

9) $x = 2t^2 + 3$, $y = t^4$, $t = -1$

$\frac{dy}{dx} \Big|_{t=-1} ?$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4t^3}{4t} = t^2 = 1$

$\frac{d^2y}{dx^2} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t}{4t} = \frac{1}{2}$

$t = -1 \rightarrow x = 5$
 $y = 1$

$y - 1 = x - 5$
 $y = x - 4$

15) $x^3 + 2t^2 = 9$, $2y^3 - 3t^2 = 4$, $t = 2$

$3x^2x' + 4t = 0$, $6y^2y' - 6t = 0$

$x' = \frac{-4t}{3x^2}$, $y' = \frac{t}{y^2}$

$\frac{dy}{dx} \Big|_{t=2} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{t}{y^2}}{\frac{-4t}{3x^2}} = \frac{\frac{2}{24}}{\frac{-8}{16}} = \frac{-3}{16}$

$t = 2 \rightarrow x^3 + 8 = 9$
 $x = 1$

$t = 2 \rightarrow 2y^3 - 12 = 4$
 $y = 2$

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23) $x = a \cos t$, $y = b \sin t$, $0 \leq t \leq 2\pi$

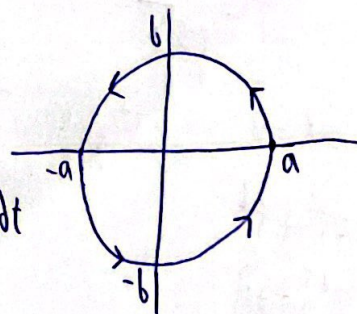
$\cos t = \frac{x}{a}$, $\sin t = \frac{y}{b}$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$t_0 = 0 \rightarrow (a, 0)$

$t_1 = 2\pi \rightarrow (a, 0)$

$dx = -a \sin t dt$



$A = 2 \int_0^\pi y dx = 2 \int_0^\pi -ab \sin^2 t dt = 2ab \int_0^\pi \left(\frac{1 - \cos 2t}{2} \right) dt = ab \int_0^\pi (1 - \cos 2t) dt$
 $= ab \left[t - \frac{\sin 2t}{2} \right]_0^\pi = \pi ab$

$$x = \sec t, \quad y = \tan t, \quad t = \frac{\pi}{6}$$

$$\frac{dy}{dx} \bigg|_{t=\frac{\pi}{6}} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sec^2 t}{\sec t \tan t} = \frac{\sec t}{\tan t} = \csc t = \csc \frac{\pi}{6} = \frac{1}{\sin \frac{\pi}{6}} = 2$$

$$\frac{d^2y}{dx^2} \bigg|_{t=\frac{\pi}{6}} = \frac{\frac{d}{dt} \left(\frac{dy}{dx} \right)}{\frac{dx}{dt}} = \frac{-\csc t \cot t}{\sec t \tan t} = -\cot^3 t \bigg|_{\frac{\pi}{6}} = -3\sqrt{3}$$

$$t = \frac{\pi}{6} \rightarrow x = \frac{2}{\sqrt{3}}$$

$$y = \frac{1}{\sqrt{3}}$$

$$y - \frac{1}{\sqrt{3}} = 2\left(x - \frac{2}{\sqrt{3}}\right)$$

$$y = 2x - \frac{4}{\sqrt{3}} + \frac{1}{\sqrt{3}}$$

$$y = 2x - \frac{3}{\sqrt{3}}$$

$$y = 2x - \sqrt{3}$$

$$14) \quad x = t + e^t, \quad y = 1 - e^t, \quad t = 0$$

$$\frac{dy}{dx} \bigg|_{t=0} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-e^t}{1+e^t} = -\frac{1}{2}$$

$$\frac{d^2y}{dx^2} \bigg|_{t=0} = \frac{\frac{d}{dt} \left(\frac{dy}{dx} \right)}{\frac{dx}{dt}} = \frac{-e^t}{(1+e^t)^2} \cdot \frac{1}{(1+e^t)} = \frac{-e^t}{(1+e^t)^3} = -\frac{1}{8}$$

