vo = +Vsat
Transistor (2) is Off , Relay is
denergized and Heater is Off.



2)Then T < 9.44 C , Vo = -Vsat
Transistor (2) is On , Relay is
energized
and Heater is on .