Probability:

$$P[A|B]=\frac{P[A\cap B]}{P[B]} \text{ for } P[B]>0$$; Conditional probability

$$P[\omega_j|x]=\frac{P[x|\omega_j]\cdot P[\omega_j]}{\sum_{k=1}^{N}P[x|\omega_k]\cdot P[\omega_k]}=\frac{P[x|\omega_j]\cdot P[\omega_j]}{P[x]}$$; Bayes theorem

$P[\omega_j]$: **Prior probability**; $P[\omega_j|x]$: **Posterior Probability**; $P[x|\omega_j]$: **Likelihood**;

$P[x]$: A normalization constant that does not affect the decision

Random variable:

$$F_X(x)=P[X\le x] \text{ for } -\infty<x<+\infty$$; cdf

$$f_X(x)=\frac{\Delta F_X(x)}{\Delta x}$$; pmf

$$E[X]=\mu=\int_{-\infty}^{+\infty}xf_X(x)dx$$

$$VAR[X]=E[(X-E[X])^2]=\int_{-\infty}^{+\infty}(x-\mu)^2 f_X(x)dx$$

$$STD[X]=VAR[X]^{1/2}$$

$$E[X^N]=\int_{-\infty}^{+\infty}x^N f_X(x)dx$$

Random vectors:

$$F_{\underline{X}}(\underline{x})=P_{\underline{X}}[\{X_1\le x_1\}\cap\{X_2\le x_2\}\cap\ldots\cap\{X_N\le x_N\}]$$; joint cdf

$$f_{\underline{X}}(\underline{x})=\frac{\partial^N F_{\underline{X}}(\underline{x})}{\partial x_1\partial x_2\ldots\partial x_N}$$; joint pdf

$$f_{X_1}(x_1)=\int_{x_2=-\infty}^{x_2=+\infty}f_{X_1X_2}(x_1x_2)dx_2$$; marginal pdf

$$E[X]=[E[X_1]E[X_2]\ldots E[X_N]]^T=[\mu_1\mu_2\ldots\mu_N]=\mu$$; Mean vector

$$COV[X] = \Sigma = E[(X-\mu)(X-\mu)^T] \quad ; \text{Covariance matrix}$$

$$= \begin{bmatrix} E[(x_1-\mu_1)(x_1-\mu_1)] & \cdots & E[(x_1-\mu_1)(x_N-\mu_N)] \\ \cdots & \cdots & \\ E[(x_N-\mu_N)(x_1-\mu_1)] & \cdots & E[(x_N-\mu_N)(x_N-\mu_N)] \end{bmatrix} = \begin{bmatrix} \sigma_1^2 & \cdots & c_{1N} \\ \cdots & \cdots & \\ c_{1N} & \cdots & \sigma_N^2 \end{bmatrix}$$

$$\boxed{f_X(x) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left[-\frac{1}{2}(X-\mu)^T \Sigma^{-1}(X-\mu)\right]} \quad ; \text{Normal distribution}$$

<u>Linear algebra:</u>

$$\langle x, y \rangle = x^T y = y^T x = \sum_{k=1}^{d} x_k y_k \quad ; \textbf{Dot product}$$

$$|x| = \sqrt{x^T x} = \left[\sum_{k=1}^{d} x_k x_k\right]^{1/2} \quad ; \textbf{Vector magnitude}$$

$$\langle y^T u_x \rangle u_x \quad ; \textbf{Orthogonal projection}$$

$$\cos\theta = \frac{\langle x, y \rangle}{|x| \cdot |y|} \quad ; \textbf{Angle between x and y}$$

$$|A| = \sum_{k=1}^{d} a_{ik} |A_{ik}| (-1)^{k+i} \quad ; \textbf{Determinant of } A_{d \times d}$$

$$tr(A) = \sum_{k=1}^{d} a_{kk} \quad ; \textbf{Trace of } A_{d \times d}$$

<u>Bayesian decision theory:</u>

$$\boxed{\Lambda(x) = \frac{P(x|\omega_1)}{P(x|\omega_2)} \underset{\omega_2}{\overset{\omega_1}{\gtrless}} \frac{P(\omega_2)}{P(\omega_1)}} \quad ; \textbf{Likelihood ratio}$$

$$P[error] = \sum_{i=1}^{C} P[error|\omega_i] P[\omega_i]$$

$$P[error|\omega_i] = P[choose \ \omega_j|\omega_i] = \int_{R_j} P(x|\omega_i)dx$$

$$\Re = \int_{R_1} [C_{11} \cdot P[\omega_1] \cdot P(x|\omega_1) + C_{12} \cdot P[\omega_2] \cdot P(x|\omega_2)]dx + \int_{R_2} [C_{21} \cdot P[\omega_1] \cdot P(x|\omega_1) + C_{22} \cdot P[\omega_2] \cdot P(x|\omega_2)]dx \quad ; \textbf{Bayes risk}$$

$$\Lambda(x) = \frac{P(x \mid \omega_1)}{P(x \mid \omega_2)} \mathop{\gtrless}_{\omega_2}^{\omega_1} \frac{(C_{12} - C_{22})\,P[\omega_2]}{(C_{21} - C_{11})\,P[\omega_1]}$$ **Bayes criterion**

$$C_{ij} = \begin{cases} 0 & i = j \\ 1 & i \neq j \end{cases} \Rightarrow \Lambda(x) = \frac{P(x \mid \omega_1)}{P(x \mid \omega_2)} \mathop{\gtrless}_{\omega_2}^{\omega_1} \frac{P(\omega_2)}{P(\omega_1)} \Leftrightarrow \frac{P(\omega_1 \mid x)}{P(\omega_2 \mid x)} \mathop{\gtrless}_{\omega_2}^{\omega_1} 1$$ **Maximum A Posteriori (MAP) Criterion**

$$\left.\begin{array}{l} C_{ij} = \begin{cases} 0 & i = j \\ 1 & i \neq j \end{cases} \\ P(\omega_i) = \dfrac{1}{C} \quad \forall i \end{array}\right\} \Rightarrow \Lambda(x) = \frac{P(x \mid \omega_1)}{P(x \mid \omega_2)} \mathop{\gtrless}_{\omega_2}^{\omega_1} 1$$ **Maximum Likelihood (ML) Criterion**

$$P[\text{correct}] = \sum_{i=1}^{C} P(\omega_i) \int_{R_i} P(x \mid \omega_i)\,dx = \sum_{i=1}^{C} \int_{R_i} P(x \mid \omega_i) P(\omega_i)\,dx = \sum_{i=1}^{C} \underbrace{\int_{R_i} P(\omega_i \mid x) P(x)\,dx}_{\Im_i}$$

Optical sensors:

$C = \lambda.f$; **C :light speed** = 3 x 10^8 m/s ; λ **:wavelength ; f : frequency**

$$\Theta = 2\arctan\left(\frac{D_f - D_i}{2l}\right)$$; Divergence of a beam

$$P_{\text{net}} = A\sigma\varepsilon\left(T^4 - T_0^4\right)$$; Stefan Boltzmann law ; $\sigma = 5.67 \text{X} 10^{-8}$ w m^{-2} K^{-4}