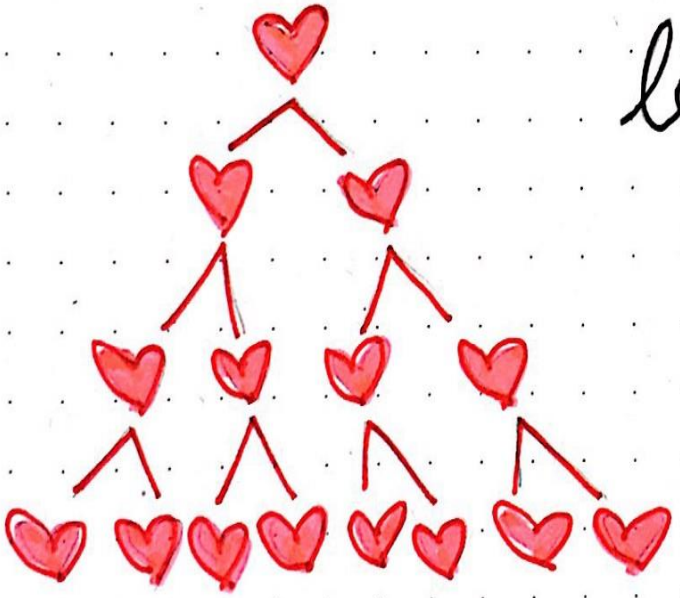

COMPUTER SCIENCE DEPARTMENT FACULTY OF ENGINEERING AND TECHNOLOGY

COMP2321

# Data Structures

# Chapter 6: Heap



learning to love

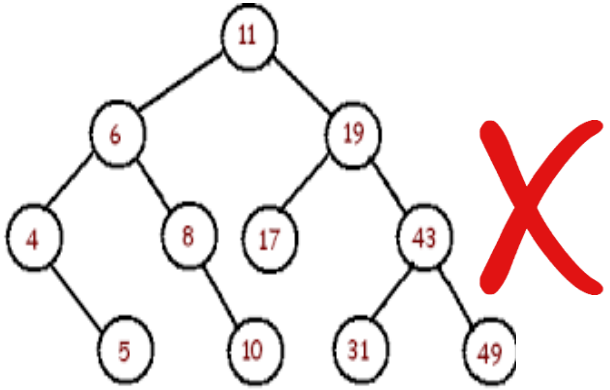
# HEAPS

HEAPIFY

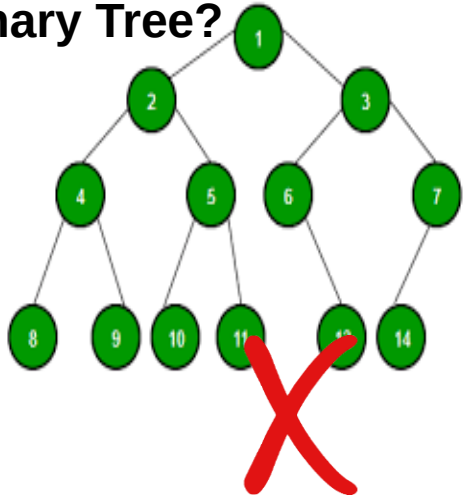
ALL THE THINGS!

(with heap sort)

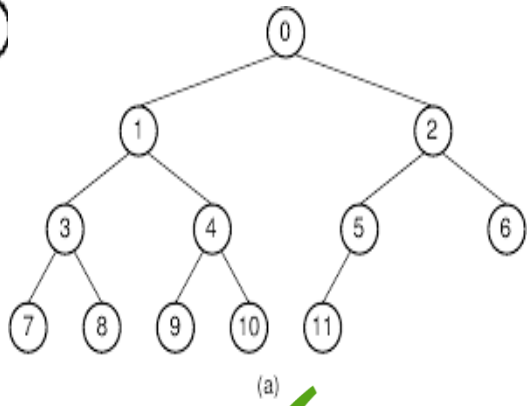
## Which of the following is Complete Binary Tree?



