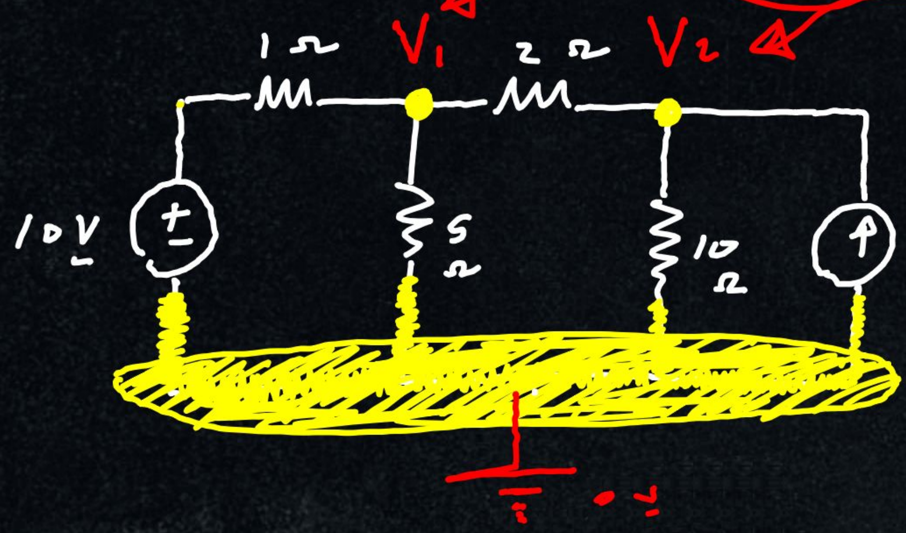
CH4: Techniques of Circuit Analysis

① Node-Voltage Method



step 1: Determine the number of essential nodes, n_e.

$n_e = 3$

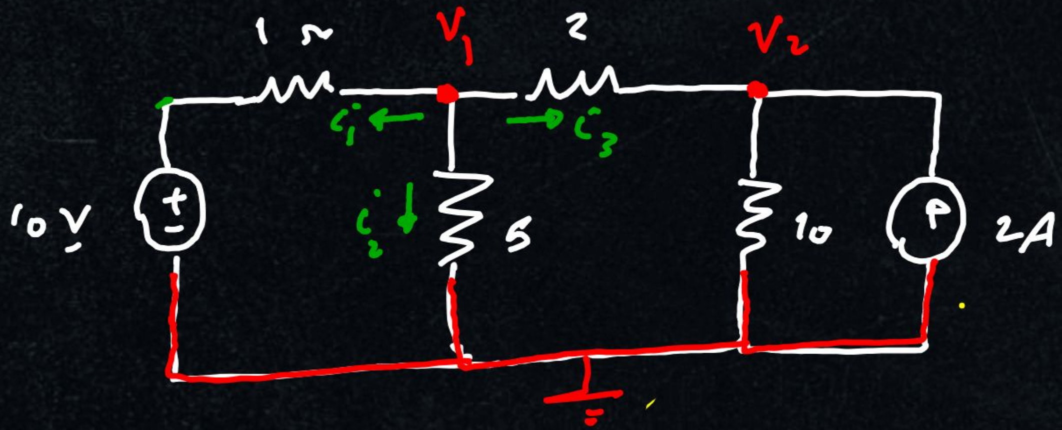
step 2: select one of these nodes as a reference node.

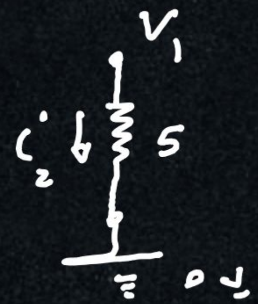
ref — 1st, 2nd, ---

ground — 0V

(Note: Node with the most branches is usually a good choice)

step 3: Define the node voltages on the circuit

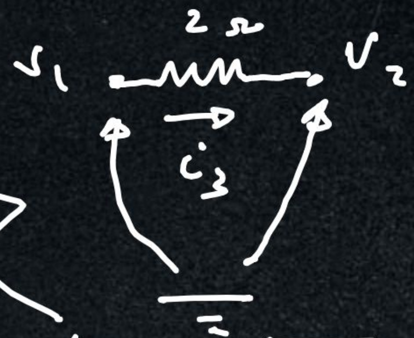




i_2 ??

$$i_2 = \frac{V_1 - 0}{5}$$

Step 4: Apply KCL at each node

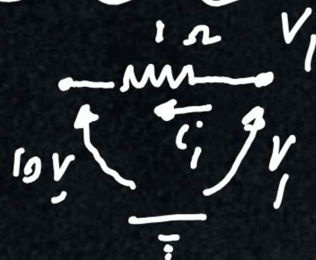


i_3 ??

$$i_3 = \frac{V_1 - V_2}{2}$$

KCL at node V_1:

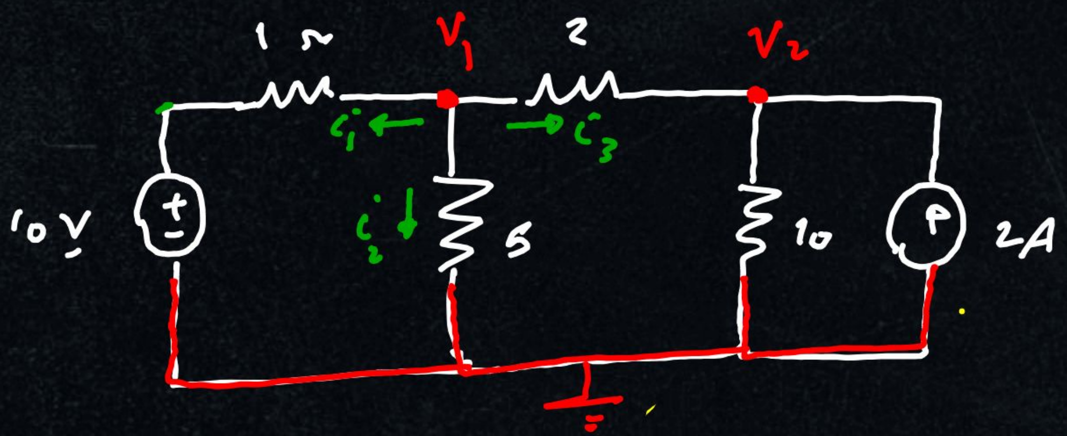
$$i_1 + i_2 + i_3 = 0$$



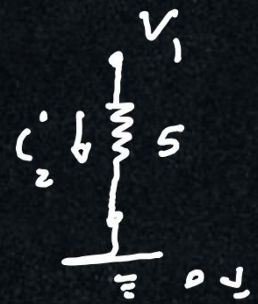
i_1 ??

$$i_1 = \frac{V_1 - 10}{1\,\Omega}$$

$$\boxed{\left(\frac{V_1 - 10}{1}\right) + \left(\frac{V_1 - 0}{5}\right) + \left(\frac{V_1 - V_2}{2}\right) = 0 \quad \text{①}}$$



i_2 ??



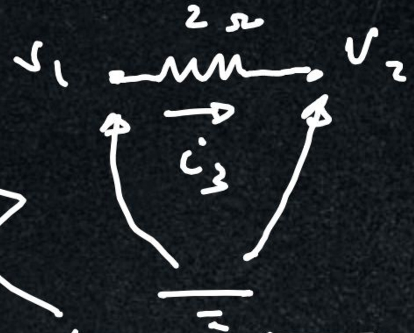
$$i_2 = \frac{V_1 - 0}{5}$$

Step 4: Apply KCL at each node

KCL at node V_1:

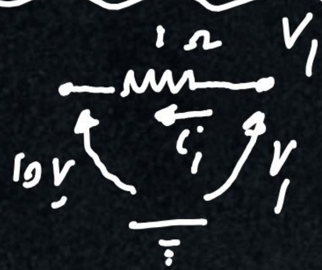
$$i_1 + i_2 + i_3 = 0$$

i_3 ??



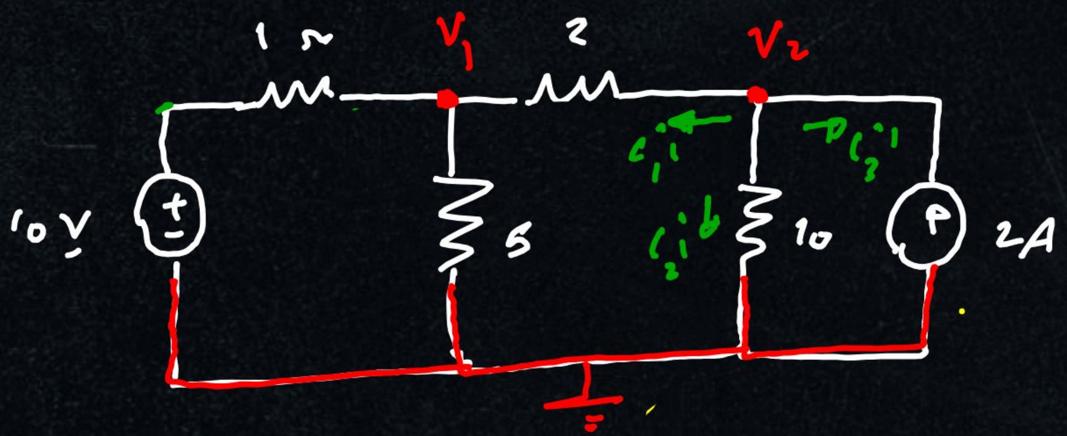
$$i_3 = \frac{V_1 - V_2}{2}$$

i_1 ??



$$i_1 = \frac{V_1 - 10}{1\,\Omega}$$

$$\left(\frac{V_1 - 10}{1}\right) + \left(\frac{V_1 - 0}{5}\right) + \left(\frac{V_1 - V_2}{2}\right) = 0 \quad (1)$$



$$\frac{V_1 - 10}{1} + \frac{V_1 - 0}{5} + \frac{V_1 - V_2}{2} = 0$$

$$\frac{V_2 - V_1}{2} + \frac{V_2 - 0}{10} + (-2) = 0$$

Step 4: Apply KCL at each node

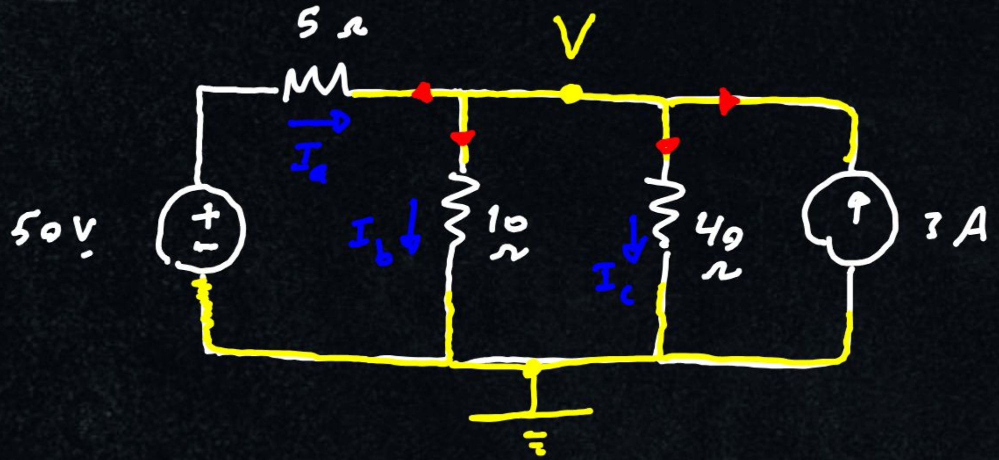
$$\boxed{V_1 = 9.09\ V \qquad V_2 = 10.91\ V}$$

KCL at node V_2

$$i_1 + i_2 + i_3 = 0$$

$$\boxed{\left(\frac{V_2 - V_1}{2}\right) + \left(\frac{V_2 - 0}{10}\right) + (-2) = 0} \quad \text{--- } (2)$$

Ex : Find I_a, I_b, & I_c using Node-voltage method



KCL at V :-

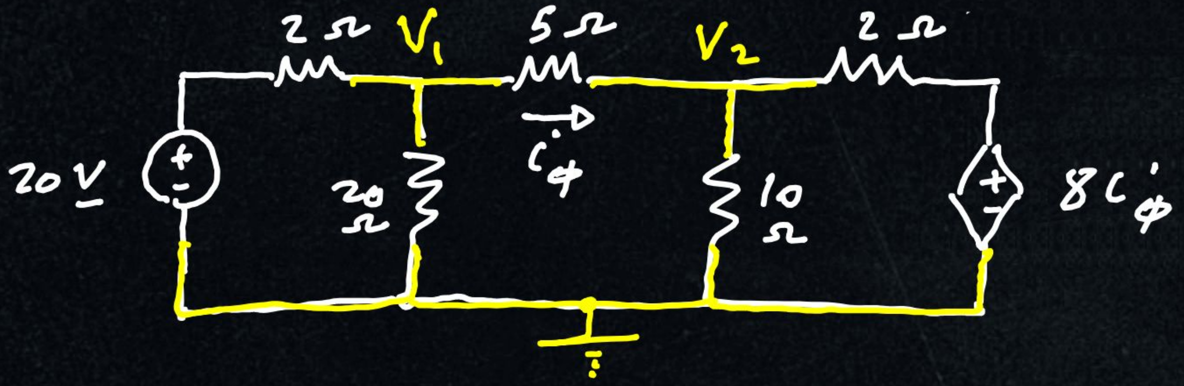
$$\left(\frac{V-50}{5}\right)+\left(\frac{V-0}{10}\right)+\left(\frac{V-0}{40}\right)+(-3)=0$$

$$8V-400+4V+V=120$$

$$V=40\,\underline{V}$$

$$\boxed{\begin{aligned} I_a &= \frac{50-V}{5} = 2\,A \\ I_b &= \frac{V-0}{10} = 4\,A \\ I_c &= \frac{V-0}{40} = 1\,A \end{aligned}}$$

EX



$$P = i_\phi^2 (5) = (1.2)^2 (5) = \boxed{7.2\ W}$$

Find the power absorbed by the 5 Ω resistor ?

