Exercises:

Q48: if $\phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{w^2}{2}} \, dw$, show that $\phi(-z) = 1 - \phi(z)$.

$$\phi(-z) = F(-x) = P(X < -x)$$

$$= \int_{-\infty}^{-x} f(z) \, dz$$

$$= \int_{x}^{\infty} f(-t) \, dt$$

$$= \int_{x}^{\infty} f(t) \, dt$$

$$= P(X > x)$$

$$= 1 - P(X \leq x)$$

$$= 1 - F(x)$$

Hence $\phi(-z) = 1 - \phi(z)$.

Q49: if X is $N(75, 100)$, find $Pr(X < 60)$ and $Pr(70 < X < 100)$.

$\mu = 75$, $\sigma^2 = 100$

$$Z = \frac{X - \mu}{\sigma} = \frac{X - 75}{10}$$

$Z \sim N(0, 1)$

$\rightarrow$

$\rightarrow \Pr(X<60) = \Pr\left(\frac{X-75}{10} < \frac{60-75}{10}\right)$

$= \Pr(Z < -1.5)$

$= 1-\phi(1.5)$

$= 1- 0.933$

$= 0.067$.

$\rightarrow \Pr(70<X<100) = \Pr(-0.5 < Z < 2.5)$

$= \phi(2.5) - \phi(-0.5)$

$= \phi(2.5) - (1-\phi(0.5))$

$= 0.994 - (1- 0.691)$

$= 0.994 - 0.309$

$= 0.685$.

Q51: let X be $N(\mu, \sigma^2)$ so that $\Pr(X<89) = 0.9$ and $\Pr(X<94) = 0.95$

Find μ and σ^2.

$\Pr\left(Z < \frac{89-\mu}{\sigma}\right) = 0.9$

$\phi\left(\frac{89-\mu}{\sigma}\right) = 0.9$.

$\phi^{-1}(0.9) = 1.282$

$\Rightarrow \frac{89-\mu}{\sigma} = 1.282 \rightarrow 89-\mu = 1.282\,\sigma$. ——①

$\rightarrow$

$\rightarrow Pr(X<94) = 0.95$

$$\phi\left(\frac{94-\mu}{\sigma}\right) = 0.95$$

$$\phi^{-1}(0.95) = 1.645$$

$$\Rightarrow \frac{94-\mu}{\sigma} = 1.645$$

$$94-\mu = 1.645\sigma \quad (2)$$

$$94-\mu = 1.645\sigma$$

$$-\quad 89-\mu = 1.282\sigma$$

$$5 = 0.363\sigma \rightarrow \sigma = 13.77 \rightarrow \sigma^2 = 189.7$$

subsitute $\sigma \rightarrow \mu = 94 - 1.645(13.77)$

$$= 71.3$$

So $X \sim N(71.3, 189.7)$.

Q57: If e^{3t+8t^2} is the m.g.f of the Random variable X, Find $Pr(-1 < X < 9)$.

$\boxed{\mu = 3}$, $\frac{1}{2}\sigma^2 = 8 \rightarrow \sigma^2 = 16 \rightarrow \boxed{\sigma = 4}$

$X \sim (3, 4^2)$

$Z = \frac{X-3}{4}$

$Pr(-1 < Z < 1.5) = \phi(1.5) - \phi(-1)$

$= 0.933 - (1 - \phi(1))$

$= 0.933 - 0.159$

$= 0.774$

Q59: let X be $N(5, 10)$ [μ, σ^2], Find $Pr(0.04 < (X-5)^2 < 38.4)$.

$\frac{X-5}{\sqrt{10}} \sim N(0,1)$ " Thm1 "

so $\left(\frac{X-5}{\sqrt{10}}\right)^2 \sim \chi^2(1)$ " Thm 2 "

Now, $Pr(0.04 < (X-5)^2 < 38.4) = Pr(0.004 < \frac{(X-5)^2}{10} < 3.84)$

$= Pr(0.004 < V < 3.84)$

By chi-table with $r=1$ $= Pr(V < 3.84) - Pr(V < 0.004)$

$= 0.95 - 0.05$

$= 0.9$

Q6: is X be $N(1,4)$, compute the $Pr(\sqrt{1} < \sqrt{X^2} < \sqrt{9})$.

$Pr(1 < X < 9) = Pr(-3 < X < -1) + Pr(1 < X < 3)$

$\rightarrow Z = \frac{X-1}{2}$

$\Rightarrow Pr(1 < X < 9) = Pr(-2 < Z < -1) + Pr(0 < Z < 1)$

By Normal table

$= \phi(-1) - \phi(-2) + \phi(1) - \phi(0)$

$= (1-\phi(1)) - (1-\phi(2)) + \phi(1) - \phi(0)$

$= (1-0.841) - (1-0.977) + 0.841 - 0.5$

$= 0.477$.

Done.