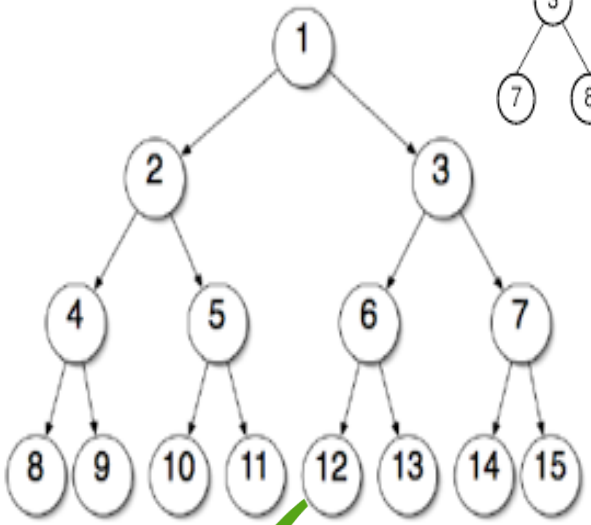

A



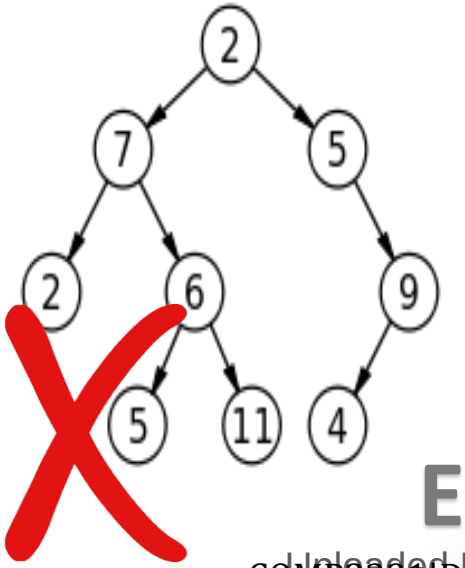

B




C



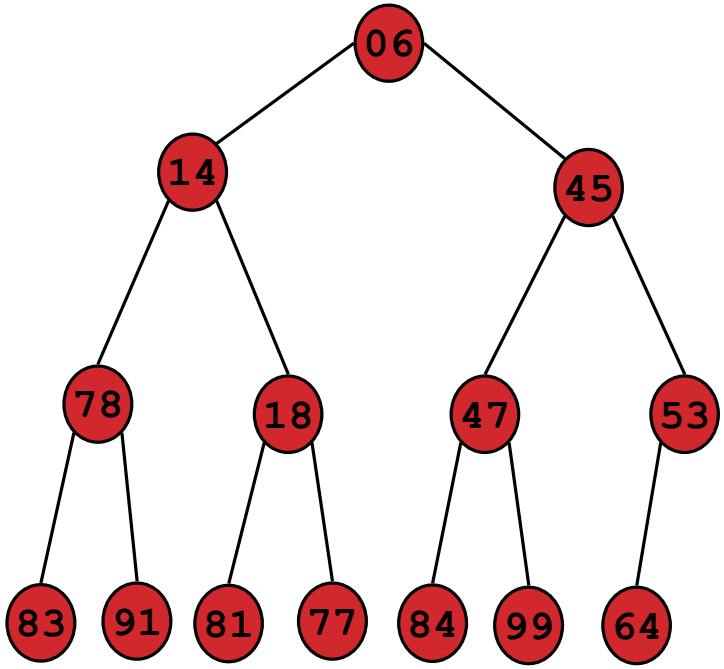

D




E

# Binary Heap: Definition

- Binary heap: it must follow the two properties:
  - Complete binary tree.
    - filled on all levels, except last, where filled from left to right
  - Order property, e.g. Min-heap or Max-heap
    - Min-heap: parent is less than children
    - Max-heap: parent is greater than children

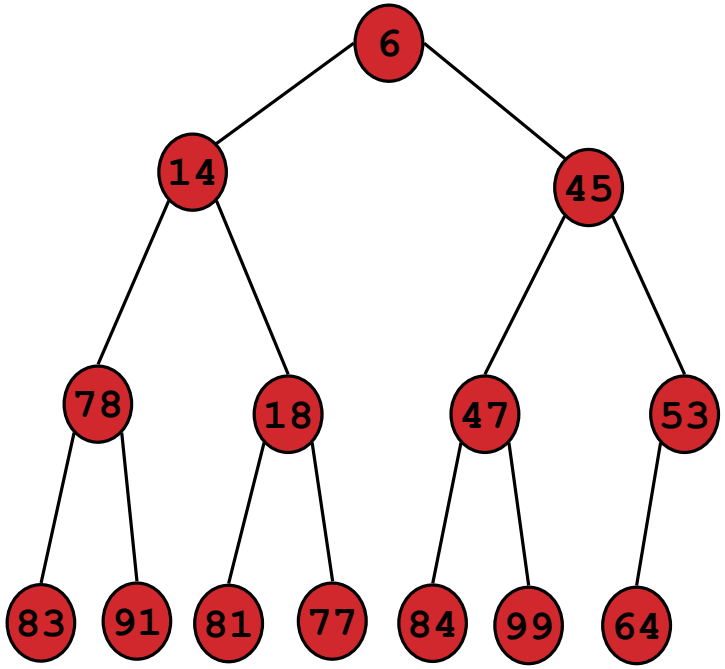



**Notes :**

1) Node keys could be repeated
2) Left child may be greater then right c and vice versa

# Binary Heap: Properties

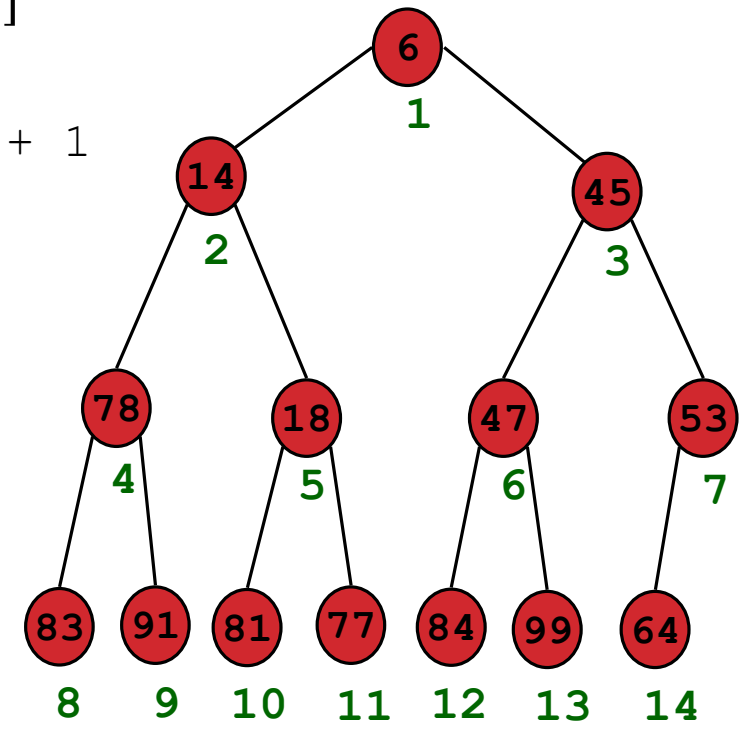
- Properties.
  - Minimum element is in the root.
  - Heap with N elements has height = $\lfloor log_2 N \rfloor$.




