CHAPTER 13

THE MECHANICS OF NON-VISCOUS FLUIDS

Three (common) phases of matter:

- ♦ **Solid:** Maintains shape & size (approx.), even under large forces.
- ♦ **Liquid:** No fixed shape. Takes shape of container.
- ♦ **Gas:** Neither fixed shape, nor fixed volume. Expands to fill container

Density and Pressure

♦**Density**

In fluids, we are interested in properties that can vary from point to point. Thus, it is more useful to speak of density and pressure than of mass and force.

$\rho = m/v$ (uniform density)

•Density is a scalar, the SI unit is kg/m^3.

♦ **Pressure**

$$P \equiv \frac{F}{A}$$

(pressure of uniform force on flat area)

- ♦ F is the magnitude of the normal force on area A.
- ♦ The SI unit of pressure is N/m^2, called the Pascal (Pa).
- ♦ The tire pressure of cars are in kilopascals.

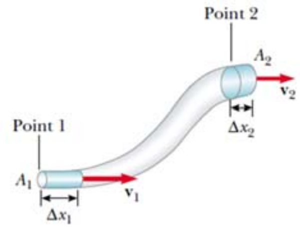
1 atm = 1.013x10^5 Pa = 76 cm Hg = 760mm Hg

13.2. The Equation of Continuity

A fluid moving with steady flow through a pipe of varying cross-sectional area. The volume of fluid flowing through area A_1 in a time interval t must equal the volume flowing through area A_2 in the same time interval.

Therefore,

$$A_1 v_1 = A_2 v_2 = \text{constant}$$



- ♦ This expression is called the equation of continuity for fluids.
- ♦ It states that the product of the area and the fluid speed at all points along a pipe is constant for an incompressible fluid.

Example

A water pipe leading up to a hose has a radius of 1 cm. water leaves the hose at a rate of 3 litres per minute.

a) Find the velocity of the water in the pipe.
b) The hose has a radius of 0.5 cm. What is the velocity of the water in the hose?

Fluid Flow

1. The fluid is **nonviscous**. In a nonviscous fluid, internal friction is neglected. An object moving through the fluid experiences no viscous force.

2. The flow is **steady**. In steady (**laminar**) flow, the velocity of the fluid at each point remains constant.

3. The fluid is **incompressible**. The density of an incompressible fluid is constant.

4. The flow is **irrotational**. In irrotational flow, the fluid has no angular momentum about any point.

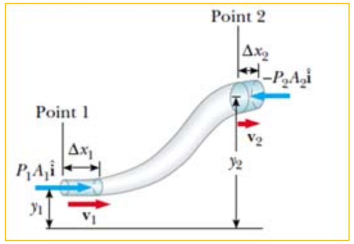
5. If a small paddle wheel placed anywhere in the fluid does not rotate about the wheel's center of mass, then the flow is **irrotational**.

6. The path taken by a fluid particle under steady flow is called a **streamline.**

13.3. Bernoulli's Equation

This is Bernoulli's equation as applied to an ideal fluid. It is often expressed as

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$



$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{a constant} \quad \text{(Bernoulli's equation)}$$

- If $y_1 = y_2$, then
$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$
- If the speed of a fluid element increases as it travels along a horizontal streamline, the pressure of the fluid must decrease, and conversely.
- Large speed means small pressure.

4- Static Consequences of Bernoulli's Equation

When the fluid is at rest v=0, p + ρgh is constant:

$$P_1 + \rho g h_1 = P_2 + \rho g h_2$$

The pressure at the same depth at two places in a fluid at rest is the same:

$$P_1 = P_2$$

Example 1

What is the pressure on a swimmer 5 m below the surface of a lake?

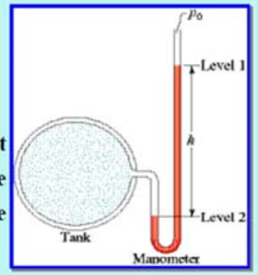
Example 2

The pressure 1 m above a floor is measured to be normal atmospheric pressure, 1.013×10^5 Pa. How much greater is the pressure at the floor if the temperature is 0^0C? $\rho = 1.29$ kg.m^{-3}

The Open Tube Manometer

The **gauge pressure** is the difference between the absolute pressure and the atmospheric pressure.

$$p_g = p_A - p_0 = \rho g h$$

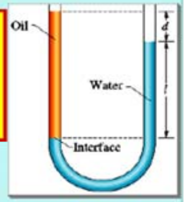


The gauge pressure is directly proportional to h. It can be positive or negative depending on whether the absolute pressure is greater or less than the atmospheric pressure.