$$t=0 \rightarrow x=1, \quad y=0$$

$$y = -\frac{1}{2}(x-1)$$

$$y = -\frac{x}{2} + \frac{1}{2}$$

$$20) \quad t = \ln(x-t), \quad y = te^t, \quad t=0$$

$$1 = \frac{x'-1}{x-t} \quad y' = te^t + e^t$$

$$x-t = x'-1$$

$$x' = x-t+1$$

$$\frac{dy}{dx} \bigg|_{t=0} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{e^t[t+1]}{x-t+1} = \frac{1[0+1]}{1-0+1} = \frac{1}{2}$$

$$t=0 \rightarrow 0 = \ln x, \quad e^0 = e^{\ln x}, \quad x=1$$

$$22) \quad x = t - t^2, \quad y = 1 + e^{-t}$$

$$\int_0^1 x dy = \left| \int_0^1 (t - t^2) - e^{-t} dt \right|$$

$$= \left| \int_0^1 (t - t^2) e^{-t} dt \right|$$

$$dy = -e^{-t} dt$$

$$t - t^2 = 0$$

$$t(1-t) = 0$$

$$t = 0, 1$$

$$= \left[e^{-t} (t - t^2) + e^{-t} (1 - 2t) - 2e^{-t} \right]_0^1$$

$$= \left[\frac{1}{e} (0) + \frac{1}{e} (-1) - \frac{2}{e} \right]$$

$$[0 + 1 - 2]$$

$$= -\frac{1}{e} - \frac{2}{e} + 1 = \left| -\frac{3}{e} + 1 \right|$$

$$\begin{array}{rcl} t-t^2 & + & -e^{-t} \\ 1-2t & - & +e^{-t} \\ -2 & + & -e^{-t} \\ 0 & + & +e^{-t} \end{array}$$

$$23) \quad x = \cos t, \quad y = t + \sin t \quad 0 \leq t \leq \pi$$

$$L = \int_0^\pi \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} dt$$

$$= \int_0^\pi \sqrt{2 + 2\cos t} dt = \sqrt{2} \int_0^\pi \sqrt{(1 + \cos t) \cdot \frac{1 - \cos t}{1 - \cos t}} dt$$

$$= \sqrt{2} \int_0^\pi \sqrt{\frac{1 - \cos^2 t}{1 - \cos t}} dt = \sqrt{2} \int_0^\pi \frac{\sin t}{\sqrt{1 - \cos t}} dt$$

$$u = 1 - \cos t$$

$$du = \sin t dt$$

$$= \sqrt{2} \int \frac{du}{\sqrt{u}} = 2\sqrt{2} \left[\sqrt{1 - \cos t} \right]_0^\pi = 2\sqrt{2} [\sqrt{2} - 0] = 2(2) = 4$$

$$\frac{dx}{dt} = -\sin t \rightarrow \left(\frac{dx}{dt}\right)^2 = \sin^2 t$$

$$\frac{dy}{dt} = 1 + \cos t \rightarrow \left(\frac{dy}{dt}\right)^2 = (1 + \cos t)^2$$

$$= 1 + 2\cos t + \cos^2 t$$

$$27] \quad x = \frac{t^2}{2}, \quad y = \frac{(2t+1)^{\frac{3}{2}}}{3}, \quad 0 \leq t \leq 4$$

$$\frac{dx}{dt} = t \Rightarrow \left(\frac{dx}{dt}\right)^2 = t^2$$

$$\frac{dy}{dt} = \frac{1}{2} (2t+1)^{\frac{1}{2}} \cdot 2 = \sqrt{2t+1} \Rightarrow \left(\frac{dy}{dt}\right)^2 = 2t+1$$

$$L = \int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_0^4 \sqrt{t^2 + 2t + 1} dt = \int_0^4 \sqrt{(t+1)^2} dt = \int_0^4 (t+1) dt$$

$$= \left[\frac{t^2}{2} + t \right]_0^4 = 12 - 0 = 12$$