**N = 14**
**Height = 3**

# Binary Heaps: Array Implementation

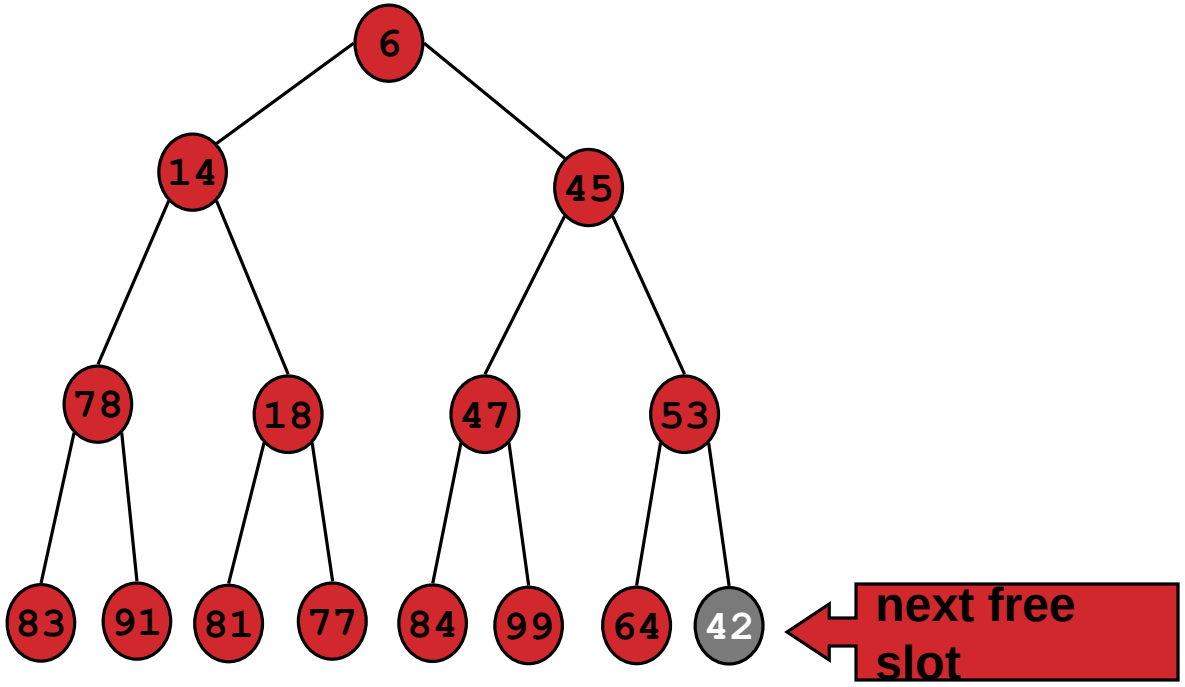
- Implementing binary heaps.
  - Use an array: no need for explicit parent or child pointers.
    - `Parent(i)` = $\lfloor i/2 \rfloor$
    - `Left(i)` = `2i`
    - `Right(i)` = `2i + 1`




| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 14 | 45 | 78 | 18 | 47 | 53 | 83 | 91 | 81 | 77 | 84 | 99 | 64 |