We can suck fluids up a straw because at that time the absolute pressure in the lungs is **less** than the atmospheric pressure.

Example ;

The U-tube in Figure contains two liquids in static equilibrium: Water of density ρ_w (= 998 kg/m^3) is in the right arm, and oil of unknown density ρ_x is in the left. Measurement gives l = 135 mm and d = 12.3 mm. What is the density of the oil?



SOLUTION:

We equate the pressure in the two arms at the level of the interface :

$$P_{int} = p_0 + \rho_w g l \quad (right\ arm)$$

$$P_{int} = p_0 + \rho_x g (l + d) \quad (left\ arm)$$

$$\rho_x = \rho_w \frac{1}{l+d} = (998\ kg/m^3)\frac{135\ mm}{135\ mm + 12.3\ mm} = 915\ kg/m^3$$

13.7- Dynamic Consequences of Bernoulli's equation

Since the flow is horizontal, the terms in Bernoulli's equation containing y are equal and cancel, then

$$P_1 + 1/2\rho v_1^2 = P_2 + 1/2\rho v_2^2$$

a very simple example is tear a piece of paper in half and, holding the halves side by side about 2 cm apart, blow between them the pressure difference $P_1 - P_2 = 1/2\rho(v_2^2 - v_1^2)$ results in the sheets moving toward one another, as you will observe if you try it.

The fact that the pressure drops when the velocity increases for a fluid moving at a constant height is a consequence of energy conservation.

Review Questions

Q1

Instead of describing fluids in terms of masses and forces, we use the............... and the ....

Q2

In static fluids, the pressure difference between two points in the fluid is determined by the fluid density, g, and the ....

Q3

In fluid motion, regions of high average velocity tend to have................ Pressures

Example 13.7

Water enters a basement through a pipe 2 cm in radius at an absolute pressure of 3 atm. A hose with a 0.5 cm radius is used to water plants 10 m above the basement. Find the velocity of the water as it leaves the hose.

$$P_h + \rho g y_h + \frac{1}{2}\rho v_h^2 = P_p + \rho g y_p + \frac{1}{2}\rho v_p^2$$

$$\pi R_h^2 v_h = \pi R_p^2 v_p$$

Example 13.4

A water pipe leading up to a hose has a radius of 1 cm. Water leaves the hose at a rate of 3 litres per minute. (a) Find the velocity of the water in the pipe. (b) The hose has a radius of 0.5 cm. What is the velocity of the water in the hose?

(a) The velocity (strictly speaking, the average velocity) can be found from the flow rate and the area. The flow rate is the same in the hose and in the pipe. Using 1 litre = 0.001 m³ and 1 min = 60 s, the flow rate is

$$Q = \frac{\Delta V}{\Delta t} = \frac{0.003\ \text{m}^3}{60\ \text{s}}$$
$$= 5 \times 10^{-5}\ \text{m s}^{-1}$$

We will call the velocity and area in the pipe v_1 and A_1, respectively. Then, with $Q = Av$, we have

$$v_1 = \frac{Q}{A_1} = \frac{Q}{\pi r_1^2}$$
$$= \frac{5 \times 10^{-5}\ \text{m s}^{-1}}{\pi (0.01\ \text{m})^2} = 0.159\ \text{m s}^{-1}$$

(b) The flow rate is constant, so $A_1 v_1 = A_2 v_2$, and the velocity v_2 in the hose is

$$v_2 = v_1 \frac{A_1}{A_2} = v_1 \frac{\pi r_1^2}{\pi r_2^2} = v_1 \left(\frac{r_1}{r_2}\right)^2$$
$$= (0.159\ \text{m s}^{-1}) \frac{1}{(0.5)^2} = 0.636\ \text{m s}^{-1}$$

The water flows faster in the narrower channel.

