* Amplitude modulation

AM

$$S_{AM}(t) = A_c\,[1 + k_a m(t)]\cos(2\pi f_c t)$$

k_a ↳ amplitude sensitivity

⇒ when single tone

$$S_{AM}(t) = A_c\,[1 + k_a A_m \cos(2\pi f_m t)]\cos(2\pi f_c t)$$

modulation index ↳ $\mu = k_a A_m$

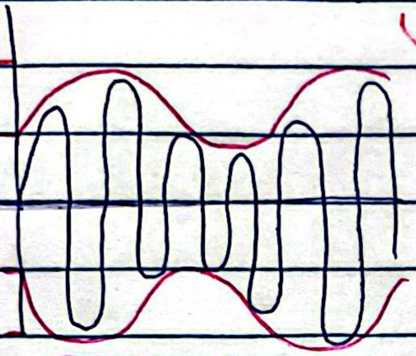
Amp. for $\cos(2\pi f_c t)$

$A_{max} = A_c(1+\mu)$

$A_{min} = A_c(1-\mu)$

$-A_{min} = -A_c(1-\mu)$

$-A_{max} = -A_c(1+\mu)$



↳ from the graph we can get μ

$$\mu = \left|\frac{A_{max} - A_{min}}{A_{max} + A_{min}}\right|$$

* when $\mu < 1$ ⇒ undermodulation (✓)
  - $\mu = 1$ normal (✓)
  - $\mu > 1$ overmodulation (X)

- ✓ can be retrieved
- X can't retrieve the message

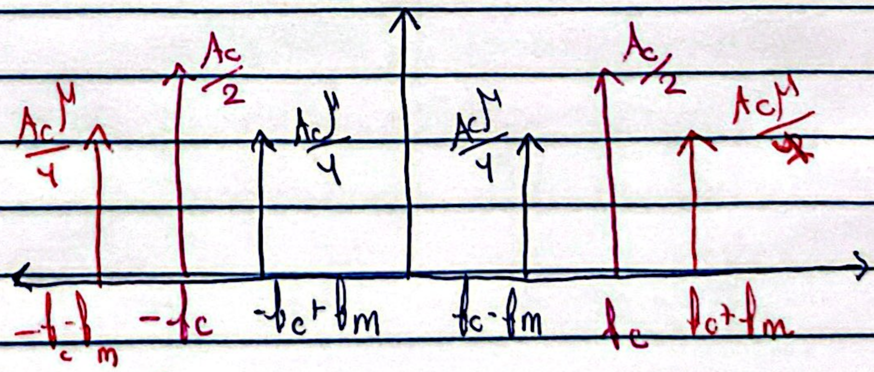
$$-\ \mu_{\%} = \frac{P_{USB} + P_{LSB}}{P_c + P_{USB} + P_{LSB}}$$

P_c ↳ $\frac{A_c^2}{2}$, P_{USB}, P_{LSB} ↳ $\left(\frac{A_c\mu}{2}\right)^2/2$

$$\mu = \frac{\mu^2}{\mu^2+2} * 100\%$$

$$S_{AM}(t) = A_c\cos(2\pi f_c t) + \frac{A_c\mu}{2}\cos(2\pi(f_m+f_c)t) + \frac{A_c\mu}{2}\cos(2\pi(f_c-f_m)t)$$

$\boxed{BW = 2f_m}$ ⇐



red: upper side band
blue: lower side band
pink: carrier

⇒ in Normal Am max $\mu_{\%} = 33\%$

Double Sideband Suppressed carrier (DSB-SC)