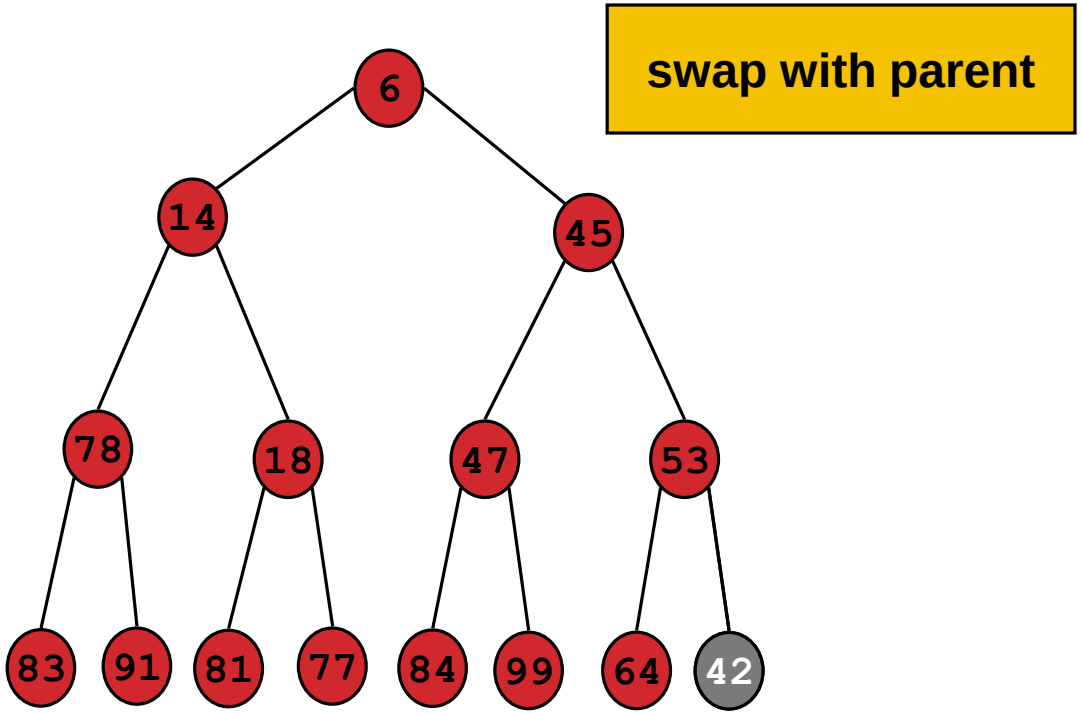
# Binary Heap: Insertion

- Insert element x into heap.
  - Insert into next available slot.
  - Bubble up until it's heap ordered.
    - Peter principle: nodes rise to level of incompetence



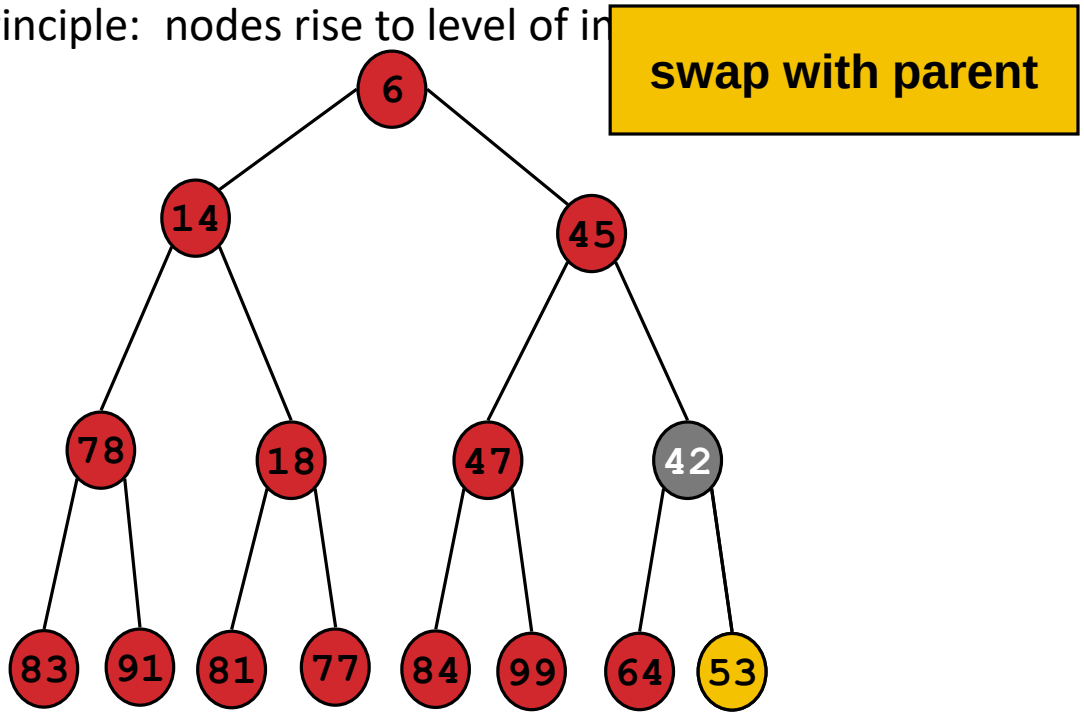
# Binary Heap: Insertion

- Insert element x into heap.
  - Insert into next available slot.
  - Bubble up until it's heap ordered.
    - Peter principle: nodes rise to level of incompetence



