Exercises of 4.9 :

Q105 : let X_1 and X_2 be two independent Random Variables so that the Variances of X_1 and X_2 are $\sigma_1^2 = K$ and $\sigma_2^2 = 2$ respectively . given that the variance of $Y = 3X_2 - X_1$ is 25 . Find K .

$$Var(y) = \sum_{i=1}^{n} k_i^2 \sigma_i^2 = \sum_{i=1}^{2} k_i^2 \sigma_i^2$$

$$25 = k_1^2 \sigma_1^2 + k_2^2 \sigma_2^2$$

$$25 = (-1)^2 (K) + (3)^2 (2)$$

$$25 = K + 18$$

$$-18 \qquad - 18$$

$$\boxed{K = 7 = \sigma_1^2}$$

Q108: Determine the mean and variance of the mean $\bar{X}$ of a Random sample of size 9 from a distribution having p.d.f $f(x) = \begin{cases} 4x^3 & , 0 < x < 1 \\ 0 & , \text{elsewhere} \end{cases}$.

$\rightsquigarrow$ $E(x) = \int_{\mathbb{R}} x\, f(x)\, dx = \int_0^1 4x^4\, dx$

$$= \frac{4}{5} x^5 \Big|_0^1 = \frac{4}{5}$$

$$\Rightarrow E(\bar{X}) = E(x) = \frac{4}{5}$$

$\rightsquigarrow$ $Var(x) = E(x^2) - (E(x))^2$

. $E(x^2) = \int_0^1 4x^5\, dx = \frac{4}{6} x^6 \Big|_0^1 = \frac{4}{6} = \frac{2}{3}$

$$\Rightarrow Var(x) = \frac{2}{3} - \frac{16}{25}$$

$$= \frac{50}{75} - \frac{48}{75} = \frac{2}{75}$$

$$\Rightarrow Var(\bar{X}) = \frac{Var(x)}{n} = \frac{2}{75(9)} = \frac{2}{675}.$$

Q112 : Find the mean and the variance of $Y = X_1 - 2X_2 + 3X_3$ where X_1, X_2, X_3 are observations of a Random sample from a chi-square distribution with 6 degrees of freedom .

since it chi-square :

$\rightarrow E(X_1) = E(X_2) = E(X_3) = 6$

$\rightarrow Var(X_1) = Var(X_2) = Var(X_3) = 2(6) = 12$.

$$\Rightarrow E(Y) = \sum_{i=1}^{3} k_i \mu_i$$

$$= (1)(6) + (-2)(6) + (3)(6)$$

$$= \underline{12}$$

$$\Rightarrow Var(Y) = \sum_{i=1}^{3} k_i^2 \sigma_i^2$$

$$= 12(1 + 4 + 9)$$

$$= \underline{168} .$$

Q116 : let U and V be two independent chi-square variables with r_1 and r_2 df , respectively . Find the mean and variance of $F = \frac{r_2 U}{r_1 V}$. what restriction is needed on the parameters r_1 and r_2 in order to ensure the existence of both the mean and the variance of F .

بدو القيد المطلوب على df عشان يكون موجود Mean + Var

$$E(F) = E\left(\frac{r_2 U}{r_1 V}\right) = \frac{r_2}{r_1} E(U) \, E\left(\frac{1}{V}\right)$$

$\star \; E\left(\frac{1}{V}\right) = \frac{1}{r_2 - 2}$, $r_2 > 2$

$\star \; E(U) = r_1$

$$= \frac{r_2}{\cancel{r_1}} \cdot \cancel{r_1} \cdot \frac{1}{r_2 - 2} = \frac{r_2}{r_2 - 2} , \quad r_2 > 2 .$$

$$\rightarrow E(F) = \frac{r_2}{r_2 - 2}$$

cont. 116 :

$$\text{var}(F) = E(F^2) - (E(F))^2$$

$$\rightarrow E(F^2) = E\left(\frac{r_2^2 U^2}{r_1^2 V^2}\right) = \frac{r_2^2}{r_1^2} E(U^2)\, E\left(\frac{1}{V}\right)$$

$$\cdot\ E(U^2) = \int U^2 \frac{1}{\Gamma(\frac{r_1}{2}) 2^{\frac{r_1}{2}}} U^{\frac{r_1}{2}} e^{-\frac{U}{2}}\, dU$$

$$= \frac{1}{\Gamma(\frac{r_1}{2}) 2^{\frac{r_1}{2}}} \Gamma\left(\frac{r_1}{2} + 2\right) 2^{\frac{r_1}{2}+2}$$

$$= \left(\frac{r_1}{2} + 1\right)\left(\frac{r_1}{2}\right) 4$$

$$= r_1^2 + 2r_1 .$$

$$\cdot\ E\left(\frac{1}{V^2}\right) = \int \frac{1}{V^2} \frac{1}{\Gamma(\frac{r_2}{2}) 2^{\frac{r_2}{2}}} V^{\frac{r_2}{2}} e^{-\frac{V}{2}}\, dV$$

$$= \frac{1}{\Gamma(\frac{r_2}{2}) 2^{\frac{r_2}{2}}} \Gamma\left(\frac{r_2}{2} - 2\right) 2^{\frac{r_2}{2}-2}$$

$$= \frac{1}{(\frac{r_2}{2} - 1)(\frac{r_2}{2} - 2) 4}$$

$$= \frac{1}{r_2^2 - 6r_2 + 8} \quad , \quad r_2^2 - 6r_2 + 8 > 0 \Rightarrow r > 4$$

$$\Rightarrow E(F^2) = \frac{r_2^2}{\cancel{r_1^2}} \cdot \frac{r_1^2 + 2r_1}{\cancel{r_1}(r_1+2)} \cdot \frac{1}{r_2^2 - 6r_2 + 8}$$

$$= \frac{r_2^2 (r_1 + 2)}{r_1 (r_2^2 - 6r_2 + 8)} \quad , \quad r_2 > 4$$

$$\Rightarrow r_2 > 4$$