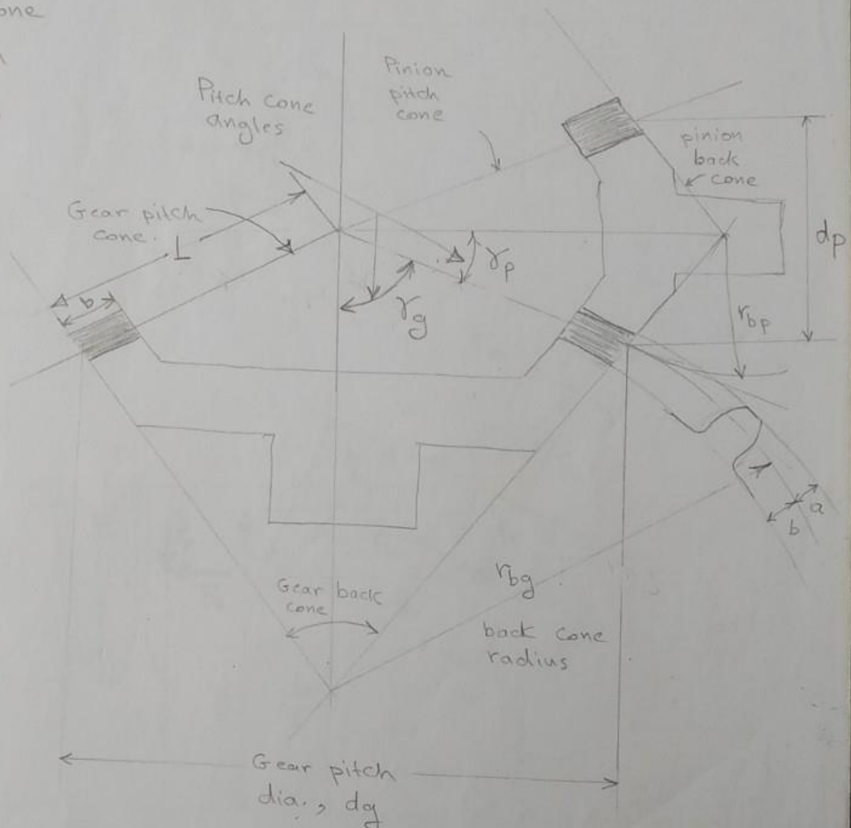
Bevel Gear Geometry:

Bevel gears are used to connect non parallel shafts.
pitch cones (Analogous to pitch cylinder of spur gears) are tangent along the elements, with apexes at the interse ction of shafts.

Tooth profile at the back cone is the same of spur gear with pitch radius r_p = Developed back cone radius (r_b)



No. of teeth in the imaginary circles :-

$$N'_p = \frac{2\pi r_{bp}}{p}$$

$$N'_g = \frac{2\pi r_{bg}}{p}$$

N' = virtual no. of teeth
p = circular pitch
[for imaginary spur and bevel gears].

$$N'_p = 2 r_{bp} \underline{P}$$

$$N'_g = 2 r_{bg} \underline{P}$$

pitch dia. of Bevel gears is measured at the large end of tooth.
Circular pitch and pitch dia is measured as spur gears.

Gear ratio:

$$\text{Gear ratio} = \frac{w_p}{w_g} = \frac{N_g}{N_p} = \frac{d_g}{d_p} = \tan(\gamma_g) = \cot(\gamma_p)$$

Face width: practice imposed two limits on face width:

$$b \leq \frac{10}{\underline{P}} \quad \text{and} \quad b \leq \frac{L}{3}$$, [L = pitch cone length].