# Binary Heap: Insertion

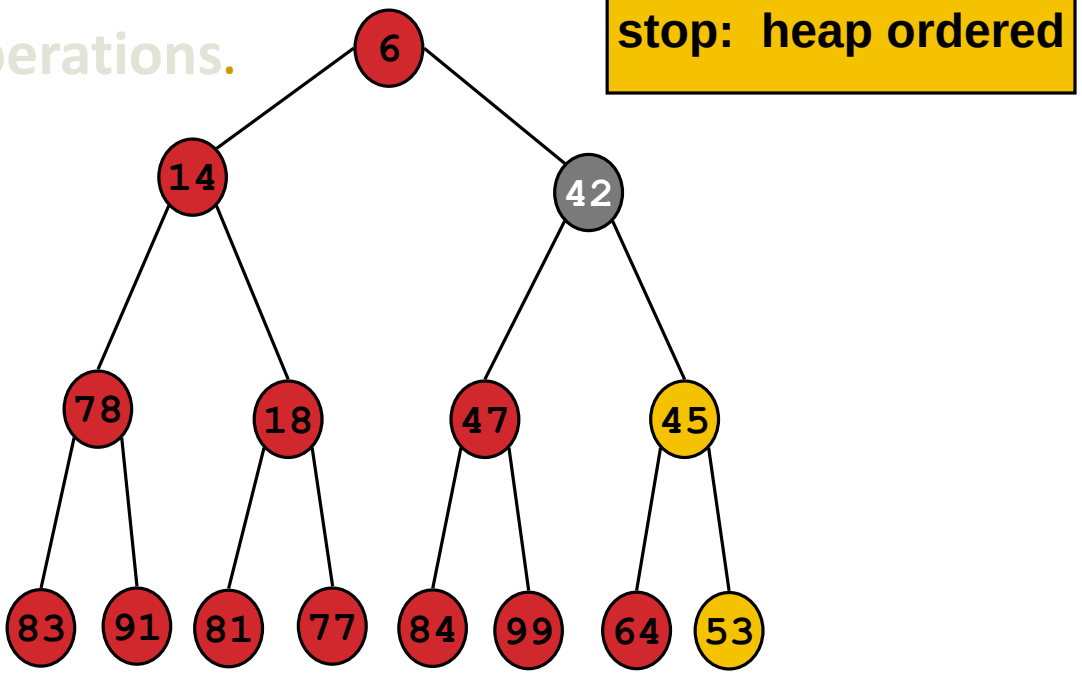
- Insert element x into heap.
  - Insert into next available slot.
  - Bubble up until it's heap ordered.
    - Peter principle: nodes rise to level of in



# Binary Heap: Insertion

**Insert element x into heap.**

- **Insert into next available slot.**
- **Bubble up until it's heap ordered.**
  - **Peter principle: nodes rise to level of incompetence**
- **O(log N) operations.**



Given Data:

Solved At board

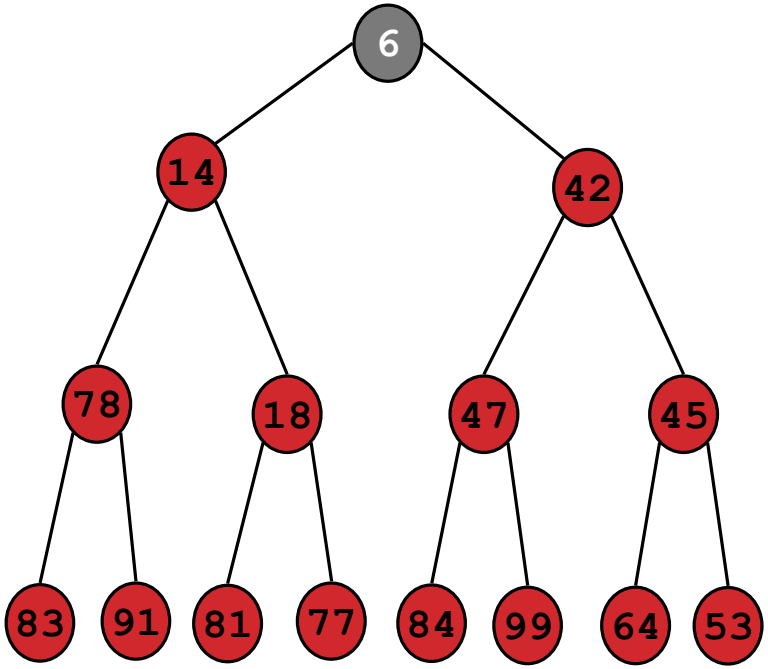
44, 33, 77 ,11, 55, 88, 66, 22

Build max heap tree ? (show all works).

# Binary Heap: Delete Min

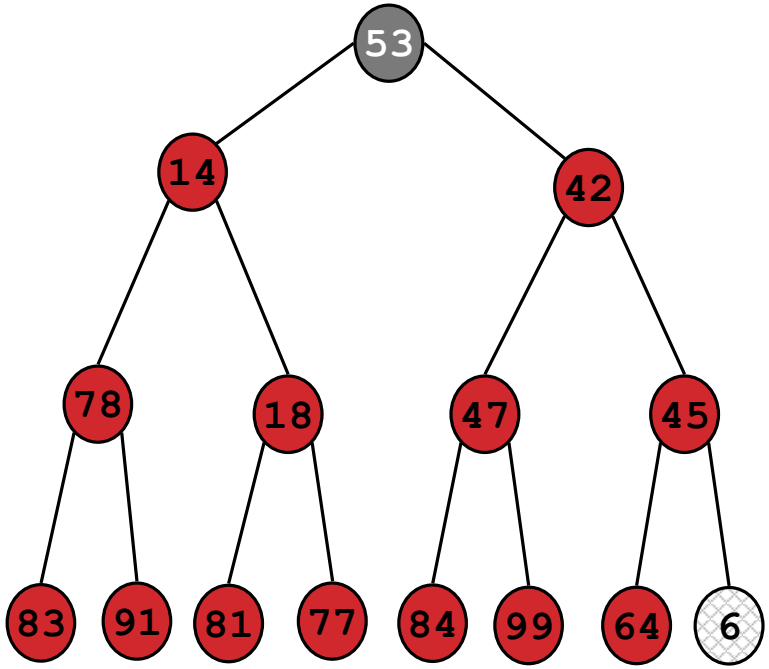
**Delete minimum element from heap.**

- **Exchange root with rightmost leaf.**
- **Bubble root down until it's heap ordered.**
  - **power struggle principle: better subordinate is promoted**