"Using Node voltage method"

$$\left(\frac{V_1 - 20}{2}\right) + \left(\frac{V_1 - 0}{20}\right) + \left(\frac{V_1 - V_2}{5}\right) = 0 \quad \cdots (1)$$

$$\left(\frac{V_2 - V_1}{5}\right) + \left(\frac{V_2 - 0}{10}\right) + \left(\frac{V_2 - 8 i_\phi}{2}\right) = 0 \quad \cdots (2)$$

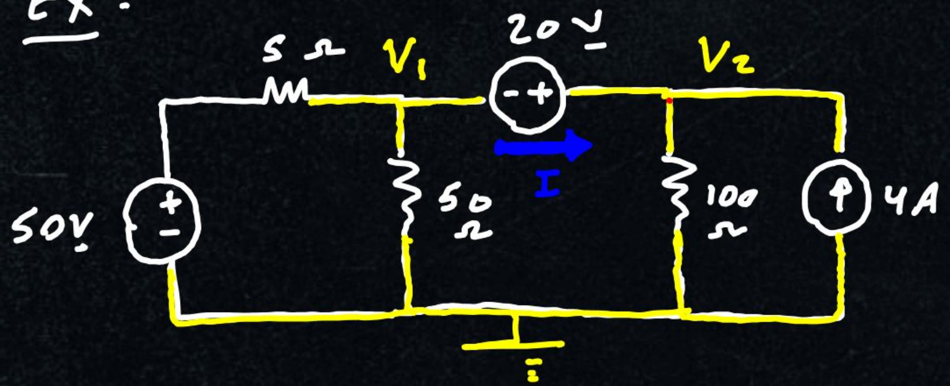
$$i_\phi = \frac{V_1 - V_2}{5} \quad \cdots (3)$$

$V_1 = 16\ V$

$V_2 = 10\ V$

$i_\phi = 1.2\ A$

EX :



$$\frac{V_1 - 50}{5} + \frac{V_1 - 0}{50} + \cancel{I} = 0 \quad ---①$$

$$-\cancel{I} + \frac{V_2 - 0}{100} + (-4) = 0 \quad ---②$$

$$\boxed{\frac{V_1 - 50}{5} + \frac{V_1}{50} + \frac{V_2}{100} + (-4) = 0} \quad ---③ = ① + ②$$

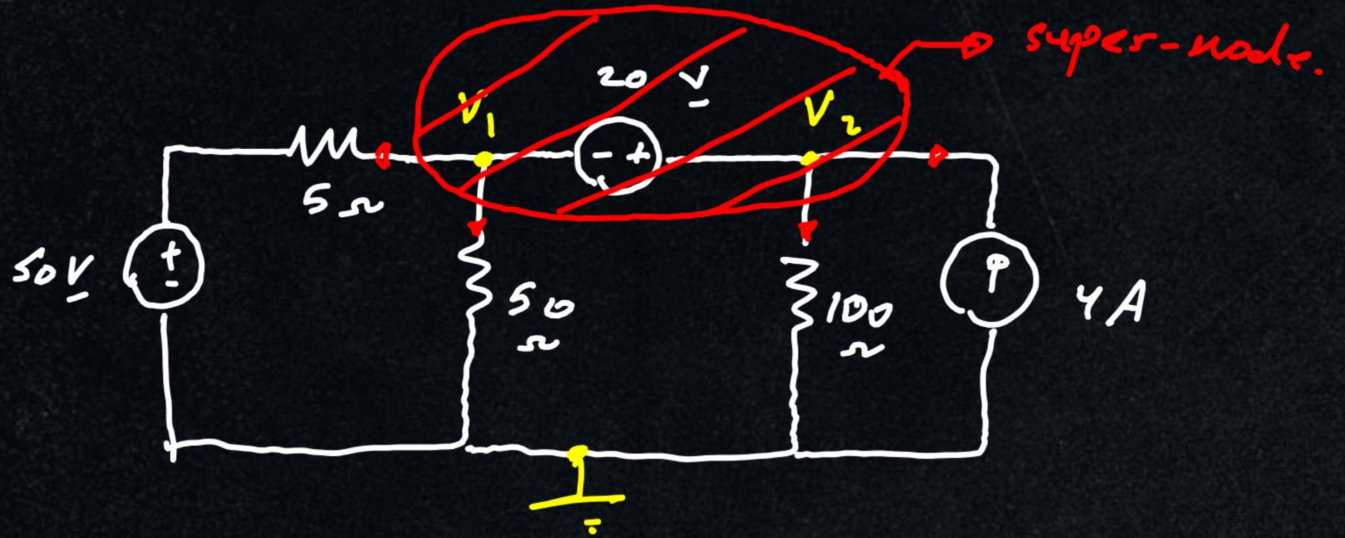
$$\boxed{V_2 - V_1 = 20\ V} \quad ---④$$

solve for V_1 & V_2 using equations ③ & ④

$V_1 = 60\ V$

$V_2 = 80\ V$

When a voltage source is between two essential nodes, we can combine these nodes to form a super-node.



KCL at super-node :-

$$\frac{V_1 - 50}{5} + \frac{V_1 - 0}{50} + \frac{V_2 - 0}{100} + (-4) = 0$$

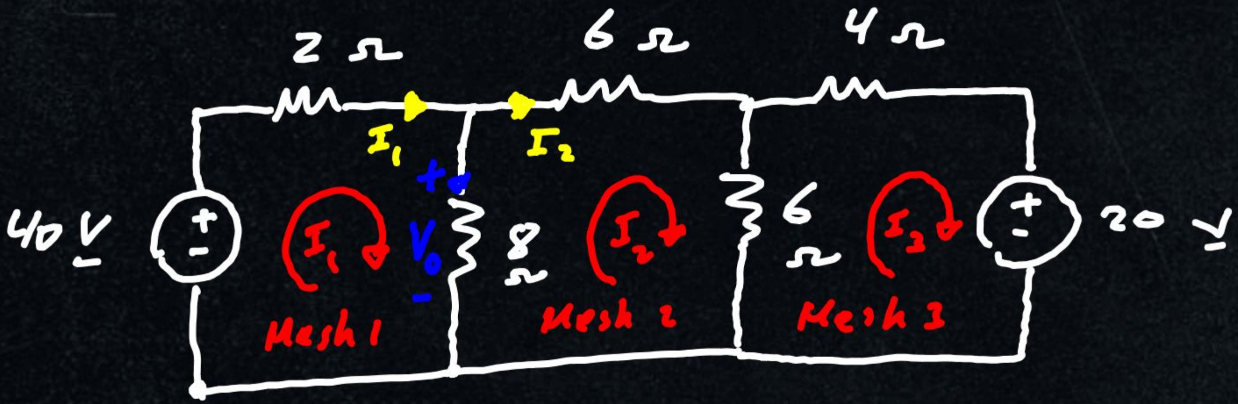
$$V_2 - V_1 = 20$$

solve for V_1 + V_2

$V_1 = 60$ V

$V_2 = 80$ V

(2) Mesh-current Method



KVL in mesh 1 :-

$$-40 + 2I_1 + 8(I_1 - I_2) = 0 \quad \cdots\cdots (1)$$

KVL in mesh 2 :-

$$8(I_2 - I_1) + 6I_2 + 6(I_2 - I_3) = 0 \quad \cdots (2)$$

KVL in mesh 3 :-

$$6(I_3 - I_2) + 4I_3 + 20 = 0 \quad \cdots (3)$$

By solving (1), (2), + (3)

$I_1 = 5.6\,A$

$I_2 = 2\,A$

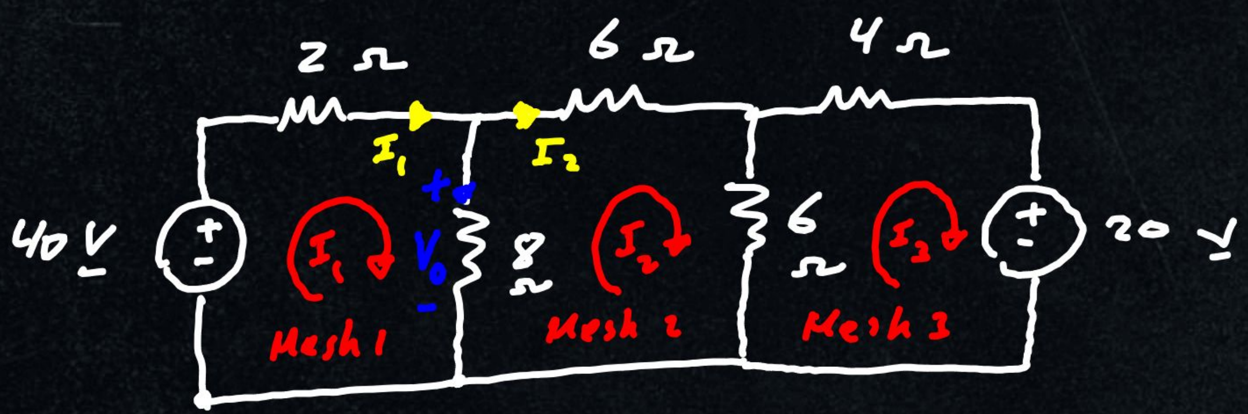
$I_3 = -0.8\,A$

Find V_o using mesh-current method :-

$$V_o = 8(I_1 - I_2)$$

$$= \boxed{28.8}\ V$$

② Mesh-current method



KVL in mesh 1 :-

$$-40 + 2I_1 + 8(I_1 - I_2) = 0 \quad \cdots\cdots (1)$$

KVL in mesh 2 :-

$$8(I_2 - I_1) + 6I_2 + 6(I_2 - I_3) = 0 \quad \cdots (2)$$

KVL in mesh 3 :-

$$6(I_3 - I_2) + 4I_3 + 20 = 0 \quad \cdots (3)$$

By solving (1), (2), + (3)

$I_1 = 5.6\ A$

$I_2 = 2\ A$

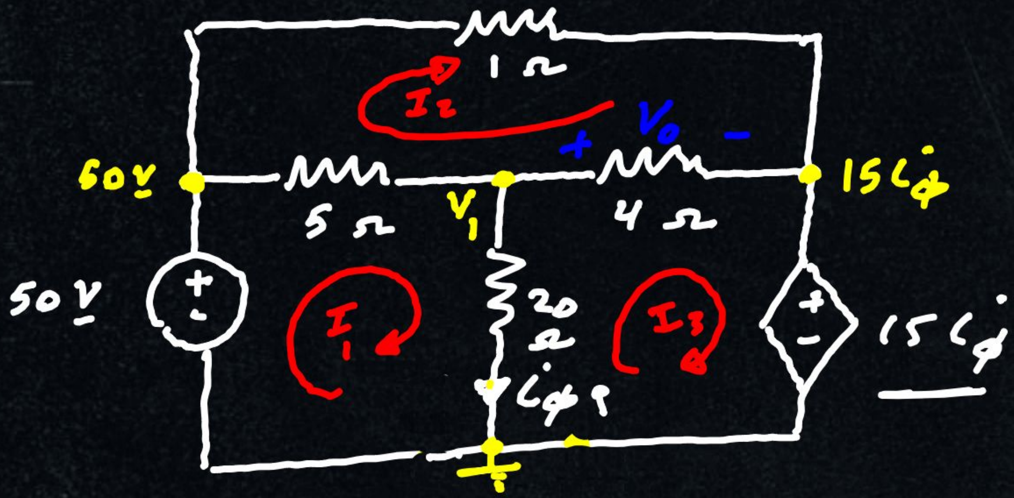
$I_3 = -0.8\ A$

Find V_o using mesh-current method :-

$$V_0 = 8(I_1 - I_2)$$

$$= \boxed{28.8}\ V$$

EX: Find V_0 using Mesh-current method :-



$$-50 + 5(I_1 - I_2) + 20(I_1 - I_3) = 0$$

$$1 I_2 + 4(I_2 - I_3) + 5(I_2 - I_1) = 0$$

$$20(I_3 - I_1) + 4(I_3 - I_2) + 15 i_\phi = 0$$

$$i_\phi = I_1 - I_3$$

solve for I_2 & I_3

$$I_2 = 26\ A$$

$$I_3 = 28\ A$$

$$V_0 = 4(I_3 - I_2) = \boxed{8\ V}$$

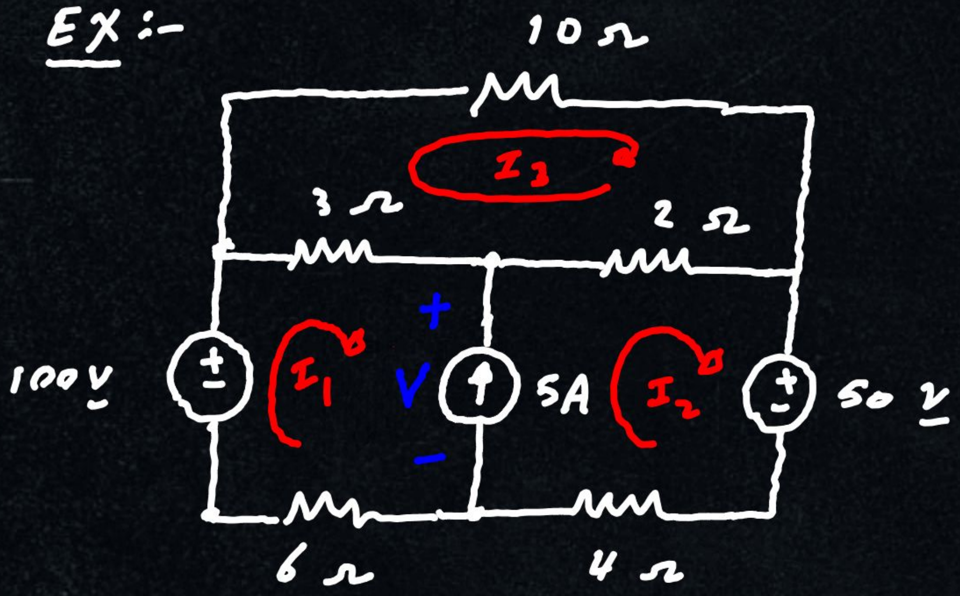
Node-voltage method

$$\frac{V_1 - 50}{5} + \frac{V_1}{20} + \frac{V_1 - 15 i_\phi}{4} = 0$$

$$i_\phi = \frac{V_1}{20}$$

Find V_1 & i_ϕ $\Rightarrow$ $V_0 = V_1 - 15 i_\phi$

EX :-



Mesh : It is a loop with no loops inside it

$$\left.\begin{aligned} -100 + 3(I_1 - I_3) + V + 6I_1 = 0 \\ -V + 2(I_2 - I_3) + 50 + 4I_2 = 0 \end{aligned}\right] \Rightarrow -100 + 3(I_1 - I_3) + 6I_1 + 2(I_2 - I_3) + 50 + 4I_2 = 0 \quad \text{--- (1)}$$