$$L = \frac{d_g/2}{\sin\gamma_g} = \frac{d_p/2}{\sin\gamma_p}$$

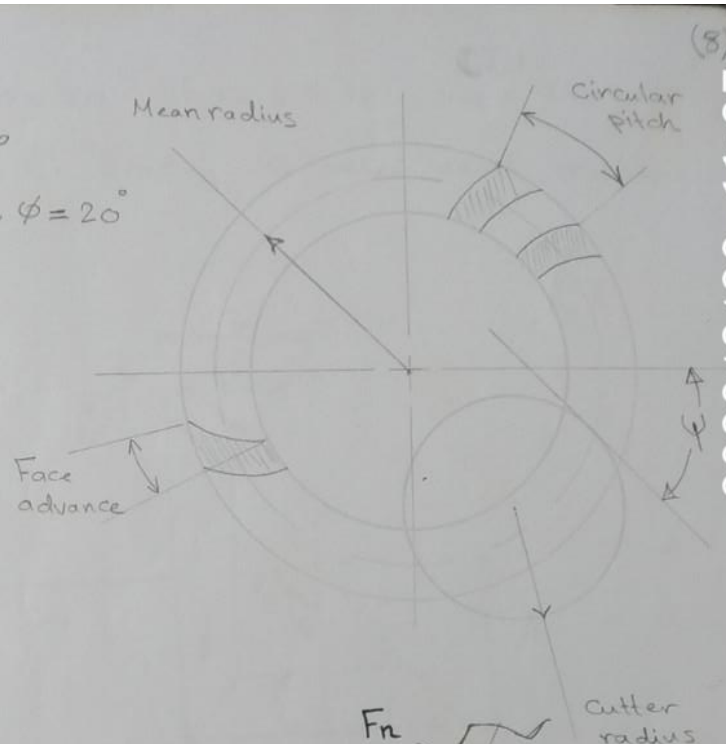
2020/3/28 11:26

Spiral angle of bevel gear = ψ

Spiral bevel gear normally have, $\psi = 35^\circ$

Bevel gear normally have pressure angle, $\phi = 20^\circ$

Spiral Bevel gear:
used for
1- high speed
2 - Quiter operation
(low noise level)



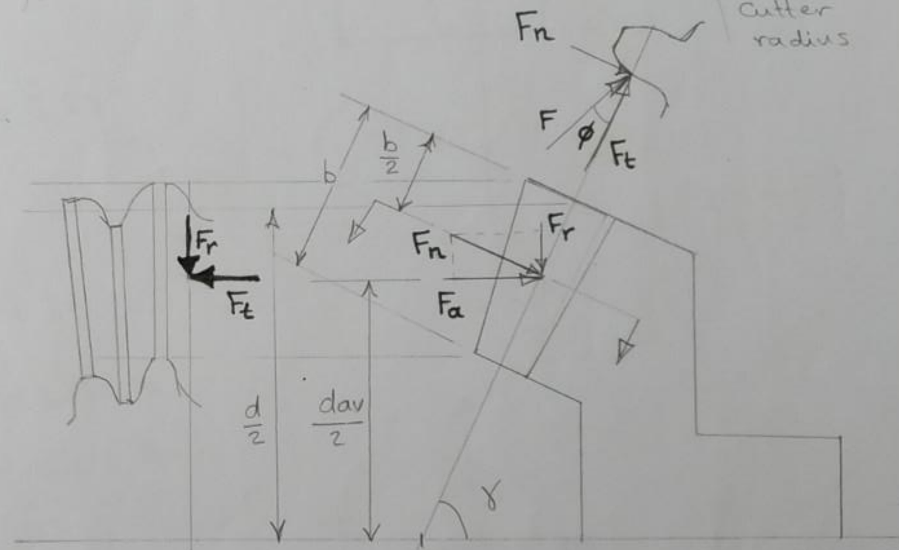
Bevel gear force analysis:

Force components:

F_t = tangential force
(Torque producing).

F_r = radial

F_a = axial (thrust).