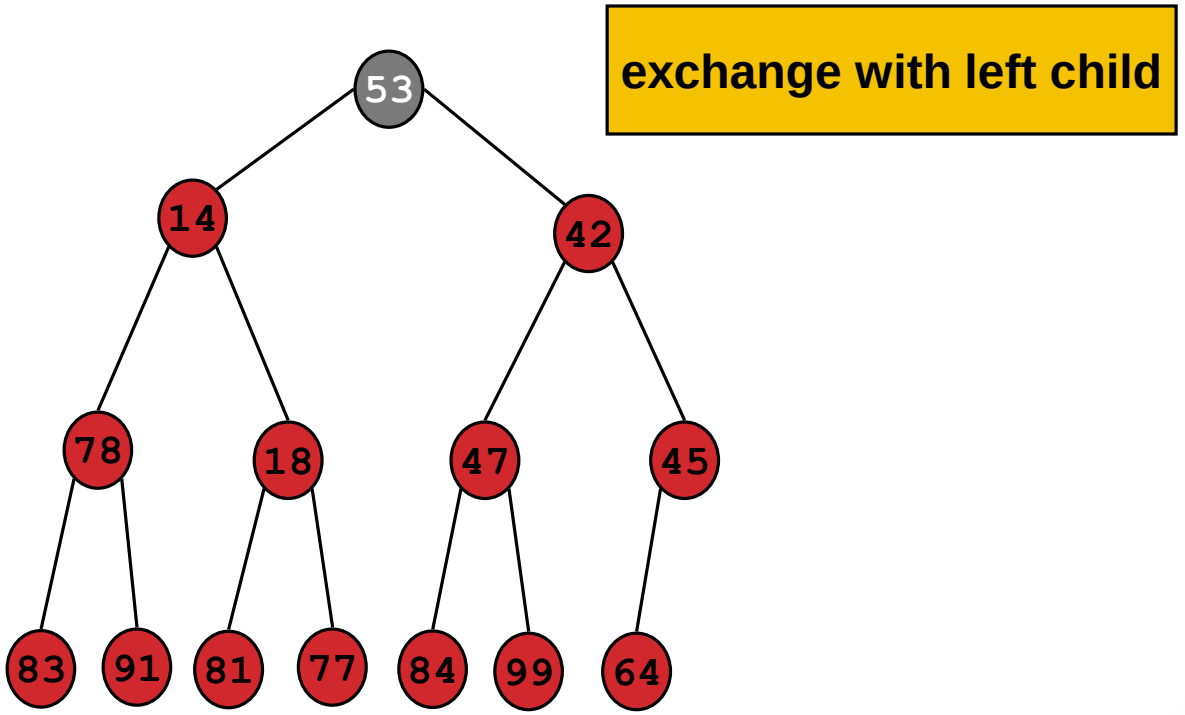
# Binary Heap: Delete Min

- Delete minimum element from heap.
  - Exchange root with rightmost leaf.
  - Bubble root down until it's heap ordered.
    - power struggle principle: better subordinate is promoted