$S_{DSB-SC}(t) = m(t)\,c(t)$

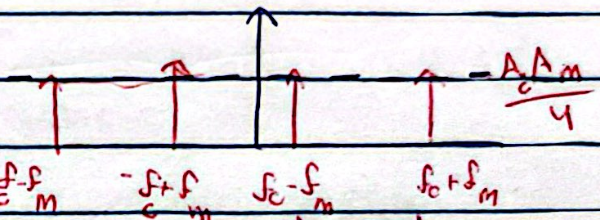
$\Rightarrow$ if $m(t)$ is single tone

$\Rightarrow S_{DSB}(t) = A_m \cos(2\pi f_m t)\, A_c \cos(2\pi f_c t)$

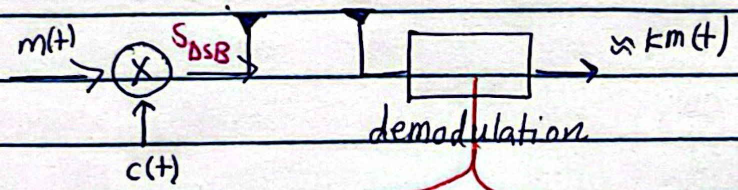
$$\Rightarrow S_{DSB}(t) = \underbrace{\frac{A_m A_c}{2}\cos(2\pi(f_m + f_c)t)}_{\text{upper}} + \underbrace{\frac{A_m A_c}{2}\cos(2\pi(f_c - f_m)t)}_{\text{Lower}}$$

in DSB-SC $M\% = 100\%$

* $P_c = 0$
* $P_{USB} = P_{LSB} = \left(\frac{A_c A_m}{2}\right)^2 / 2$

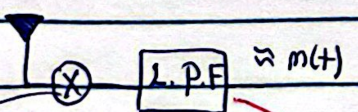


$BW = 2f_m$



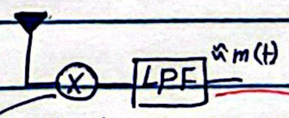
coherent detector → amplitude distortion

non-coherent detector → phase difference, freq difference



$c'(t) = A'_c \cos(2\pi f_c t)$

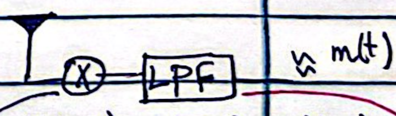
$f_m \le f_{3db} \le 2f_c$ $\quad k = \frac{2}{A_c A'_c}$



$c'(t) = A'_c \cos(2\pi f_c t + \theta)$

$f_m \le f_{3db} \le 2f_c$

$k = \frac{2}{A_c A'_c}$



$c'(t) = A'_c \cos(2\pi(f_c + \Delta f)t)$

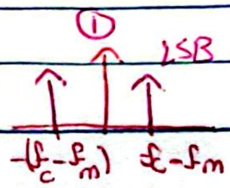
$k = \frac{2}{A_c A'_c}$

$f_m \le f_{3db} \le 2f_c$

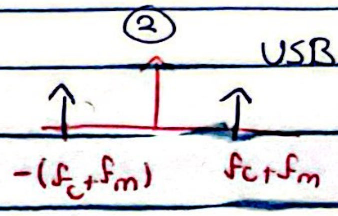
SSB-SC

- → LSB
- → USB

① LSB



② USB



$\mu = 100\%$ $BW = f_m$

SSB Generation

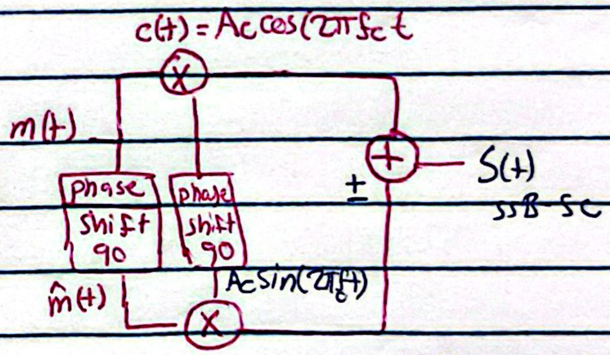
DSB-SC → $\boxed{B.P.F}$ → SSB

- center f_c+f_m ⇒ USB
- center f_c-f_m ⇒ LSB

$A_{mp} = \frac{A_m A_c}{2}$

$$S_{SSB_{LSB}} = A_c m(t)\cos(2\pi f_c t) + A_c \hat{m}(t)\,\overset{\sin}{\cancel{\cos}}(2\pi f_c t)$$

$$S_{USB} = A_c m(t)\cos(2\pi f_c t) - A_c \hat{m}(t)\,\overset{\sin}{\cancel{\cos}}(2\pi f_c t)$$