Force analysis are made at pitch conce surface at half face width ($\frac{b}{2}$)

$\Rightarrow$ load uniformly distributed along tooth width.

$$d_{av} = d - b \sin\gamma$$

$$V_{av} = \frac{\pi \, d_{av} \, n}{12} \text{ (rpm, rps).}$$

$$F_t \text{ (lb)} = \frac{33{,}000 \text{ (hp)}}{V_{av}} \quad , V_{av} \text{ (ft/min)}$$

$$F_t \text{ (Newtons)} = \frac{W \text{ (watt)}}{V_{av}} \quad [V_{av}, m/s].$$

$$F = \frac{F_t}{\cos\phi}$$

$$F_n = F \sin\phi = F_t \tan\phi$$

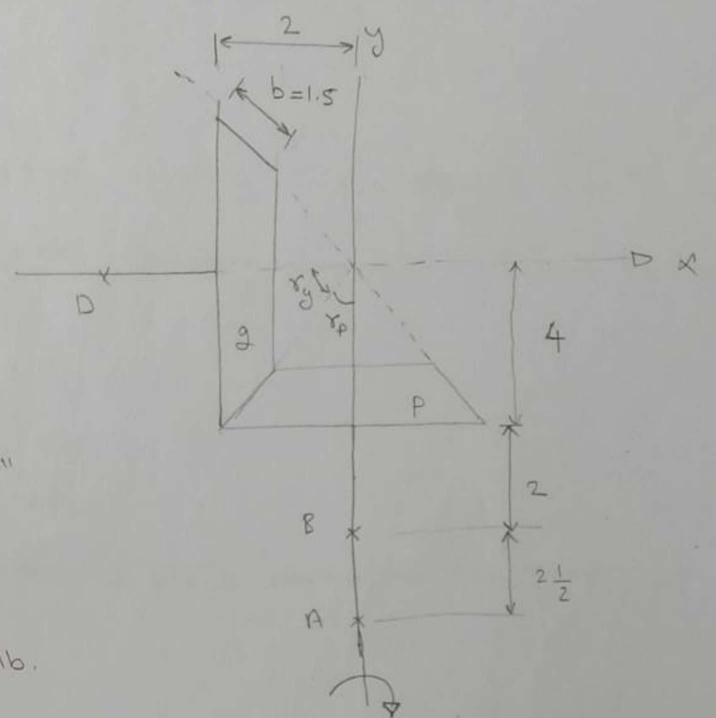
$$F_a = F_n \sin\gamma = F_t \tan\phi \sin\gamma$$

$$F_r = F_n \cos\gamma = F_t \tan\phi \cos\gamma$$

13-32 : $N_p = 16$, $N_g = 32$, $\phi = 20$, $hp = 2.5\ hp$, $\omega_p = 240\ rpm$

determine reaction at A, B. Bearing A takes axial reaction.

Solution:



$V_{av} = \frac{\pi d_{av} n}{12}$

$d_{avp} = d_p - b \sin \gamma_p$

$\gamma_p = \tan^{-1} \frac{d_p}{d_g} = \tan^{-1} \frac{N_p}{N_g}$

$= \tan^{-1}\left(\frac{16}{32}\right) = 26.56°$

$d_p = 4''$

$b = 1.5''$

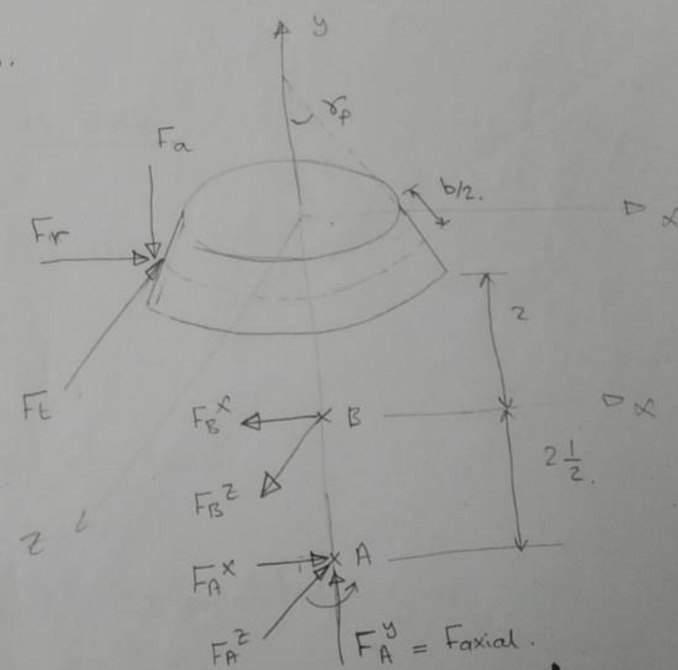
$d_{avp} = 4 - 1.5 \sin(26.56) = 3.33''$

$V_{av} = 209.18$ ft/min.