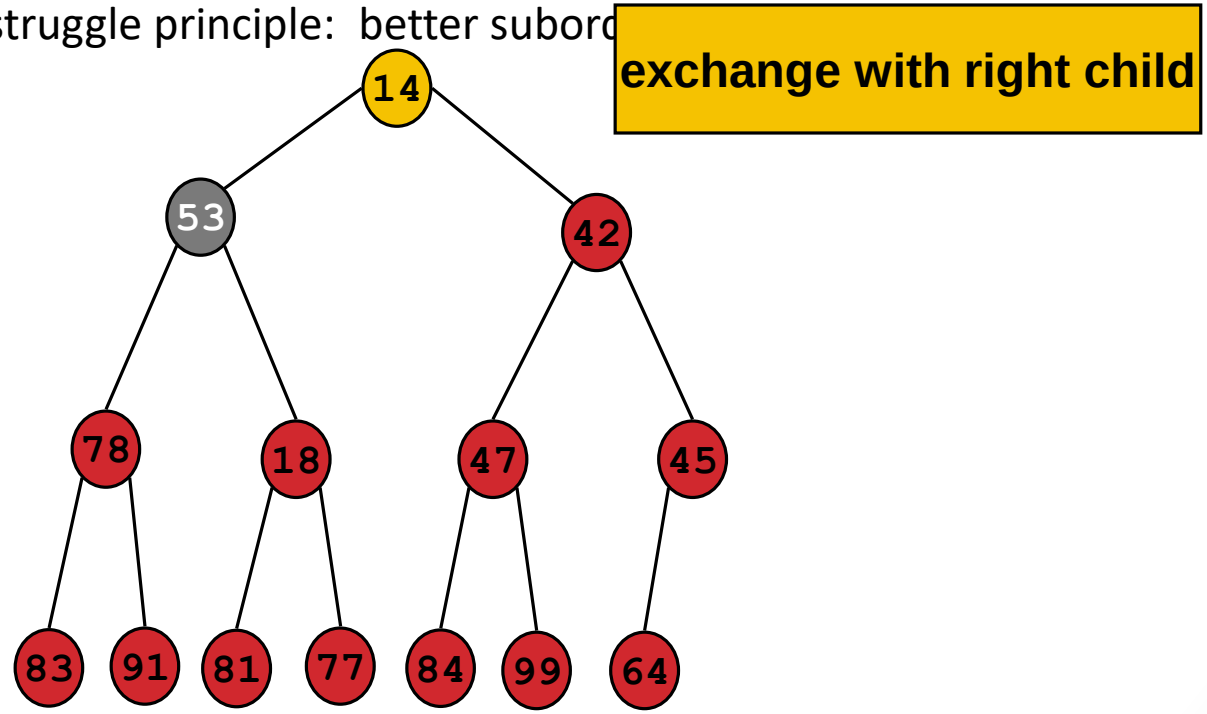
# Binary Heap: Delete Min

- Delete minimum element from heap.
  - Exchange root with rightmost leaf.
  - Bubble root down until it's heap ordered.
    - power struggle principle: better subordinate is promoted



# Binary Heap: Delete Min

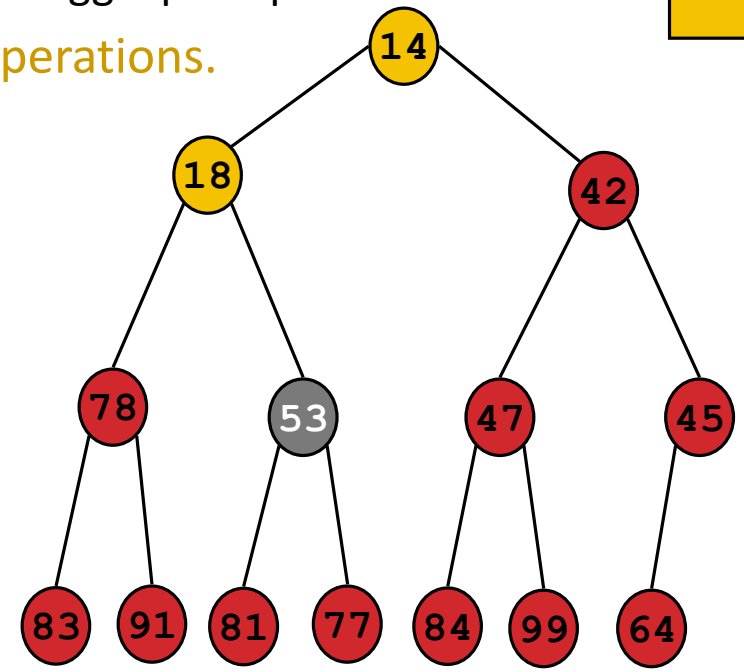
- Delete minimum element from heap.
  - Exchange root with rightmost leaf.
  - Bubble root down until it's heap ordered.
    - power struggle principle: better subord



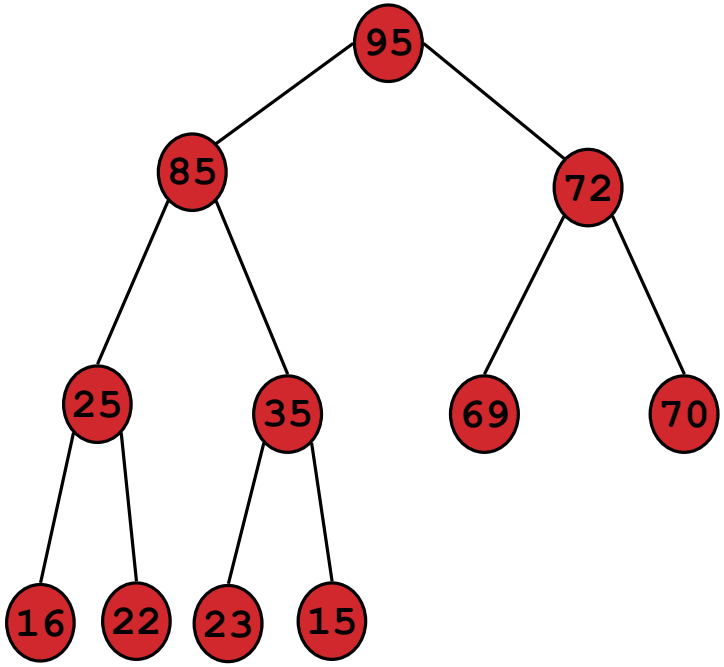
# Binary Heap: Delete Min

- Delete minimum element from heap.
  - Exchange root with rightmost leaf.
  - Bubble root down until it's heap ordered.
    - power struggle principle: better subordin
  - O(log N) operations.

