

Remark about Example 3 page 421

Assume:

$$X \sim N(\theta_1, \theta_3)$$

$$Y \sim N(\theta_2, \theta_4)$$

X, Y independent

Take independent random samples

$$X_1, \dots, X_n \text{ iid } N(\theta_1, \theta_3)$$

$$Y_1, \dots, Y_m \text{ iid } N(\theta_2, \theta_4)$$

with means and variances $\bar{X}, \bar{Y}, S_1^2, S_2^2$

$$H_0: \theta_3 = \theta_4, \quad \theta_1 \in \mathbb{R}, \theta_2 \in \mathbb{R}$$

H_1 : All Alternative

Sample of observation
 $x_1, \dots, x_n$, and $y_1, \dots, y_m$
with means and variances
 $\bar{x}, \bar{y}, s_1^2, s_2^2$

LRT Reject H_0 if $\lambda \leq \lambda_0$ where $\lambda = \frac{L(\hat{w})}{L(\hat{\omega})}$

$\Leftrightarrow$ Reject H_0 if $f \geq c_2$ or $f \leq c_1$

where $f = \frac{\frac{n S_1^2}{n-1}}{\frac{m S_2^2}{m-1}}$ is the observed value of

$$F = \frac{\frac{n S_1^2}{n-1}}{\frac{m S_2^2}{m-1}} \sim F(r_1, r_2)$$

and $r_1 = n-1$ and $r_2 = m-1$ degrees of freedom.

Note The constants c_1 and c_2 are usually selected such that
 $\Pr(F \leq c_1) = \Pr(F \geq c_2) = \frac{\alpha}{2}$

where α is the desired significance level of the test.