$F_t = \frac{hp\ 33000}{V_{av}} = 394.38$ lb.

$F_r = F_t \tan\phi \cos\gamma_p = 128.39$ lb

$F_a = F_t \tan\phi \sin\gamma_p = 64.2$ lb.



$\Sigma M_A = 0$.

$\Sigma M_A^x = 0$

$F_t \left[2.5 + 2 + \frac{b}{2}\cos\gamma_p\right] = F_B^z \left(2\frac{1}{2}\right)$

$F_B^z = 815.7$ lb $(+k)$

$\Sigma M_A^z = 0$

$-F_a\left(\frac{d_{av}}{2}\right) + F_r\left[2.5 + 2 + \frac{b}{2}\cos\gamma_p\right] =$

$F_B^x\,(2.5)$

$F_B^x = 222.8$ lb $(-i)$

$F_A^x = F_r - F_B^x = 94.4$ $(+i)$

$F_A^z = 421.3$ lb $= F_B^z - F_t$ $(-k)$

$F_A^y = F_a = 64.2$ lb $(+j)$

$F_{Radial} = \sqrt{(F_A^x)^2 + (F_A^z)^2}$

$F_{axial} = 64.2$

2020/3/28 11:27

For spiral bevel gear:

$$F_a = \frac{F_t}{\cos\psi}\left[\tan\phi_n \sin\gamma \mp \sin\psi \cos\gamma\right] \quad \text{—(1)}$$

$$F_r = \frac{F_t}{\cos\psi}\left[\tan\phi_n \cos\gamma \pm \sin\psi \sin\gamma\right] \quad \text{—(2)}$$

Upper sign in eqs (1) and (2):

For driving pinion with RH - spiral rotating cw, viewed from its large end.

For driving pinion with LH - spiral rotating ccw, viewed from its large end.

Lower sign: Applied for.

- LH driving pinion, rotating cw.
- RH driving pinion, rotating ccw

ϕ_n = pressure angle measured in plane normal to the tooth.

Bevel gear Design:

1- Bending Stress:

$$\sigma = \frac{F_t P}{bJ} \frac{K_s K_o K_m}{K_v} \quad \text{U.S. Units}$$

$$\sigma = \frac{F_t}{bJm} \frac{K_s K_o K_m}{K_v} \quad \text{SI Units.}$$

F_t = tangential load.

P = diametral pitch at large end of tooth.

b = face width

J = geometry factor Fig. (15-7) straight bevel gear with

$\Sigma = 90, \ \phi = 20$

For spiral gear of $\psi = 35°$; $\phi = 20°$, $\Sigma = 90°$ → Notes

$$K_s = \begin{cases} 0.4867 + \frac{0.213}{P}, & 0.5 \leq P \leq 16 \\ 0.5 & P > 16 \end{cases} = \begin{cases} 0.5, & m < 1.6\text{ mm} \\ 0.4867 + \text{[illegible]} & \end{cases}$$

K_v = Velocity factor, Fig (15-5)

$K_v = \left(\frac{A+\sqrt{V}}{A}\right)^B$ U.S. $\quad K_v = \left(\frac{A+\sqrt{200V}}{A}\right)^B \quad A = 50+56(1-B),\ B = \frac{(12-Q_v)^{2/3}}{4}$