**stop: heap ordered**



# Example: Delete root (95)

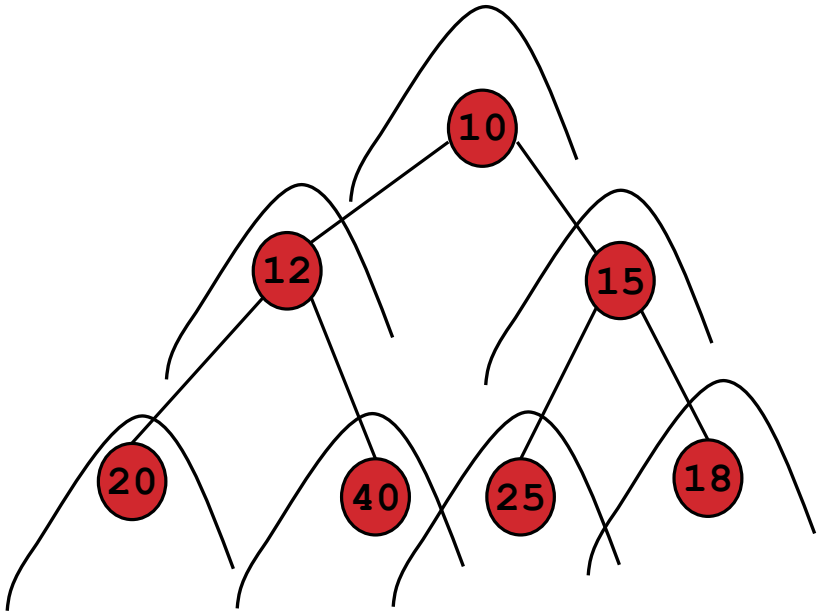



Solved At board

# Heapify

**Looking at every node as it's heap from bottom to up**

**This take less time O(log n), since creation of heap is costing O(n log n)**



```c
#include <stdio.h>
#include <limits.h>


int FRONT=1;
int size=0;
//C implementation of Min Heap
struct MinHeap {
    int maxsize;
    int Heap[];
} heap;

struct MinHeap heap;
void MinHeap(int maxsize)
{

    heap.maxsize = maxsize;
    int Heap[maxsize + 1];
    Heap[0] =INT_MIN;
}

// Function to return the position of
// the parent for the node currently
// at pos

int parent(int pos)
{
    return pos / 2;
}

// Function to return the position of the
// left child for the node currently at pos
int leftChild(int pos)
{
    return (2 * pos);
}

// Function to return the position of
// the right child for the node currently
// at pos
int rightChild(int pos)
{
    return (2 * pos) + 1;
}

// Function that returns true if the passed
// node is a leaf node
```

```
46  // Function that returns true if the passed
47  // node is a leaf node
48  int isLeaf(int pos)
49  {
50      if (pos >= (size / 2) && pos <=size) {
51          return 1;
52      }
53      return 0;
54  }
55
56  // Function to swap two nodes of the heap
57  void swap(int fpos, int spos)
58  {
59      int tmp;
60      tmp = heap.Heap[fpos];
61      heap.Heap[fpos] = heap.Heap[spos];
62      heap.Heap[spos] = tmp;
63  }
64
65  // Function to heapify the node at pos
```

```
95  // Function to insert a node into the heap
96  void insert(int element)
97  {
98
99      if (size >= heap.maxsize) {
100         return;
101     }
102     heap.Heap[++size] = element;
103     int current = size;
104
105     while (heap.Heap[current] < heap.Heap[parent(current)]) {
106         swap(current, parent(current));
107         current = parent(current);
108     }
109 }
110
```

```
65  // Function to heapify the node at pos
66  void minHeapify(int pos)
67  {
68
69      // If the node is a non-leaf node and greater
70      // than any of its child
71      if (!isLeaf(pos)) {
72          if (heap.Heap[pos] > heap.Heap[leftChild(pos)]
73              || heap.Heap[pos] > heap.Heap[rightChild(pos)]) {
74
75              // Swap with the left child and heapify
76              // the left child
77              if (heap.Heap[leftChild(pos)] < heap.Heap[rightChild(pos)]) {
78                  swap(pos, leftChild(pos));
79                  minHeapify(leftChild(pos));
80              }
81
82                  // Swap with the right child and heapify
83                  // the right child
84              else {
85                  swap(pos, rightChild(pos));
86                  minHeapify(rightChild(pos));
87              }
88          }
89      }
90  }
91
```

```
120  // Function to build the min heap using
121  // the minHeapify
122  void minHeap()
123  {
124      for (int pos = (size / 2); pos >= 1; pos--) {
125          minHeapify(pos);
126      }
127  }
128
```

# Heapsort

**Heap Sort Algorithm for sorting :**

**1.** Build a max heap from the input data.

**2.** At this point, the largest item is stored at the root of the heap. Replace it with the last item of the heap followed by reducing the size of heap by 1.

Finally, heapify the root of tree.

**3.** Repeat above steps while size of heap is greater than 1.

# Heapsort

**Time Complexity:**

- Time complexity of heapify is O(log N).
- Time complexity of create And BuildHeap() is O(N)
- overall time complexity of Heap Sort is O(N log N).

  - Heapsort.
    - Insert N items into binary heap.
    - Perform N delete-min operations.
    - O(N log N) sort.
    - No extra storage.

# Heapsort

```
void sort(int arr[], int n)
    {

        // Build heap (rearrange array)
       //for (int i = n / 2 - 1; i >= 0; i--)
           //heapify(arr, n, i);

       // One by one extract an element from heap
       for (int i=n-1; i>0; i--)
       {
           // Move current root to end
           int temp = arr[0];
           arr[0] = arr[i];
           arr[i] = temp;

           // call max heapify on the reduced heap
           heapify(arr, i, 0);
       }
    }
```