$$10I_3 + 2(I_3 - I_2) + 3(I_3 - I_1) = 0 \quad \text{--- (2)}$$

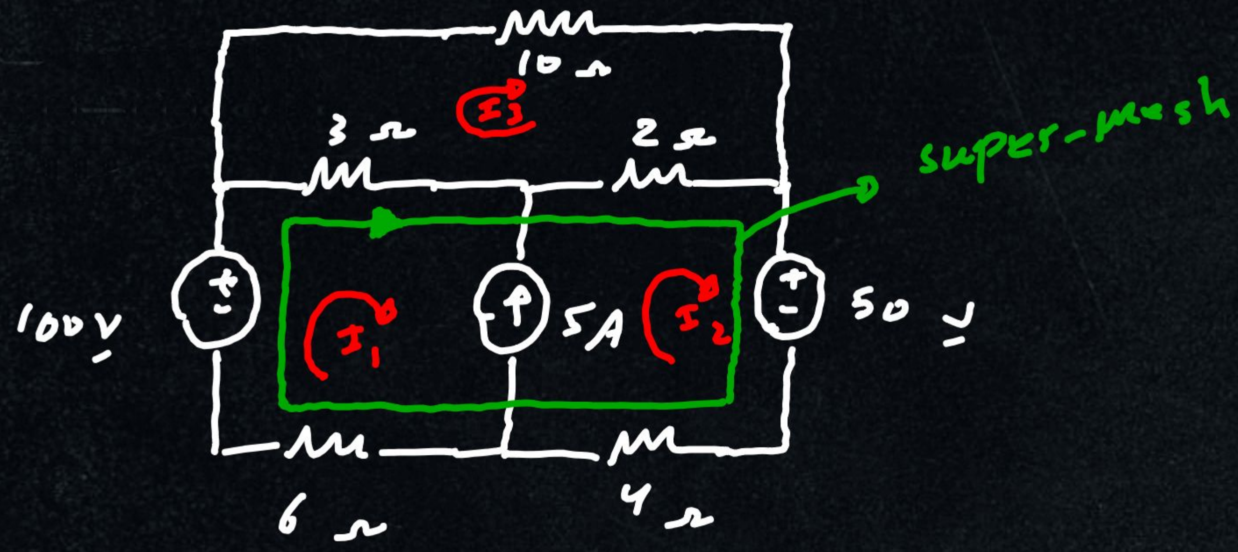
$$5 = I_2 - I_1 \quad \text{--- (3)}$$

$I_1 = 1.75\ A$

$I_2 = 6.75\ A$

$I_3 = 1.25\ A$

The concept of supermesh



$$-100 + 3(I_1 - I_3) + 2(I_2 - I_3) + 50 + 4I_2 + 6I_1 = 0 \quad \text{--- (1)}$$

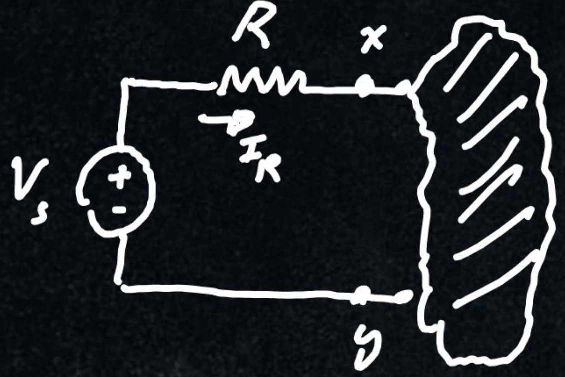
$$I_2 - I_1 = 5A \quad \text{... (2)}$$

$$10I_3 + 2(I_3 - I_2) + 3(I_3 - I_1) = 0 \quad \text{--- (3)}$$

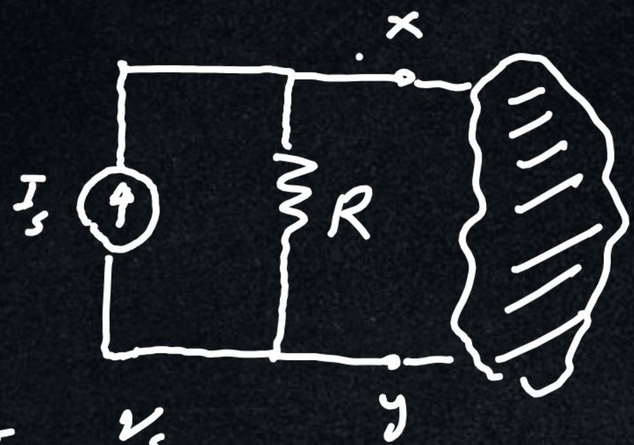
$I_1 = 1.75\ A$, $I_2 = 6.75\ A$, & $I_3 = 1.25\ A$.

③ Source Transformation

It allows a voltage source in series with a resistor to be replaced by a current source in parallel with the same resistor.



$\langle \equiv \rangle$

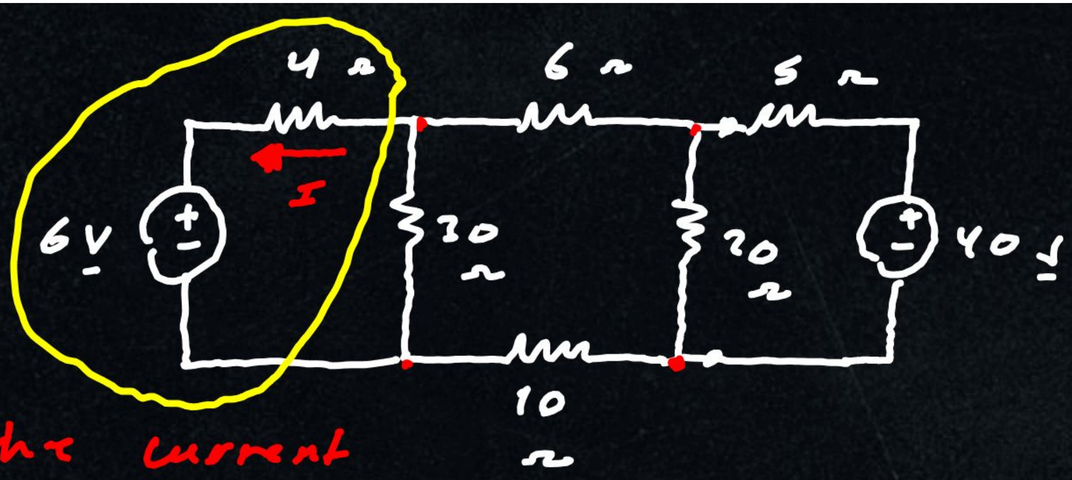


$V_s = R I_R + V_{xy}$ KVL

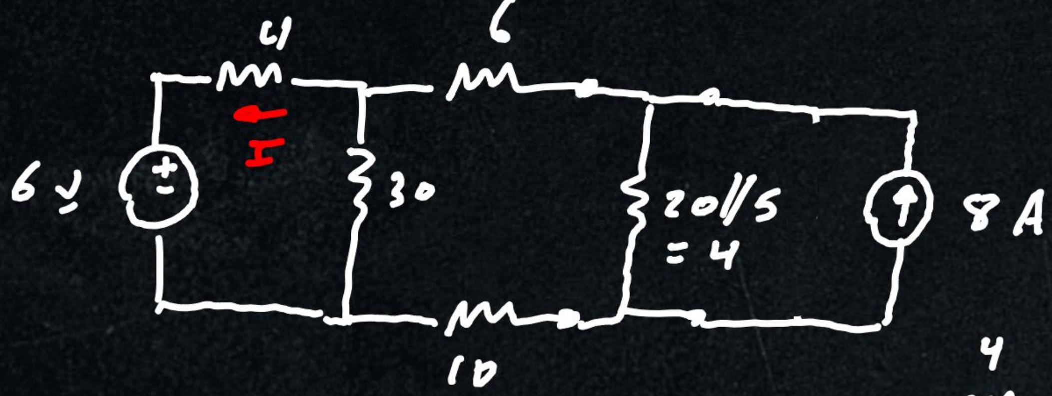
$\frac{V_s}{R} = I_R + \frac{V_{xy}}{R}$ KCL

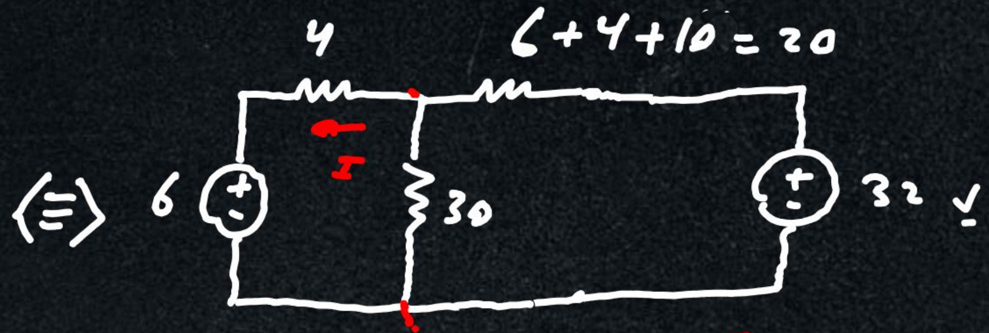
$I_s = \frac{V_s}{R}$

EX:-



Find the current I in the circuit using source transformation method.

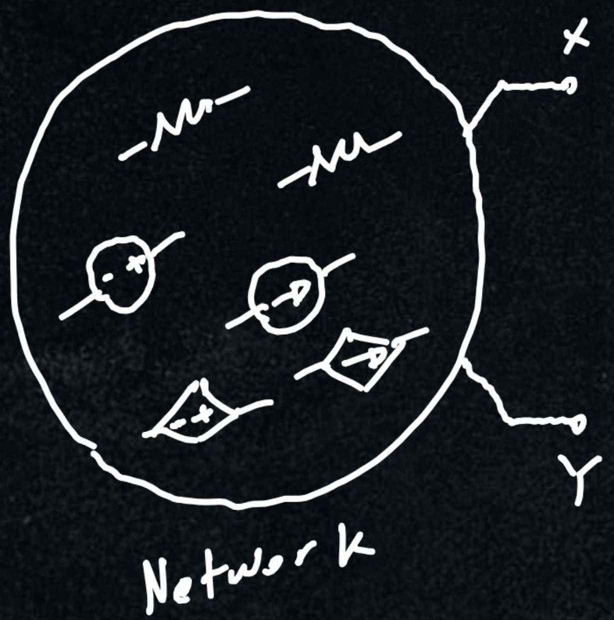




$$I = \frac{19.2 - 6}{16} = \boxed{0.825\ A}$$

④ Thevenin Method

Thevenin Equivalent circuit : It is an independent voltage source (V_{Th}) in series with a resistor (R_{Th}) which replace an interconnection of sources + resistors



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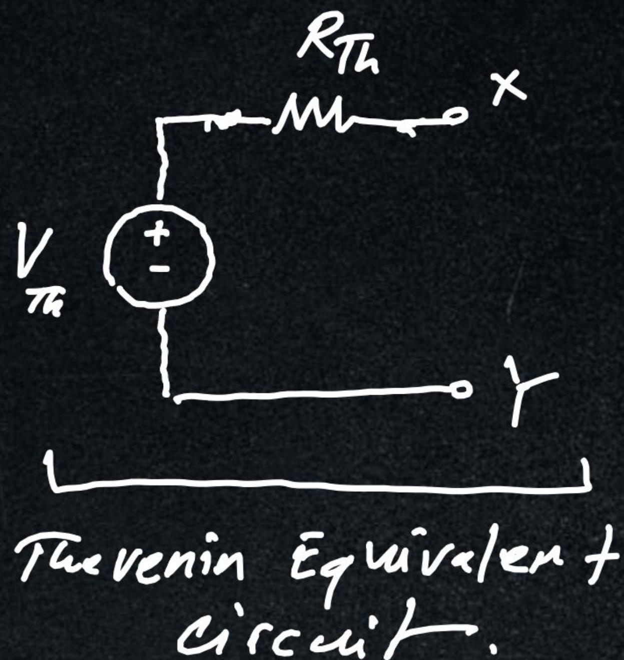
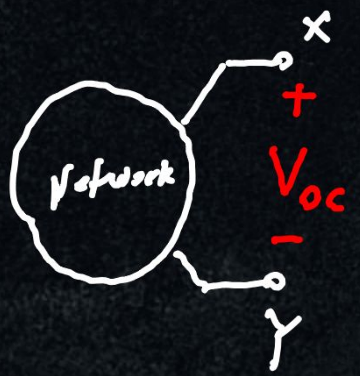