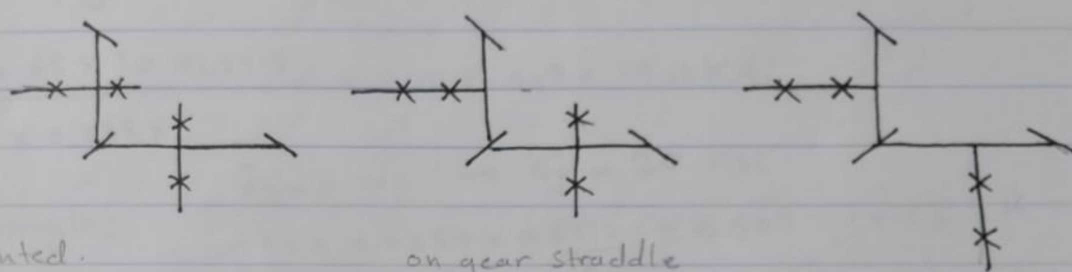
K_o = overload factor, Table [15-2]

K_m = mounting factor, depends on type of shaft mouting.

- Straddle mounting [between two bearings].
- Overhung " [out board of both bearing].
- Degree of mounting rigidity.

$K_m = K_{mb} + 0.0036\, b^2$ (us)

$K_m = K_{mb} + 5.6\times10^{-6}\, b^2$ (SI)



Both gears Straddle mounted.

$K_{mb} = 1.0$

on gear straddle and the other overhung

$K_{mb} = 1.1$

Both gears overhung.

$K_{mb} = 1.25$

Bending endurance limit:

$$S_e = S_t \frac{K_L\, K_{ms}\, K_t}{K_r} \tag{15-6}$$

S_t = AGMA bending strength of bevel gears → Table(15-2) Fig [15-13]

K_{ms} = mean stress factor.

$K_{ms} = 1.0$ for two way bending gear (Idler)

$= 1.4$ for one way bending gear (Input, out put).

K_t = tempreture factor.

T = lubricant temp.

K_L = life factor Fig. [14-9] [15-9], $K_t = \begin{cases} \frac{710}{460+T}, & t, > 250\ F^o, \quad \frac{393}{273+T}, \quad t > 120 \\ 1.0, & \leq 250\ F^o, \quad t \leq 120\ C^o \end{cases}$

K_r = Reliability factor eq [15-15]

Table [14-7] Table [15-3]

2020/3/28 11:33

Example:

A catalog rating of bevel gear → $hp = 5.2$, at $\omega_p = 1200$ rpm.
of Straight bevel gear, $N_p = 20$, $N_g = 40$, $\phi = 20°$, $b = 0.71$ in
$P = 10$ teeth/in, $\Sigma = 90°$, Uniform drive – uniform driven.
- General Industrial application, both gear out board mounted.
- Bhn = 250, hardened teeth (both gears), grade 1.
- Cutting quality correspond to → $Q_v = 5$, Life $= 1\times10^7$, $R = 99\%$

What do you think about power rating? Uncrowned teeth.

Solution: Since $(K_L)_{pinion} > (K_L)_{gear}$ ⇒ Design based on pinion

a) Based on bending:

$b = 0.71$

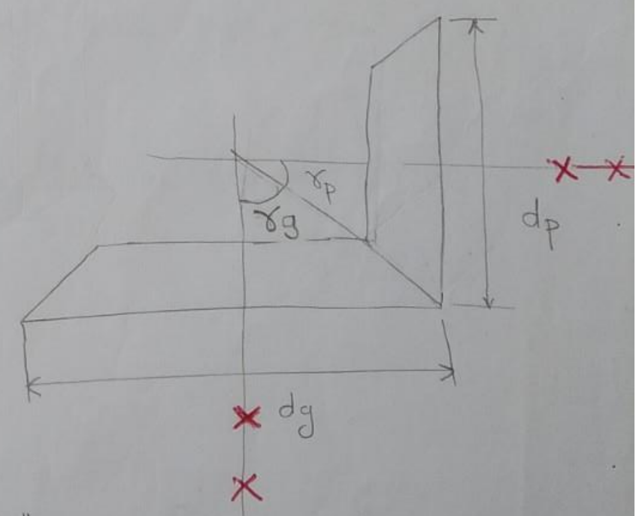
$S_t = 44\,H_B + 2100 = 13.1$ Kpsi Fig [15-13] grade 1.