$\hat{m}(t) = m(t) * \frac{1}{\pi t} \Rightarrow \hat{m}(f) = -j\,\text{sgn}(f)\,m(f)$

$A_{mp} = A_m A_c$

$\Rightarrow S_{USB} = A_c A_m \cos(2\pi(f_c+f_m)t)$

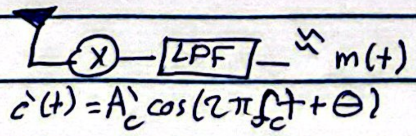
$S_{LSB} = A_c A_m \cos(2\pi(f_c-f_m)t)$

Demodulation

① coherent detector

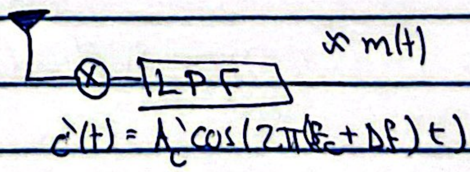
② non-coherent detector

- phase shift



* more distortion than in DSB

- frequency shift



Angle Modulation

- FM
- PM

① Frequency Modulation

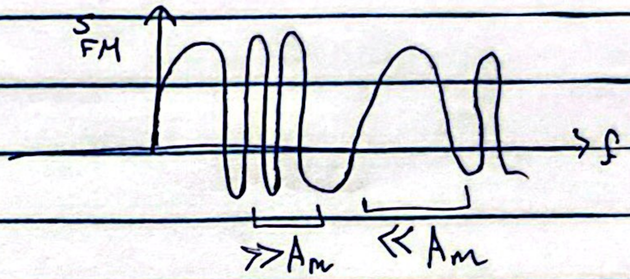
$$w_i(t) = 2\pi f_i(t) = \frac{d\phi}{dt} \qquad f_i(t) = f_c + \underline{\underline{k_f}}\, m(t) \qquad f_i(t) = \frac{1}{2\pi}\frac{d\theta}{dt}$$

k_f → freq. sensitivity

$$S_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right)$$

Freq. deviation: $\Delta f = f_i(t) - f_c = k_f m(t)$

$$|\Delta f|_{max} = |k_f m(t)|_{max}$$



$$\beta = \frac{\Delta f_{max}}{f_m}$$

when $m(t) = A_m \cos(2\pi f_m t)$

$$S_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi K_f \int A_m \cos(2\pi f_m t)\right)$$

$$= A_c \cos\left(2\pi f_c t + \frac{\cancel{2\pi} K_f A_m}{\cancel{2\pi} f_m} \sin(2\pi f_m t)\right)$$

$$= A_c \cos\left(2\pi f_c t + \frac{K_f A_m}{f_m} \sin(2\pi f_m t)\right)$$

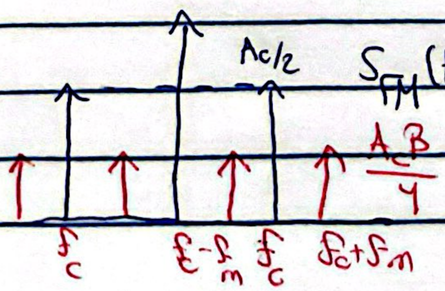
$\frac{K_f A_m}{f_m} \rightarrow \beta$

$$= A_c \cos(2\pi f_c t)\cos(\beta \sin(2\pi f_m t)) - A_c \sin(2\pi f_c t)\sin(\beta \sin(2\pi f_m t))$$