Methods to find the Thevenin equivalent circuit

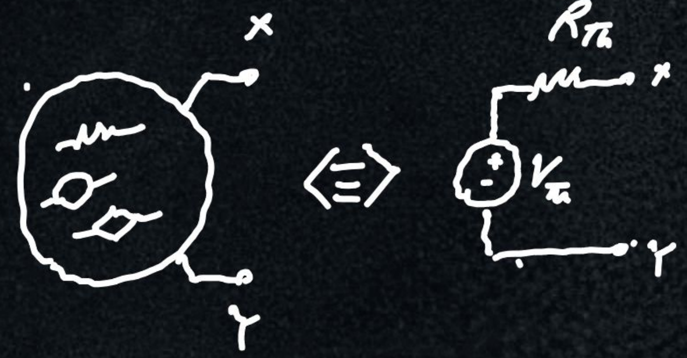
Method 1

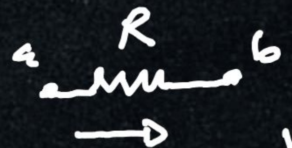
step1: calculate the open-circuit voltage across the terminals XY



V_{oc} : open-circuit voltage

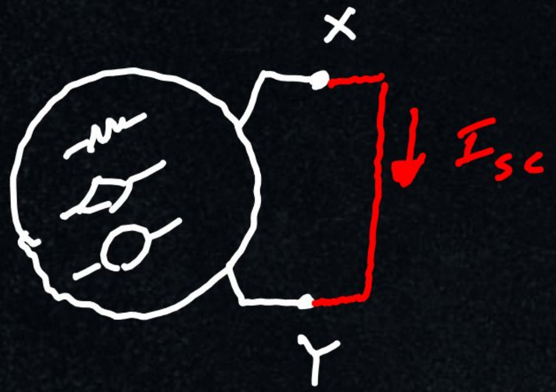
$\Rightarrow$ $V_{oc} = V_{Th}$



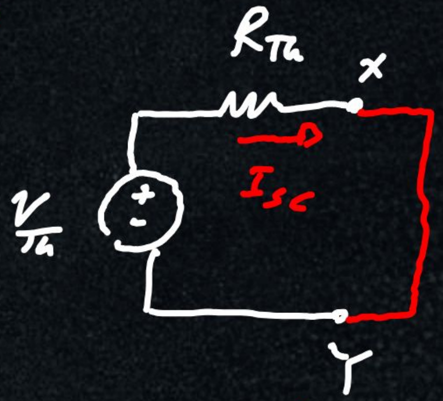


$$I = 0 = \frac{V_a - V_b}{R}$$

step 2: calculate the short circuit current between the terminals X & Y.



I_{sc} : short-circuit current.



$$I_{sc} = \frac{V_{Th}}{R_{Th}}$$

step 3: Calculate R_{Th}

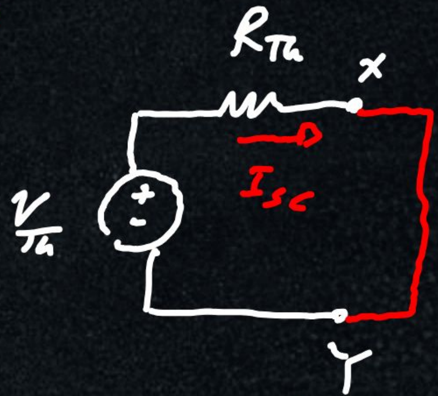
$$R_{Th} = \frac{(V_{Th} = V_{oc})}{I_{sc}}$$

← step 1

← step 2

step 2: calculate the short circuit current between the terminals X & Y.

I_{sc} : short-circuit current.



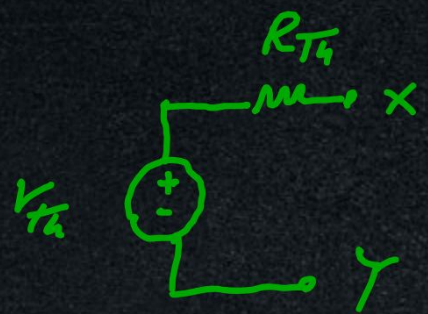
$$I_{sc} = \frac{V_{Th}}{R_{Th}}$$

step 3: Calculate R_{Th}

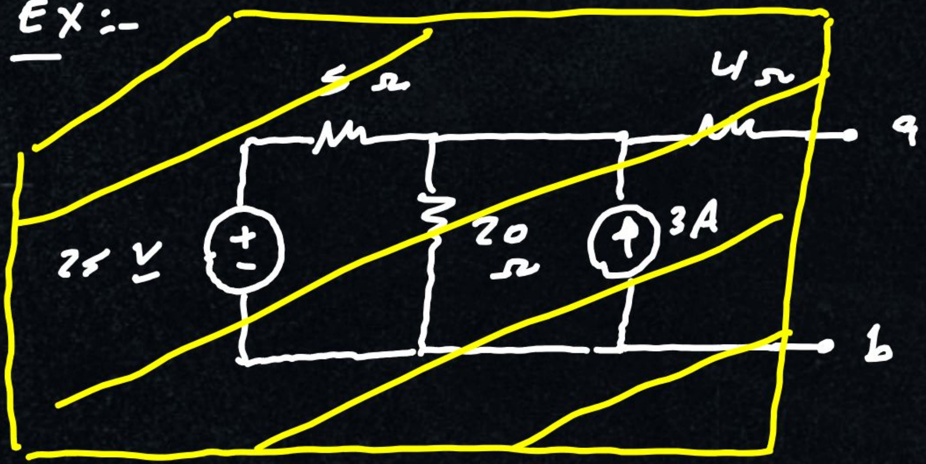
$$R_{Th} = \frac{(V_{Th} = V_{oc})}{I_{sc}}$$

← step 1

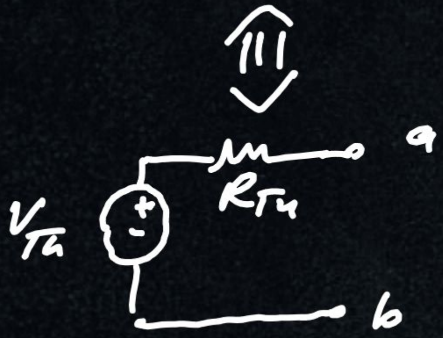
← step 2

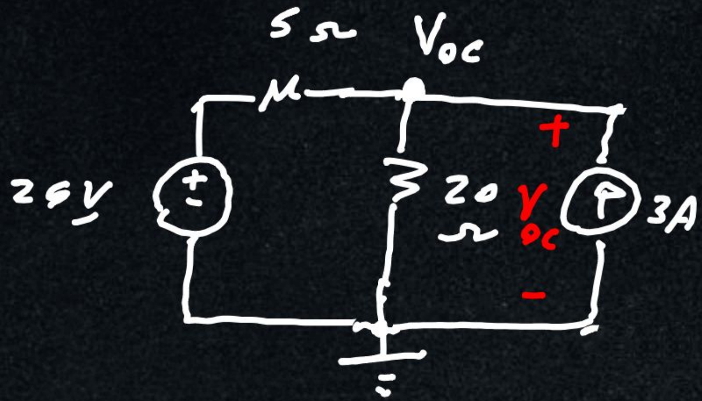


Ex:-



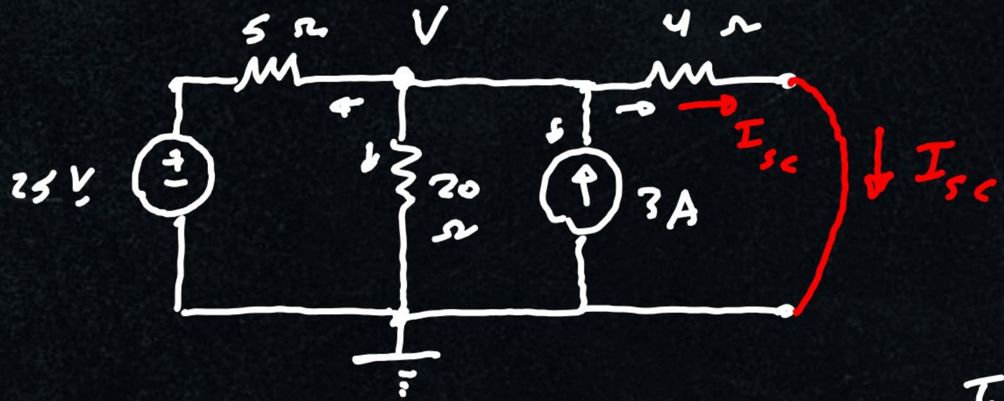
Find the Thevenin equivalent circuit across ab.





KCL at V_{oc} :

$$\frac{V_{oc}-25}{5}+\frac{V_{oc}}{20}-3=0 \Rightarrow \boxed{V_{oc}=V_{Th}=32\ \text{V}}$$

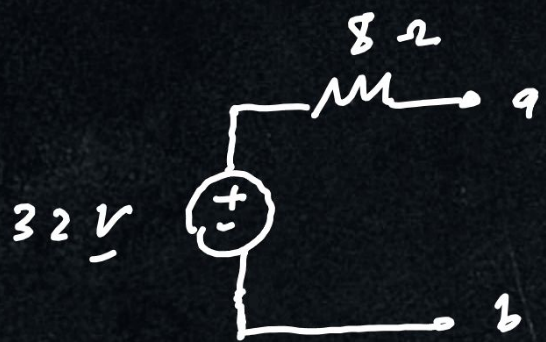


$$\frac{V-25}{5} + \frac{V}{20} - 3 + \frac{V}{4} = 0$$

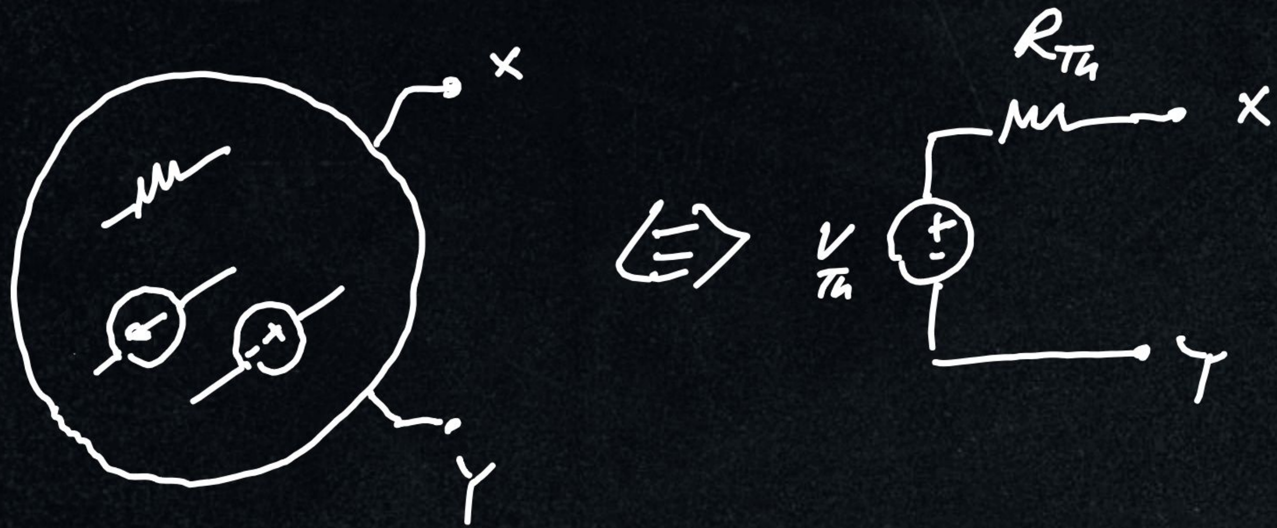
$$V = 16\ V$$

$$I_{sc} = \frac{V}{4} = \frac{16}{4} = 4\ A.$$

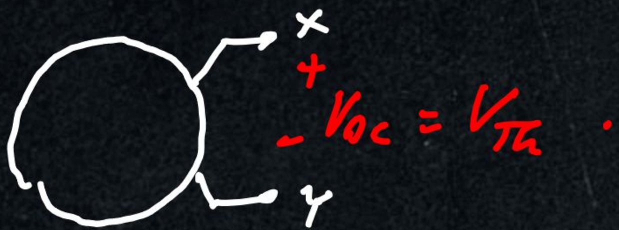
$$R_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{32}{4} = 8\ \Omega$$