$$V_{av} = \frac{\pi (d_p)_{av} n_p}{12}$$

$$d_p = \frac{N_p}{P} = \frac{20}{10} = 2''$$

$$d_g = \frac{N_g}{10} = \frac{40}{10} = 4''$$

$$\gamma_p = \tan^{-1}\frac{d_p}{d_g} = 26.56°$$



$$\Rightarrow (d_{av})_p = d_p - b\sin\gamma_p$$

$$(d_{av})_p = 2 - 0.71\sin(26.56) = 1.68''$$

$$\Rightarrow V_{av} = \frac{\pi(1.68)(1200)}{12} = 528.56 \text{ ft/min.}$$

$$hp = \frac{F_t V_{av}}{33000} \Rightarrow F_t = \frac{5.2\times 33000}{528.56} = 324.65 \text{ lb}$$

2020/3/28 11:34

$$\sigma_b = \frac{F_t\, P}{bJ} K_o K_m K_v K_s$$

$K_o = 1.0$ $\quad B = \frac{(12-Q_v)^{2/3}}{4} = 0.915$ $\quad A = 50+56(1-B) = 54.77$

$K_v = (A+\sqrt{V}/A)^B =$

$K_{mb} = 1.25$, both gears out bord mounted, $K_m = K_{mb} + 0.0036\, b^2 = 1.2518$

$K_v = 1.411$ — Fig. [15–5], $V = \frac{\pi d_p n_p}{12} = 628$ fpm

Bending stress is based on largest pitch diameter.

$F_t \rightarrow$ is based on d_{av} — since we assume that F is uniformly distributed on the full face width.

$J_p = 0.238$ — Fig.(15-5) for $N_p = 20$ — $N_g = 40$ $\qquad J_g = 0.202$

$$\Rightarrow \sigma_p = \frac{324.65 \times 10 \times 1.2518}{0.71 \times 0.238 \times} \times 1 \times 1.411 \times 0.508 = 19.68\ ksi$$

$J_g = 0.202 \rightarrow \sigma_g = 20$ ksi

Bending strength:

$K_s = 0.4867 + \frac{0.2132}{P} = 0.508 \qquad 1 \le P < 16$

$$S_e = S_t \frac{K_L K_t K_{ms}}{K_r}, \quad K_t = K_L = K_r = 1.0$$

For R = 99% and life = 1×10^7 cycles.

For pinion and gear:

$K_{ms} = 1.4$

$S_e = 13.1$ kpsi

based on hp = 5.2 $\rightarrow n = \frac{S_{eg}}{\sigma_{bg}} = \frac{13.1}{20} < 1.0 \Rightarrow$ failure.

So these gears should be rated:

<u>or:</u>

$$hp = \frac{S_e}{\sigma_b}(5.2) = 3.4\ hp.$$

$$\sigma = \frac{\sigma_b}{324.65} = \frac{37{,}990\ (psi)}{324.65} = 117.02\ F_t$$

$$\Rightarrow F_t = \frac{S_e}{117.02} = \frac{26.6\times10^3}{117.02} = 227.31\ (lb)$$

$$hp = \frac{F_t V_{av}}{33000} = \frac{227.31 \times 528}{33000} = 3.6\ (hp)$$

2020/3/28 11:34

Bevel gear contact stress

Contact stress:

$$\sigma_c = c_p \sqrt{\frac{F_t \; C_o C_m C_v C_s C_c}{b I d_p}} \quad , \quad \sigma_c = c_p \sqrt{\frac{F_t \; C_o C_m C_v C_s C_c}{b I d_p}}$$

C_p = elastic coefficient for bevel gears.

For steel, $C_p = 190 \sqrt{MPa} = 2290 \sqrt{psi}$

$$c_p = \sqrt{\frac{1}{\pi \left[\frac{(1-\nu_p^2)}{E_p} + \frac{(1-\nu_g^2)}{E_g} \right]}}$$

I = geometry factor, Fig. [15-6] I for straight bevel gear with $\phi = 20°$, $\Sigma = 90°$.

$C_o = K_o \rightarrow$ overload factor

C_m = mounting factor $= K_m$

$C_v = K_v$ = velocity factor.