* Narrow band $\beta \ll 1$

$\cos(\theta) \approx 1 \qquad \sin(\theta) \approx \theta$

$$S_{FM}(t) = A_c \cos(2\pi f_c t) - A_c \beta \sin(2\pi f_c t)\sin(2\pi f_m t)$$



$$S_{FM}(t) = A_c \cos(2\pi f_c t) - \underbrace{\frac{A_c\beta}{2}\cos(2\pi(f_c - f_m)t)}_{LSB} + \underbrace{\frac{A_c\beta}{2}\cos(2\pi(f_c + f_m)t)}_{USB}$$

$$P_c = \frac{A_c^2}{2}$$

$$P_{USB} = P_{LSB} = \left(\frac{A_c\beta}{2}\right)^2 / 2 \qquad BW = 2f_m$$

* Wide band $B \gg 1$

$$S_{FM}(t) = \sum_{n=-\infty}^{\infty} J_n(B) A_c \cos(2\pi(f_c + n f_m)t)$$

$$BW = 2n f_m$$

FM

$$S_{FM} = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right)$$

if $m(t) = A_m \cos(2\pi f_m t)$

$$\Rightarrow S_{FM}(t) = A_c \cos(2\pi f_c t + B \sin(2\pi f_m t))$$

$B \ll 1$ Narrow

$B \gg 1$ wide

$$S_{FM}(t) = A_c \cos(2\pi f_c t) + \frac{A_c B}{2} \cos(2\pi(f_c + f_m)t) + -\frac{A_c B}{2} \cos(2\pi(f_c - f_m)t)$$

$$BW = 2 f_m$$

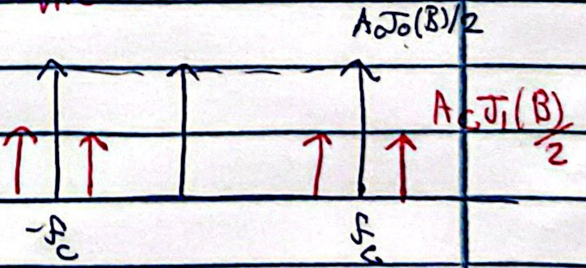
$$S_{FM}(t) = \sum_{n=-\infty}^{\infty} J_n(B) A_c \cos(2\pi(f_c + n f_m)t)$$

* to evaluate X% Power BW

$$P_{avg} = \frac{A_c^2}{2}\left[J_0^2(B) + 2J_1^2(B) + 2J_2^2(B) + \cdots\right]$$

n is when value exceeds X%.

$$B = 2n f_m$$



* tot power $= \frac{A_c^2}{2}$
* P of carrier $= \frac{(A_c J_0(B))^2}{2}$

* if BW for 100% power required

carson's rule $\Rightarrow BW = 2(B+1) f_m$

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Phase Modulation

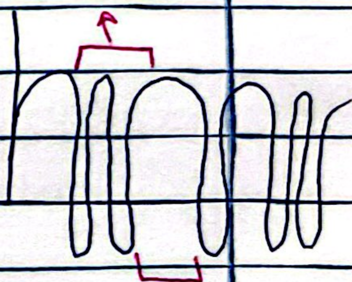
$$c(t) = A_c \cos(\phi_i(t)) = A_c \cos(2\pi f_c t + \theta)$$

$$\theta = k_p m(t)$$

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$$

$$f_i(t) = f_c + \frac{k_p}{2\pi} \frac{dm(t)}{dt}$$

$$\Delta f_{max} = f_i(t)_{max} - f_c = \frac{k_p}{2\pi} \left(\frac{dm(t)}{dt}\right)_{max} \rightarrow *$$



$$\boxed{S_{PM}(t) = A_c \cos(2\pi f_c t + k_p m(t))}$$