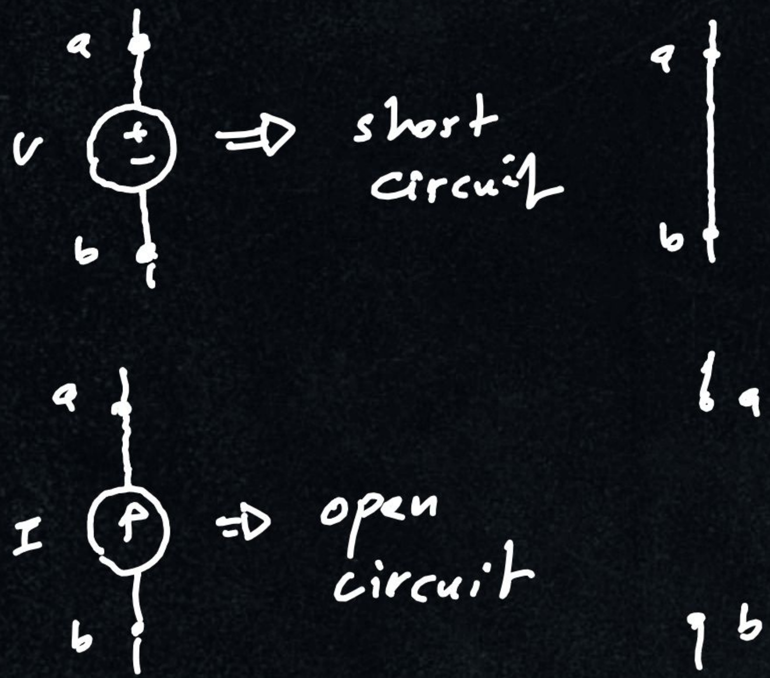
Method 2 :- It is useful if the network contains only indep. sources and resistors

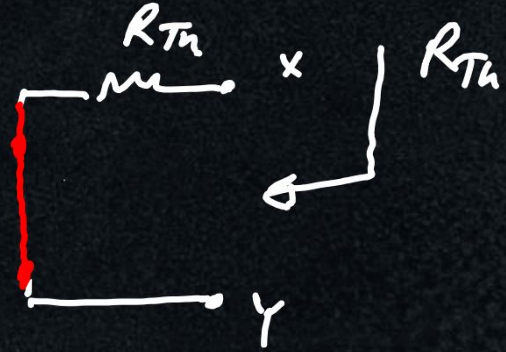


step 1: calculate the open circuit voltage across the terminals X & Y

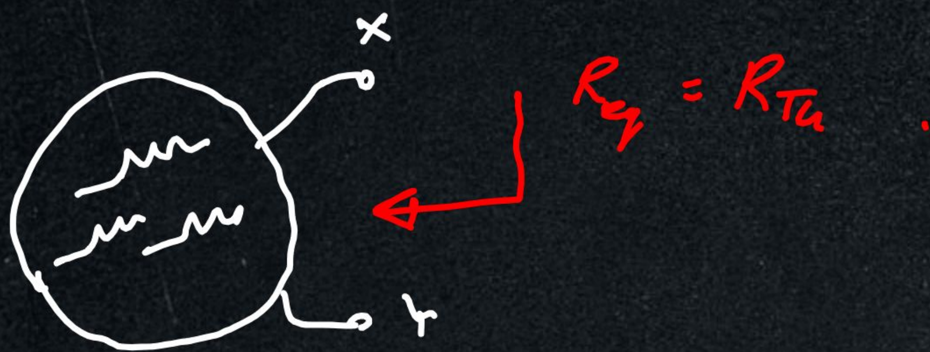


step 2: Kill all indep. sources in the circuit

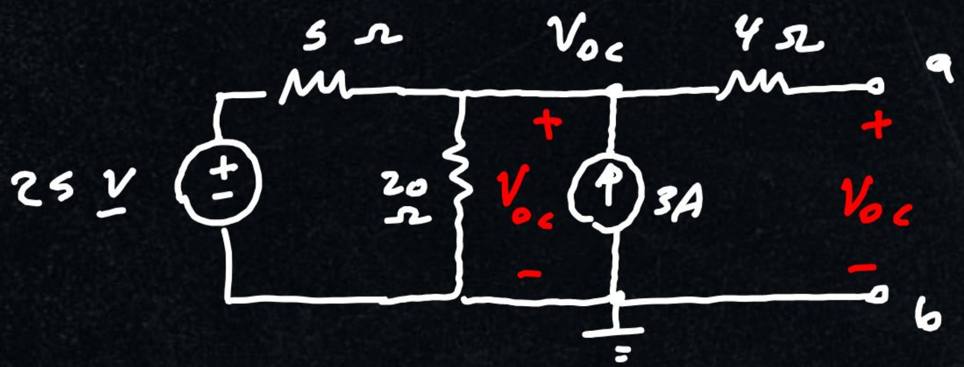




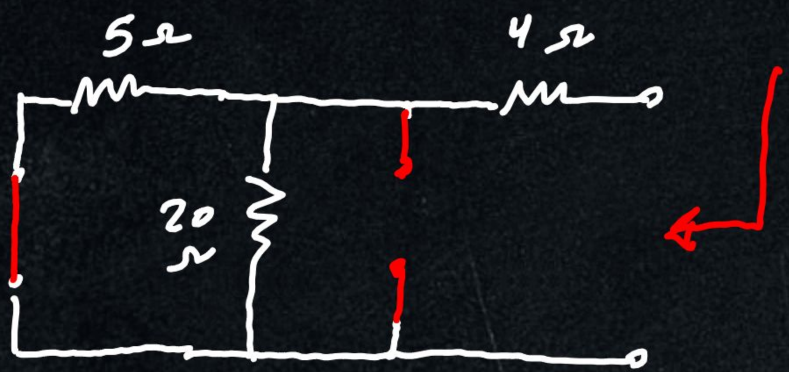
step 3:- Calculate the resistance seen at terminals x & Y



Find the Thevenin Equivalent circuit across the terminals ab



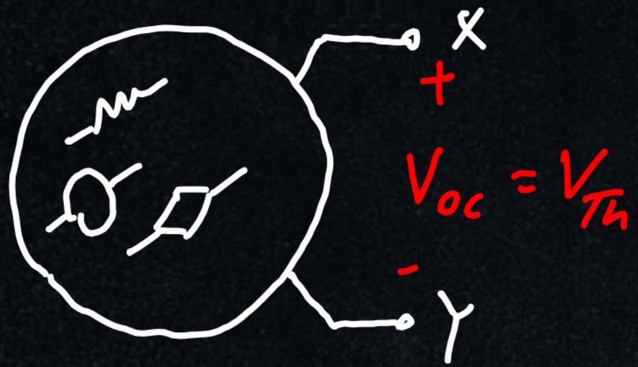
$$\frac{V_{oc} - 25}{5} + \frac{V_{oc}}{20} - 3 = 0 \Rightarrow V_{Th} = V_{oc} = 32 \text{ V}$$



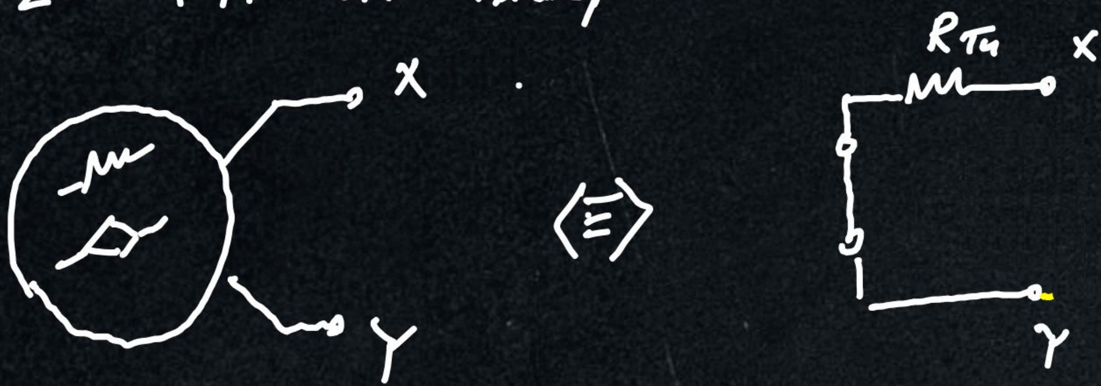
$$R_{Th} = [5 \parallel 20] + 4 = 8 \ \Omega$$

<u>Method 3</u> :- It is useful if the circuit contains dep. sources

step 1: Calculate the open circuit voltage across X + Y



step 2: Kill all indep. sources in the circuit

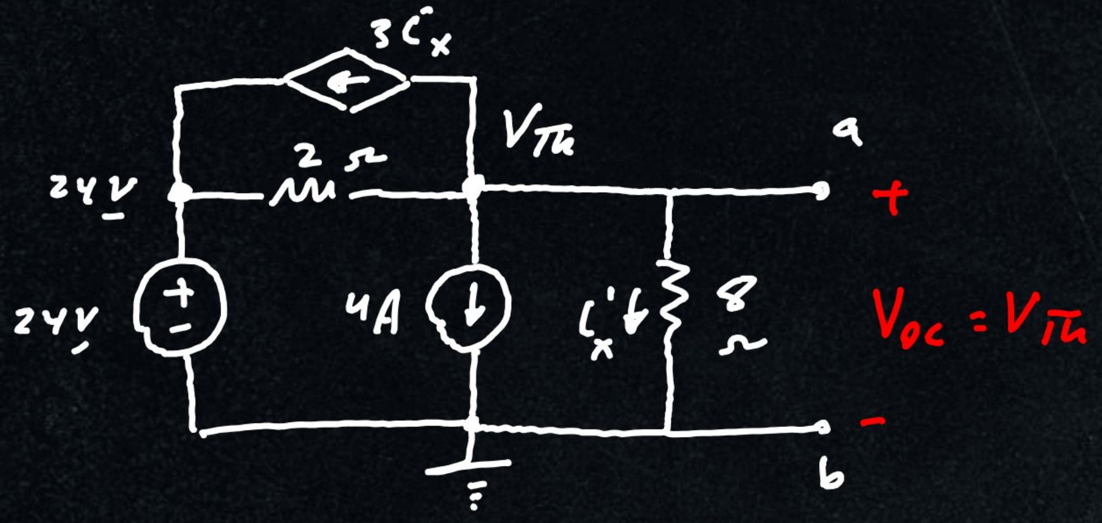


step 3 :- Apply a test source on the terminals XY.

step 4: Calculate R_{Th}.

$$R_{Th} = \frac{V_T}{I_T}$$

EX :- Find the Thevenin equivalent circuit with respect to terminals ab



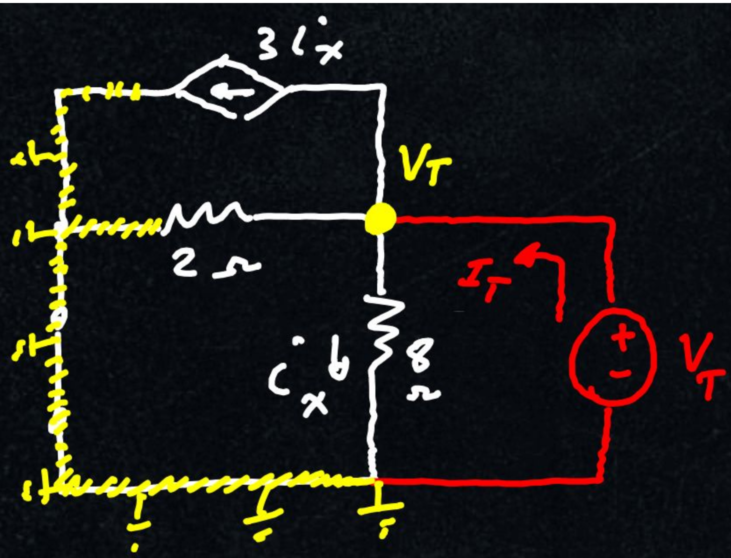
KCL at V_{Th} :

$$3i_x + 4 + \frac{V_{Th}}{8} + \frac{V_{Th} - 24}{2} = 0$$

$$i_x = \frac{V_{Th}}{8}$$

$$\frac{3}{8}V_{Th} + 4 + \frac{V_{Th}}{8} + \frac{V_{Th}}{2} - 12 = 0$$

$$\boxed{V_{Th} = 8\ \underline{V}}$$



KCL at V_T

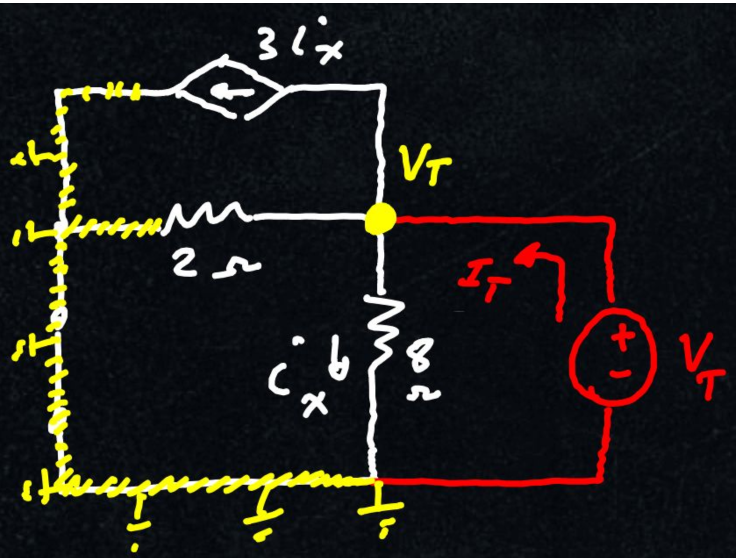
$$-I_T + \frac{V_T}{8} + \frac{V_T}{2} + 3i_x = 0$$