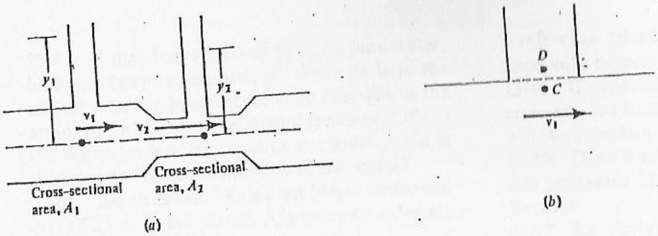


Figure 13.13. (a) Venturi tube. (b) Enlarged view of the region where column 1 connects to the flow tube.

columns is at rest while the liquid in the tube is moving. Bernoulli's equation cannot be applied directly to relate the pressures at points C and D in Fig. 13.13b because the fluid at the two points is not in the same streamline. However, if the pressures were unequal, fluid would flow from one point to the other. Since this does not occur, $P_C = P_D$. Thus the pressure in the columns is the same as the pressure in the streamline.

Bernoulli's equation requires that $P + \rho g y + \frac{1}{2}\rho v^2$ is the same everywhere in a flow tube. Applying Bernoulli's equation to points at the same height in the flow stream just below the columns,

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

From the continuity equation, $A_1 v_1 = A_2 v_2$, or

$$v_2 = \frac{A_1}{A_2} v_1 \qquad (13.15)$$

Using this expression for v_2, the preceding equation can be written as

$$P_1 - P_2 = \frac{1}{2}\rho v_1^2 \left[\frac{A_1^2}{A_2^2} - 1\right] \qquad (13.16)$$

Thus, a measurement of $P_1 - P_2$ and knowledge of the areas determines v_1; v_2 can also be found using Eq. 13.15.

The following example shows how this flow meter can be used to measure the velocity of the blood in an artery.

Example 13.8

The flow of blood through a large artery in a dog is diverted through a venturi flow meter. The wider part of the flow meter has an area $A_1 = 0.08\ \text{cm}^2$, which equals the cross-sectional area of the artery. The narrower part of the flow meter has an area $A_2 = 0.04\ \text{cm}^2$. The pressure drop in the flow meter is 25 Pa. What is the velocity v_1 of the blood in the artery?

The ratio of the areas A_1/A_2 is dimensionless and has the value $0.08/0.04 = 2$. From Table 13.1, the density of whole blood is $1059.5\ \text{kg m}^{-3}$. Dropping the units, Eq. 13.16 becomes

$$25 = \tfrac{1}{2}(1059.5)v_1^2(2^2 - 1)$$

Solving for v_1,

$$v_1 = \sqrt{\frac{(2)(25)}{(1059.5)(2^2 - 1)}} = 0.125\ \text{m s}^{-1}$$

The Prandtl Tube | Figure 13.14 shows a *Prandtl tube* inserted in a flow stream. It interrupts the flow pattern very little except at point A, where the fluid has zero velocity. At point B the velocity is assumed to be the streamline flow velocity v. From

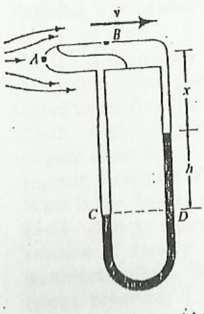


Figure 13.14. A Prandtl tube in a constant velocity flow stream. The right arm of the U-tube connects to the chamber opening at B. The left connects to the opening at A, where the fluid has zero velocity.

still have it lift off the ground?

Section 13.2 | The Equation of Continuity; Streamline Flow

13-7 A water main with a radius of 0.15 m contains water with an average velocity of 3 m s^{-1}. What is the flow rate in the water main?

13-8 In a decorative fountain in a garden, water is shot nearly vertically from a pipe. The stream of water broadens out as it rises. Explain why.

13-9 A hose delivers 20 litres of water per minute. The diameter of its nozzle is 1 cm. What is the average velocity of the water as it leaves the nozzle?

13-10 The radius of a water pipe decreases from 0.2 to 0.1 m. If the average velocity in the wider portion is 3 m s^{-1}, find the average velocity in the narrower region.

13-11 A garden hose with a cross-sectional area of 2 cm^2 has a flow of 200 cm^3 s^{-1}. What is the average velocity of the water?

13-12 A blood vessel of radius r splits into four vessels, each with radius $r/3$. If the average velocity in the larger vessel is v, find the average velocity in each of the smaller vessels.

Section 13.3 | Bernoulli's Equation