

0.7 Algebraic Fractions.

Algebraic Fractions are : $\frac{\text{algebraic expression}}{\text{algebraic expression}}$

Example: ① $\frac{6}{8}$

② $\frac{3x^2 - 14x + 8}{x^2 - 16}$

③ $\frac{a^2 b}{c}$

* Simplifying fractions:-

Example: ① $\frac{6}{8} = \frac{2 \cdot 3}{4 \cdot 2} = \frac{3}{4}$

② $\frac{3x^2 - 12x + 8}{x^2 - 16} = \frac{(x-4)(x+2)}{(x-4)(x+4)} = \frac{x+2}{x+4}$

* Products of fractions:

Example: ① $\frac{4}{5} \cdot \frac{10}{12}$

$$\frac{4}{5} \cdot \frac{10}{12} = \frac{(4)(10)}{(5)(12)} = \frac{40}{60} = \frac{4}{6} = \frac{2 \cdot 2}{3 \cdot 2} = \frac{2}{3}$$

OR

$$\frac{4}{5} \cdot \frac{10}{12} = \frac{2 \cdot 2}{5} \cdot \frac{2 \cdot 5}{2 \cdot 2 \cdot 3} = \frac{2 \cdot \cancel{2}}{3}$$

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Example: $\frac{-4x+8}{3x+6} \cdot \frac{2x+4}{4x+12}$

$$= \frac{4(-x+2)}{3(x+2)} \cdot \frac{2(x+2)}{4(x+3)} = \frac{2(-x+2)}{3(x+3)}$$

* Quotients of fractions:

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Example: ① $\frac{4}{7} \div \frac{5}{21} = \frac{4}{7} \cdot \frac{21}{5}$

$$= \frac{4}{7} \cdot \frac{3 \cdot 7}{5} = \frac{4 \cdot 3}{5} = \frac{12}{5}$$

② $\frac{a^2b}{c} \div \frac{ab}{c^2} = \frac{a^2b}{c} \cdot \frac{c^2}{ab}$

$$= \frac{a \cdot ab}{c} \cdot \frac{c \cdot c}{ab} = \frac{ac}{1} = ac$$

③ $\frac{6x^2-6}{x^2+3x+2} \div \frac{x-1}{x^2+4x+4}$

$$= \frac{6x^2-6}{x^2+3x+2} \cdot \frac{x^2+4x+4}{x-1}$$

$$= \frac{6(x^2-1)}{(x+2)(x+1)} \cdot \frac{(x+2)(x+2)}{(x-1)}$$

$$= \frac{6(x-1)(x+1)}{(x+2)(x+1)} \cdot \frac{(x+2)(x+2)}{(x-1)} = 6(x+2)$$

* Addition and subtraction:

to add or subtract fractions, find a common denominator then add or subtract the numerators.

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Example:

$$\textcircled{1} \quad \frac{3}{12} + \frac{7}{8}$$

find the least common denominator LCD:

$$\begin{array}{l} 12 = 3(2)(2) \\ 8 = 2(2)(2) \end{array} \left. \vphantom{\begin{array}{l} 12 = 3(2)(2) \\ 8 = 2(2)(2) \end{array}} \right\} \text{different factors: } 2, 3$$

$$\text{LCD} = 2(2)(2)(3) = 24$$

$$\frac{3}{12} + \frac{7}{8} = \frac{3(2)}{12(2)} + \frac{7(3)}{8(3)} =$$

$$= \frac{6}{24} + \frac{21}{24} = \frac{6+21}{24} = \frac{27}{24} = \frac{9}{8}$$

Example: $\frac{3x}{a^2} + \frac{4}{ax}$

$$\text{LCD:- } \begin{array}{l} a^2 = a(a) \\ ax = a(x) \end{array} \left. \vphantom{\begin{array}{l} a^2 = a(a) \\ ax = a(x) \end{array}} \right\} \text{different factors: } a, x$$

$$\text{LCD} = a(a)(x) = a^2x$$

$$\frac{3x(x)}{a^2(x)} + \frac{4(a)}{ax(a)} = \frac{3x^2}{a^2x} + \frac{4a}{a^2x} = \frac{3x^2 + 4a}{a^2x}$$

Example: $\frac{y-3}{(y-5)^2} - \frac{y-2}{y^2-4y-5} =$

LCD:- $(y-5)^2 = (y-5)(y-5)$
 $y^2-4y-5 = (y-5)(y+1)$ } different factors: $(y-5)(y+1)$

LCD = $(y-5)^2(y+1)$

$$\begin{aligned} & \frac{y-3}{(y-5)^2} - \frac{y-2}{y^2-4y-5} \\ &= \frac{(y-3)(y+1)}{(y-5)^2(y+1)} - \frac{(y-2)(y-5)}{(y-5)(y+1)(y-5)} \\ &= \frac{y^2-2y-3}{(y-5)^2(y+1)} - \frac{y^2-7y+10}{(y-5)^2(y+1)} \\ &= \frac{y^2-2y-3-(y^2-7y+10)}{(y-5)^2(y+1)} \\ &= \frac{y^2-2y-3-y^2+7y-10}{(y-5)^2(y+1)} \\ &= \frac{5y-13}{(y-5)^2(y+1)} \end{aligned}$$

Question: Find the LCD for the following fractions $\frac{1}{(x^2-x)}$, $\frac{1}{x^2-1}$, $\frac{1}{x^2}$

$$\text{LCD:- } x^2 - x = x(x-1)$$

$$x^2 - 1 = (x-1)(x+1)$$

$$x^2 = x \cdot x$$

different factors: x , $(x-1)$, $(x+1)$

$$\text{LCD} = x^2(x-1)(x+1)$$