$$i_x = \frac{V_T}{8}$$

$$-I_T + \frac{V_T}{8} + \frac{V_T}{2} + \frac{3}{8}V_T = 0$$

$$-I_T + V_T = 0 \Rightarrow \frac{V_T}{I_T} = 1$$

$$\boxed{R_{Th} = \frac{V_T}{I_T} = 1\,\Omega}$$



KCL at V_T

$$-I_T + \frac{V_T}{8} + \frac{V_T}{2} + 3i_x = 0$$

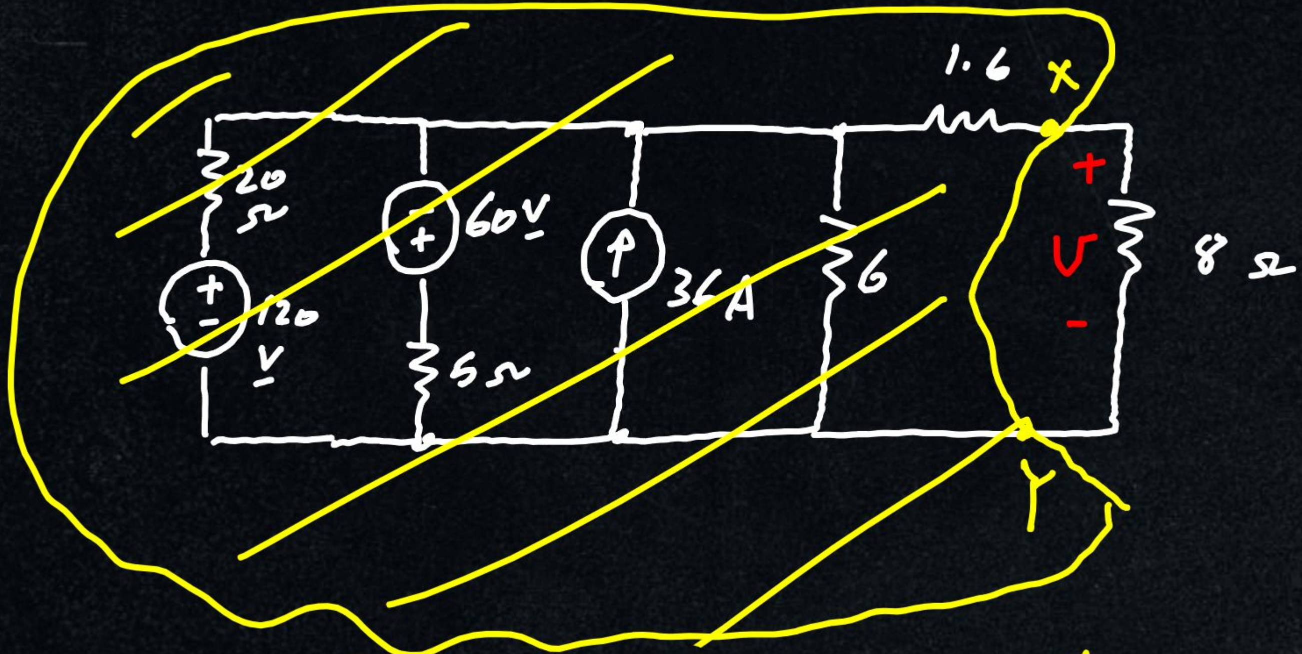
$$i_x = \frac{V_T}{8}$$

$$-I_T + \frac{V_T}{8} + \frac{V_T}{2} + \frac{3}{8}V_T = 0$$

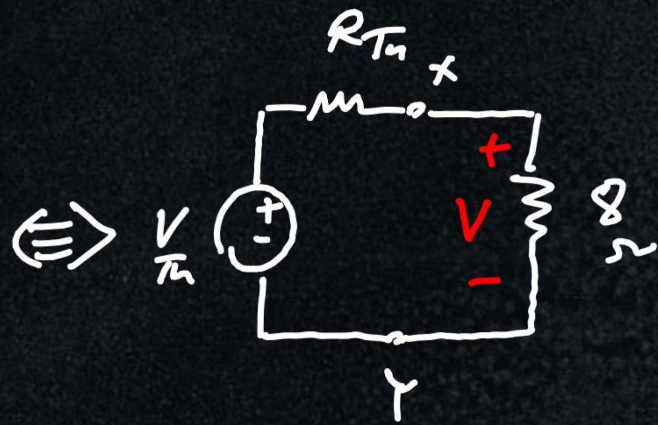
$$-I_T + V_T = 0 \Rightarrow \frac{V_T}{I_T} = 1$$

$$\boxed{R_{Th} = \frac{V_T}{I_T} = 1\,\Omega}$$

EX: Calculat V using Thevenin method

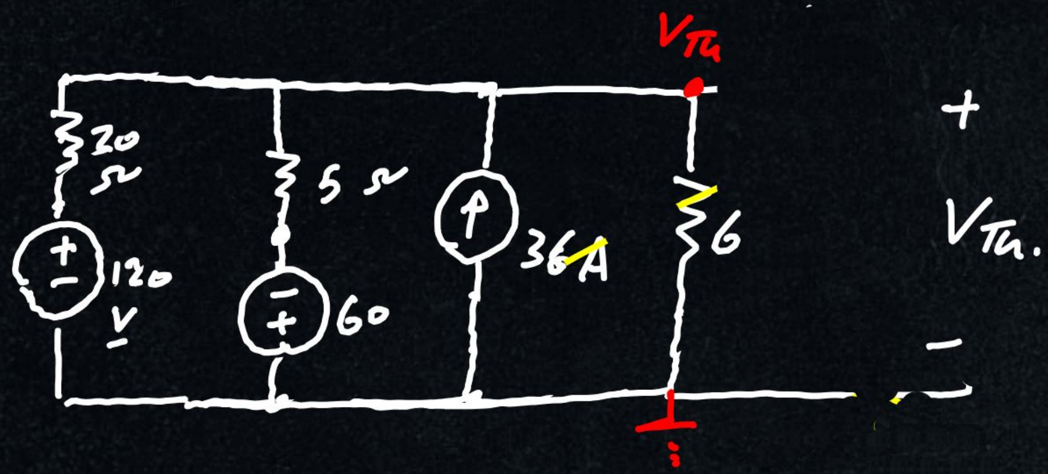


Thevenin Equivalent circuit



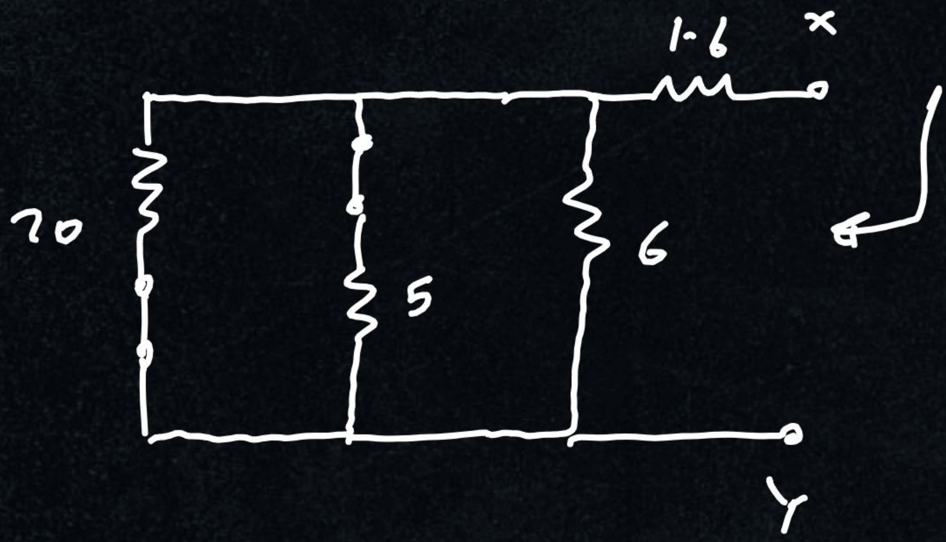
$$V = \frac{8}{8+R_{Th}} \cdot V_{Th}$$

EX: Calculat V using Thevenin method

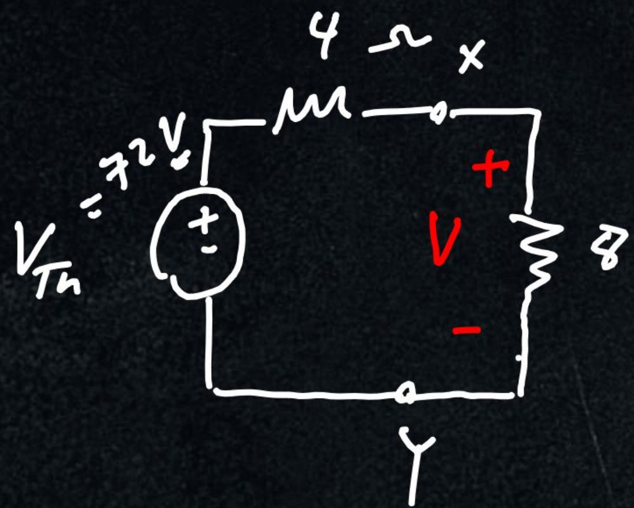


$$\frac{V_{Th}}{6} + (-36) + \frac{V_{Th} - 120}{20} + \frac{V_{Th} + 60}{5} = 0$$

solve for V_{Th}



$R_{Th} = (20 \parallel 5 \parallel 6) + 1.6 = 4\ \Omega$

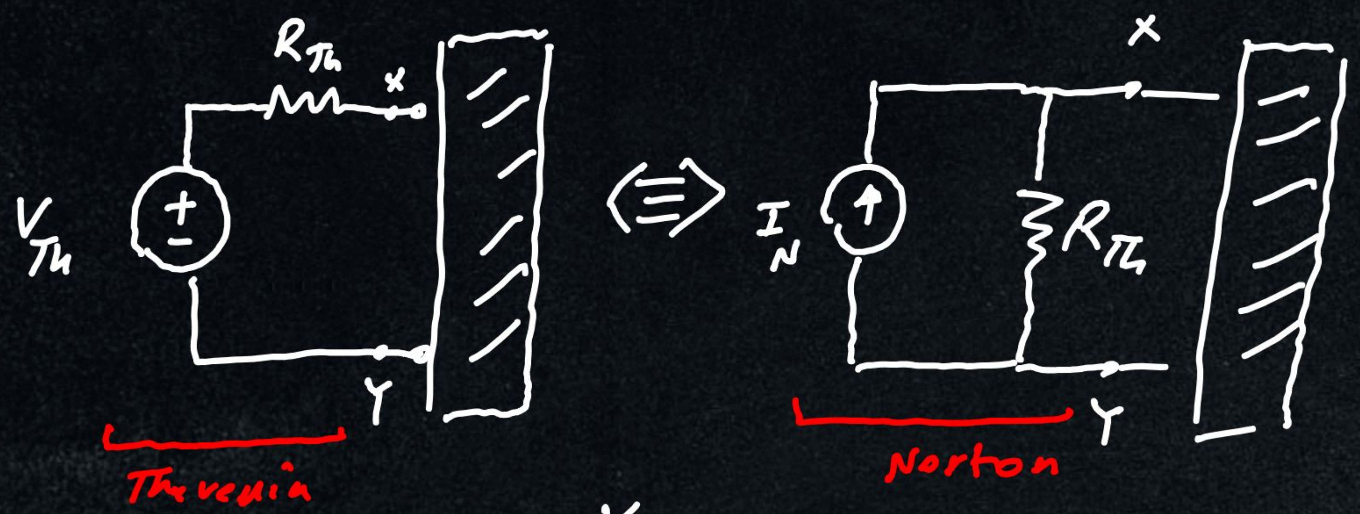


$$V = \frac{8}{8+4} V_{Th} = 48\ \underline{V}$$

voltage divider

Norton Equivalent circuit

It is the source transformation of Thevenin equivalent circuit.



$$\frac{V_{Th} = V_{oc}}{I_{sc}} = R_{Th}$$

$$I_N = \frac{V_{Th}}{R_{Th}} = I_{sc}$$

$\Downarrow$ short circuit current between x & Y

Methods to find the Norton equivalent circuit

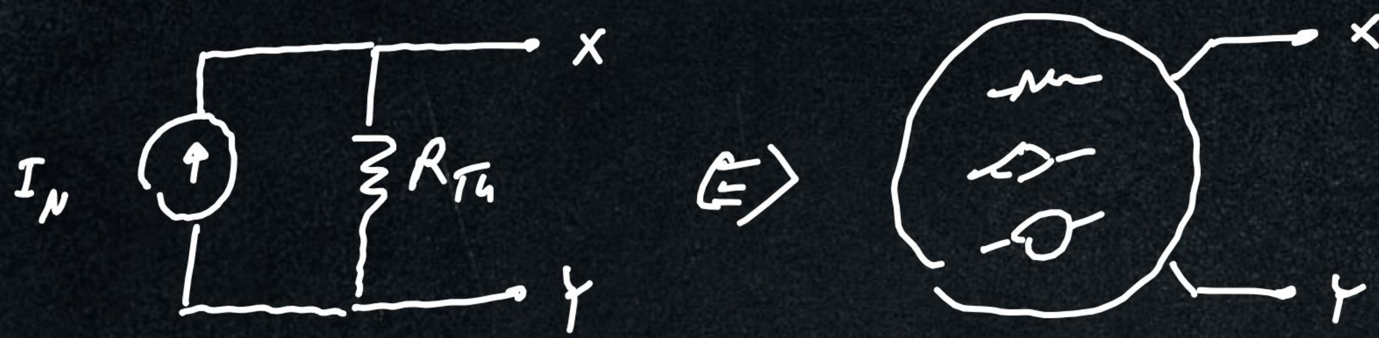
1] step 1: calculate the open circuit voltage, V_{oc}

$$V_{Th} = V_{oc}$$

step 2: Calculate the short circuit current, I_{sc}

$$I_N = I_{sc}$$

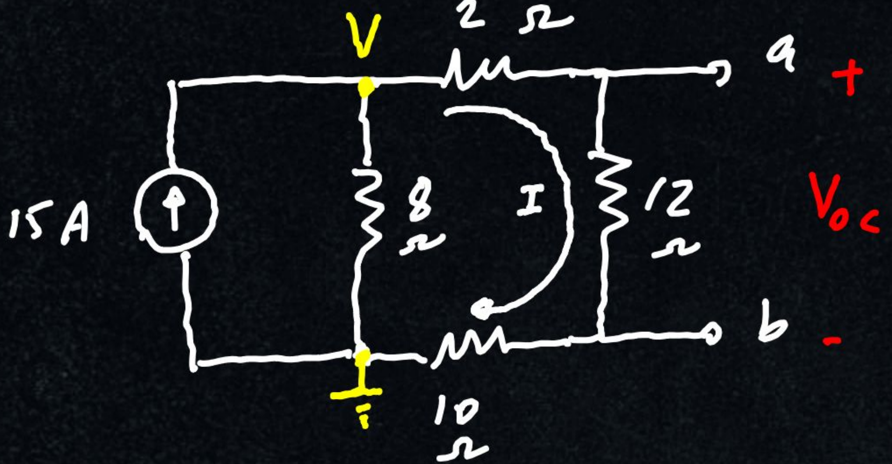
step 3: calculate $R_{Th} = \dfrac{V_{Th}}{I_N}$



