

8.4 Integration Using Partial Fraction

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Exp 2 $\int \frac{dx}{x^2 - x - 2} = \int \frac{dx}{(x-2)(x+1)}$ Partial Fraction

كسر جزئية بسيطة كسر معقد

$$= \int \left(\frac{A}{x-2} + \frac{B}{x+1} \right) dx$$

كثير مررنا في الامتحان

$$\frac{1}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$$

المقام خطي linear
Poly. of degree 1

Cover Method $\Rightarrow A = \frac{1}{\boxed{2}+1} = \frac{1}{3}$

$$B = \frac{1}{\boxed{-1}-2} = -\frac{1}{3}$$

$$\begin{aligned} \frac{1}{(x-2)(x+1)} &= \frac{\frac{1}{3}}{x-2} + \frac{-\frac{1}{3}}{x+1} \\ &= \frac{\frac{1}{3}(x+1) - \frac{1}{3}(x-2)}{(x-2)(x+1)} \\ &= \frac{\frac{1}{3}x + \frac{1}{3} - \frac{1}{3}x + \frac{2}{3}}{(x-2)(x+1)} \\ &= \frac{1}{(x-2)(x+1)} \end{aligned}$$

$$\int \frac{dx}{(x-2)(x+1)} = \int \left(\frac{\frac{1}{3}}{x-2} + \frac{-\frac{1}{3}}{x+1} \right) dx$$

$$= \frac{1}{3} \int \frac{dx}{x-2} - \frac{1}{3} \int \frac{dx}{x+1}$$

$$= \frac{1}{3} \ln |x-2| - \frac{1}{3} \ln |x+1| + C$$

Ex 10 $\int \frac{x+4}{x^2-1} dx = \int \frac{x+4}{(x-1)(x+1)} dx$

$$\begin{aligned}
 \text{Ex} \int \frac{x+4}{x^2+5x-6} dx &= \int \frac{x+4}{(x+6)(x-1)} dx \\
 &= \int \left(\frac{A}{x+6} + \frac{B}{x-1} \right) dx \\
 A = \frac{[-6]+4}{[-6]-1} &= \frac{-2}{-7} = \frac{2}{7} \quad \left| \quad B = \frac{[-1]+4}{[-1]+6} = \frac{3}{5} \right. \\
 &= \int \left(\frac{\frac{2}{7}}{x+6} + \frac{\frac{3}{5}}{x-1} \right) dx \\
 &= \frac{2}{7} \ln|x+6| + \frac{3}{5} \ln|x-1| + C
 \end{aligned}$$

$$\begin{aligned}
 \frac{x+4}{(x+6)(x-1)} &= \frac{A}{x+6} + \frac{B}{x-1} \\
 \frac{x+4}{(x+6)(x-1)} &= \frac{A(x-1) + B(x+6)}{(x+6)(x-1)}
 \end{aligned}$$

$$\Leftrightarrow x+4 = A(x-1) + B(x+6)$$

↑

$$\begin{aligned}
 x=1 &\Rightarrow 1+4 = 0 + B(1+6) \\
 5 &= 7B \\
 B &= \frac{5}{7}
 \end{aligned}$$

$$\begin{aligned}
 x=-6 &\Rightarrow -6+4 = A(-6-1) + 0 \\
 -2 &= -7A \\
 A &= \frac{2}{7}
 \end{aligned}$$

Cover Method

لازم يكون المقام عبارة عن حاصل ضرب عوامل مختلفة

Cover Method

نستخدم هذا على حاصل ضرب
عوامل مختلفة ومختلفة

$$\checkmark \frac{x+7}{x(x-1)(x+3)(x+5)}$$



x

$$\frac{\quad}{(x^2-1)(x-1)(x-1)}$$

x

Ex P

$$\int \frac{x^2+4x+1}{(x-1)(x+1)(x+3)} dx$$

$$\int \left(\frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+3} \right) dx$$

$$\begin{aligned} C &= \frac{(-3)^2 + 4(-3) + 1}{(-3-1)(-3+1)} \\ &= \frac{9-12+1}{(-4)(-2)} \\ &= \frac{-2}{8} = -\frac{1}{4} \end{aligned}$$

Cover Method

$$A = \frac{1^2 + 4(1) + 1}{(1+1)(1+3)} = \frac{1+4+1}{(2)(4)} = \frac{6}{8} = \frac{3}{4}$$

$$B = \frac{(-1)^2 + 4(-1) + 1}{(-1-1)(-1+3)} = \frac{1-4+1}{(-2)(2)} = \frac{-2}{-4} = \frac{1}{2}$$

$$= \frac{3}{4} \ln|x-1| + \frac{1}{2} \ln|x+1| - \frac{1}{4} \ln|x+3| + C$$