C_s = Size factor eq. [15-9]

$$C_s = \begin{cases} 0.5, & b < 0.5'' \\ 0.125\,b + 0.4375, & 0.5 \leq b \leq 4.5'' \\ 1.0, & b > 4.5'' \end{cases} = \begin{cases} 0.5, & b \leq 12.7 \text{ mm} \\ 0.00492(b) + 0.4375, & 12.7 \leq b \leq 114.3 \\ 1.0, & b > 114.3 \text{ mm} \end{cases}$$

C_c = crowning factor.

$$C_c = \begin{cases} 1.5, & \text{crowned teeth} \\ 2.0, & \text{Uncrowned teeth.} \end{cases}$$

Contact fatigue strength:

$$S_{fe} = S_c \frac{C_L C_t C_H}{C_r}$$

S_c = AGMA contact strength. $\rightarrow$ Fig. [15-12]

C_L = life factor $\rightarrow$ Fig [15-8] $\quad C_L = \begin{cases} 2 & 10^3 \leq N < 10^4 \\ 3.4822\, N^{-0.0602}, & 10^4 \leq N \leq 10^{10} \end{cases}$

C_H = hardness ratio $\rightarrow$ Fig [15-10], [15-11]

$C_H = 1 + B\left(\frac{N_g}{N_p} - 1\right)$, $B = 0.00898\left[\frac{H_{BP}}{H_{Bg}}\right] - 0.0$[illegible] $\leq \frac{H_{BP}}{H_{Bg}} \leq$ [illegible]

2020/3/28 11:35

(b) Based on contact stress

$$\sigma_H = C_p \sqrt{\frac{F_t \; C_o C_m C_v C_s C_c}{I \, d_p \, b}}$$

$C_o = 1.0$

$C_m = 1.2518$

$C_v = 1.411$

$C_s = 0.125\, b + 0.4375 = 0.52$, $0.5'' \leq b \leq 4.5$

$C_c = 2.0$ Uncrowned.

$C_p = 2290 \sqrt{psi}$ [P: steel, g: steel]

$d_p = 2$ in [largest pitch dia.]

$I = 0.078$, Fig (15-6), $N_p = 20$, $N_g = 40$.

$$\sigma_H = 2290 \sqrt{\frac{324.65 \times 1.0 \times 1.411 \times 2 \times 0.52 \times 1.2518}{0.078 \times 0.71 \times 2}} = 169.\ ksi$$

$$S_{fe} = S_c \frac{C_L C_T}{C_R} C_H$$

$C_L = C_T = C_R = 1.0$

$C_H = \frac{H_{Bp}}{H_{Bg}} = 1.0 < 1.2 \rightarrow C_H = 1.0$

$S_c = 108.8$ ksi — Fig [15-12], Bhn = 250 grade 1.

$S_c = 341\, H_B + 2360 = 108.87$ ksi

$$n = \frac{S_{fe}}{\sigma_H} = \frac{108}{169.7} < 1.0 \Rightarrow$$ The [pinion, gear] will not carry 5.2 hp

$\Rightarrow$ Gears rating:

$$H = 5.2 \left(\frac{108}{169.7}\right)^2 = 2.15\ hp$$

or

$$\sigma_H = 2800 \sqrt{\frac{F_t \quad 1.4}{0.078 \times 2 \times 0.71 \times 0.708}} = S_{fe} = 120.$$

$\Rightarrow S_{fe}^2 = 892.445\, F_t \Rightarrow F_{t_R} = \frac{S_f^2}{K}$

based on hp = 5.2 $\Rightarrow (\sigma_H)^2 = K\, F_t \Rightarrow F_t = \frac{\sigma_H^2}{K}$

$\Rightarrow$ Rated power $(H) = \frac{F_{tR} V_{av}}{33{,}000} \Rightarrow F_{tR} = F_t \left(\frac{S_{fe}}{\sigma_H}\right)^2 \Rightarrow H = 5.2 \left(\frac{S_{fe}}{\sigma_H}\right)^2$ hp

2020/3/28 11:35