Transformation

2] Using the methods of Thevenin equivalent circuit with source

Ex:- Find the Norton equivalent circuit with respect to the terminals ab



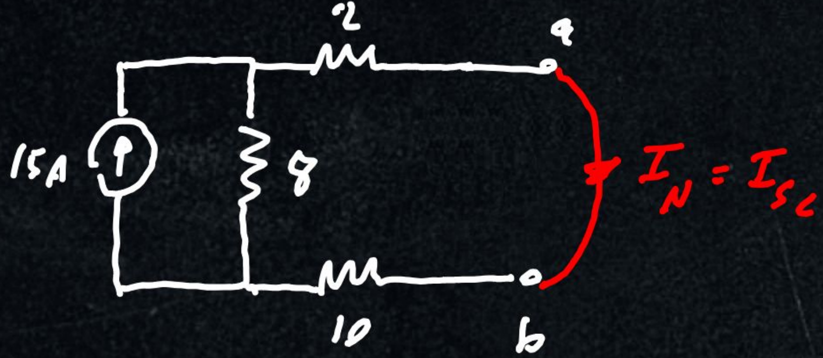
$$I = \frac{8}{8+24} 15 = 3.75 A$$

$$V_{Th} = V_{oc} = 12 I = 45 \underline{V}$$

$$\frac{V}{8} + \frac{V}{24} + (-15) = 0$$

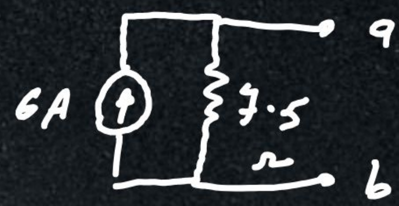
Find V

$$V_{oc} = \frac{12}{12+2+10} V = 45 \underline{V}$$

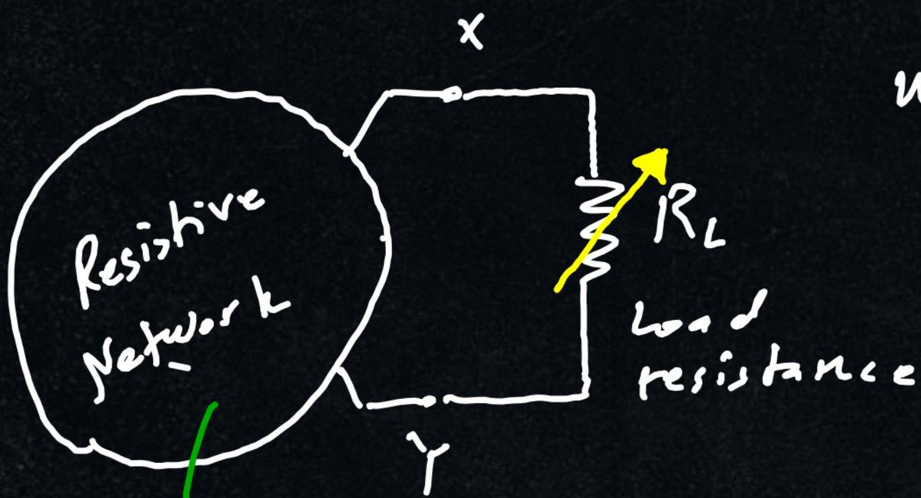


$$I_N = \frac{8}{8+12} \cdot 15 = 6 \quad A$$

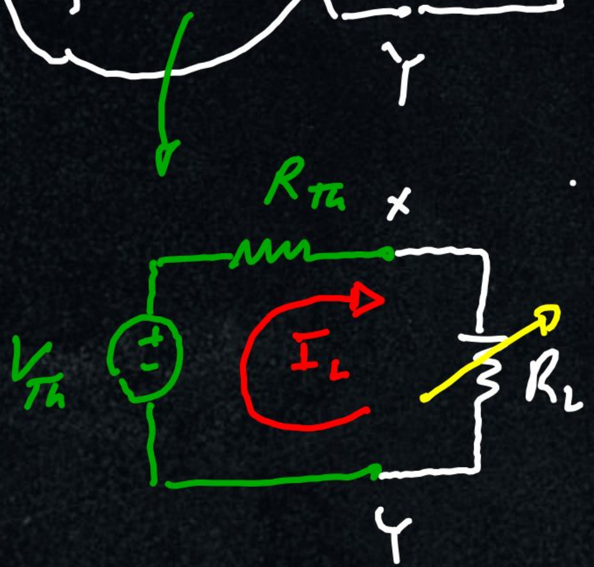
$$R_{Th} = \frac{V_{Th}}{I_N} = 7.5 \ \Omega$$



Maximum Power Transfer



What is the value of R_L such that the power absorbed by R_L is maximum?



I_L : Load current

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

The power absorbed by R_L:

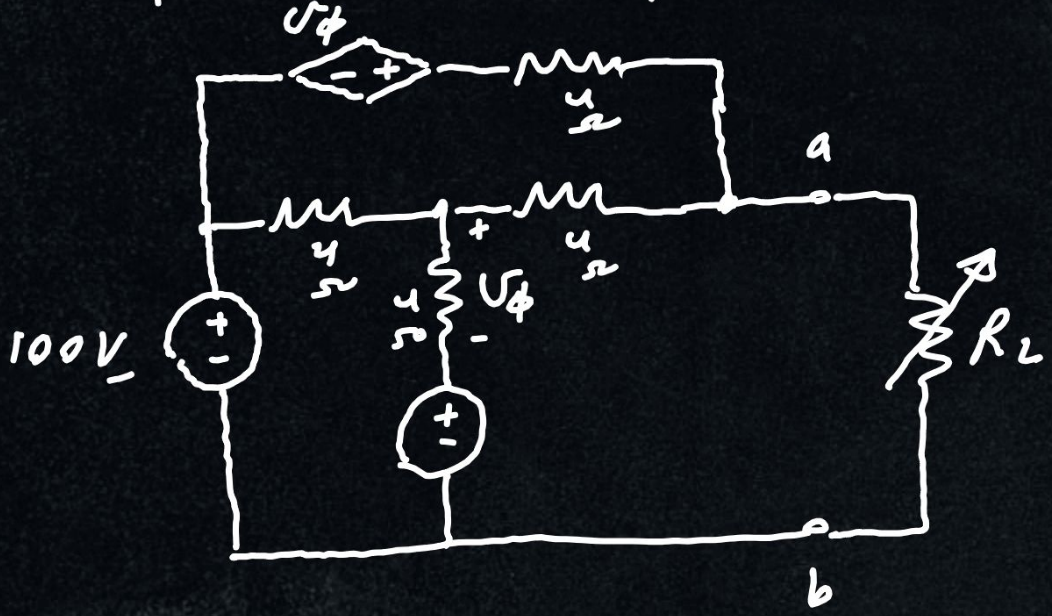
$$P_L = I_L^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 \cdot R_L$$

$$P_L = \left(\frac{V_{Th}}{R_L + R_{Th}}\right)^2 \cdot R_L \quad , \quad f(x) = \left(\frac{V_{Th}}{x + R_{Th}}\right)^2 \cdot x$$

$$\frac{\partial P_L}{\partial R_L} = 0 \Rightarrow \boxed{R_L = R_{Th} = x}$$

$$\boxed{P_{L,max} = \frac{V_{Th}^2}{4R_{Th}}}$$

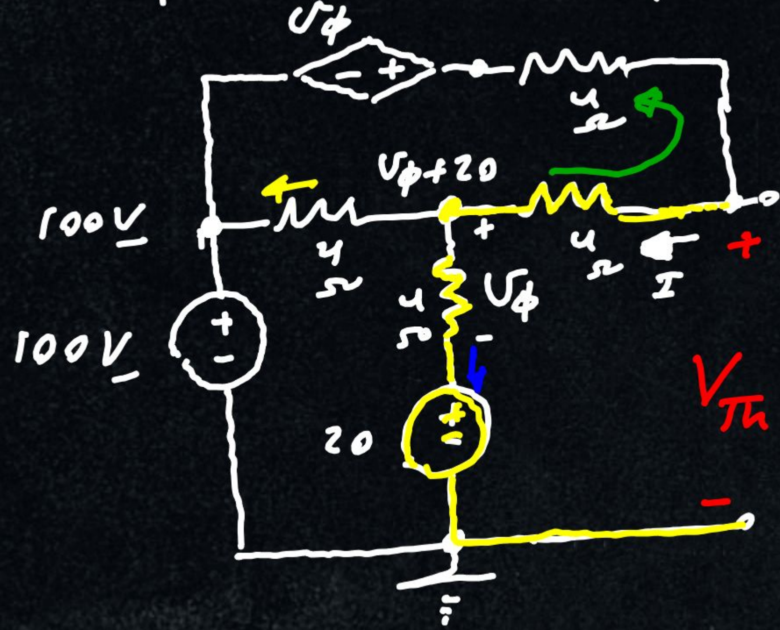
Ex:- Find the value of R_L to enable the circuit to deliver maximum power and find the maximum power delivered to R_L.



$R_L = R_{Th}$

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

Ex:- Find the value of R_L to enable the circuit to deliver maximum power and find the maximum power delivered to R_L.



$$R_L = R_{Th}$$

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

$$\frac{V_\phi}{4} + \frac{(V_\phi+20)-(V_\phi+100)}{8} + \frac{(V_\phi+20)-100}{4} = 0$$

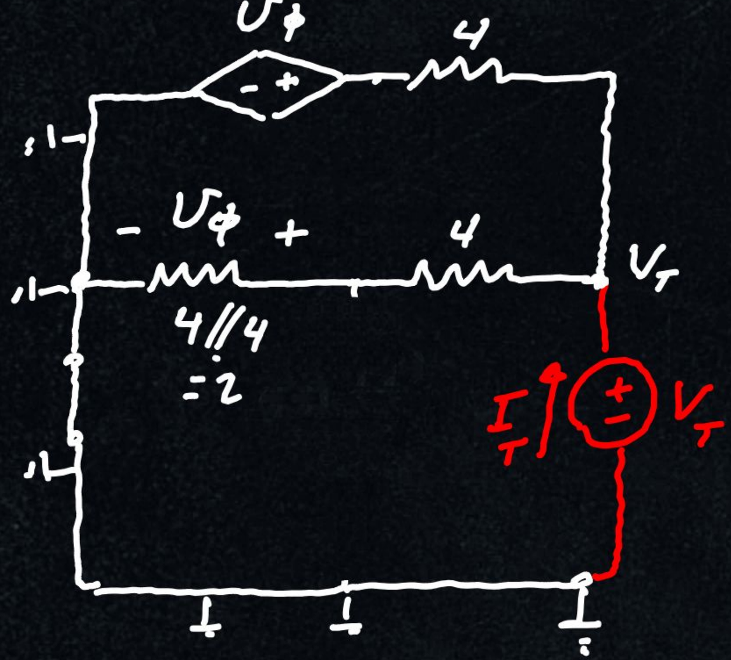
$$\frac{V_\phi}{2} - 10 - 20 = 0 \Rightarrow V_\phi = 60\,V$$

$$V_{Th} = 4I + V_\phi + 20$$

$$I = \frac{(V_\phi+100)-(V_\phi+20)}{8} = 10\,A$$

$$\Rightarrow \boxed{V_{Th} = 120\,V}$$

R_{Th} " Method 3 "



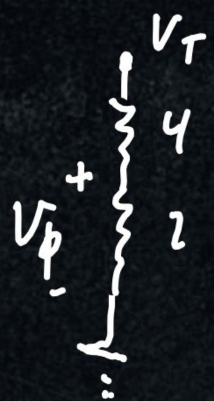
KCL at V_T

$$-I_T + \frac{V_T}{6} + \frac{V_T - V_\phi}{4} = 0$$

$$V_\phi = \frac{2}{2+4} V_T = \frac{1}{3} V_T$$

$$-I_T + \frac{V_T}{6} + \frac{1}{6} V_T = 0$$

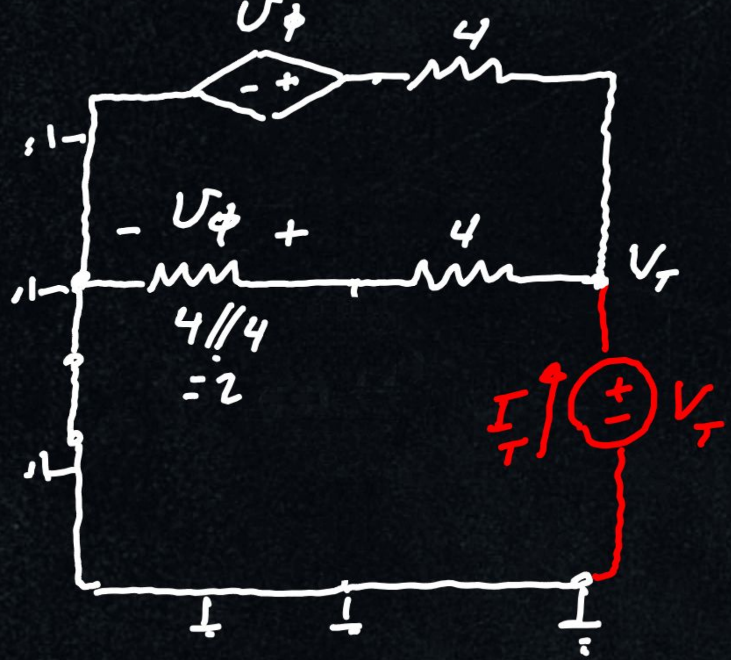
$$I_T = \frac{V_T}{3} \Rightarrow \boxed{R_{Th} = \frac{V_T}{I_T} = 3\,\Omega}$$



$$\underline{R_L = 3\,\Omega}$$

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(120)^2}{4(3)} = \boxed{1.2\ kW}$$

R_{Th} " Method 3 "



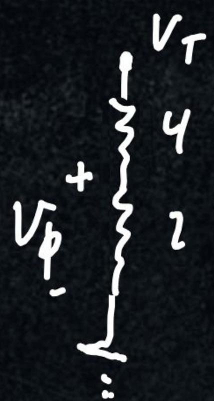
KCL at V_T

$$-I_T + \frac{V_T}{6} + \frac{V_T - V_\phi}{4} = 0$$

$$V_\phi = \frac{2}{2+4} V_T = \frac{1}{3} V_T$$

$$-I_T + \frac{V_T}{6} + \frac{1}{6} V_T = 0$$

$$I_T = \frac{V_T}{3} \Rightarrow \boxed{R_{Th} = \frac{V_T}{I_T} = 3\,\Omega}$$

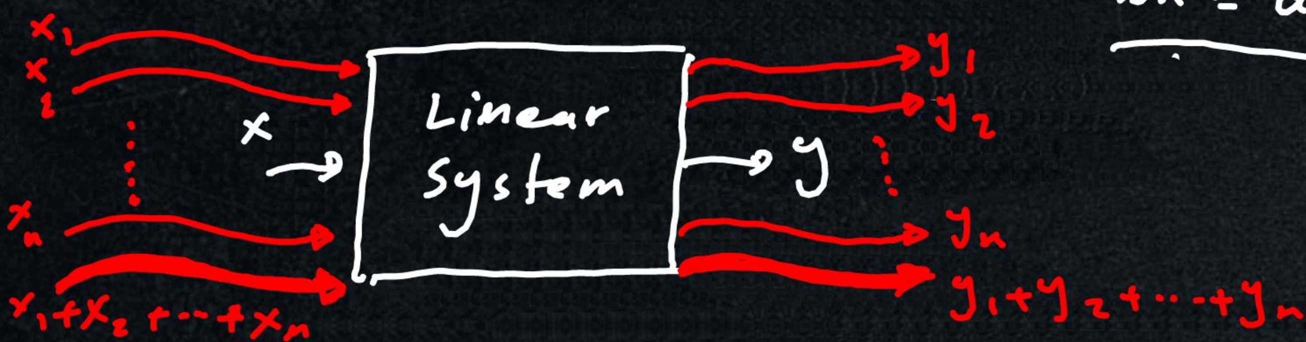


$R_L = 3\,\Omega$

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(120)^2}{4(3)} = \boxed{1.2\text{ kW}}$$

⑤ Superposition

- A linear system obeys the principle of superposition, which states that when a linear system is excited, or driven, by more than one independen source, the total response is the sum of the individual responses.
- The electric circuit is made up of interconnected linear - circuit elements. Therefore, we can apply the superposition to the analysis of the circuit



$$\underbrace{bx = a_n y^{(n)} + a_{n-1} y^{(n-1)} + \dots + a_2 y'' + a_1 y' + a_0 y}_{\text{Linear differential equation}}$$

Assume that we have a network with n indep. sources

$$Set = \{u_1, u_2, u_3, \ldots, u_n\}$$

We can take subsets $A_1, A_2, \ldots, A_k$ $k > 1$

such that

$$A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n = set$$

$$A_i \cap A_j = \phi \qquad i \neq j$$

<u>EX</u> :- $set = \{u_1, u_2, u_3\}$

$A_1 = \{u_1\}$

$A_2 = \{u_2\}$

$A_3 = \{u_3\}$

$A_1 \cup A_2 \cup A_3 = set$

$A_i \cap A_j = \phi$

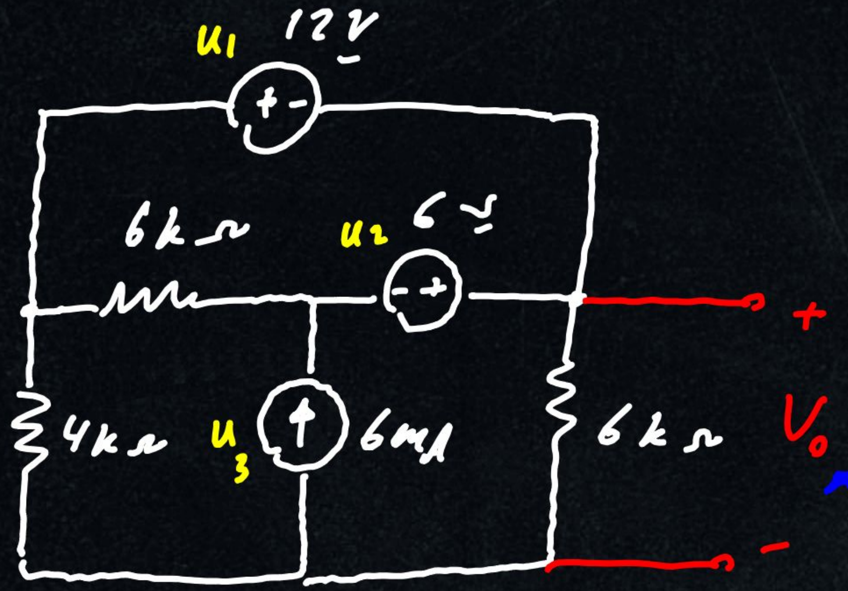
$A_1 = \{u_1, u_2\}$ kill u_3

$A_2 = \{u_3\}$ kill $u_1 + u_2$

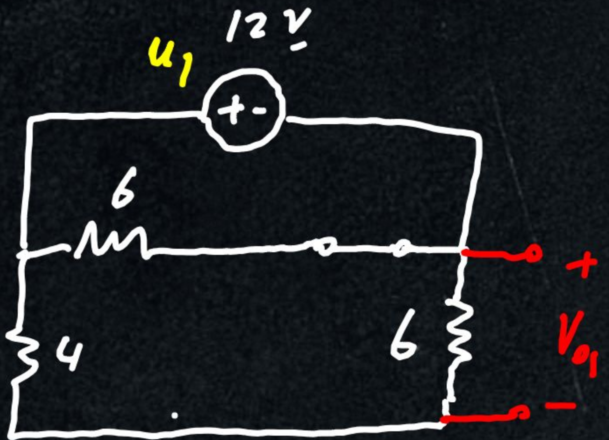
$A_1 \cup A_2 = set$

$A_1 \cap A_2 = \phi$

EX:- Find V_o using superposition :-

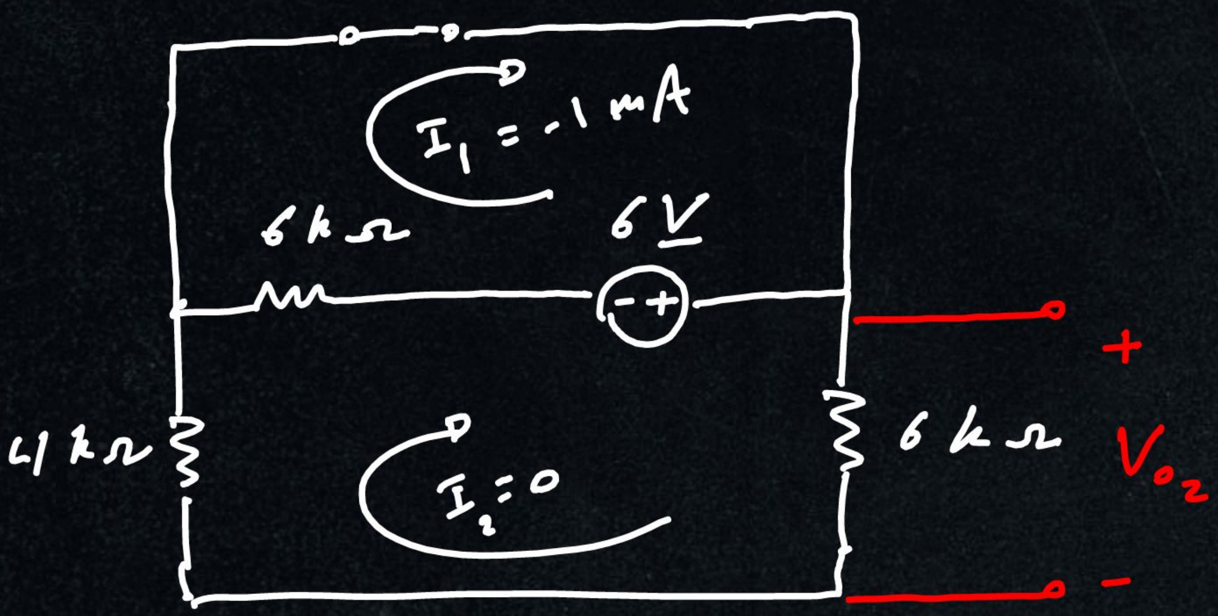


$A_1 = \{u_1\}$ kill $u_2 + u_3$



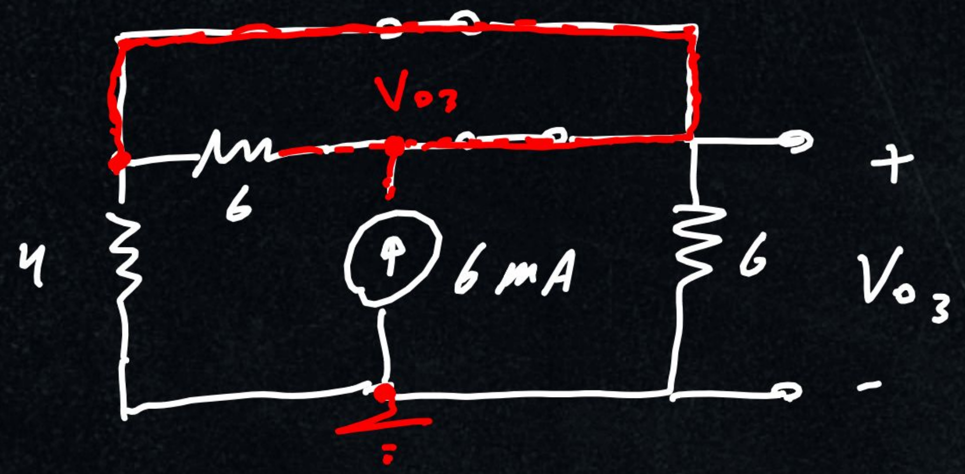
$$V_{o_1} = \frac{6}{6+4} \cdot (-12) = -6\ V$$

$A_2 = \{ U_2 \}$



$V_{o_2} = 0$

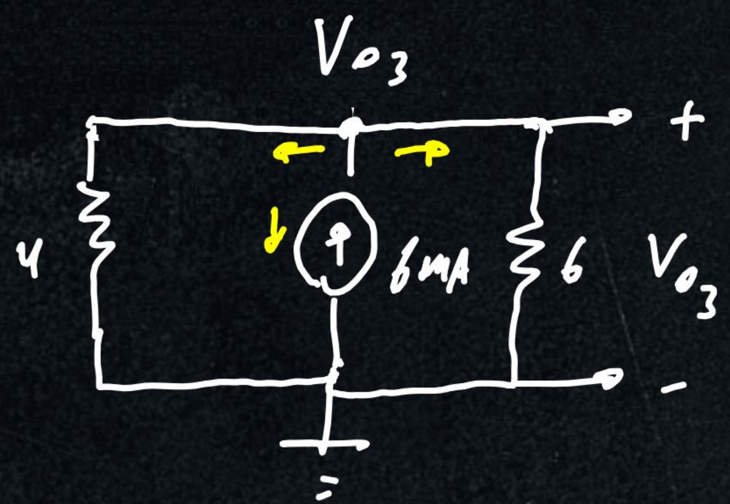
$A_3 : \{u_3\}$ kill $u_1 + u_2$



$$-6 + \frac{V_{o3}}{4} + \frac{V_{o3}}{6} = 0$$

$$5V_{o_3} = 6 \times 12$$

$$V_{o_3} = 14.4\ \underline{V}$$



$$V_o = V_{o_1} + V_{o_2} + V_{o_3}$$

$$= -6 + 0 + 14.4$$

$$= 8.4\ \underline{V}$$